Financial Data Science Python Notebooks

Terence Lim

Apr 01, 2025

CONTENTS

As financial markets produce vast volumes of structured and unstructured data, the ability to extract insights and develop predictive models has become increasingly important. Financial Data Science Python Notebooks provide a practical guide for analysts, researchers, and data scientists looking to apply Python and its broad ecosystem of libraries, tools, frameworks, and community resources to financial analysis, econometrics, and machine learning.

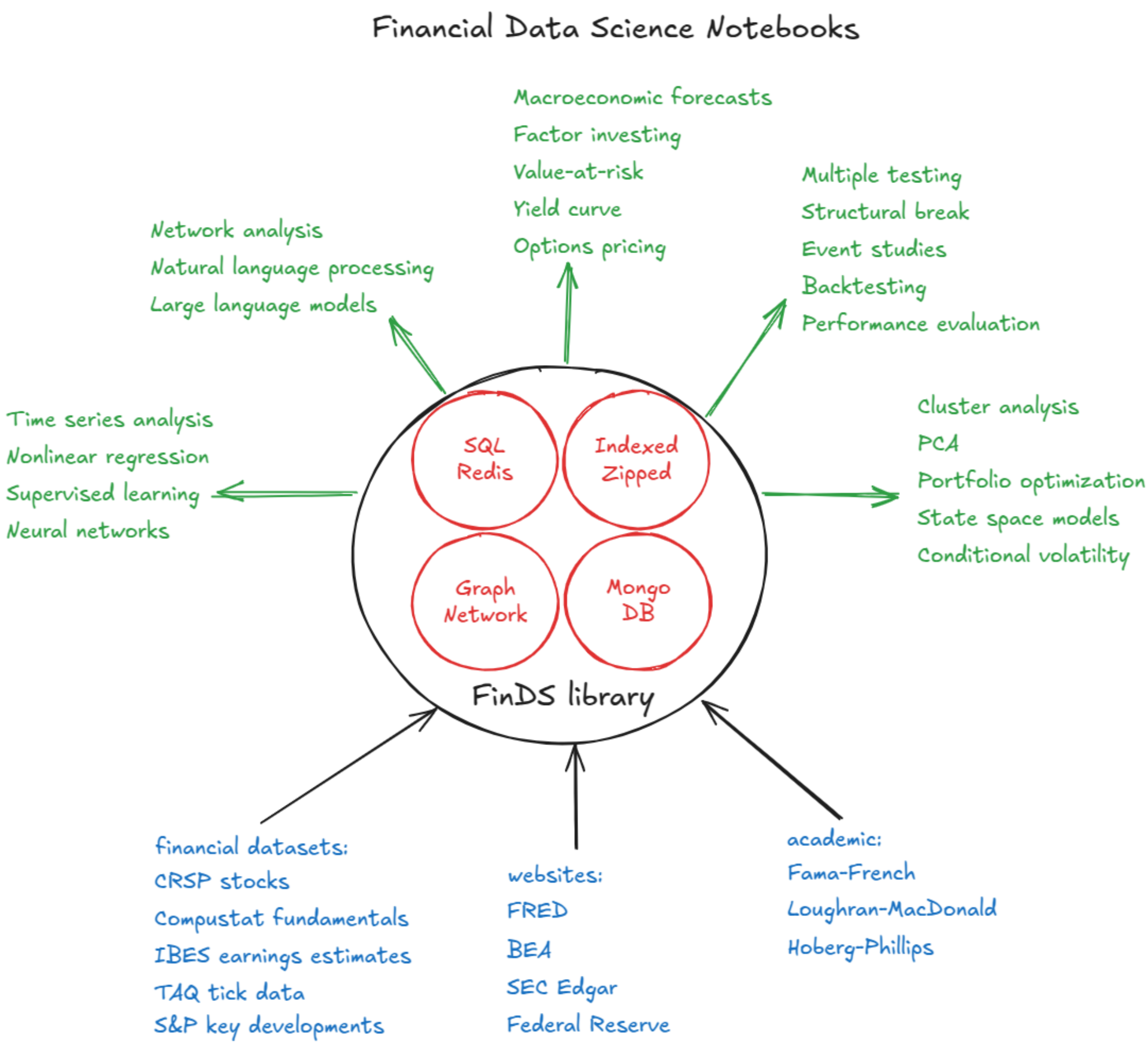
Designed to support financial data science workflows, the companion FinDS Python package demonstrates how to use database engines such as SQL, Redis, and MongoDB to manage and access large datasets, including:

- Core financial databases such as CRSP, Compustat, IBES, and TAQ
- Public economic data APIs from sources like FRED and the Bureau of Economic Analysis (BEA)
- Structured and unstructured data from academic and research websites

In addition to data access, it provides practical examples and templates for applying:

- Financial econometrics and time series modeling
- Graph analytics, event studies, and backtesting strategies
- Machine learning for predictive analytics
- Natural language processing (NLP) to extract insights from financial text
- Neural networks and large language models (LLMs) for advanced decision-making

March 2025: Updated with data through early 2025 and incorporated the latest LLMs –Microsoft Phi-4-multimodal (released Feb 2025), Google Gemma-3-12B (March 2025), DeepSeek-R1-14B (January 2025), Meta Llama-3.1-8B (July 2024), GPT-4o-mini (July 2024).



Topics

notebook	Financial	Data	Science
1.1_stock_prices	Stock price properties	CRSP stocks	Statistical moments
1.2_jegadeesh_titman	Price momentum	CRSP stocks	Hypothesis testing, Newey-West estimator
1.3_fama_french	Value and size	CRSP stocks, Compustat	Linear regression
1.4_fama_macbeth	CAPM	Fama-French	Non-linear regression, Quadratic optimization
1.5_contrarian_trading	Mean reversion, Implementation shortfall	CRSP stocks	Structural breaks
1.6_quant_factors	Factor investing, Backtesting	CRSP stocks, Compustat, IBES	Cluster analysis

continues on next page

notebook	Financial	Data	Science
1.7_event_study	Event studies	S&P key developments	Multiple testing, Fourier transforms
2.1_economic_indicators	Economic data revisions, Employment payrolls	ALFRED	Outlier detection
2.2_regression_diagnostics	Consumer and producer prices	FRED	Linear regression diagnostics
2.3_time_series	Industrial production and inflation	FRED	Time series analysis
2.4_approximate_factors	Approximate factor models	FRED-MD	Unit root test, EM Algorithm
2.5_economic_states	State space models	FRED-MD	Gaussian mixture, hidden Markov models
3.1_term_structure	Interest rates	FRED yield curve	Low-rank approximation
3.2_bond_returns	Bonds risk factors	FRED bond returns	Principal component analysis
3.3_options_pricing	Binomial tree, Black-Scholes-Merton	simulated	Monte Carlo simulations
3.4_value_at_risk	Value-at-risk	FRED crypto-currencies	Conditional volatility
3.5_covariance_matrix	Portfolio risk	Fama-French industries	Covariance matrix estimation
3.6_market_microstructure	Market liquidity	TAQ tick data	High frequency volatility
3.7_event_risk	Earnings expectations	IBES	Poisson regression, generalized linear model
4.1_network_graphs	Supply chain	Compustat principal customers	Network graphs
4.2_community_detection	Industry taxonomy	Hoberg-Phillips	Community detection
4.3_graph_centrality	Input-output uses	Bureau of Economic Analysis	Graph centrality
4.4_link_prediction	Product markets	Hoberg-Phillips	Link prediction
4.5_spatial_regression	Earnings surprises	IBES, Hoberg-Phillips	Spatial regression
5.1_fomc_topics	FOMC meetings	Federal Reserve	Topic modeling
5.2_management_sentiment	Management discussions	SEC Edgar, Loughran-Macdonald	Sentiment analysis
5.3_business_textual	Business descriptions	SEC Edgar	Part-of-speech, Density-based clustering
6.1_classification_models	Industry classification	SEC Edgar	Classification
6.2_regression_models	Macroeconomic forecasts	FRED-MD	Regression
6.3_deep_learning	Industry classification	SEC Edgar	Neural networks, word embeddings
6.4_convolutional_net	Macroeconomic forecasts	FRED-MD	Convolutional neural nets, vector autoregression
6.5_recurrent_net	Macroeconomic forecasts	FRED-MD	Recurrent neural nets, dynamic factor models
6.6_reinforcement_learning	Retirement spending	SBBI	Reinforcement learning
6.7_language_modeling	Fedspeak	Federal Reserve	Language modeling, Transformers
7.1_large_language_models	Market risk disclosures	SEC Edgar	Text summarization
7.2_llm_finetuning	Industry classification	SEC Edgar	LLM fine-tuning
7.3_llm_prompting	Financial news sentiment	Kaggle	Prompt engineering

continues on next page

notebook	Financial	Data	Science
7.4_llm_agents	Corporate philanthropy	MVCP textbook	Multi-agents, chatbots, retrieval-augmented generation

Documentation

- Financial Data Science Notebooks
- Download PDF
- FinDS API reference

Github repos

- FinDS package
- Jupyter notebooks

Contact

https://terence-lim.github.io

CHAPTER

ONE

STOCK PRICES

In physics it takes three laws to explain 99% of the data; in finance it takes more than 99 laws to explain about 3% - Andrew Lo

Stock price data encompasses historical prices as well as corporate actions such as stock splits, dividends, and delistings. The CRSP database is a standard resource in academic research due to its comprehensive coverage of both active and delisted stocks, which supports unbiased and representative analysis. Efficient storage, retrieval, and processing of such structured financial data are enabled by tools such as SQL, SQLAlchemy, Redis, and Pandas. We examine statistical moments, assume log-normal distributions, and explore alternative correlation measures for modeling stock return behavior and dependencies.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import scipy
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import warnings
from tqdm import tqdm
from finds.database import SQL, RedisDB
from finds.structured import BusDay, CRSP, CRSPBuffer, Benchmarks
from finds.utils import Finder
from secret import credentials, CRSP_DATE, paths
VERBOSE = 0

# %matplotlib qt
```

1.1 FinDS package

Developed over a journey to support financial data science workflows, our *FinDS* Python package integrates:

- Use of database engines like SQL, Redis, and MongoDB to manage large structured and unstructured datasets
- Tools for accessing key financial datasets, including CRSP, Compustat, IBES, and TAQ
- Interfaces to public data sources such as FRED, SEC EDGAR, and the BEA
- Utilities for extracting data from academic and research websites
- Recipes for applying a range of statistical and machine learning methods, including network graphs, natural language processing and large language models.

1.1.1 SQL

Structured Query Language (SQL) is a popular tool for storing and managing relational data organized into tables with columns (fields) and rows (records). Open-source systems like **MySQL** are widely used to implement relational databases, and Python libraries such as **SQLAlchemy** provide convenient interfaces for interacting with them. Additionally, the **Pandas** library allows users to run SQL queries and load the results directly into DataFrames for efficient analysis.

1.1.2 Redis

Redis is an open-source, in-memory data store commonly used as a caching layer. It helps improve performance by storing frequently accessed data in memory, thereby reducing the load on slower, primary databases.

```
# open database connections
sql = SQL(**credentials['sql'], verbose=VERBOSE)
rdb = RedisDB(**credentials['redis'])
bd = BusDay(sql, endweek=3, verbose=VERBOSE)
crsp = CRSP(sql, bd, rdb=rdb, verbose=VERBOSE)
bench = Benchmarks(sql, bd, verbose=VERBOSE)
find = Finder(sql)
outdir = paths['scratch']
```

1.2 Stock price data

Besides the historical prices of stocks, their adjustments such as identifier changes, stock splits, dividends, mergers, and delistings must also be recorded to accurately analyze performance over time.

1.2.1 CRSP stocks

The Center for Research in Security Prices (CRSP) provides the most widely used data for academic research into US stocks. It includes both successful and unsuccessful entities, rather than just those that have "survived" over time. This helps avoid the pitfalls of focusing only on surviving entities which can lead to an overestimation of average performance and underestimation of risk. It also captures corporate actions by standardizing data on events such as name changes, distributions and delistings. This information is recorded and validated from official sources, integrated into its historical databases along with details like announcement and effective dates, adjustment factors, and impact on stock performance calculations.

```
# describe the database schema of the CRP Stocks `names` table
DataFrame(**sql.run('describe names'))
```

```
       Field         Type Null  Key Default Extra
0       date          int   NO  PRI    None
1     comnam  varchar(32)  YES         None
2     ncusip   varchar(8)  YES  MUL    None
3     shrcls   varchar(1)  YES         None
4     ticker   varchar(5)  YES         None
5     permno          int   NO  PRI    None
6   nameendt          int  YES         None
7      shrcd     smallint  YES         None
8     exchcd     smallint  YES         None
9      siccd     smallint  YES         None
```

(continues on next page)

(continued from previous page)

```
10   tsymbol   varchar(7)  YES         None
11     naics          int  YES         None
12  primexch   varchar(1)  YES         None
13   trdstat   varchar(1)  YES         None
14   secstat   varchar(4)  YES         None
15    permco          int  YES         None
```

SQL `select` and `join` statements to retrieve Apple Computer's identifiers and corporate actions, such as stock splits and dividends. Commonly used SQL commands listed at the end of this notebook.

```
# double up the % when passing sql command stringo to pandas
names_df = pd.read_sql("select * from names where comnam like '%%APPLE COMPUTER%%'",␣
↪con=sql.engine)
names_df
```

```
       date             comnam    ncusip shrcls ticker  permno  nameendt  \
0  19801212  APPLE COMPUTER INC  03783310          AAPL   14593  19821031
1  19821101  APPLE COMPUTER INC  03783310          AAPL   14593  20040609
2  20040610  APPLE COMPUTER INC  03783310          AAPL   14593  20070110

  shrcd  exchcd  siccd tsymbol   naics primexch trdstat secstat  permco
0    11       3   3573               0        Q       A       R       7
1    11       3   3573    AAPL       0        Q       A       R       7
2    11       3   3573    AAPL  334111        Q       A       R       7
```

```
# inner join of identifiers (names) and distributions (dist) tables
cmd = """
select distinct names.permno, divamt, facpr, exdt, comnam, ticker from names
inner join dist
on names.permno = dist.permno
where names.comnam like '%%APPLE COMPUTER%%'
"""
dist_df = pd.read_sql(cmd, con=sql.engine)
dist_df
```

```
    permno  divamt  facpr      exdt              comnam ticker
0    14593    0.12    0.0  19870511  APPLE COMPUTER INC   AAPL
1    14593    0.06    0.0  19870810  APPLE COMPUTER INC   AAPL
2    14593    0.08    0.0  19871117  APPLE COMPUTER INC   AAPL
3    14593    0.08    0.0  19880212  APPLE COMPUTER INC   AAPL
4    14593    0.08    0.0  19880516  APPLE COMPUTER INC   AAPL
..     ...     ...    ...       ...                 ...    ...
86   14593    0.00    1.0  19870616  APPLE COMPUTER INC   AAPL
87   14593    0.00    1.0  20000621  APPLE COMPUTER INC   AAPL
88   14593    0.00    1.0  20050228  APPLE COMPUTER INC   AAPL
89   14593    0.00    6.0  20140609  APPLE COMPUTER INC   AAPL
90   14593    0.00    3.0  20200831  APPLE COMPUTER INC   AAPL

[91 rows x 6 columns]
```

1.2.2 Stock splits and dividends

An investor' s total holding returns (`ret` in CRSP) include gains from appreciated stock prices (`prc`), adjusted for stock splits (`facpr`), plus ordinary cash dividends (`divamt`). Specifically, on ex-dates t:

$$ret_t = \frac{prc_t\ (1 + facpr_t) + div_t}{prc_{t-1}}$$

The Factor to Adjust Price (`facpr`) values over time can be used to adjust prices for distributions such as stock dividends and splits so that stock prices before and after one or more distributions are comparable. Historical cumulative adjust factors are computed by additing 1 to and then taking cumulative product from current to earlier time periods. This cumulative factor between two dates is divided into the earlier raw stock price to derive comparable split-adjusted prices. Hence to split-adjust CRSP raw prices

- apply cumulative factor to raw prices before corresponding ex-date
- back-fill to dates prior to ex-date
- default factor after latest ex-date is 1

yfinance

The yfinance Python library enables users to access current financial data from Yahoo Finance.

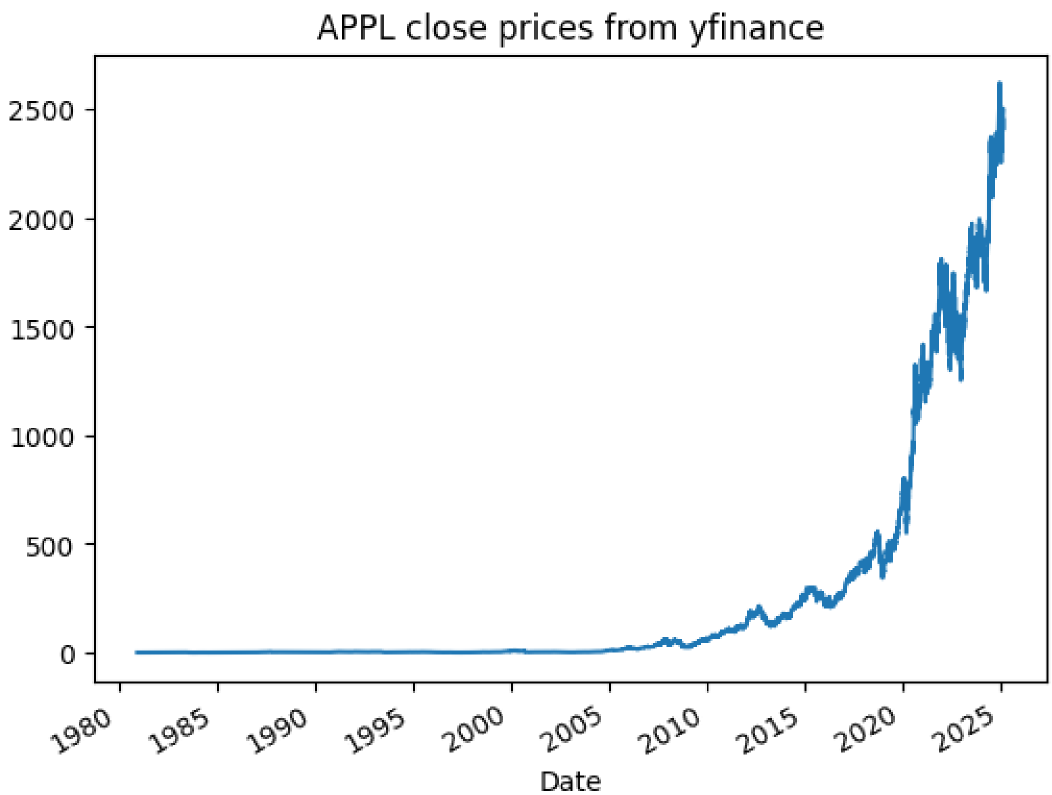
```
import yfinance as yf
ticker = yf.Ticker('AAPL')
df = ticker.history(period='max')
df[df['Dividends'].gt(0) | df['Stock Splits'].ne(0)]
```

```
                                 Open        High         Low       Close  \
Date
1987-05-11 00:00:00-04:00    0.264817    0.273415    0.263957    0.264817
1987-06-16 00:00:00-04:00    0.285452    0.287171    0.261378    0.285452
1987-08-10 00:00:00-04:00    0.332310    0.332310    0.315091    0.332310
1987-11-17 00:00:00-05:00    0.253658    0.255384    0.241579    0.241579
1988-02-12 00:00:00-05:00    0.280957    0.287009    0.280093    0.283551
...                               ...         ...         ...         ...
2024-02-09 00:00:00-05:00  187.763406  189.097120  187.116467  187.962479
2024-05-10 00:00:00-04:00  184.280653  184.470019  181.519943  182.436859
2024-08-12 00:00:00-04:00  215.595507  219.027939  215.126538  217.052292
2024-11-08 00:00:00-05:00  226.920500  228.408869  226.161340  226.710739
2025-02-10 00:00:00-05:00  229.570007  230.589996  227.199997  227.649994

                              Volume  Dividends  Stock Splits
Date
1987-05-11 00:00:00-04:00  197276800   0.000536           0.0
1987-06-16 00:00:00-04:00  342720000   0.000000           2.0
1987-08-10 00:00:00-04:00   77996800   0.000536           0.0
1987-11-17 00:00:00-05:00  268800000   0.000714           0.0
1988-02-12 00:00:00-05:00  137760000   0.000714           0.0
...                              ...        ...           ...
2024-02-09 00:00:00-05:00   45155200   0.240000           0.0
2024-05-10 00:00:00-04:00   50759500   0.250000           0.0
2024-08-12 00:00:00-04:00   38028100   0.250000           0.0
2024-11-08 00:00:00-05:00   38328800   0.250000           0.0
2025-02-10 00:00:00-05:00   33115600   0.250000           0.0

[91 rows x 7 columns]
```

The daily `Close` prices from yFinance have been adjusted for stock splits and dividend payments. As a result, the plotted values directly represent cumulative total holding returns.

```
df['Close'].div(df['Close'].iloc[0]).plot(title="APPL close prices from yfinance")
```

```
<Axes: title={'center': 'APPL close prices from yfinance'}, xlabel='Date'>
```



To retrieve unadjusted historical prices accounting for stock splits and dividends, first apply the split factors to determine the original dividends per unadjusted share. The resulting values closely match the CRSP `divamt` figures, with minor differences likely due to cumulative numerical precision errors.

```
split = df['Stock Splits'].where(df['Stock Splits'] != 0.0, 1)\
    .shift(-1).fillna(1).iloc[::-1].cumprod().iloc[::-1]       # cumulate the split␣
↪factors back in time
df['unadj_div'] = df['Dividends'] * split.shift(1).fillna(split.iloc[0])    # apply␣
↪the split factors to dividends
df.set_index(df.index.strftime('%Y-%m-%d'))[(df['unadj_div'] > 0).values]
```

```
                  Open        High         Low       Close     Volume  \
Date
1987-05-11    0.264817    0.273415    0.263957    0.264817  197276800
1987-08-10    0.332310    0.332310    0.315091    0.332310   77996800
1987-11-17    0.253658    0.255384    0.241579    0.241579  268800000
1988-02-12    0.280957    0.287009    0.280093    0.283551  137760000
1988-05-16    0.280647    0.286711    0.277183    0.285845   74760000
...                ...         ...         ...         ...        ...
2024-02-09  187.763406  189.097120  187.116467  187.962479   45155200
```

(continues on next page)

(continued from previous page)

```
2024-05-10  184.280653  184.470019  181.519943  182.436859   50759500
2024-08-12  215.595507  219.027939  215.126538  217.052292   38028100
2024-11-08  226.920500  228.408869  226.161340  226.710739   38328800
2025-02-10  229.570007  230.589996  227.199997  227.649994   33115600

            Dividends  Stock Splits  unadj_div
Date
1987-05-11   0.000536           0.0   0.120064
1987-08-10   0.000536           0.0   0.060032
1987-11-17   0.000714           0.0   0.079968
1988-02-12   0.000714           0.0   0.079968
1988-05-16   0.000714           0.0   0.079968
...               ...           ...        ...
2024-02-09   0.240000           0.0   0.240000
2024-05-10   0.250000           0.0   0.250000
2024-08-12   0.250000           0.0   0.250000
2024-11-08   0.250000           0.0   0.250000
2025-02-10   0.250000           0.0   0.250000

[86 rows x 8 columns]
```

Next, we work backward through time to reconstruct the original stock prices using daily holding returns and closing prices. Stock splits and dividends are accounted for on ex-dates by applying the rearranged formula:

$$prc_{t-1} = \frac{(1 + facpr_t)(prc_t + div_t)}{ret_t}$$

```python
rets = df['Close'] / df['Close'].shift(1)
prc = df['Close'].iloc[-1]
div = 0
fac = 0
ret = 1
for i, t in enumerate(df.index[::-1]):      # iterate over days in reverse
    df.loc[t, 'unadj_prc'] =  (((fac if fac else 1) * prc) + div) / ret
    prc = df.loc[t, 'unadj_prc']
    div = df.loc[t, 'unadj_div']
    fac = df.loc[t, 'Stock Splits']
    ret = rets.loc[t]
df.set_index(df.index.strftime('%Y-%m-%d'))
```

```
                  Open        High         Low       Close     Volume  \
Date
1980-12-12    0.098726    0.099155    0.098726    0.098726  469033600
1980-12-15    0.094005    0.094005    0.093575    0.093575  175884800
1980-12-16    0.087136    0.087136    0.086707    0.086707  105728000
1980-12-17    0.088853    0.089282    0.088853    0.088853   86441600
1980-12-18    0.091429    0.091858    0.091429    0.091429   73449600
...                ...         ...         ...         ...        ...
2025-02-24  244.929993  248.860001  244.419998  247.100006   51326400
2025-02-25  248.000000  250.000000  244.910004  247.039993   48013300
2025-02-26  244.330002  244.979996  239.130005  240.360001   44433600
2025-02-27  239.410004  242.460007  237.059998  237.300003   41153600
2025-02-28  236.949997  242.089996  230.199997  241.839996   56796200

            Dividends  Stock Splits  unadj_div   unadj_prc
```

(continues on next page)

(continued from previous page)

```
Date
1980-12-12        0.0          0.0         0.0   28.757305
1980-12-15        0.0          0.0         0.0   27.257003
1980-12-16        0.0          0.0         0.0   25.256412
1980-12-17        0.0          0.0         0.0   25.881523
1980-12-18        0.0          0.0         0.0   26.631895
...               ...          ...         ...         ...
2025-02-24        0.0          0.0         0.0  247.100006
2025-02-25        0.0          0.0         0.0  247.039993
2025-02-26        0.0          0.0         0.0  240.360001
2025-02-27        0.0          0.0         0.0  237.300003
2025-02-28        0.0          0.0         0.0  241.839996

[11144 rows x 9 columns]
```

The unadjusted price computer and the original closing prices from CRSP during the early days of AAPL stock are nearly identical, with only small differences due to cumulative numerical precision errors.

```
price_df = pd.read_sql(f"select permno, date, abs(prc) from daily where permno={names_
↪df['permno'][0]}", con=sql.engine)
price_df.head(10)
```

```
   permno      date  abs(prc)
0   14593  19801212   28.8125
1   14593  19801215   27.3125
2   14593  19801216   25.3125
3   14593  19801217   25.9375
4   14593  19801218   26.6875
5   14593  19801219   28.3125
6   14593  19801222   29.6875
7   14593  19801223   30.9375
8   14593  19801224   32.5625
9   14593  19801226   35.5625
```

1.2.3 Market capitalization

Following Fama and French (1992), academic research typically focuses on U.S.-domiciled stocks, specifically those with a share code (`shrcd`) of 10 or 11, that are listed on major exchanges (exchange code (`exchcd`) of 1, 2, or 3).

Stocks are often sorted into ten deciles based on market capitalization, with the smallest stocks placed in the 10th decile. These decile breakpoints are determined using only NYSE-listed stocks. For companies with multiple classes of securities, total market value is calculated by summing the market capitalization of all classes. Since CRSP reports shares outstanding (shrout) in thousands, all market capitalization values must be multiplied by 1,000 to reflect their actual size.

Plot the number of stocks in and the market cap breakpoints of each size decile by year:

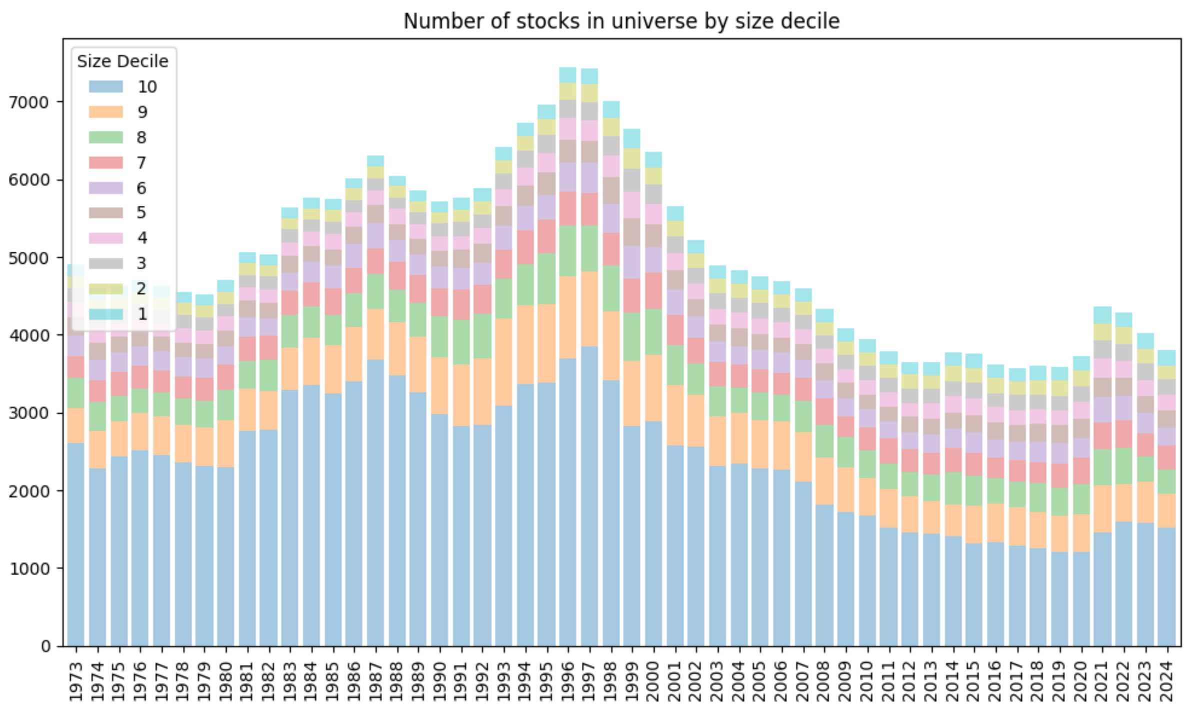
```
# retrieve universe of stocks annually from 1981
start = bd.endyr(19731231)
rebals = bd.date_range(start, CRSP_DATE, freq=12)
univs = {rebal: crsp.get_universe(date=rebal) for rebal in rebals}
num = dict()
for date, univ in univs.items():
    num[str(date//10000)] = {decile: sum(univ['decile']==decile)
```

(continues on next page)

(continued from previous page)

```
                              for decile in range(10, 0, -1)}
num = DataFrame.from_dict(num, orient='index')
```

```
# plot number of stocks in each size decile
fig, ax = plt.subplots(figsize=(10, 6))
ax.set_title('Number of stocks in universe by size decile')
num.plot.bar(stacked=True, ax=ax, width=.8, alpha=0.4)
#set_xtickbins(ax=ax, nbins=len(cap)//10)
plt.legend(title='Size Decile', loc='upper left')
plt.tight_layout()
```




```
# If a company has multiple share classes, sum up its total market capitalization
# instead of separate capital values for each share class
names = find('ALPHABET', 'comnam').groupby('permno').tail(1).set_index('permno')
names
```

```
            date        comnam    ncusip shrcls ticker  nameendt  shrcd  \
permno
14542   20231013  ALPHABET INC  02079K10      C   GOOG  20241231     11
90319   20231013  ALPHABET INC  02079K30      A  GOOGL  20241231     11

        exchcd  siccd tsymbol   naics primexch trdstat secstat  permco
permno
14542        3   7375    GOOG  541511        Q       A       R   45483
90319        3   7375   GOOGL  541511        Q       A       R   45483
```

```
univ.loc[names.index]  # market caps of share class (cap) and total company (capco)
```

```
                  cap          capco  decile   nyse  siccd     prc   naics
permno
14542   1.053895e+09  2.159975e+09       1  False   7375  190.44  541511
90319   1.106080e+09  2.159975e+09       1  False   7375  189.30  541511
```

1.2.4 Stock delistings

An important feature of the CRSP database is that it is free of survivorship-bias. It includes the historical records of stocks that have delisted from trading on the exchanges.

In CRSP Monthly, the Delisting Return is calculated from the last month ending price to the last daily trading price if no other delisting information is available. In this case the delisting payment date is the same as the delisting date. If the return is calculated from a daily price, it is a partial-month return. The partial-month returns are not truly Delisting Returns since they do not represent values after delisting, but allow the researcher to make a more accurate estimate of the Delisting Returns.

Following Bali, Engle, and Murray (2016) and Shumway (1997): we can construct returns adjusted for delistings, which result when a company is acquired, ceases operations, or fails to meet exchange listing requirements. The adjustment reflects the partial month of returns to investors who bought the stock in the month before the delisting. For certain delisting codes ([500, 520, 551..574, 580, 584]) where the delisting return is missing, a delisting return of -30% is assumed which reflects the average recovery amount after delisting.

```
# Show sample of original CRSP Monthly ret and dlret before and after adjustment
pd.read_sql('select * from monthly where dlstcd>100 and date=20041130', sql.engine)\
  .set_index('permno')\
  .rename(columns={'ret': 'original_ret'})\
  .join(crsp.get_ret(beg=20041101, end=20041130), how='left')\
  .round(4)
```

```
            date    prc  original_ret    retx  dlstcd   dlret     ret
permno
10275   20041130    NaN           NaN     NaN     584 -0.2703 -0.2703
10418   20041130    NaN           NaN     NaN     520  0.0509  0.0509
11194   20041130    NaN           NaN     NaN     233  0.0066  0.0066
12010   20041130    NaN           NaN     NaN     587  0.0490  0.0490
20459   20041130    NaN           NaN     NaN     231  0.0724  0.0724
32897   20041130    NaN           NaN     NaN     584 -0.2357 -0.2357
55589   20041130    NaN           NaN     NaN     233 -0.0114 -0.0114
64290   20041130  13.70        0.0178  0.0178     233  0.0000  0.0178
67708   20041130    NaN           NaN     NaN     233  0.0164  0.0164
69593   20041130    NaN           NaN     NaN     331  0.0998  0.0998
69681   20041130    NaN           NaN     NaN     584 -0.4853 -0.4853
70447   20041130   5.80       -0.2246 -0.2246     570     NaN -0.4572
75606   20041130    NaN           NaN     NaN     520 -0.2466 -0.2466
75684   20041130    NaN           NaN     NaN     233  0.0094  0.0094
76306   20041130    NaN           NaN     NaN     241 -0.0329 -0.0329
76691   20041130    NaN           NaN     NaN     582  0.0127  0.0127
77838   20041130    NaN           NaN     NaN     520 -0.0041 -0.0041
79149   20041130    NaN           NaN     NaN     574 -0.2088 -0.2088
79523   20041130    NaN           NaN     NaN     233  0.0043  0.0043
80211   20041130    NaN           NaN     NaN     470  0.0186  0.0186
80714   20041130    NaN           NaN     NaN     332  0.0337  0.0337
82491   20041130   4.21       -0.0644 -0.0644     551  0.0689  0.0000
```

(continues on next page)

(continued from previous page)

```
83583   20041130  125.10         0.2810  0.2810     241  0.0624  0.3608
83702   20041130     NaN            NaN     NaN     587  0.2667  0.2667
83995   20041130   26.58         0.2374  0.2374     231  0.0394  0.2861
84047   20041130   16.35         0.0467  0.0467     241 -0.2661 -0.2318
86123   20041130     NaN            NaN     NaN     233  0.0059  0.0059
86307   20041130     NaN            NaN     NaN     233  0.0223  0.0223
86388   20041130     NaN            NaN     NaN     584 -0.0337 -0.0337
86991   20041130     NaN            NaN     NaN     233  0.0106  0.0106
87126   20041130     NaN            NaN     NaN     231  0.0784  0.0784
87158   20041130     NaN            NaN     NaN     233  0.0086  0.0086
87247   20041130     NaN            NaN     NaN     233  0.0046  0.0046
88670   20041130     NaN            NaN     NaN     470  0.0013  0.0013
89186   20041130     NaN            NaN     NaN     331  0.0696  0.0696
89385   20041130     NaN            NaN     NaN     231  0.2959  0.2959
89936   20041130     NaN            NaN     NaN     231  0.0719  0.0719
89939   20041130     NaN            NaN     NaN     233  0.0059  0.0059
```

1.3 Properties of stock returns

1.3.1 Long-run market averages

Calculate and plot the time series of annual cross-sectional averages of stock returns, where each year’s average is cap-weighted, and the final time series is equal-weighted.

Over time, the contribution of dividend yield to total average stock returns has decreased, while share trading turnover has increased.

```
# Loop over the 20-year eras, and compute means of annual cap-weighted averages
years = range(1925, 2025, 20)
results = DataFrame(columns=['divyld', 'turnover', 'means'])
for era in tqdm(years):
    label = f"{era+1}-{era+20}"
    divyld, means, turnover = {}, {}, {}

    for year in bd.date_range(era, min(CRSP_DATE//10000 -1 , era+19), freq=12):
        # universe stocks at end of year
        univ = crsp.get_universe(bd.endyr(year))

        # retrieve cap-weighted average of next year's returns
        cmd = f"""
select permno, SUM(LOG(1+ret)) AS ret FROM daily
WHERE date > {bd.endyr(year)} AND date <= {bd.endyr(year, 1)}
GROUP BY permno
""".strip()
        data = pd.read_sql(cmd, sql.engine)
        df = data.set_index('permno').join(univ['cap'], how='right').dropna()
        means[year] = (np.exp(df['ret'])-1).dot(df['cap']) / df['cap'].sum()

        # retrieve cap-weighted average of annualized turnover
        cmd = f"""
select permno, 252*AVG(vol/(shrout*1000)) AS turnover FROM daily
WHERE date > {bd.endyr(year)} AND date <= {bd.endyr(year, 1)}
GROUP BY permno
```

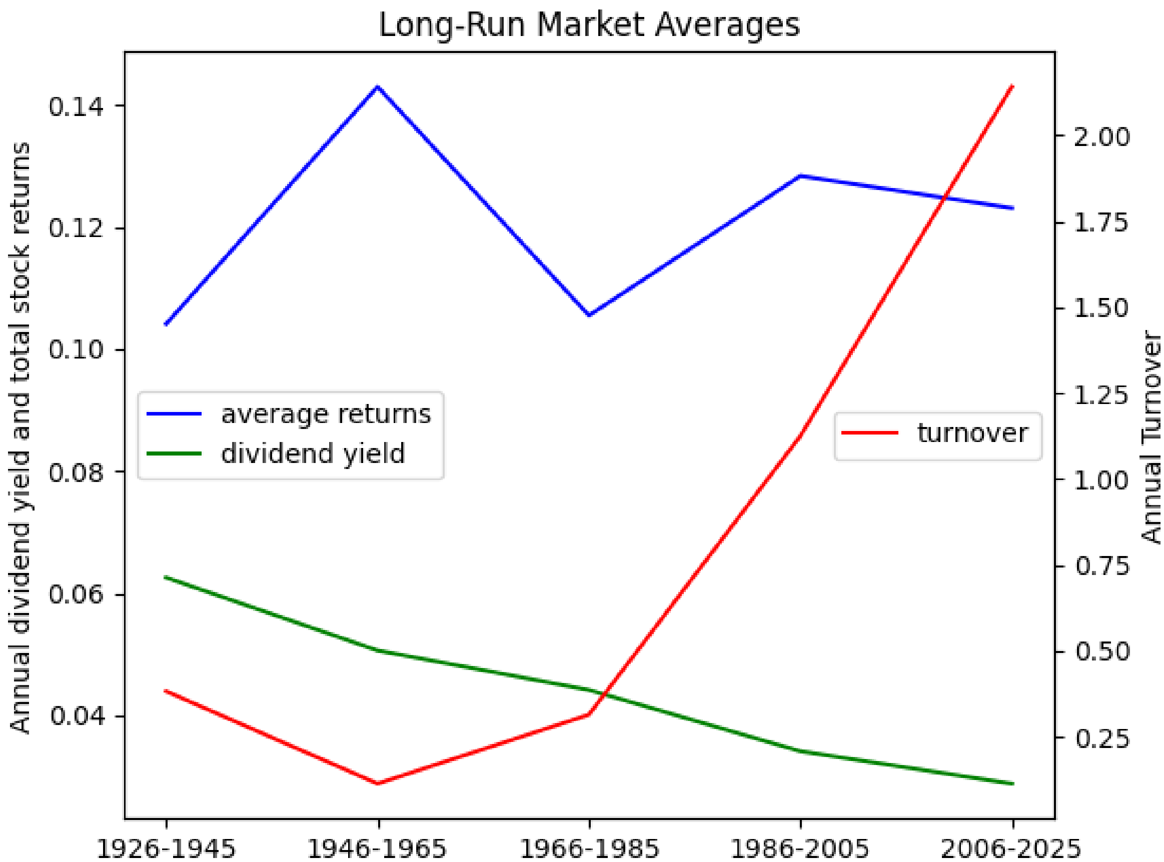
(continues on next page)

(continued from previous page)

```
""".strip()
        data = pd.read_sql(cmd, sql.engine)
        df = data.set_index('permno').join(univ['cap'], how='right').dropna()
        turnover[year] = df['turnover'].dot(df['cap']) / df['cap'].sum()

        # retrieve cap-weighted average of annual dividend amounts
        cmd = f"""
SELECT dist.permno as permno, SUM(daily.shrout * dist.divamt) AS divamt
FROM dist INNER JOIN daily
ON daily.permno = dist.permno AND daily.date = dist.exdt
WHERE dist.divamt > 0 AND dist.exdt > {bd.endyr(year)}
  AND dist.exdt <= {bd.endyr(year, 1)}
GROUP BY permno
""".strip()
        data = pd.read_sql(cmd, sql.engine)
        df = data.set_index('permno').join(univ['cap'], how='right').dropna()
        divyld[year] = df['divamt'].sum() / df['cap'].sum()

    results.loc[label, 'turnover'] = np.mean(list(turnover.values()))
    results.loc[label, 'divyld'] = np.mean(list(divyld.values()))
    results.loc[label, 'means'] = np.mean(list(means.values()))
```

```
100%|██████████| 5/5 [1:03:37<00:00, 763.46s/it]
```

```
# Plot mean returns, dividend yield and turnover using both y-axes
fig, ax = plt.subplots()
ax.plot(results['means'], color="blue")
ax.plot(results['divyld'], color="green")
ax.legend(['average returns', 'dividend yield'], loc="center left")
ax.set_ylabel("Annual dividend yield and total stock returns")
bx = ax.twinx()
bx.plot(results['turnover'], color="red")
bx.set_ylabel("Annual Turnover")
bx.legend(['turnover'], loc="center right")
plt.title("Long-Run Market Averages")
plt.tight_layout()
```



1.3.2 Statistical moments

The volatility of an asset is usually measured using the standard deviation of the returns. The common practice is to report the annualized volatility using the square-root rule which assumes that variance scales linearly with time: e.g. daily by $\sqrt{252}$, weekly by $\sqrt{52}$, monthly by $\sqrt{12}$. Mean returns are annualized by multiplying by the respective number of periods in a year.

A normal distribution is symmetric and thin-tailed, and so has no skewness or excess kurtosis. However, many return series are both skewed and fat-tailed (kurtosis in excess of 3). A left-skewed distribution is longer on the left side of its peak than on its right. In other words, a left-skewed distribution has a long tail on its left side, where the mean typically lies to the left of its median. Left skew is also referred to as negative skew. Right or positive skew has the opposite properties.

Stock prices are often modeled with a log-normal distribution because prices cannot be negative. Large positive jumps are possible, but extreme negative moves are bounded at zero, hence the distribution of log-normal returns is positively-skewed with a long right tail. The skewness of a log-normal distribution is given by: $(e^{\sigma^2}+2)\sqrt{e^{\sigma^2}-1}$

By Jensen' s inequality, the arithmetic mean is greater than the geometric mean. Under the assumption of log-normality, the amount by which the arithmetic mean exceeds the geometric means of returns is half the volatility.

- Suppose the continuously compounded (log) returns (r_t) are normally distributed with mean (\mu) and variance (\sigma^2): $r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \sim N(\mu, \sigma^2)$
- The geometric mean return is given by $\mu_G = \frac{1}{T}\sum_{t=1}^{T} r_t$, with expectation $\mathbb{E}[r_t] = \mu$
- The arithmetic mean of the simple returns is approximately given by $\mu_A \approx e^{\mu+\frac{1}{2}\sigma^2} - 1 \approx \mu_G + \frac{1}{2}\sigma^2$, using the first-order approximation $e^x \approx 1 + x$ for small x.

Jarque-Bera test

The Jarque-Bera test statistic can be used to test whether the sample skewness and kurtosis are compatible with an assumption that the returns are normally distributed. When returns are normally distributed, the skewness is asymptotically normally distributed with a variance of 6, so (skewness)$^2/6$ has a χ^2_1 distribution. Meanwhile, the kurtosis is asymptotically normally distributed with mean 3 and variance of 24, and so (kurtosis - 3)$^2/24$ also has a χ^2_1 distribution. These two statistics are asymptotically independent (uncorrelated), and so their sum is χ^2_2

```
# Compute the stock returns sampled at various frequencies
intervals = {'annual': (12, 1), 'monthly': ('e', 12), 'weekly': ('w', 52), 'daily': (
↪'d', 252)}
moments = []
begdate, enddate = bd.begyr(CRSP_DATE//10000 - 19), bd.endyr(CRSP_DATE//10000)

for dataset, (freq, annualize) in intervals.items():

    # If annual or monthly frequency, use StocksBuffer to pre-load all monthly returns
    if dataset not in ['daily', 'weekly']:
        stocks = CRSPBuffer(stocks=crsp, dataset='monthly', fields=['ret'],
                            beg=bd.begyr(begdate), end=enddate)
    if dataset in ['weekly']:  # weekly returns already computed from CRSP Daily, and␣
↪cached
        stocks = crsp

    univ_year = bd.endyr(begdate - 1)  # universe as of end of previous calendar year␣
↪

    dates = bd.date_range(bd.endyr(begdate), bd.endyr(enddate), freq=freq)
    allstocks = []
    for beg, end in tqdm(bd.date_tuples(dates)):

        # Update the investment universe every calendar year                          ␣
↪
        if bd.endyr(beg) != univ_year:
            univ = crsp.get_universe(univ_year)
            univ_year = bd.endyr(beg)

            # Use StocksBuffer to cache daily returns for the new calendar year       ␣
↪
            if dataset in ['daily']:
                stocks = CRSPBuffer(stocks=crsp, dataset='daily', fields=['ret'],
                                    beg=bd.offset(beg, -1), end=bd.endyr(end))

        # retrieve returns for universe stocks                                        ␣
↪
        ret = stocks.get_ret(beg=beg, end=end).reindex(univ.index)
        allstocks.append(ret.rename(end))

    # combine all years' stock returns, require stock in all years                    ␣
↪
    allstocks = pd.concat(allstocks, axis=1, join='inner')

    # compute annualized moments, ignoring small sample warnings
    with warnings.catch_warnings():
        warnings.simplefilter("ignore")
        out = {
            f"MeanAnnualized": np.nanmedian(
                np.nanmean(np.log(1 + allstocks), axis=1)) * annualize,
            f"VolAnnualized": np.nanmedian(
```

(continues on next page)

(continued from previous page)

```
                np.nanstd(allstocks, axis=1, ddof=0)) * np.sqrt(annualize),
            f"Skewness": np.nanmedian(
                scipy.stats.skew(allstocks, nan_policy='omit', axis=1)),
            f"ExcessKurtosis": np.nanmedian(
                scipy.stats.kurtosis(allstocks, nan_policy='omit', axis=1)),
            f"Count": len(allstocks),
        }
    df = DataFrame({dataset: Series(out)}).T
    moments.append(df)
moments = pd.concat(moments, axis=0)  # accumulate df to results
moments.reset_index().to_json(outdir / 'moments.json')
```

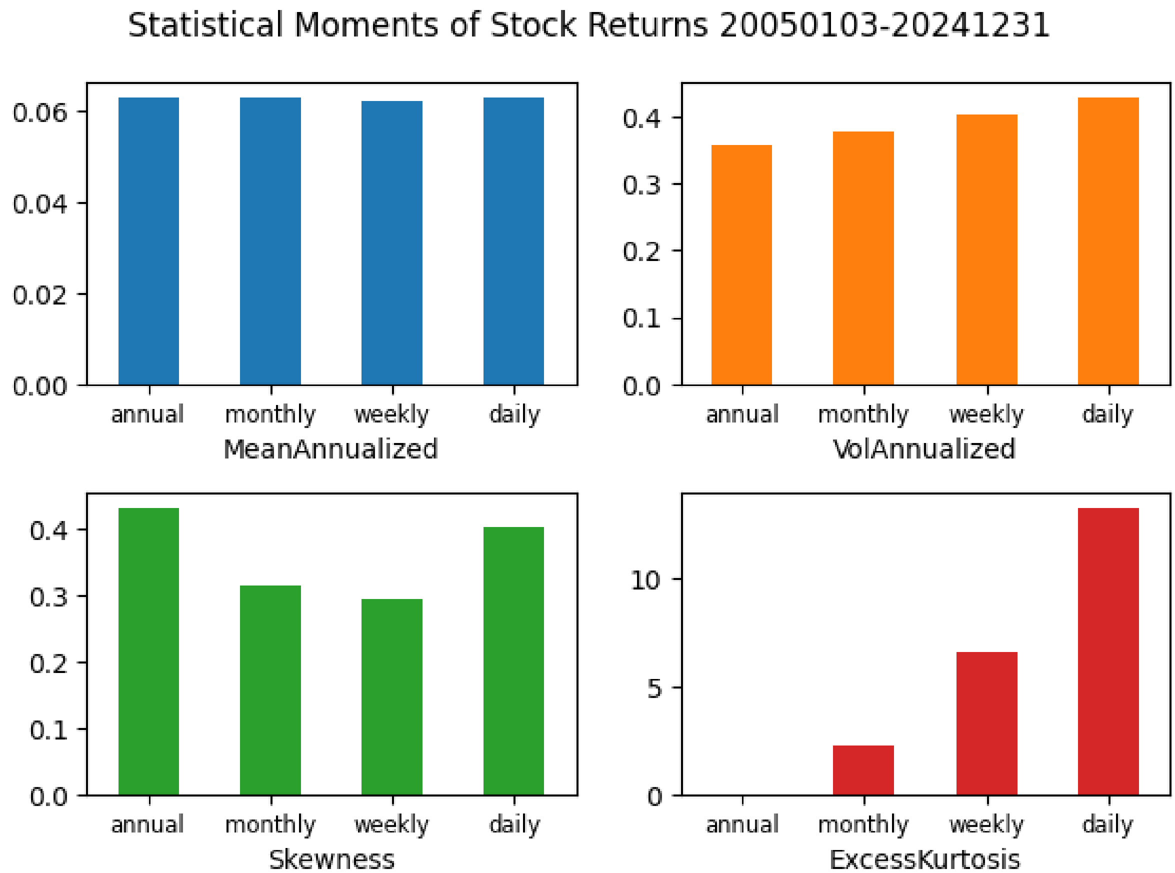
```
100%|██████████| 19/19 [00:00<00:00, 37.61it/s]
100%|██████████| 228/228 [00:04<00:00, 53.61it/s]
100%|██████████| 991/991 [00:01<00:00, 517.26it/s]
100%|██████████| 4781/4781 [07:05<00:00, 11.23it/s]
```

As we move from annual to daily sampling, stock returns exhibit greater kurtosis (i.e. fat tails) and annualized standard deviation. Skewness is positive –the right tail of the distribution is longer than the left tail –but are U-shaped with daily and annual returns featuring more positive-skewness than weekly or monthly.

```
moments
```

	MeanAnnualized	VolAnnualized	Skewness	ExcessKurtosis	Count
annual	0.063249	0.356397	0.434219	0.002332	1519.0
monthly	0.063249	0.376568	0.314758	2.242009	1519.0
weekly	0.062119	0.402871	0.293850	6.612277	1519.0
daily	0.063325	0.429472	0.404691	13.332600	1519.0

```
fig, axes = plt.subplots(2,2)
for ix, ax in enumerate(axes.flatten()):
    moments[moments.columns[ix]].plot.bar(ax=ax, color=f"C{ix}")
    ax.set_xticklabels(moments.index, rotation=0, fontsize='small')
    ax.set_xlabel(moments.columns[ix])
plt.suptitle(f"Statistical Moments of Stock Returns {begdate}-{enddate}")
plt.tight_layout()
plt.show()
```



1.3.3 Correlations

Pearson's correlation, also known as the linear correlation estimator, measures the strength of a linear relationship between two variables. However, alternative methods can better capture nonlinear dependencies:

- **Spearman's rank correlation** applies Pearson's correlation to the ranked values of observations, measuring monotonic relationships while being less sensitive to outliers.
- **Kendall's** τ quantifies the relationship between two variables by comparing the number of concordant and discordant pairs –pairs that agree or disagree on ordering. It is robust to outliers and effective for skewed or non-normally distributed data.

Monthly SP500 and 30-year Treasury total market returns show positive correlation, which is fairly robust across the three methods.

```
# retrieve SP500 and 30-Year Treasury total market returns from CRSP Indexes
ret = bench.get_series(['sprtrn', 'b30ret(mo)'], field='ret').dropna()
ret
```

```
permno       sprtrn  b30ret(mo)
date
19620731  0.006917   -0.008187
19620831  0.007498    0.031939
19620928  0.008965    0.015465
```

(continues on next page)

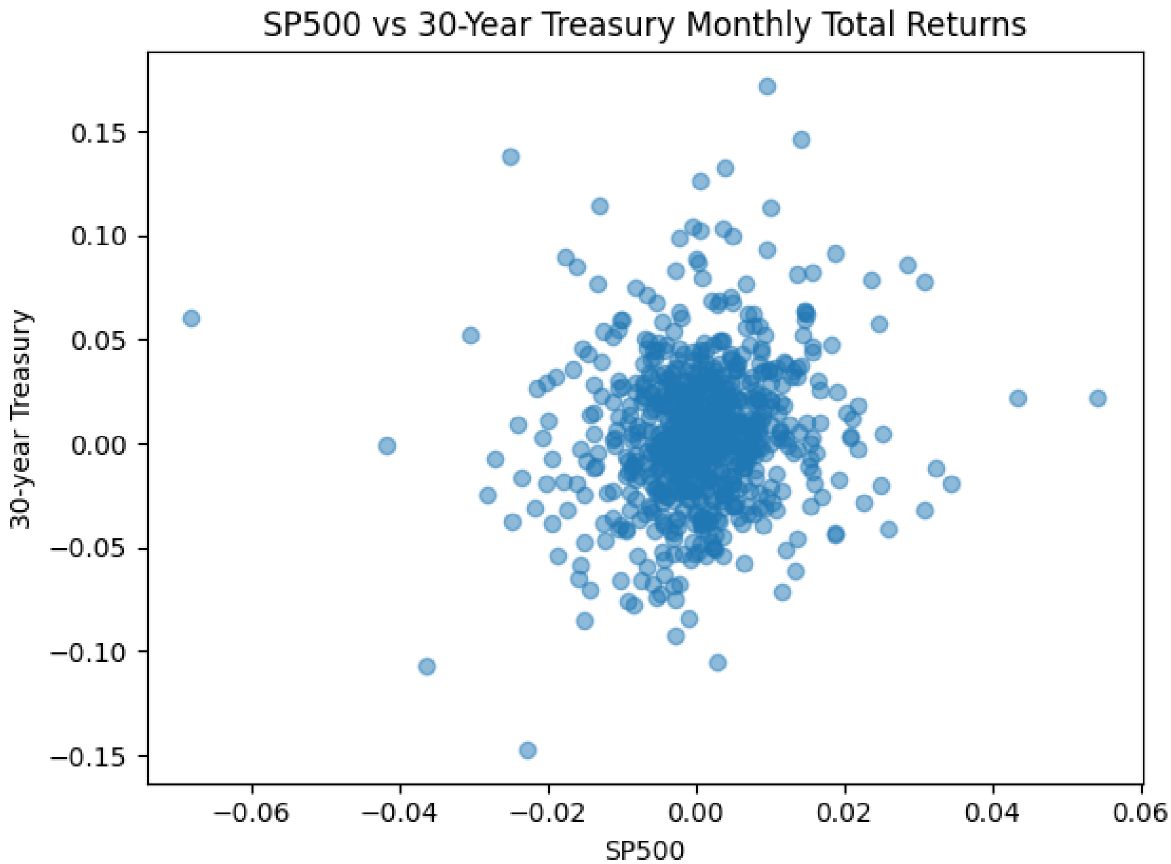
(continued from previous page)

```
19621031 -0.000354    0.012171
19621130 -0.002403    0.003414
...            ...         ...
20240830  0.010093    0.020778
20240930  0.004237    0.015521
20241031 -0.018615   -0.053895
20241129  0.005608    0.021467
20241231 -0.004285   -0.063115

[749 rows x 2 columns]
```

```python
# Compute alternative measures of correlation
DataFrame(dict(spearman=scipy.stats.spearmanr(ret['sprtrn'], ret['b30ret(mo)'])[0],
               kendall=scipy.stats.kendalltau(ret['sprtrn'], ret['b30ret(mo)'])[0],
               pearson=scipy.stats.pearsonr(ret['sprtrn'], ret['b30ret(mo)'])[0]),
               index=['Correlation of SP500 vs 30-Year Treasury Total Returns']).
↪round(4)
```

```
                                                    spearman  kendall  pearson
Correlation of SP500 vs 30-Year Treasury Total ...    0.1348   0.0933   0.1275
```

```python
# Scatter plot of SP500 and and 30-year Treasury total market returns
fig, ax = plt.subplots()
ax.scatter(ret['sprtrn'], ret['b30ret(mo)'], alpha=0.5)
ax.set_xlabel('SP500')
ax.set_ylabel('30-year Treasury')
plt.title('SP500 vs 30-Year Treasury Monthly Total Returns')
plt.tight_layout()
```



APPENDIX

SQL commands

- Manage tables
  - CREATE DATABASE –Creates a new database.
  - CREATE TABLE –Creates a new table.
  - DELETE –Delete data from a table.
  - DROP COLUMN –Deletes a column from a table.
  - DROP DATABASE –Deletes the entire database.
  - DROP TABLE –Deletes a table from a database.
  - TRUNCATE TABLE –Deletes the data but does not delete the table.
- **Querying a table**
  - SELECT –Used to select data from a database, which is then returned in a results set.
  - SELECT DISTINCT –Sames as SELECT, except duplicate values are excluded.
  - SELECT INTO –Copies data from one table and inserts it into another.
  - UNIQUE –This constraint ensures all values in a column are unique.
  - FROM –Specifies which table to select or delete data from.
  - AS –Renames a table or column with an alias value which only exists for the duration of the query.

- **Query conditions**
  - WHERE –Filters results to only include data which meets the given condition.
  - AND –Used to join separate conditions within a WHERE clause.
  - BETWEEN –Selects values within the given range.
  - IS NULL –Tests for empty (NULL) values.
  - IS NOT NULL –The reverse of NULL. Tests for values that aren' t empty / NULL.
  - LIKE –Returns true if the operand value matches a pattern.
  - NOT –Returns true if a record DOESN' T meet the condition.
  - OR –Used alongside WHERE to include data when either condition is true.
- **Organize results**
  - ORDER BY –Used to sort the result data in ascending (default) or descending order through the use of ASC or DESC keywords.
  - GROUP BY –Used alongside aggregate functions (COUNT, MAX, MIN, SUM, AVG) to group the results.
- **Join tables**
  - INNER JOIN returns rows that have matching values in both tables.
  - LEFT JOIN returns all rows from the left table, and the matching rows from the right table.-
  - RIGHT JOIN returns all rows from the right table, and the matching records from the left table
  - FULL OUTER JOIN returns all rows when there is a match in either left table or right table.

References:

Fama, Eugene F., and Kenneth R. French. 1992. "The cross-section of expected stock returns." The Journal of Finance 47 (2): 427–65.

Shumway, Tyler. 1997. The Delisting Bias in CRSP Data. The Journal of Finance, 52, 327-340.

Bali, Turan G, Robert F Engle, and Scott Murray. 2016. Empirical asset pricing: The cross section of stock returns. John Wiley & Sons.

Stulz, René M., 2018, "The Shrinking Universe of Public Firms: Facts, Causes, and Consequences", National Bureau of Economics, 2 (June 2018)

FRM Exam Book Part I Quantative Analysis Chapter 12

Wharton Research Data Services.

CHAPTER TWO

JEGADEESH-TITMAN ROLLING PORTFOLIOS

The future is just more of the past waiting to happen - Fred D' Aguiar.

The Jegadeesh-Titman rolling portfolios approach explores the phenomenon of price momentum in financial markets, focusing on strategies that involve buying stocks with recent strong performance and selling stocks with weak performance. Univariate spread portfolios are constructed, which help isolate the return differences between high- and low-ranked stocks. The following analysis covers key aspects such as overlapping and non-overlapping portfolio returns, the impact of autocorrelation on variance estimation, and statistical hypothesis testing. Additionally, it discusses the Newey-West correction for standard errors and evaluates the power of hypothesis tests.

```
import math
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import scipy
from scipy.stats import kurtosis, skew, norm
import statsmodels.formula.api as smf
import statsmodels.api as sm
from tqdm import tqdm
from finds.database import SQL, RedisDB
from finds.structured import BusDay, CRSP, CRSPBuffer
from finds.recipes import fractile_split
from finds.utils import plot_date
from secret import credentials, CRSP_DATE
```

```
VERBOSE = 0
#%matplotlib qt
```

```
# open connections
sql = SQL(**credentials['sql'], verbose=VERBOSE)
rdb = RedisDB(**credentials['redis'])
bd = BusDay(sql, verbose=VERBOSE)
crsp = CRSP(sql, bd, rdb=rdb, verbose=VERBOSE)
```

```
# date range and parameters to construct momentum portfolios
begrebal = 19260630                    # first price date is 19251231
endrebal = bd.endmo(CRSP_DATE, -1)     # last rebal is one month before last CRSP
rebaldates = bd.date_range(begrebal, endrebal, 'endmo')
percentiles = [20, 80]    # quintile spread percentile breakpoints
maxhold = 6    # hold each monthly-rebalanced portfolio for 6 months
```

```
# preload monthly dataset into memory
monthly = CRSPBuffer(stocks=crsp, dataset='monthly',
                     fields=['ret', 'retx', 'prc'],
                     beg=bd.begmo(rebaldates[0], -6),
                     end=bd.endmo(rebaldates[-1], 1))
```

2.1 Price momentum

2.1.1 Overlapping portfolio returns

First, we estimate the six-month returns of a momentum strategy by averaging monthly observations.

At the end of each month t, we calculate the sorting variable as the past six-month return of all stocks in the investment universe. The 20th and 80th percentiles of NYSE-listed stocks serve as thresholds: we go long on stocks in the top fractile and short those in the bottom fractile. Each fractile is weighted by market capitalization, while the spread portfolio return is the equal-weighted difference between the two sub-portfolios. The spread portfolio's returns over the next six months are recorded on a monthly basis by dividing by six.

A stock is eligible for inclusion if it meets the usual investment universe criteria at the end of the rebalance month and has a non-missing month-end price from six months prior.

```
stocks = monthly
mom = []
for rebaldate in tqdm(rebaldates):
    # determine pricing dates relative to rebaldate
    beg = bd.endmo(rebaldate, -6)   # require price at beg date
    end = bd.endmo(rebaldate, 0)    # require price at end date
    start = bd.offset(beg, 1)       # starting day of momemtum signal

    # retrieve universe, prices, and momentum signal
    p = [crsp.get_universe(rebaldate),
         stocks.get_ret(beg=start, end=end).rename('mom'),
         stocks.get_section(fields=['prc'], date=beg)['prc'].rename('beg')]
    df = pd.concat(p, axis=1, join='inner').dropna()

    # quintile breakpoints are determined from NYSE subset
    tritile = fractile_split(values=df['mom'],
                             pct=percentiles,
                             keys=df.loc[df['nyse'], 'mom'])

    # construct cap-wtd tritile spread portfolios
    porthi, portlo = [df.loc[tritile==t, 'cap'] for t in [1, 3]]
    port =  pd.concat((porthi/porthi.sum(), -portlo/portlo.sum()))

    # compute and store cap-weighted average returns over (up to) maxhold periods
    begret = bd.offset(rebaldate, 1)
    nhold = min(maxhold, len(rebaldates) - rebaldates.index(rebaldate))
    endret = bd.endmo(begret, nhold - 1)    # if maxhold is beyond end date
    rets = monthly.get_ret(begret, endret)
    ret = rets.reindex(port.index).fillna(0.).mul(port, axis=0).sum()
    mom.append(float(ret) / nhold)
```

```
  0%|          | 0/1182 [00:00<?, ?it/s]
```

```
100%|██████████| 1182/1182 [11:41:22<00:00, 35.60s/it]
```

```
DataFrame({'mean': np.mean(mom), 'std': np.std(mom)}, index=['Overlapping Returns'])
```

```
                         mean       std
Overlapping Returns  0.004526  0.024599
```

2.1.2 Non-overlapping portfolio returns

A spread portfolio is constructed at the end of each month in the same manner. However, instead of overlapping returns, the return recorded is the equal-weighted average of the following month's returns from six distinct portfolios formed between t and $t-5$. Each month, the weights of stocks in the spread portfolios adjust according to their price changes, following a "buy-and-hold" approach over six months.

```
ports = []  # to roll 6 past portfolios
jt = []
stocks = monthly
for rebaldate in tqdm(rebaldates):

    # determine returns dates relative to rebaldate
    beg = bd.endmo(rebaldate, -6)    # require price at beg date
    end = bd.endmo(rebaldate, 0)     # require price at end date
    start = bd.offset(beg, 1)        # starting day of momemtum signal

    # retrieve universe, prices, and momentum signal
    p = [crsp.get_universe(rebaldate),
         stocks.get_ret(beg=start, end=end).rename('mom'),
         stocks.get_section(fields=['prc'], date=beg)['prc'].rename('beg')]
    df = pd.concat(p, axis=1, join='inner').dropna()

    # quintile breakpoints determined from NYSE subset
    tritile = fractile_split(values=df['mom'],
                             pct=percentiles,
                             keys=df.loc[df['nyse'], 'mom'])

    # construct cap-wtd tritile spread portfolios
    porthi, portlo = [df.loc[tritile==t, 'cap'] for t in [1, 3]]
    port =  pd.concat((porthi/porthi.sum(), -portlo/portlo.sum()))

    # retain up to 6 prior months of monthly-rebalanced portfolios
    ports.insert(0, port)
    if len(ports) > maxhold:
        ports.pop(-1)

    # compute all 6 portfolios' monthly capwtd returns, and store eqlwtd average
    begret = bd.offset(rebaldate, 1)
    endret = bd.endmo(begret)
    rets = stocks.get_ret(begret, endret)
    ret = np.mean([rets.reindex(p.index).fillna(0.).mul(p, axis=0).sum()
                   for p in ports])
    jt.append(ret)

    # adjust stock weights by monthly capital appreciation
```

(continues on next page)

(continued from previous page)

```
    retx = stocks.get_ret(begret, endret, field='retx')
    ports = [(1 + retx.reindex(p.index).fillna(0.)).mul(p, axis=0)
             for p in ports]

DataFrame({'mean': np.mean(jt), 'std': np.std(jt)}, index=['Non-overlapping Returns'])
```

```
  0%|          | 0/1182 [00:00<?, ?it/s]
```

```
100%|██████████| 1182/1182 [03:30<00:00,  5.62it/s]
```

```
                             mean       std
Non-overlapping Returns  0.004502  0.051419
```

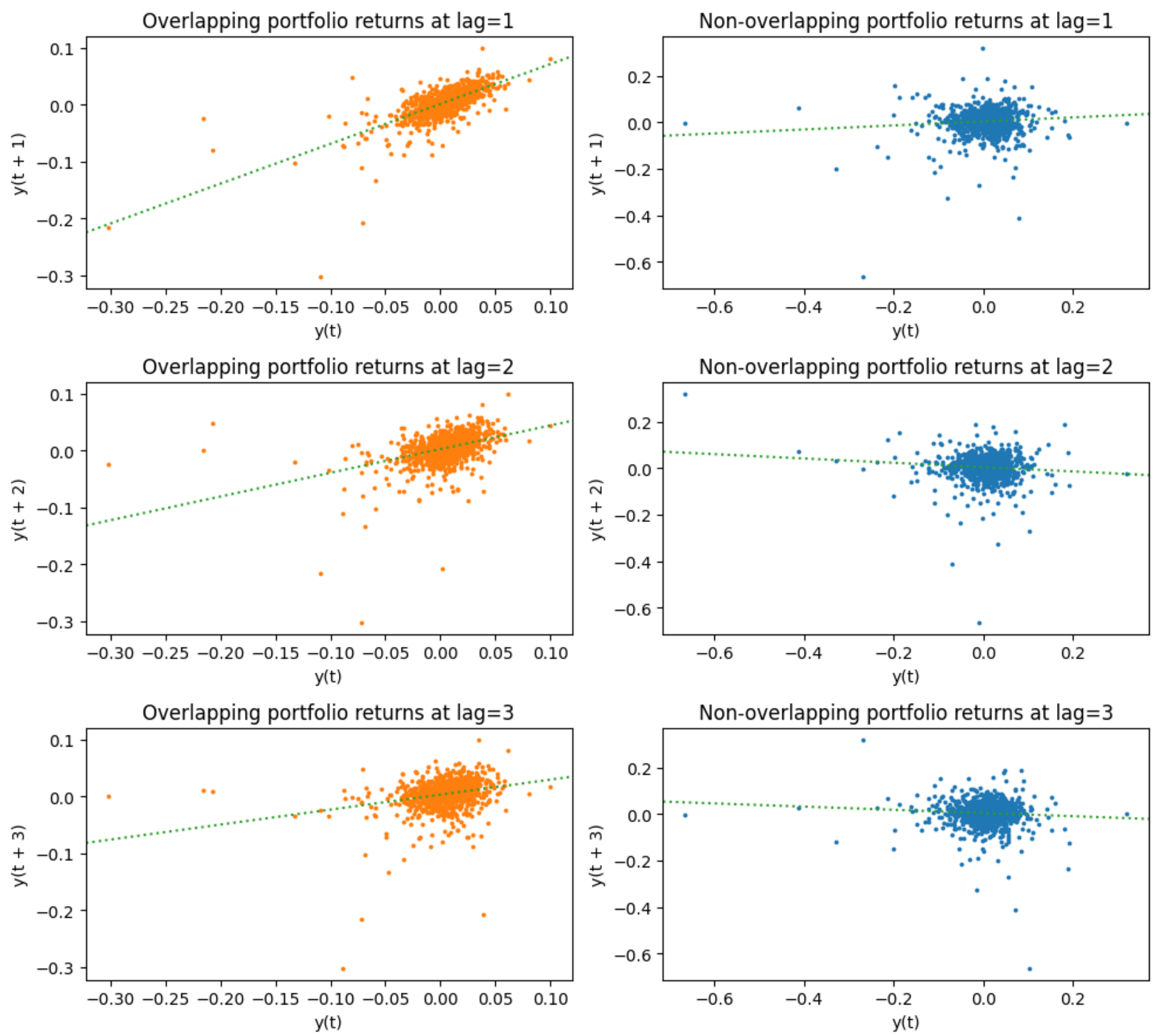
Correlation with lagged returns

For the overlapping portfolios, each month's recorded return is (one-sixth of) a six-month return. Let r_t be the return at time t. The 6-month return at time t, denoted as R_t, is the sum of the past 6 monthly returns: $R_t = r_t + r_{t-1} + r_{t-2} + r_{t-3} + r_{t-4} + r_{t-5}$

Since we sample monthly, consecutive returns R_t and R_{t+1} overlap significantly. Up to 5/6 of adjacent months' returns actually reflect the same month's stock returns. Even returns recorded five months apart share one month of stock returns in common. Ignoring this overlap when estimating variance leads to underestimation of the true variance.

The Jegadeesh-Titman non-overlapping portfolio approach eliminates this issue.

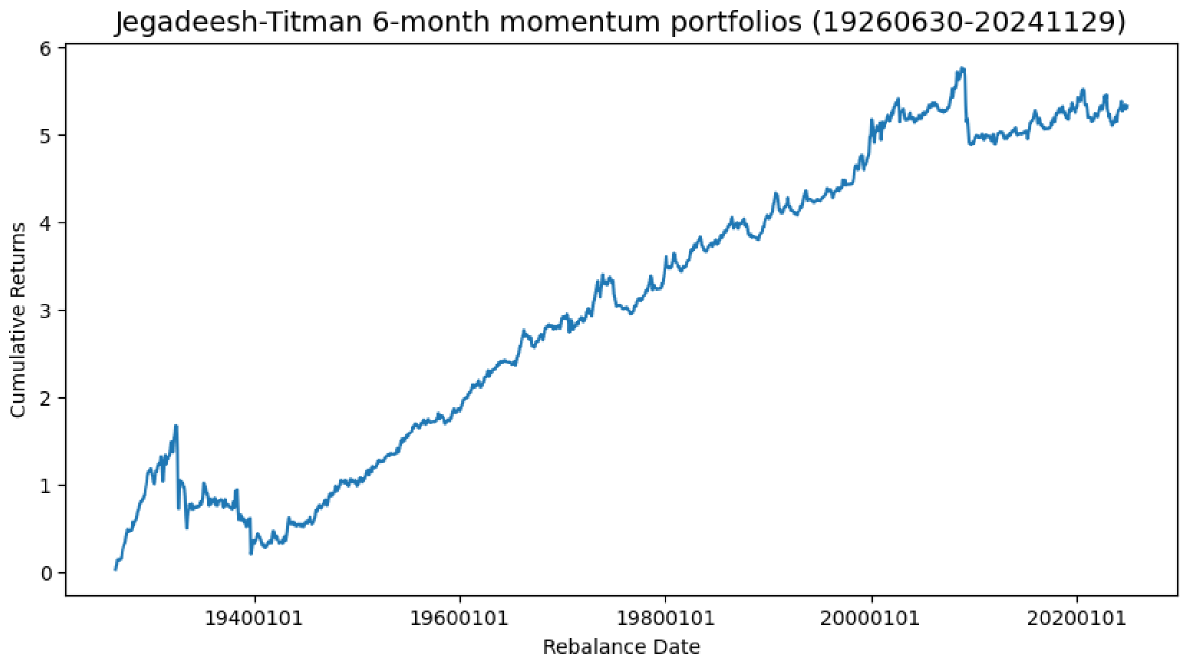
```
fig, axes = plt.subplots(nrows=3, ncols=2, figsize=(10, 9))
for lag, ax in zip(range(1, axes.shape[0]+1, 1), axes):
    pd.plotting.lag_plot(Series(mom), lag=lag, ax=ax[0], s=3, c="C1")
    ax[0].set_title(f"Overlapping portfolio returns at lag={lag}")
    r = scipy.stats.linregress(mom[lag:], mom[:-lag])
    ax[0].axline((0, r.intercept), slope=r.slope, ls=':', color="C2")

    pd.plotting.lag_plot(Series(jt), lag=lag, ax=ax[1], s=3, c="C0")
    ax[1].set_title(f"Non-overlapping portfolio returns at lag={lag}")
    r = scipy.stats.linregress(jt[lag:], jt[:-lag])
    ax[1].axline((0, r.intercept), slope=r.slope, ls=':', color="C2")
plt.tight_layout()
```



Plot cumulative monthly average returns

Jegadeesh-Titman non-overlapping 6-month momentum portfolio cumulative returns

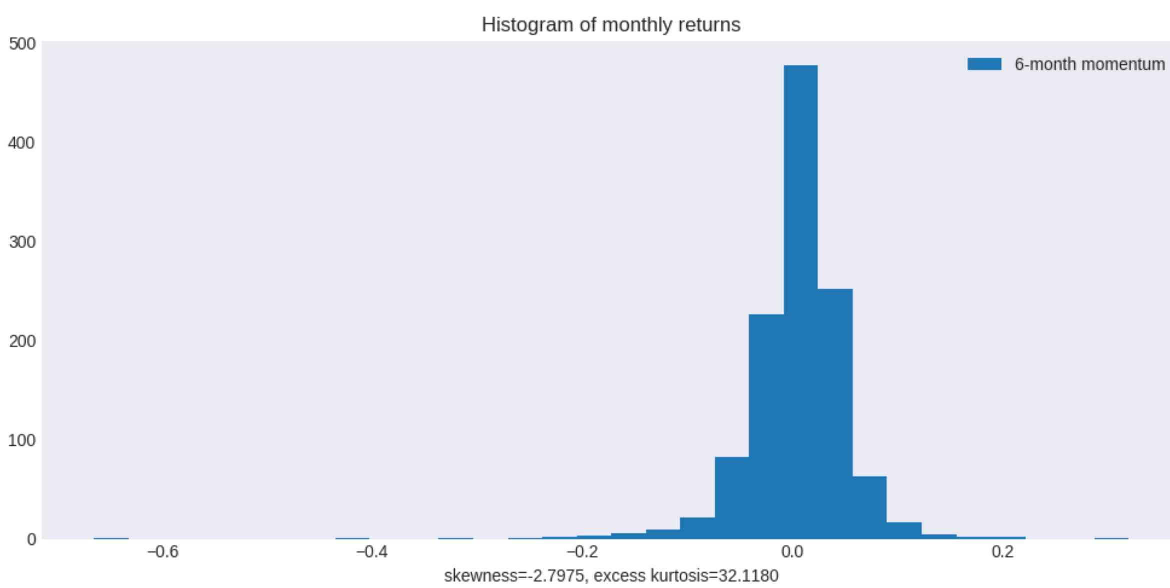
```
fig, ax = plt.subplots(figsize=(10, 5), clear=True)
plot_date(DataFrame(index=rebaldates, data=np.cumsum(jt), columns=['momentum']),
          ax=ax, fontsize=10, rotation=0,
          ylabel1='Cumulative Returns', xlabel='Rebalance Date',
          title=f'Jegadeesh-Titman 6-month momentum portfolios ({rebaldates[0]}-
↪{rebaldates[-1]})')
plt.show()
```



Plot histogram of monthly returns

Distribution of Jegadeesh-Titman non-overlapping 6-month momentum portfolio returns

```
fig, ax = plt.subplots(1, 1, clear=True, figsize=(10, 5))
ax.hist(jt, bins=30)
ax.set_title(f"Histogram of monthly returns")
ax.legend(['6-month momentum'])
kurt = kurtosis(jt, bias=True, fisher=True)  # excess kurtosis
skewness = skew(jt, bias=True)
ax.set_xlabel(f"skewness={skewness:.4f}, excess kurtosis={kurt:.4f}")
plt.tight_layout()
```



2.2 Hypothesis testing

A hypothesis test makes a precise statement about population parameters and evaluates the likelihood of observing the data under a given assumption.

- The *null hypothesis* specifies the true value of a parameter to be tested, often $H_0 : \hat{\mu} = \mu_0$
- The *test statistic* is a summary of the observed data that has a known distribution when the null hypothesis is true, e.g. $T - \dfrac{\hat{\mu} - \mu_0}{\sqrt{\sigma^2/n}} \sim N(0, 1)$
- The *alternative hypothesis* defines the range of values of the parameter where the null should be rejected, e.g. $H_a : \hat{\mu} \neq \mu_0$
  - In some testing problems, the alternative hypothesis is not the full complement of the null, for example, a *one-sided alternative* $H_a : \hat{\mu} > \mu_0$, which is used whtn the outcome of interest is only above or below the value assumed by the null.
- The *critical value* C_α marks the start of a range of values where the test statistic is unlikely to fall in, if the null hypthesis were true, e.g. $C_\alpha = \Phi^{-1}(1 - \alpha/2) = 1.96$ when $\alpha = 5\%$ for a two-sided test. This range is known as the *rejection region*.
- The *size* of the test is the probability of making a *Type I error* of rejecting null hypothesis that is actually true. A test is said to have *significance level* α if its *size* is less than or equal to α. This reflects the aversion to rejecting a null hypothesis that is, in fact, true.
- The *p-value* is the probability of obtaining a test statistic at least as extreme as the one we observed from the sample, if the null hypothesis were true, e.g. $p = 2(1 - \Phi(|T|))$ for a two-sided test.

2.2.1 Confidence Interval

A $1 - \alpha$ *confidence interval* contains the values surrounding the test statistic that cannot be rejected when using a test size of α, e.g. $[\hat{\mu} - C_\alpha \dfrac{\sigma^2}{\sqrt{n}}, \hat{\mu} + C_\alpha \dfrac{\sigma^2}{\sqrt{n}}]$ for a two-sided interval

2.2.2 Newey-West corrected t-stats

Standard errors are underestimated when assuming independent observations, as this assumption does not hold for overlapping returns. The **Newey-West (1987) estimator** corrects for heteroskedasticity and autocorrelation by specifying a "maximum lag" for autocorrelation control. A common choice is $L =$ the fourth root of the number of observations (e.g., Greene, *Econometric Analysis*, 7th ed., p. 960).

Applying the Newey-West correction nearly doubles the estimated standard error for overlapping portfolios, but it has a minimal effect on non-overlapping returns.

```
print('n =', len(mom), '   L =', math.ceil(len(mom)**(1/4)))
results = []
for rets, label in zip([mom, jt], ['Overlapping', 'Non-overlapping']):
    data = DataFrame(rets, columns=['ret'])

    # raw t-stats
    reg = smf.ols('ret ~ 1', data=data).fit()
    uncorrected = Series({stat: round(float(getattr(reg, stat).iloc[0]), 6)
                          for stat in ['params','bse','tvalues','pvalues']},
                         name='uncorrected')  # coef, stderr, t-value, P>|z|
```

(continues on next page)

(continued from previous page)

```
    # Newey-West correct t-stats
    reg = smf.ols('ret ~ 1', data=data)\
             .fit(cov_type='HAC', cov_kwds={'maxlags': 6})
    corrected = Series({stat: round(float(getattr(reg, stat).iloc[0]), 6)
                        for stat in ['params','bse','tvalues','pvalues']},
                       name='NeweyWest')    # coef, stderr, t-value, P>|z|

    # merge into intermediate dataframe with multicolumn index
    df = pd.concat([uncorrected, corrected], axis=1)
    df.columns = pd.MultiIndex.from_product([[label], df.columns])
    results.append(df)

pd.concat(results, axis=1).rename_axis('Standard Errors')
```

```
n = 1182    L = 6
```

```
                Overlapping           Non-overlapping
                uncorrected NeweyWest     uncorrected NeweyWest
Standard Errors
params             0.004526  0.004526        0.004502  0.004502
bse                0.000716  0.001285        0.001496  0.001463
tvalues            6.322644  3.522659        3.009069  3.078432
pvalues            0.000000  0.000427        0.002676  0.002081
```

2.2.3 Power of Test

A **Type II error** occurs when the alternative hypothesis is true but the null is not rejected. The probability of a Type II error is denoted by β, while **power** $(1-\beta)$ represents the probability of correctly rejecting a false null hypothesis.

Unlike test size, the power of a test depends on:

1. Sample size
2. Test size (α)
3. The distance between the true parameter value and the null hypothesis value

For a one-sided test $H_a : \hat{\mu} > \mu_0$, power is given by:

$$1-\beta(\alpha) = \Phi\left(C_\alpha \frac{\sigma^2}{\sqrt{n}} \middle| \mu_a, \frac{\sigma^2}{\sqrt{n}}\right)$$

```
DataFrame(data={"True Null": ['correct', '(1 - alpha)', 'Type I Error', 'Size: (alpha)
↪'],
                "False Null": ['Type II Error', '(beta)', 'correct', 'Power: (1-beta)
↪']},
          index=['Accept Null', '', 'Reject Null', ''])\
          .rename_axis(index='Decision')
```

```
                 True Null       False Null
Decision
Accept Null        correct    Type II Error
```

(continues on next page)

(continued from previous page)

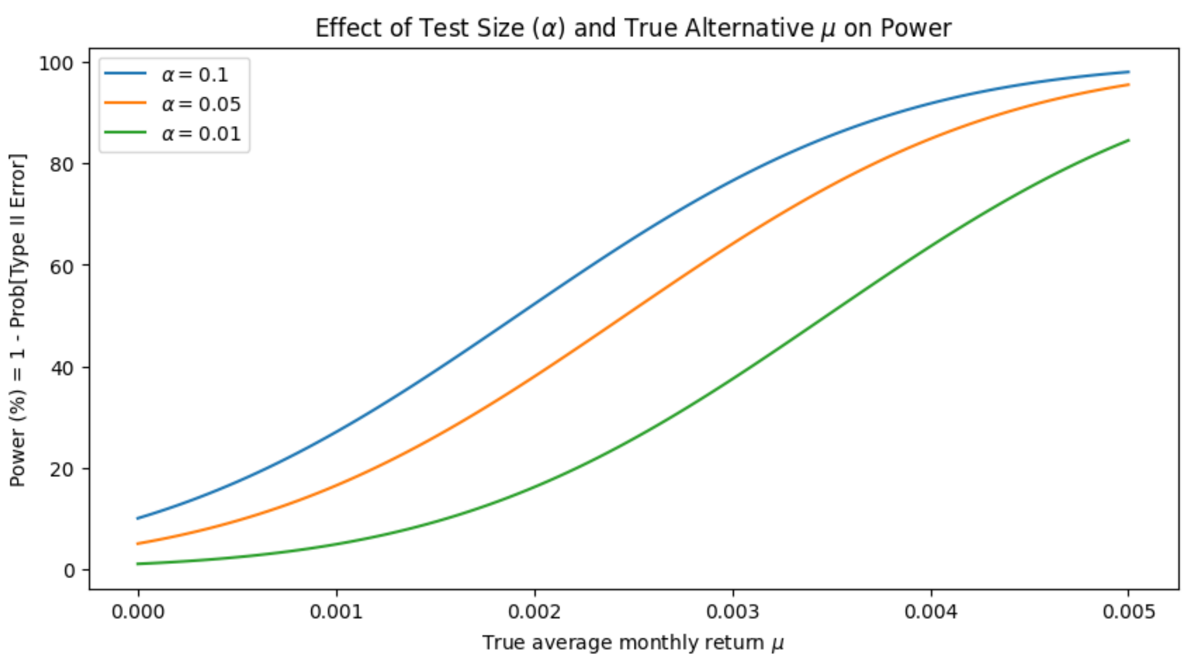
```
                 (1 - alpha)            (beta)
Reject Null    Type I Error           correct
              Size: (alpha)   Power: (1-beta)
```

Effect of Test Size (alpha) and True Alternative (mu) on Power

```
# Assumtions
alternative = 0.06   # alternative hypothesis that annualized mean is as large as 6%
scale = np.std(jt) / np.sqrt(len(jt))  # assumed scale (std dev or volatility)
```

```
# Vary test size (alpha) and true mean (mu)
mu = np.linspace(0, alternative/12, 100)    # vary true mean
plt.figure(figsize=(10, 5))
for alpha in [0.1, 0.05, 0.01]:             # vary test size
    power = 1 - norm.cdf(norm.ppf(1 - alpha) * scale, loc=mu, scale=scale)
    plt.plot(mu, 100*power, label=f"$\\alpha=${alpha}")
plt.title("Effect of Test Size ($\\alpha$) and True Alternative $\\mu$ on Power")
plt.ylabel('Power (%) = 1 - Prob[Type II Error]')
plt.xlabel('True average monthly return $\\mu$')
plt.legend()
```

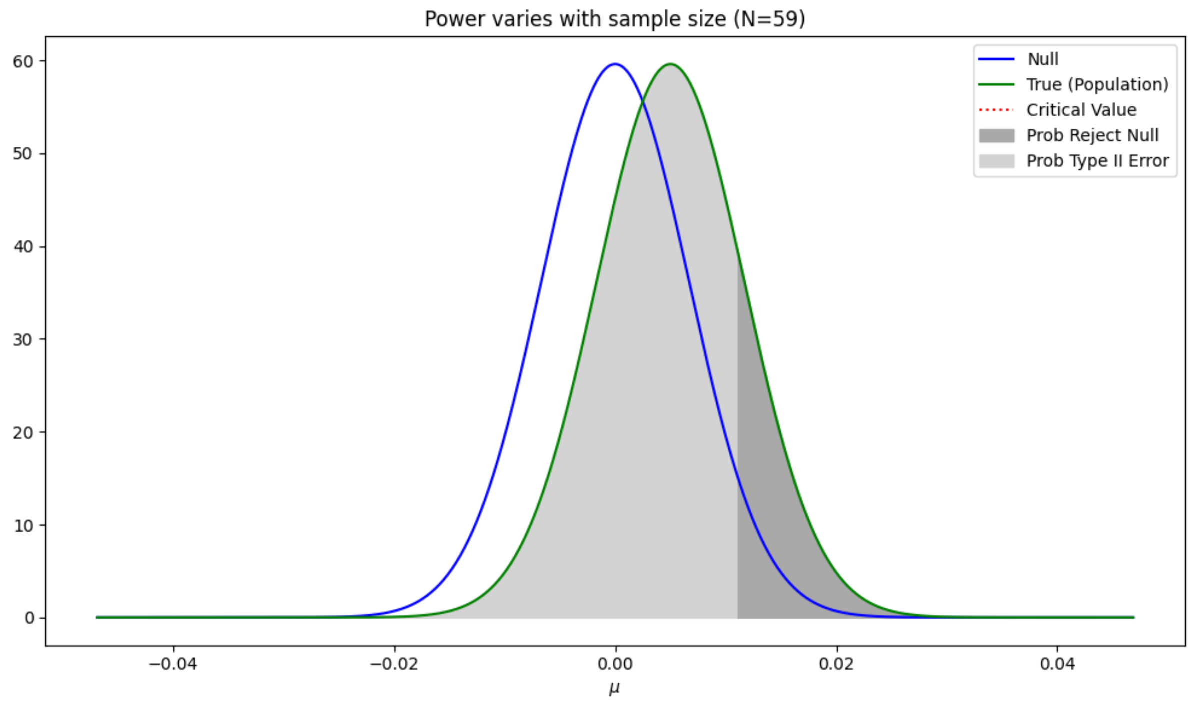
```
<matplotlib.legend.Legend at 0x7f12c13d0950>
```

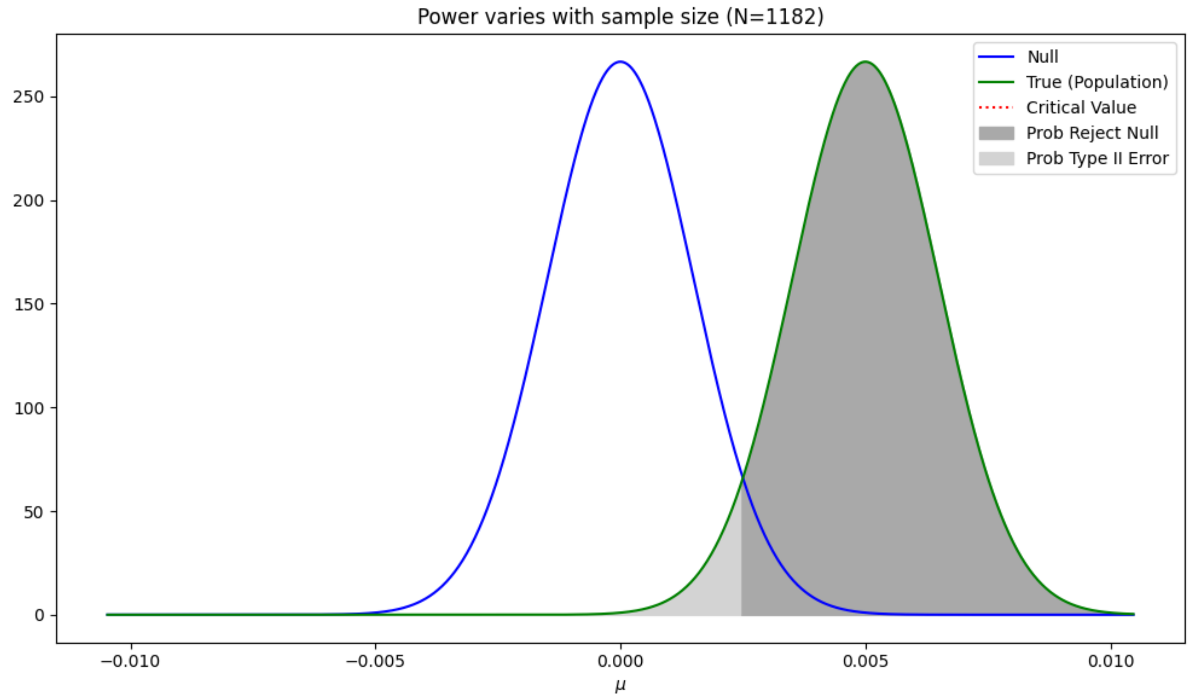


Effect of Sample Size on Power

```
# Assumptions
volatility = np.std(jt)
alternative = 0.06/12      # mean of the alternate hypotehsis
alpha = 0.05               # desired size of the test
```

```
# Compare large and small sample sizes
for N in [len(jt) // 20, len(jt)]:

    # define null and alternate distributions given sample size
    scale = volatility/np.sqrt(N)    # scaled by square root of sample size
    null_dist = norm(0, scale)
    alt_dist = norm(alternative, scale)
    critical_val = null_dist.ppf(1-alpha)  # critical value to reject null

    fig, ax = plt.subplots(figsize=(10, 6))
    x = np.linspace(-7 * scale, 7 * scale, 1000)
    ax.plot(x, null_dist.pdf(x), color='blue')  # plot null distribution
    ax.plot(x, alt_dist.pdf(x), color='green')  # plot alt distribution
    ylim = plt.ylim()[0]
    ax.axvline(x=critical_val, ymax=ylim, ls=':', color='r') # critical value
    px = x[x > critical_val]
    ax.fill_between(px, alt_dist.pdf(px), color='darkgrey')  # rejection region
    px = x[x < critical_val]
    ax.fill_between(px, alt_dist.pdf(px), color='lightgrey') # acceptance region
    ax.set_title(f"Power varies with sample size (N={N})")
    ax.set_xlabel("$\mu$")
    plt.legend(['Null', 'True (Population)', 'Critical Value',
                'Prob Reject Null', 'Prob Type II Error'])
    plt.tight_layout()
```





References:

Jegadeesh, Narasimhan, and Sheridan Titman (1993), "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency". Journal of Finance. March 1993, Volume 48, Issue 1, Pages 65-91.

Newey, Whitney K, West, Kenneth D (1987). "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix". Econometrica. 55 (3): 703–708.

Hong, Harrison, Terence Lim, Jeremy C. Stein, 2000, "Bad News Travels Slowly: Size, Analyst Coverage, and the Profitability of Momentum Strategies", Volume 55, Issue 1, Pages 265-295. https://doi.org/10.1111/0022-1082.00206

FRM Exam Book Part I Quantative Analysis Chapter 6

CHAPTER

THREE

FAMA-FRENCH PORTFOLIO SORTS

The way to become rich is to put all your eggs in one basket and then watch that basket - Andrew Carnegie

The Fama-French portfolio sorting methodology is widely used in empirical asset pricing research, particularly in understanding the cross-section of stock returns. By classifying stocks based on fundamental characteristics such as book-to-market ratio and firm size, this approach provides insights into the risk and return dynamics of different investment strategies. This notebook also includes linear regression analysis to assess factor exposures, tests for the value and small-firm effects, and a structural break analysis using the Chow test.

```
import numpy as np
import scipy
from scipy.stats import skew, kurtosis
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import statsmodels.formula.api as smf
from finds.database import SQL, RedisDB
from finds.structured import BusDay, CRSP, CRSPBuffer, Signals, Benchmarks, PSTAT
from finds.utils import plot_date
from finds.backtesting import bivariate_sorts, BackTest
from finds.utils import plot_date, plot_scatter, plot_hist
from tqdm import tqdm
from secret import credentials, CRSP_DATE
```

```
VERBOSE = 0
# %matplotlib qt
```

```
sql = SQL(**credentials['sql'], verbose=VERBOSE)
user = SQL(**credentials['user'], verbose=VERBOSE)
rdb = RedisDB(**credentials['redis'])
bd = BusDay(sql, verbose=VERBOSE)
crsp = CRSP(sql, bd, rdb=rdb, verbose=VERBOSE)
pstat = PSTAT(sql, bd, verbose=VERBOSE)
signals = Signals(user, verbose=VERBOSE)
bench = Benchmarks(sql, bd, verbose=VERBOSE)
backtest = BackTest(user, bench, rf='RF', max_date=CRSP_DATE, verbose=VERBOSE)
LAST_DATE = bd.endmo(CRSP_DATE, -1)  # last monthly rebalance date
```

3.1 Stock fundementals data

3.1.1 Compustat

Compustat is a database containing financial statements and market data for both active and inactive U.S. and international companies. It is commonly used in academic and industry research.

To compute book-to-price ratios from financial statements, we:

- Extract balance sheet items from the Compustat Annual dataset.
- Construct the High Minus Low (HML) factor by calculating book equity as shareholders' equity plus investment tax credits, minus preferred stock, divided by the market capitalization at the end of December.
- Apply a six-month reporting lag and require at least two years of history in Compustat.
- Exclude deferred taxes and investment tax credits from book equity for fiscal years ending in 1993 or later, following FASB 109, which improved the accounting treatment for deferred income taxes.

```
label = 'hml'
lag = 6               # number of months to lag fundamental data
# retrieve data fields from compustat, linked by permno
df = pstat.get_linked(dataset = 'annual',
                      date_field = 'datadate',
                      fields = ['seq', 'pstk', 'pstkrv', 'pstkl', 'txditc'],
                      where = ("indfmt = 'INDL'"
                               "  AND datafmt = 'STD'"
                               "  AND curcd = 'USD' "
                               "  AND popsrc = 'D'"
                               "  AND consol = 'C'"
                               "  AND seq > 0 "))
```

```
# subtract preferred stock
df[label] = np.where(df['pstkrv'].isna(), df['pstkl'], df['pstkrv'])
df[label] = np.where(df[label].isna(), df['pstk'], df[label])
df[label] = np.where(df[label].isna(), 0, df[label])
```

```
# do not add back deferred investment tax credit for fiscal years in 1993 or later
df[label] = (df['seq'] - df[label]
             + df['txditc'].fillna(0).where(df['datadate'] // 10000 <= 1993, 0))
df.dropna(subset = [label], inplace=True)
df = df[df[label] > 0][['permno', 'gvkey', 'datadate', label]]
```

```
# count years in Compustat
df = df.sort_values(by=['gvkey','datadate'])
df['count'] = df.groupby(['gvkey']).cumcount()
```

```
# construct b/m ratio
df['rebaldate'] = 0
for datadate in tqdm(sorted(df['datadate'].unique())):
    f = df['datadate'].eq(datadate)
    rebaldate = bd.endmo(datadate, abs(lag)) # 6 month lag
    capdate = bd.endyr(datadate)    # Dec mktcap
    if rebaldate >= CRSP_DATE or capdate >= CRSP_DATE:
        continue
```

(continues on next page)

(continued from previous page)

```
    df.loc[f, 'rebaldate'] = rebaldate
    df.loc[f, 'cap'] = crsp.get_cap(capdate, use_permco=True)\
                           .reindex(df.loc[f, 'permno'])\
                           .values
df[label] /= df['cap']
df = df[df[label].gt(0) & df['count'].gt(1)]  # 2+ years in Compustat
signals.write(df, label)
```

```
100%|██████████| 758/758 [00:02<00:00, 317.69it/s]
```

```
227642
```

3.2 Bivariate sorts

Independent bivariate sorts categorize stocks based on two characteristics: book-to-market ratio and market capitalization. Portfolios are formed at the end of each June and represent the intersections of:

- Two groups sorted by size (market equity, ME).
- Three groups sorted by book-to-market ratio (BE/ME).

The size breakpoint for year t is the median NYSE market equity at the end of June in that year. Stocks within each of the six resulting portfolios are weighted by market capitalization.

The two key factors derived from these sorts are:

- **HML (High Minus Low):** The equal-weighted average return of the two value portfolios minus the average return of the two growth portfolios.
- **SMB (Small Minus Big):** The equal-weighted average return of the three small-size portfolios minus the average return of the three large-size portfolios.

Causal Analysis

This sorting approach has conceptual parallels with causal analysis techniques. Specifically, propensity score matching is often used in statistical research to mitigate confounding effects when estimating treatment effects. Propensity scores, estimated via logistic regression, allow researchers to:

- **Stratify** subjects into groups based on similar propensity scores.
- **Match** treated and control subjects with comparable propensity scores.
- **Adjust** for imbalances using regression models.

Since firm size directly influences the book-to-market ratio (as its denominator), applying bivariate sorting ensures that value returns are estimated while controlling for the small-firm effect—similar to how propensity score matching controls for confounding variables in observational studies.

3.2.1 HML

Compute High Minus Low book-to-price monthly returns and compare to Fama-French research factor

```
label, benchname = 'hml', 'HML(mo)'
rebalend = LAST_DATE
rebalbeg = 19700101
```

```
# preload monthly dataset into memory
monthly = CRSPBuffer(stocks=crsp, dataset='monthly',
                     fields=['ret', 'retx', 'prc'],
                     beg=19251201, end=CRSP_DATE)
```

```
hml, smb = bivariate_sorts(stocks=monthly,
                           label=label,
                           signals=signals,
                           rebalbeg=rebalbeg,
                           rebalend=rebalend,
                           window=12,
                           months=[6])
```

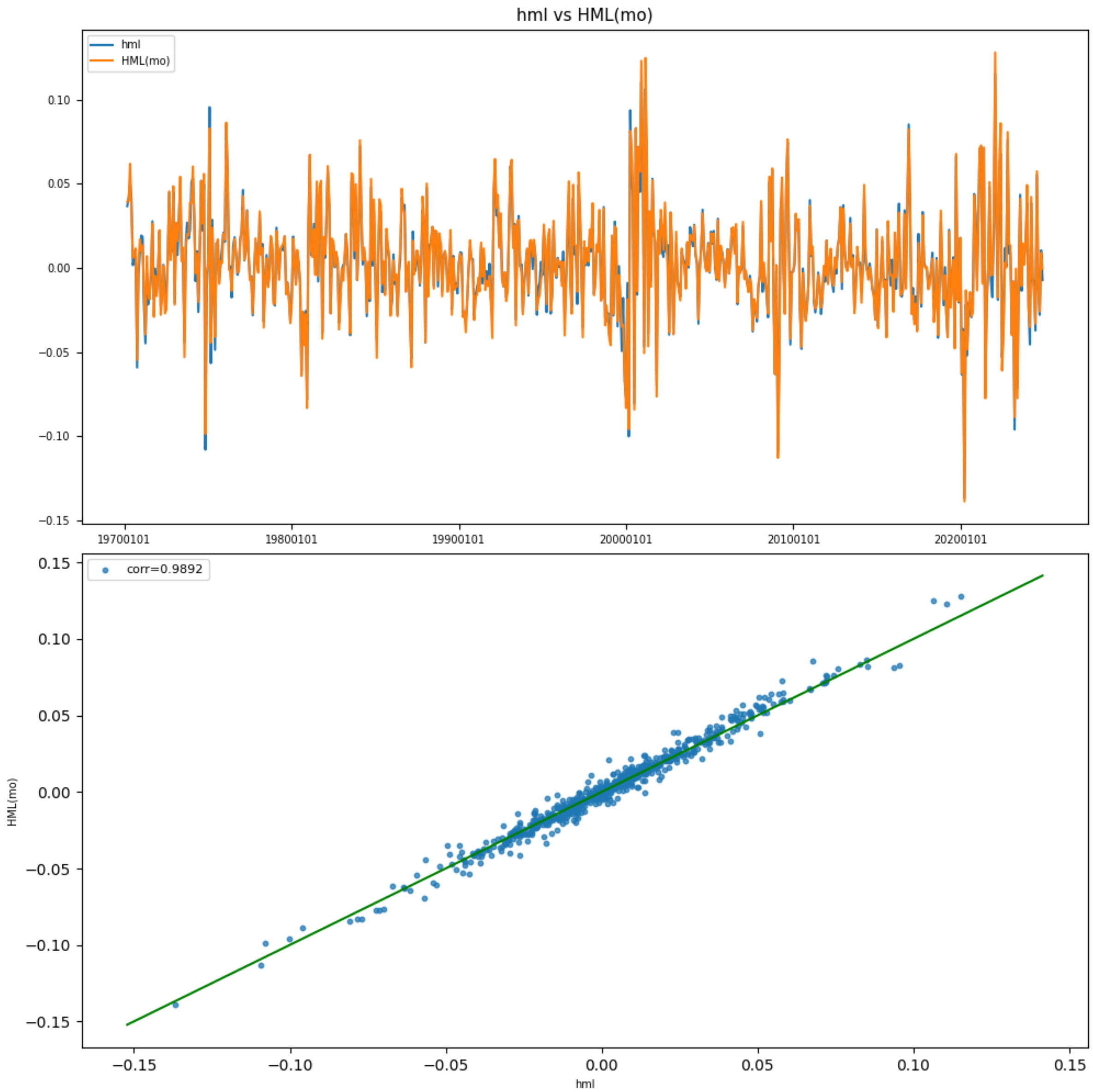
Helpers to show histograms and comparisons of portfolio returns

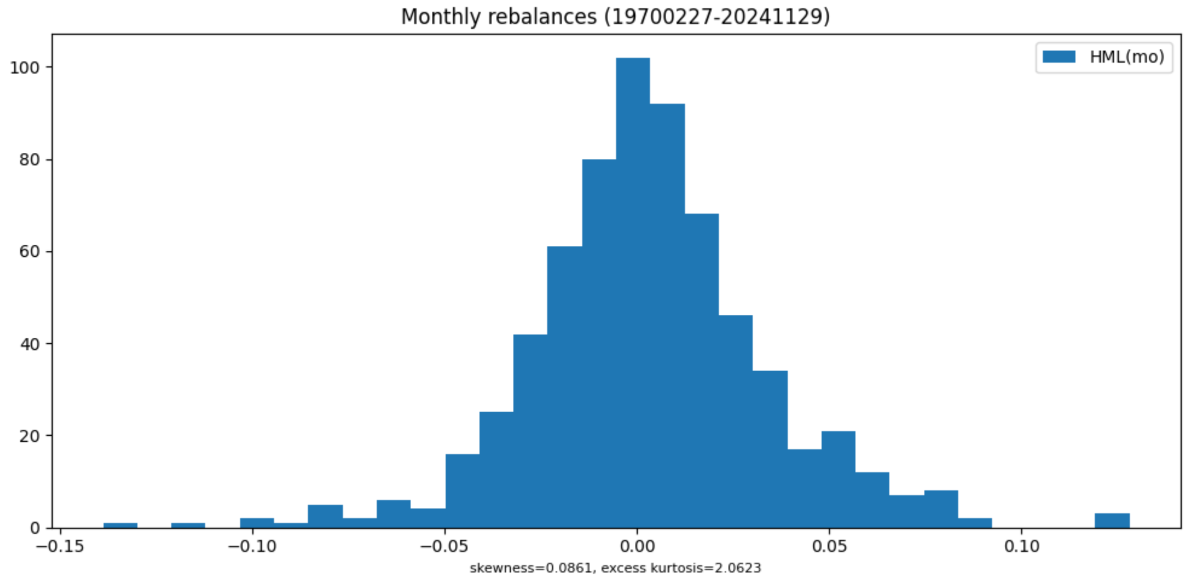
```
def plot_ff(y, label):
    """helper to scatter plot and compare portfolio returns"""
    y = y.rename(columns={'excess': label})
    corr = np.corrcoef(y, rowvar=False)[0,1]
    fig, (ax1, ax2) = plt.subplots(2, 1, clear=True, figsize=(10, 10))
    plot_date(y, ax=ax1, title=" vs ".join(y.columns), fontsize=7)
    plot_scatter(y.iloc[:,0], y.iloc[:,1], ax=ax2, abline=False, fontsize=7)
    plt.legend([f"corr={corr:.4f}"], fontsize=8)
    plt.tight_layout(pad=0.5)
    print(f"<R-squared of {label} vs {benchname}"
          f" ({y.index[0]} - {y.index[-1]}): {corr*corr:.4f}")
```

```
def plot_summary(y, label):
    """helper to plot histogram and statistics of portfolio returns"""
    y = y[label]
    kurt = kurtosis(y, bias=True, fisher=True)  # excess kurtosis
    skewness = skew(y, bias=True)
    fig, ax = plt.subplots(1, 1, clear=True, figsize=(10, 5))
    ax.hist(y, bins=30)
    ax.set_title(f"Monthly rebalances ({y.index[0]}-{y.index[-1]})")
    ax.set_xlabel(f"skewness={skewness:.4f}, excess kurtosis={kurt:.4f}",
                  fontsize=8)
    plt.legend([label])
    plt.tight_layout()
```

```
# Plot histogram and comparison of HML returns
holdings = hml
result = backtest(monthly, holdings, label)
y = backtest.fit([benchname], rebalbeg, LAST_DATE)
plot_ff(y, label)
plot_summary(y, benchname)
```

```
<R-squared of hml vs HML(mo) (19700227 - 20241129): 0.9784
```



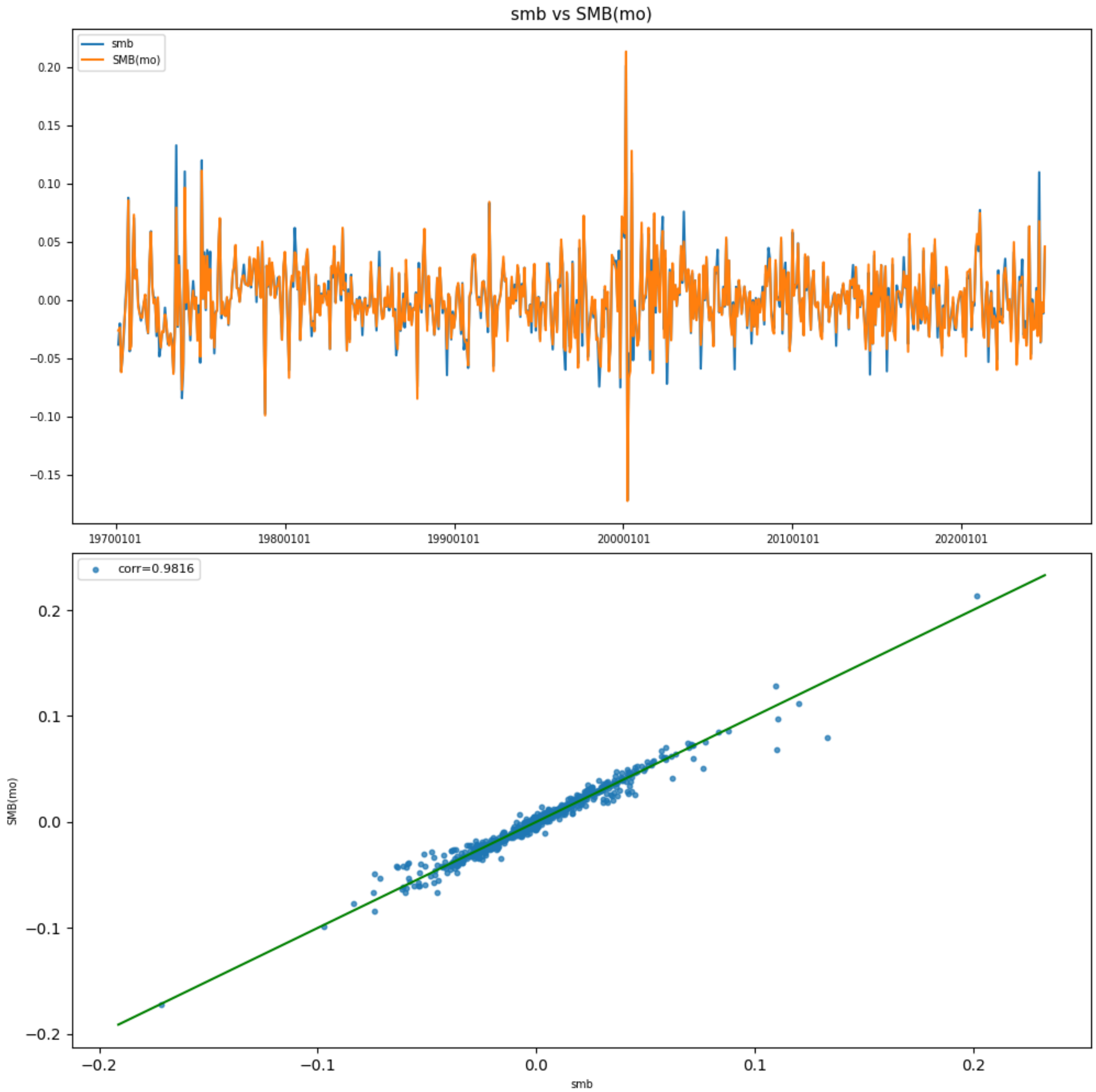


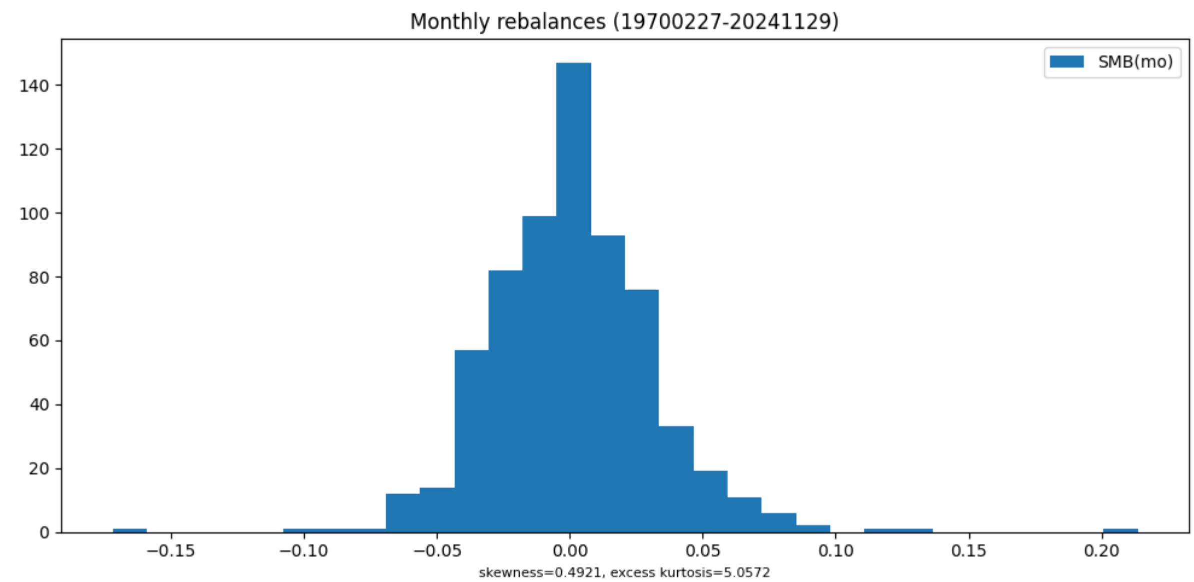
3.2.2 SMB

Compare Small-Minus-Big monthly returns and compare to Fama-French research factor

```
# Plot histogram and comparison of SMB returns
label, benchname = 'smb', 'SMB(mo)'
holdings = smb
result = backtest(monthly, holdings, label)
y = backtest.fit([benchname], rebalbeg, LAST_DATE)
plot_ff(y, label)
plot_summary(y, benchname)
```

```
<R-squared of smb vs SMB(mo) (19700227 - 20241129): 0.9636
```


smb vs SMB(mo)
smb
SMB(mo)
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
19700101
19800101
19900101
20000101
20100101
20200101
corr=0.9816
0.2
0.1
0.0
−0.1
−0.2
SMB(mo)
−0.2
−0.1
0.0
0.1
0.2
smb



3.3 Linear regression

Simple Linear Regression

The simple linear regression (SLR) model relates a continuous response (or dependent) variable y_i with one predictor (or explanatory or independent) variable x_i and an error term ϵ_i:

$$y_i = f(x_i) + \epsilon_i = a + bx_i + \epsilon_i$$

Coefficent estimates of the slope b and intercept a are chosen to minimize the residual sum of squares:

$$\hat{b} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\hat{a} = \bar{y} - \hat{b}\bar{x}$$

Key concepts:

- Residuals are the difference between the observed response values and the response values predicted by the model, $e_i = y_i - \hat{y}_i$.
- Residual sum of squares (RSS) over all observations is $RSS = e_1^2 + e_2^2 + ... + e_n^2$ or equivalently $RSS = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$.
- Mean square error (MSE) is an estimate of the variance of the residuals $s^2 = \hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$.
- Residual standard error (RSE) or residual standard deviation is the estimate of the (square root of the) variance of the residuals. Standard error tells us the average amount that the estimate differs from the actual value. The residual standard error is the estimate of the (square root of the) variance of the residuals $\hat{s} \equiv RSE = \sqrt{RSS/(n-2)}$.

Hypothesis testing and confidence intervals

The estimators of the coefficients follow a normal distribution in large samples. Therefore, tests of a hypothesis about a regression parameter are implemented using a t-test. The standard errors associated with linear regression coefficient and mean response estimates are:

- Slope: $se(\hat{b}) = \sqrt{\dfrac{s^2}{\sum_{i=1}^n (x_i - \overline{x})^2}}$
- Intercept: $se(\hat{a}) = \sqrt{s^2[\dfrac{1}{n} + \dfrac{\overline{x}^2}{\sum_{i=1}^n (x_i - \overline{x})^2}]}$
- Mean response: $se(\hat{y}) = \sqrt{s^2[\dfrac{1}{n} + \dfrac{(x - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})^2}]}$
- Confidence intervals can be computed from standard errors. A 95% confidence interval is defined as a range of values such that with 95% probability, the range will contain the true unknown value of the parameter. The confidence interval for coefficient estimates is:\ $b_j \pm t_{n-(k+1),1-\frac{\alpha}{2}}\ se(b_j)$, where k, the number of regressors, equals 1 for SLR.
- Prediction interval for a new response is $se(\hat{y}_{n+1}) = \sqrt{s^2(1 + \dfrac{1}{n} + \dfrac{(x - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})^2})}$

Multiple Linear Regression

With multiple regressors, the linear model $y = b_0 + b_1 x_1 + ... + b_k x_k + \epsilon$ has coefficient estimates: \$$\hat{b} = (X^T X)^{-1} X^T y$\$

The coefficient b_j quantifies the association between the j 'th predictor and the response. It is the average effect on Y of a one unit increase in X_j, holding all other predictors fixed.

Additional concepts:

- Sum of squares total (SST) measures the total variance in the response Y, and can be thought of as the amount of variability inherent in the response before the regression is performed. SST $= \sum_{i=1}^n (y_i - \overline{y})^2$ measures the total variance in the response Y.
- Sum of squares regression (SSR) measures the total amount variance captured by the regression model: SSR $= \sum i = 1^n (\hat{y}_i - \overline{y})^2$
- Sum of squares error (SSE) measures the total variance of the response not explained by the regression model. In the linear regression context, we may intepret total deviation to equal the deviation not explained by the explanatory variables plus deviation explained by the explanatory variables: $(y_i - \overline{y}) = (y_i - \hat{y}_i) + (\hat{y}_i - \overline{y}_i)$.
  Squaring each side and summing over all observations yields for the total sum of squared deviations $SST = \sum_{i=1}^n (y_i - \hat{y}_i)^2 + \sum_{i=1}^n (\hat{y}_i - \overline{y}_i)^2 = SSE + SSR$, where the sum of the cross-product terms turns out to be zero.
- R^2 statistic or coefficient of determination measures the proportion of variability in Y that can be explained using X. It is also identical to the squared correlation between X and Y. An R^2 statistic that is close to 1 indicates that a large proportion of the variability in the response has been explained by the regression. A number near 0 indicates that the regression did not explain much of the variability in the response. The R^2 statistic provides a relative measure of the quality of a linear regression fit $R^2 = 1 - \frac{\text{RSS}}{\text{SST}}$, and always takes on a value between 0 and 1, and is independent of the scale. R^2 is identical to the squared correlation between X and Y.
- Adjusted R^2: The usual R^2 always increases (since residual sum of squares RSS always decreases) as more variables are added. The intuition behind the adjusted R^2 is that once all of the correct variables have been included in the model, adding noise variables will lead to a decrease in the statistic. In theory, the model with the largest adjusted R^2 will have only correct variables and no noise variables.
- Partial correlation coefficients, which measure the correlation between y and the j 'th explanatory variable x_j controlling for other explanatory variables, can also be obtained by running only one regression: \$$r(y, x_j | x_1, x_2, ..., x_{j-1}, x_{j+1}, ..., x_k) = \frac{t(b_j)}{\sqrt{t(b_j)^2 + n - (k+1)}} where$t(b_j)$is the t-ratio for$b_j$from a regression of$y$on$x_1, \cdots ,x_k$(including the variable$x_j\$).

- t-statistic or t-ratio: $t(b_j) = \frac{b_j}{se(b_j)}$ can be interpreted to be the number of standard errors that b_j is away from zero. In a t-test, the null hypothesis ($H_0 : \beta_j = 0$) is rejected in favor of the alternative if the absolute value of the t-ratio $|t(b_j)|$ exceeds a t-value, denoted $t_{n-(k+1),1-\frac{\alpha}{2}}$, equal to the $(1 - \frac{\alpha}{2})$ 'th percentile from the t-distribution using $df = n - (k+1)$ degrees of freedom.

**Hypothesis tests*

The t-test is not directly applicable when testing hypotheses that involve more than one parameter, because the parameter estimators can be correlated. Instead, a common alternative called the **F-test** compares the fit of the model (measured using the RSS) when the null hypothesis is true relative to the fit of the model without the restriction on the parameters assumed by the null. To test whether all regression slope coefficients are zero $H_0 : b_1 = ... = b_p = 0$, versus the alternative H_a : at least one b_j is non-zero, compute the statistic. which has a $F(p, n-p-1)$ distribution:

$$F = \frac{(TSS - RSS)/p}{RSS/(n-p-1)}$$

Partial F-test: Sometimes, we want to test that a particular subset of q of the coefficients are zero. In this case we fit a second model that uses all the variables except those last q, then compute residual sum of squares for that model and the appropriate F-statistic, which has a $F(p-q, n-p-1)$ distribution:

$$F = \frac{(RSS_q - RSS)/(p-q)}{RSS/(n-p-1)}$$

3.3.1 Value effect

The value effect refers to the observed tendency of value stocks (low price-to-book ratios) to outperform growth stocks (high price-to-book ratios) over time. This phenomenon supports value investing, a strategy linked to Benjamin Graham and David Dodd, which focuses on identifying undervalued stocks based on fundamental analysis.

```python
# Linear regression on Mkt-Rf and intercept
x = ["HML(mo)", "Mkt-RF(mo)"]
formula = f'Q("{x[0]}") ~ ' + " + ".join(f'Q("{v}")' for v in x[1:])
data = bench.get_series(x, field='ret', beg=19620701, end=20991231)
lm = smf.ols(formula, data=data).fit(cov_type='HAC', cov_kwds={'maxlags': 6})
print(f"Period: {data.index[0]}-{data.index[-1]}")
print(lm.summary())
```

```
Period: 19620731-20241231
                            OLS Regression Results
==============================================================================
Dep. Variable:            Q("HML(mo)")   R-squared:                       0.041
Model:                             OLS   Adj. R-squared:                  0.040
Method:                  Least Squares   F-statistic:                     8.701
Date:                 Sun, 02 Mar 2025   Prob (F-statistic):            0.00328
Time:                         14:45:35   Log-Likelihood:                 1587.2
No. Observations:                  750   AIC:                           -3170.
Df Residuals:                      748   BIC:                           -3161.
Df Model:                            1
Covariance Type:                   HAC
===================================================================================
                      coef    std err          z      P>|z|      [0.025      0.975]
-----------------------------------------------------------------------------------
Intercept           0.0037      0.001      2.615      0.009       0.001       0.006
Q("Mkt-RF(mo)")    -0.1344      0.046     -2.950      0.003      -0.224      -0.045
```

(continues on next page)

(continued from previous page)

```
==============================================================================
Omnibus:                       54.218   Durbin-Watson:                   1.670
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              243.740
Skew:                          -0.030   Prob(JB):                     1.18e-53
Kurtosis:                       5.792   Cond. No.                         22.3
==============================================================================

Notes:
[1] Standard Errors are heteroscedasticity and autocorrelation robust (HAC) using
↪6 lags and without small sample correction
```

3.3.2 Small firm effect

The small firm effect, identified by Rolf Banz in 1981, describes the tendency of small-cap stocks to generate higher risk-adjusted returns than large-cap stocks. This suggests that smaller companies may offer higher expected returns as compensation for their increased risk and lower liquidity.

```python
# Linear regression on Mkt-Rf, HML and intercept
x = ["SMB(mo)", "HML(mo)", "Mkt-RF(mo)"]
formula = f'Q("{x[0]}") ~ ' + " + ".join(f'Q("{v}")' for v in x[1:])
data = bench.get_series(x, field='ret', beg=19620701, end=20991231)
lm = smf.ols(formula, data=data).fit(cov_type='HAC', cov_kwds={'maxlags': 6})
print(f"Period: {data.index[0]}-{data.index[-1]}")
print(lm.summary())
```

```
Period: 19620731-20241231
                            OLS Regression Results
==============================================================================
Dep. Variable:            Q("SMB(mo)")   R-squared:                       0.097
Model:                             OLS   Adj. R-squared:                  0.095
Method:                  Least Squares   F-statistic:                     28.64
Date:                 Sun, 02 Mar 2025   Prob (F-statistic):           1.03e-12
Time:                         14:45:35   Log-Likelihood:                 1593.2
No. Observations:                  750   AIC:                            -3180.
Df Residuals:                      747   BIC:                            -3167.
Df Model:                            2
Covariance Type:                   HAC
=================================================================================
                    coef    std err          z      P>|z|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept         0.0007      0.001      0.592      0.554      -0.002       0.003
Q("HML(mo)")     -0.0937      0.088     -1.066      0.286      -0.266       0.079
Q("Mkt-RF(mo)")   0.1901      0.028      6.672      0.000       0.134       0.246
==============================================================================
Omnibus:                      105.898   Durbin-Watson:                   2.056
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              722.449
Skew:                           0.406   Prob(JB):                    1.33e-157
Kurtosis:                       7.739   Cond. No.                         34.8
==============================================================================

Notes:
[1] Standard Errors are heteroscedasticity and autocorrelation robust (HAC) using
↪6 lags and without small sample correction
```

3.4 Structural break test

The **Chow test** is used to detect structural breaks in time-series data. This involves estimating separate regression models before and after a specified breakpoint (e.g., the publication of the HML factor in 1993). The test statistic is:

$$\text{Chow} = \frac{(RSS - (RSS_1 + RSS_2))/K}{(RSS_1 + RSS_2)/(N - 2K)}$$

where the test follows an $F(K, N - 2K)$ distribution. If the test statistic exceeds a critical value, we reject the null hypothesis that the regression coefficients remain constant across both time periods.

```
# Run restricted and unregression models
x = ["HML(mo)", "Mkt-RF(mo)"]
formula = f'Q("{x[0]}") ~ ' + " + ".join(f'Q("{v}")' for v in x[1:])
#formula = f'Q("HML(mo)")~ .'
bp = 19931231  # breakpoint date
lm1 = smf.ols(formula, data=data[data.index<=bp]).fit()
print(f'\nSub Model 1 ({data.index[0]}-{bp}):')
print(lm1.summary())
lm2 = smf.ols(formula, data=data[data.index>bp]).fit()
print(f'\nSub Model 2 ({bp}-{data.index[-1]}):')
print(lm2.summary())
lm0 = smf.ols(formula, data=data).fit()
print('\nRestricted Model (coefficient is equal):')
print(lm0.summary())
```

```
Sub Model 1 (19620731-19931231):
                            OLS Regression Results
==============================================================================
Dep. Variable:            Q("HML(mo)")   R-squared:                       0.125
Model:                             OLS   Adj. R-squared:                  0.122
Method:                  Least Squares   F-statistic:                     53.47
Date:                 Sun, 02 Mar 2025   Prob (F-statistic):           1.59e-12
Time:                         16:34:00   Log-Likelihood:                 874.79
No. Observations:                  378   AIC:                            -1746.
Df Residuals:                      376   BIC:                            -1738.
Df Model:                            1
Covariance Type:             nonrobust
=====================================================================================
                        coef    std err          t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------------
Intercept             0.0056      0.001      4.524      0.000       0.003       0.008
Q("Mkt-RF(mo)")      -0.2021      0.028     -7.312      0.000      -0.256      -0.148
==============================================================================
Omnibus:                       29.355   Durbin-Watson:                   1.600
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               55.192
Skew:                           0.462   Prob(JB):                     1.04e-12
Kurtosis:                       4.628   Cond. No.                         22.4
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
↪specified.

Sub Model 2 (19931231-20241231):
                            OLS Regression Results
```

(continues on next page)

(continued from previous page)

```
==============================================================================
Dep. Variable:            Q("HML(mo)")   R-squared:                       0.007
Model:                             OLS   Adj. R-squared:                  0.005
Method:                  Least Squares   F-statistic:                     2.752
Date:                 Sun, 02 Mar 2025   Prob (F-statistic):             0.0980
Time:                         16:34:00   Log-Likelihood:                 738.00
No. Observations:                  372   AIC:                            -1472.
Df Residuals:                      370   BIC:                            -1464.
Df Model:                            1
Covariance Type:             nonrobust
=====================================================================================
                      coef    std err          t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------------
Intercept           0.0015      0.002      0.866      0.387      -0.002       0.005
Q("Mkt-RF(mo)")    -0.0640      0.039     -1.659      0.098      -0.140       0.012
==============================================================================
Omnibus:                       25.036   Durbin-Watson:                   1.720
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               87.339
Skew:                           0.058   Prob(JB):                     1.08e-19
Kurtosis:                       5.371   Cond. No.                         22.3
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly␣
↪specified.

Restricted Model (coefficient is equal):
                            OLS Regression Results
==============================================================================
Dep. Variable:            Q("HML(mo)")   R-squared:                       0.041
Model:                             OLS   Adj. R-squared:                  0.040
Method:                  Least Squares   F-statistic:                     31.83
Date:                 Sun, 02 Mar 2025   Prob (F-statistic):           2.39e-08
Time:                         16:34:00   Log-Likelihood:                 1587.2
No. Observations:                  750   AIC:                            -3170.
Df Residuals:                      748   BIC:                            -3161.
Df Model:                            1
Covariance Type:             nonrobust
=====================================================================================
                      coef    std err          t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------------
Intercept           0.0037      0.001      3.421      0.001       0.002       0.006
Q("Mkt-RF(mo)")    -0.1344      0.024     -5.642      0.000      -0.181      -0.088
==============================================================================
Omnibus:                       54.218   Durbin-Watson:                   1.670
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              243.740
Skew:                          -0.030   Prob(JB):                     1.18e-53
Kurtosis:                       5.792   Cond. No.                         22.3
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly␣
↪specified.
```

Test statistic with K parameters and N observations follows a $F(K, N-2K)$-distribution

```
# Compute test statistic
K = len(lm0.params)
N = len(data)
RSS = lm0.resid.dot(lm0.resid)
RSS1 = lm1.resid.dot(lm1.resid)
RSS2 = lm2.resid.dot(lm2.resid)
chow = ((RSS - (RSS1 + RSS2)) / K) / ((RSS1 + RSS2) / (N - 2*K))
chow, N, K
```

```
(5.425564122014983, 750, 2)
```

p-value of Chow test statitic

```
1 - scipy.stats.f.cdf(chow, dfn=K, dfd=N - 2*K)
```

```
0.0045780451491863605
```

5% critical value to reject null

```
scipy.stats.f.ppf(q=1 - 0.05, dfn=K, dfd=N - 2*K)
```

```
3.0077945872688696
```

References:

Eugene F. Fama and Kenneth R. French (1992), "The Cross-Section of Expected Stock Returns", Journal of Finance, Volume 47, Issue 2, June 1992, pages 427-465

Eugene Fama and Kenneth French (2023), "Production of U.S. Rm-Rf, SMB, and HML in the Fama-French Data Library", Chicago Booth Paper No. 23-22

FRM Exam I Book Quantative Analysis Chapter 7-8

Wharton Research Data Services.

CHAPTER

FOUR

FAMA-MACBETH CROSS-SECTIONAL REGRESSIONS

If you don' t risk anything, you risk even more –Erica Jong

The Fama-MacBeth (1973) cross-sectional regression methodology is a fundamental tool in empirical asset pricing, used to estimate risk factor loadings and associated risk premia while accounting for cross-sectional correlation in errors. By performing two-stage regressions, the method first estimates factor loadings for individual assets and then determines the associated risk premia over time. This approach has broad applications in testing asset pricing models, including the Capital Asset Pricing Model (CAPM) and multi-factor models. The following sections also analyze efficient frontier construction, Black-Litterman implied alphas and portfolio optimization, risk factor modeling, and non-linear regressions.

```
import numpy as np
from numpy import linalg as la
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import statsmodels.formula.api as smf
from sklearn.kernel_ridge import KernelRidge
import random
from tqdm import tqdm
import cvxpy as cp
from finds.database import SQL, RedisDB
from finds.structured import (BusDay, Signals, Benchmarks, CRSP,
                              CRSPBuffer, SignalsFrame)
from finds.backtesting import RiskPremium
from finds.recipes import winsorize, least_squares
from finds.readers import FFReader
from secret import credentials, paths, CRSP_DATE
VERBOSE = 0
#%matplotlib qt
```

```
# open connections
sql = SQL(**credentials['sql'], verbose=VERBOSE)
user = SQL(**credentials['user'], verbose=VERBOSE)
rdb = RedisDB(**credentials['redis'])
bd = BusDay(sql, verbose=VERBOSE)
crsp = CRSP(sql, bd, rdb=rdb, verbose=VERBOSE)
signals = Signals(user, verbose=VERBOSE)
bench = Benchmarks(sql, bd, verbose=VERBOSE)
imgdir = paths['images']
LAST_DATE = bd.endmo(CRSP_DATE, -1)
```

```
import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)
```

4.1 Mean variance optimization

Markowitz demonstrated that, given two investments with the same expected return (measured as the mean of returns), a risk-averse investor will prefer the one with lower risk (measured by variance). His theory relies on several assumptions, including the absence of market frictions (such as taxes or transaction costs) and normally distributed returns.

The assumption of normally distributed returns implies that rational investors should evaluate potential portfolio allocations based solely on the means and variances of their return distributions. Investors generally seek higher mean returns while minimizing variance. Diversification plays a crucial role in reducing portfolio risk by incorporating assets whose price movements are not perfectly correlated.

A key challenge in implementing this framework is estimating the necessary parameters—mean returns, variances, and asset correlations—using historical data. The choice of historical period or forecast assumptions can significantly impact the resulting allocation. To address this uncertainty, techniques such as robust portfolio optimization and the Black-Litterman model have been developed.

```
# Retrieve test asset returns and risk-free rate
symbol = '6_Portfolios_2x3'
ff = FFReader(symbol)
rf = FFReader('F-F_Research_Data_Factors')[0]['RF'] / 100  # risk-free rates
mktcaps = ff[4] * ff[5]   # number of firms x average market cap
labels = [s.replace('ME1', 'BIG').replace('ME2', 'SMALL') for s in mktcaps.columns]
n = len(labels)
```

```
r = (ff[0]/100).sub(rf.fillna(0), axis=0)     # excess, of the risk-free, returns
sigma = np.cov(r, rowvar=False)
mu = np.mean(r, axis=0).values
assets = DataFrame(data={'mean': mu, 'volatility': np.sqrt(np.diag(sigma))},␣
↪index=labels)
```

```
mkt = {'weights': (mktcaps.iloc[-1]/mktcaps.iloc[-1].sum()).values}  # latest caps
mkt['mean'] = mkt['weights'].dot(mu)
mkt['variance'] = mkt['weights'].dot(sigma).dot(mkt['weights'])
pd.concat([assets.T, Series({'mean': mkt['mean'], 'volatility': np.sqrt(mkt['variance
↪'])},
                            name='Mkt')], axis=1)
```

```
            SMALL LoBM   BIG BM2  SMALL HiBM  BIG LoBM  SMALL BM2  BIG HiBM  \
mean          0.007149  0.009629    0.011528  0.006860   0.006886  0.009252
volatility    0.074748  0.069687    0.081045  0.052923   0.056215  0.071231

                 Mkt
mean        0.007152
volatility  0.053822
```

4.1.1 Global minimum variance portfolio

The Global Minimum Variance (GMV) portfolio is the allocation that achieves the lowest possible risk based on estimated asset variances and correlations while disregarding expected returns. This optimization problem is convex (quadratic) and subject to the constraint that portfolio weights sum to one.

Mathematically, the GMV portfolio is obtained by solving:
$\min_w w^T \Sigma w, \quad$ subject to $w^T 1 = 1 where$\Sigma$ represents the covariance matrix of asset returns. This can be solved numerically with the `cvxpy` Python package for convext optimization.

```
W = cp.Variable(n)    # variable to optimize over - portfolio weights
Var = cp.quad_form(W, sigma)     # objective to minimize portfolio volatility
Ret = mu.T @ W                   # objective to maximize portfolio return
```

```
obj = cp.Problem(cp.Minimize(Var), [cp.sum(W) == 1])
obj.solve()
gmv = dict(weights=W.value, variance=Var.value, mean=Ret.value,
              coords=(np.sqrt(Var.value), Ret.value))
```

The GMV portfolio weights can also be derived using a closed form solution by differentiating the (convex) objective function and setting the first-order conditions to zero: GMV $= \dfrac{\Sigma^{-1} 1}{1^T \Sigma^{-1} 1}$

```
def gmv_portfolio(sigma, mu=None):
    """Returns position weights of global minimum variance portfolio"""
    ones = np.ones((sigma.shape[0], 1))
    w = la.inv(sigma).dot(ones) / ones.T.dot(la.inv(sigma)).dot(ones)
    return {'weights': w, 'volatility': np.sqrt(w.T.dot(sigma).dot(w)),
            'mean': None if mu is None else w.T.dot(mu)}
```

```
w = gmv_portfolio(mu=mu, sigma=sigma)['weights']
pd.concat([Series(gmv['weights']).rename('numerical'),
           Series(w.flatten()).rename('formula')], axis=1)\
  .set_index(assets.index).T
```

	SMALL LoBM	BIG BM2	SMALL HiBM	BIG LoBM	SMALL BM2	BIG HiBM
numerical	-0.485571	0.668181	-0.330072	0.789931	0.83042	-0.47289
formula	-0.485571	0.668181	-0.330072	0.789931	0.83042	-0.47289

4.1.2 Efficient frontier

Each point on the efficient frontier represents a portfolio that offers the highest expected return for a given level of risk, measured by the standard deviation of returns. A line drawn from the risk-free rate becomes tangent to the efficient frontier at the **tangency portfolio**, defining the **Capital Market Line (CML)**:

$$E(R_p) = r_f + \frac{E[R_M] - r_f}{\sigma_M} \sigma_p$$

Portfolios along this line dominate all other portfolios on the efficient frontier. This leads to the **Two-Fund Separation Theorem**, which states that all investors should allocate capital between the risk-free asset and the tangency portfolio.

```
var_ticks = np.linspace(gmv['variance'], 3*np.max(np.diag(sigma)), 200)
best_slope, tangency = 0, tuple()     # to find the tangency portfolio
efficient = []
for var in var_ticks:
    obj = cp.Problem(cp.Maximize(Ret), [cp.sum(W) == 1, Var <= var])
    obj.solve(verbose=False)

    # tangency portfolio has best slope
    risk = np.sqrt(var)
    slope = Ret.value / risk
    if slope > best_slope:
        best_slope = slope
        tangency = {'coords': (risk, Ret.value), 'weights': W.value}
    efficient.append(dict(mean=Ret.value, volatility=risk))
```

```
frontier = []      # inefficient frontier
for var in var_ticks:
    obj = cp.Problem(cp.Minimize(Ret), [cp.sum(W) == 1, Var <= var])
    obj.solve(verbose=False)
    frontier.append(dict(mean=Ret.value, volatility=np.sqrt(var)))
```

The efficient and tangency portfolios can be derived as:

- Efficient portfolio (target return μ_0) = $\Sigma^{-1}M(M^T\Sigma^{-1}M)^{-1}[\mu_0\ 1]^T$, where $M = [\mu\ 1]$
- Tangency portfolio = $\dfrac{\Sigma^{-1}\mu}{1^T\Sigma^{-1}\mu}$

Any portfolio on the efficient frontier can be expressed as a linear combination of two other efficient portfolios.

```
def efficient_portfolio(mu, sigma, target):
    """Returns weights of minimum variance portfolio that exceeds target return"""
    mu = mu.flatten()
    n = len(mu)
    ones = np.ones((n, 1))
    M = np.hstack([mu.reshape(-1, 1), ones])
    B = M.T.dot(la.inv(sigma)).dot(M)
    w = la.inv(sigma).dot(M).dot(la.inv(B)).dot(np.array([[target], [1]]))
    return {'weights': w, 'volatility': np.sqrt(float(w.T.dot(sigma).dot(w))),
            'mean': float(w.T.dot(mu))}
```

```
p = random.choice(efficient)
e = efficient_portfolio(mu,sigma, p['mean'])
df = DataFrame({'random efficient portfolio': p,
                'by formula': dict(mean=e['mean'], volatility=e['volatility'])})
```

```
/tmp/ipykernel_1785309/3388186085.py:9: DeprecationWarning: Conversion of an array␣
↪with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you␣
↪extract a single element from your array before performing this operation.␣
↪(Deprecated NumPy 1.25.)
  return {'weights': w, 'volatility': np.sqrt(float(w.T.dot(sigma).dot(w))),
/tmp/ipykernel_1785309/3388186085.py:10: DeprecationWarning: Conversion of an␣
↪array with ndim > 0 to a scalar is deprecated, and will error in future. Ensure␣
↪you extract a single element from your array before performing this operation.␣
↪(Deprecated NumPy 1.25.)
  'mean': float(w.T.dot(mu))}
```

```python
def tangency_portfolio(mu, sigma):
    """Returns weights of tangency portfolio with largest slope (sharpe ratio)"""
    mu = mu.flatten()
    n = len(mu)
    ones = np.ones((n, 1))
    w = la.inv(sigma).dot(mu)/ones.T.dot(la.inv(sigma).dot(mu))
    return {'weights': w, 'mean': float(w.T.dot(mu)),
            'volatility': np.sqrt(float(w.T.dot(sigma).dot(w)))}
```

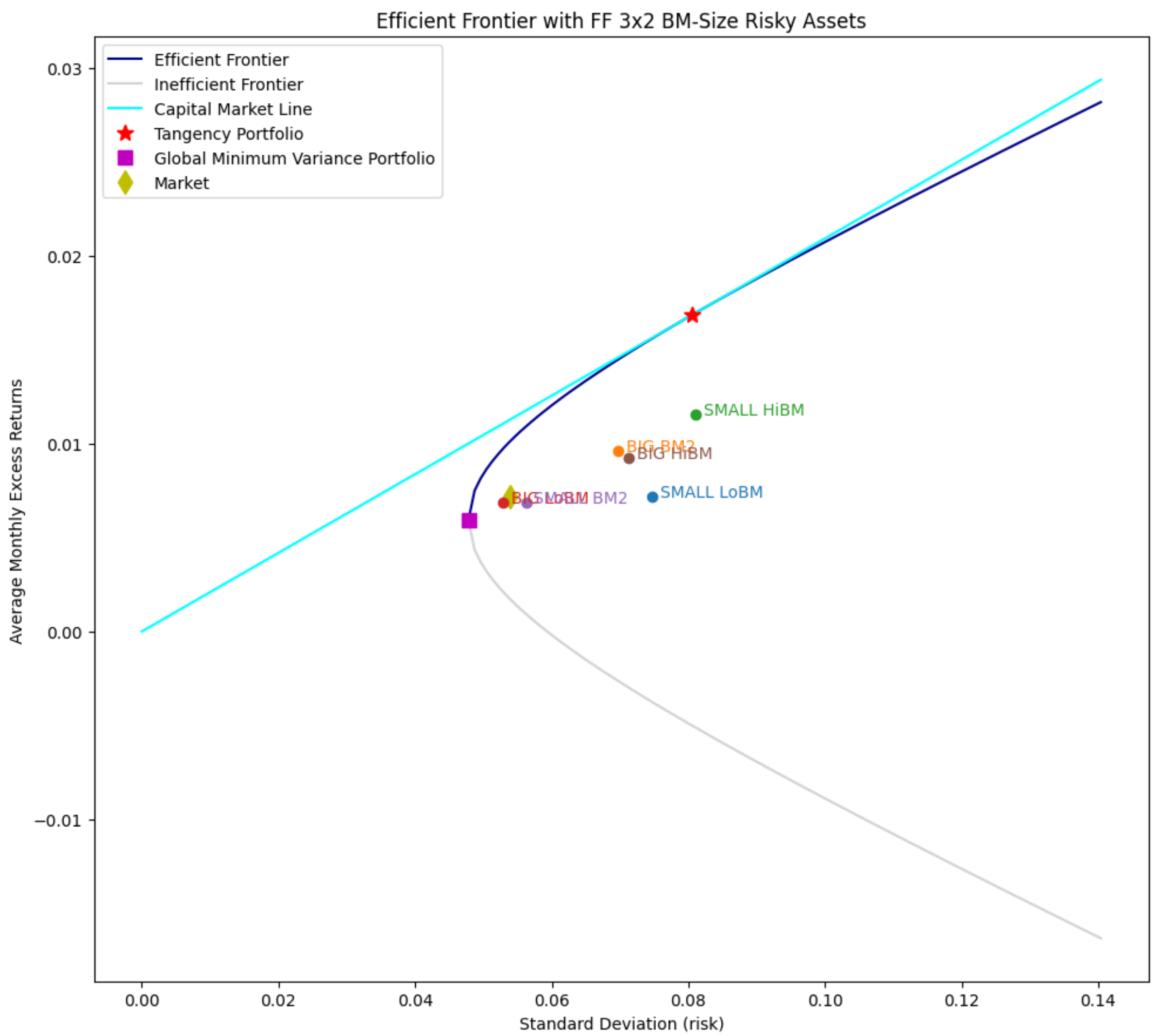
```python
s = tangency_portfolio(mu, sigma)
```

```python
# show numerical and formulas are same solution
DataFrame({'tangency portfolio': list(tangency['coords']),
           'tangency formula': [s['volatility'], s['mean']]},
          index=['volatility', 'mean']).join(df)
```

```
            tangency portfolio  tangency formula  random efficient portfolio  \
volatility            0.080541          0.080748                     0.08782
mean                  0.016848          0.016891                     0.01834

            by formula
volatility     0.08782
mean           0.01834
```

Plot efficient frontier and portfolios

```python
fig, ax = plt.subplots(figsize=(10, 9))
DataFrame(efficient).set_index('volatility').plot(ax=ax, color='darkblue')
DataFrame(frontier).set_index('volatility').plot(ax=ax, color='lightgrey')
ax.plot([0, np.sqrt(max(var_ticks))], [0, np.sqrt(max(var_ticks))*best_slope],
        color='cyan')  # capital market line
ax.plot(*tangency['coords'], "r*", ms=10)                     # tangency portfolio
ax.plot(np.sqrt(gmv['variance']), gmv['mean'], "ms", ms=8)    # GMV portfolio
ax.plot(np.sqrt(mkt['variance']), mkt['mean'], "yd", ms=10)   # market portfolio
plt.legend(['Efficient Frontier', 'Inefficient Frontier', 'Capital Market Line',
            'Tangency Portfolio', 'Global Minimum Variance Portfolio', 'Market'])
for c, r in enumerate(assets.itertuples()):    # risky assets
    ax.plot(r.volatility, r.mean, marker='o', color=f"C{c}")
    ax.annotate(text=r.Index, xy=(r.volatility, r.mean),
                xytext=(0.5, 0), textcoords="offset fontsize", color=f"C{c}")
ax.set_xlabel('Standard Deviation (risk)')
ax.set_ylabel('Average Monthly Excess Returns')
ax.set_title('Efficient Frontier with FF 3x2 BM-Size Risky Assets')
plt.tight_layout()
```



4.1.3 CAPM

Sharpe, Lintner, and Mossin developed the CAPM, an equilibrium model that describes the relationship between risk and expected return for risky assets. The model assumes market efficiency and that investors optimize portfolios based on mean-variance principles.

The CAPM posits that market equilibrium is reached when all investors hold combinations of the risk-free asset and the market portfolio. An asset' s expected return is determined by its contribution to the market portfolio' s total risk, specifically its **systematic risk**, which cannot be diversified away. This risk is measured by beta:

$$\beta_i = \frac{\text{cov}(R_i, R_M)}{\text{var}(R_M)}$$

The **Security Market Line (SML)** represents the relationship between expected returns and beta:

$$E(R_i) = r_f + \beta_i (E[R_M] - r_f)$$

4.2 Implied alphas

If a known portfolio allocation W is an optimal solution to a mean-variance objective, the implied mean return inputs can be inferred given the covariance matrix. These **implied alphas**, proportional to $w^T\Sigma$, when used as expected returns in the optimization process, yield the same portfolio W.

```
# market cap-weighted portfolio implied expected returns
capm = mkt['weights'].dot(sigma) * 2
```

```
# HML implied alphas
hml = Series(0.0, index=assets.index)
hml['BIG HiBM'] = 0.5
hml['SMALL HiBM'] = 0.5
hml['BIG LoBM'] = -0.5
hml['SMALL LoBM'] = -0.5
#hml = {'weights': hml.values}
#hml['variance'] = hml['weights'].dot(sigma).dot(hml['weights'])
#hml['coords'] = (np.sqrt(hml['variance']), hml['weights'].dot(mu))
alphas = hml.dot(sigma) * 2
```

```
pd.concat([Series(hml.values).rename('HML weights'),
           Series(alphas).rename('HML implied-alpha'),
           Series(mkt['weights']).rename('Market weights'),
           Series(capm).rename('CAPM equilibrium returns'),
           Series(mu).rename('historical mu'),
           Series(Series(alphas)/Series(capm)).rename('implied/capm')],
          axis=1, ignore_index=False)\
  .set_index(assets.index).T
```

```
                          SMALL LoBM   BIG BM2  SMALL HiBM  BIG LoBM  \
HML weights                -0.500000  0.000000    0.500000 -0.500000
HML implied-alpha           0.000742  0.001827    0.003132  0.000277
Market weights              0.010631  0.017235    0.011753  0.693932
CAPM equilibrium returns    0.007044  0.006705    0.007413  0.005624
historical mu               0.007149  0.009629    0.011528  0.006860
implied/capm                0.105320  0.272514    0.422451  0.049240

                          SMALL BM2  BIG HiBM
HML weights                0.000000  0.500000
HML implied-alpha          0.001634  0.002967
Market weights             0.190550  0.075900
CAPM equilibrium returns   0.005733  0.006865
historical mu              0.006886  0.009252
implied/capm               0.285082  0.432198
```

```
# Correlations of alphas
DataFrame({'historical mu': np.corrcoef(alphas, mu)[0][-1],
           'capm equilbrium': np.corrcoef(alphas, capm)[0][-1]},
          index=['Correlation with implied alphas'])
```

```
                                 historical mu  capm equilbrium
Correlation with implied alphas       0.836893         0.610935
```

```
# Mean-variance optimization with HML-implied alphas
MeanVariance = alphas @ W - Var
obj = cp.Problem(cp.Maximize(MeanVariance))
obj.solve(verbose=False)
DataFrame.from_records([hml.values, W.value], columns=labels,
                       index=['HML weights', 'mean-variance weights']).round(6)
```

```
                       SMALL LoBM  BIG BM2  SMALL HiBM  BIG LoBM  SMALL BM2  \
HML weights                  -0.5      0.0         0.5      -0.5        0.0
mean-variance weights        -0.5      0.0         0.5      -0.5        0.0

                       BIG HiBM
HML weights                 0.5
mean-variance weights       0.5
```

```
# Mean-variance optimization with CAPM-implied expected returns
MeanVariance = capm @ W - Var
obj = cp.Problem(cp.Maximize(MeanVariance))
obj.solve(verbose=False)
p = tangency_portfolio(mu=capm, sigma=sigma)
DataFrame.from_records([W.value, mkt['weights'], p['weights']], columns=labels,
                       index=['Market weights', 'mean-variance weights', 'formula'])
```

```
                       SMALL LoBM   BIG BM2  SMALL HiBM  BIG LoBM  SMALL BM2  \
Market weights           0.010631  0.017235    0.011753  0.693932    0.19055
mean-variance weights    0.010631  0.017235    0.011753  0.693932    0.19055
formula                  0.010631  0.017235    0.011753  0.693932    0.19055

                       BIG HiBM
Market weights           0.0759
mean-variance weights    0.0759
formula                  0.0759
```

4.2.1 Black-Litterman Model

Mean-variance portfolios are highly sensitive to input estimates, particularly expected returns. Errors in estimating expected returns have a far greater impact than errors in estimating variances and covariances. Black and Litterman (1992) proposed *shrinking* investor expectations toward equilibrium market returns to reduce sensitivity to estimation errors.

```
active = tangency['weights'] - mkt['weights']
DataFrame.from_records([tangency['weights'], mkt['weights'], active], columns=labels,
                       index=['Tangency Portfolio Weights', 'Market Weights',
                              'Active Weights']).round(6)
```

```
                            SMALL LoBM   BIG BM2  SMALL HiBM  BIG LoBM  \
Tangency Portfolio Weights   -2.807841  2.631802    1.023199  2.132014
Market Weights                0.010631  0.017235    0.011753  0.693932
Active Weights               -2.818472  2.614567    1.011446  1.438083

                            SMALL BM2  BIG HiBM
Tangency Portfolio Weights  -1.466995  -0.51218
```

(continues on next page)

(continued from previous page)

```
Market Weights                0.190550   0.07590
Active Weights               -1.657544  -0.58808
```

The Black-Litterman expected return estimates are computed as:

$$E[R] = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]$$

where:

- τ is a confidence scalar for investor views versus equilibrium returns.
- In Bayesian terms, τ represents uncertainty in equilibrium return estimation.

```
tau = 0.05  # He and Litterman (1992) for a moderate amount of active risk
k = 1
Pi = capm.reshape((n, 1))           # equilibrium views: CAPM implied excess returns
P = (tangency['weights']).reshape((k, n))          # view portfolio weights
Q = (tangency['weights'].dot(mu)).reshape((k, k))  # portfolio view
Omega = np.diag(np.array(P.dot(sigma).dot(P.T)).reshape((k,k))) # uncertainty
```

```
def black_litterman(tau, Pi, Sigma, P, Q):
    """Returns black-litterman alphas"""
    def inv(x):
        """helper wraps over la.inv to handle scalar/1d inputs"""
        try:
            return la.inv(x)
        except:
            return np.array(1/x).reshape((1,1))
    return inv(inv(tau*Sigma)+P.T.dot(inv(Omega)).dot(P))\
        .dot(inv(tau*Sigma).dot(Pi) + P.T.dot(inv(Omega)).dot(Q))
```

```
bl = {'alphas': black_litterman(tau=tau, Pi=Pi, Sigma=sigma, P=P, Q=Q)}
bl |= tangency_portfolio(mu=bl['alphas'], sigma=sigma)
bl['mean'] = bl['weights'].dot(mu)    # express mean based on original mu
bl['tilt'] = bl['weights'] - mkt['weights']
print('Active Risk:', np.sqrt(bl['tilt'].T.dot(sigma).dot(bl['tilt'])))
DataFrame.from_dict({'Black-Litterman weights': bl['weights'],
                     'Market weights': mkt['weights'], 'Active weights': bl['tilt']},
                     columns=labels, orient='index').round(6)
```

```
Active Risk: 0.0024824588797282025
```

```
                         SMALL LoBM   BIG BM2  SMALL HiBM  BIG LoBM  \
Black-Litterman weights   -0.101900  0.121624    0.052136  0.751349
Market weights             0.010631  0.017235    0.011753  0.693932
Active weights            -0.112531  0.104390    0.040383  0.057417

                         SMALL BM2  BIG HiBM
Black-Litterman weights   0.124370   0.05242
Market weights            0.190550   0.07590
Active weights           -0.066179  -0.02348
```

```
# BL tilts the optimal weights towards the active positions in the view portfolio
bl['tilt'] / active
```

```
array([0.03992611, 0.03992611, 0.03992611, 0.03992611, 0.03992611,
       0.03992611])
```

With numerical solvers, constraints like no short-selling can be incorporated, and additional constraints can be iteratively added to achieve more reasonable portfolio allocations.

```
Alpha = W @ mu
```

```
# Minimize variance to same expected return
obj = cp.Problem(cp.Minimize(Var), [cp.sum(W) == 1, W >= 0, Alpha >= bl['mean']])
obj.solve()
tilt = W.value - mkt['weights']
print('Minimize variance to achieve target return, with no short sales:')
print('Active Risk:', np.sqrt(tilt.T.dot(sigma).dot(tilt)))
DataFrame.from_dict({'Constrained weights': W.value,
                     'Market weights': mkt['weights'], 'Active weights': tilt},
                    columns=labels, orient='index').round(6)
```

```
Minimize variance to achieve target return, with no short sales:
Active Risk: 0.004263652598782702
```

```
                     SMALL LoBM   BIG BM2  SMALL HiBM  BIG LoBM  SMALL BM2  \
Constrained weights   -0.000000 -0.000000    0.145177  0.790527   0.064296
Market weights         0.010631  0.017235    0.011753  0.693932   0.190550
Active weights        -0.010631 -0.017235    0.133425  0.096595  -0.126254

                     BIG HiBM
Constrained weights    0.0000
Market weights         0.0759
Active weights        -0.0759
```

```
obj = cp.Problem(cp.Maximize(Alpha),
                 [cp.sum(W) == 1, W >= 0, Var <= bl['volatility']**2])
obj.solve()
tilt = W.value - mkt['weights']
print('Maximize return within target variance, with no short sales:')
print('Active Risk (annualized):', np.sqrt(tilt.T.dot(sigma).dot(tilt) * 12))
DataFrame.from_dict({'Constrained weights': W.value,
                     'Market weights': mkt['weights'], 'Active weights': tilt},
                    columns=labels, orient='index').round(6)
```

```
Maximize return within target variance, with no short sales:
Active Risk (annualized): 0.009003930083077559
```

```
                     SMALL LoBM   BIG BM2  SMALL HiBM  BIG LoBM  SMALL BM2  \
Constrained weights    0.000000  0.000001    0.079741  0.778467   0.141791
Market weights         0.010631  0.017235    0.011753  0.693932   0.190550
Active weights        -0.010631 -0.017234    0.067988  0.084535  -0.048759
```

(continues on next page)

(continued from previous page)

```
                     BIG HiBM
Constrained weights    0.0000
Market weights         0.0759
Active weights        -0.0759
```

4.3 Cross-sectional regressions

The Fama-MacBeth (1973) methodology estimates factor betas and risk premia in two steps:

1. **Time-Series Regressions**: Estimate each asset's beta by regressing its returns against proposed factor returns.
2. **Cross-Sectional Regressions**: Estimate factor risk premia by regressing all asset returns against their betas across multiple time periods.

This approach provides standard errors adjusted for cross-sectional correlation.

4.3.1 Testing the CAPM

We retrieve test asset returns and the risk-free rate. To test whether beta is priced, we examine whether higher-beta assets earn proportionally higher risk premia. We also test for **non-linearity** (beta-squared) and for the pricing of residual risk.

```
factors = FFReader('F-F_Research_Data_Factors')[0] / 100  # risk-free rates
test_assets = FFReader('25_Portfolios_ME_BETA_5x5')
df = test_assets[1] / 100
df = df.sub(factors['RF'], axis=0).dropna().copy()
```

```
# unpivot the wide table to a long one
rets = df.stack()\
         .reset_index(name='ret')\
         .rename(columns={'level_1':'port', 'level_0':'Date'})
```

```
# estimate test assets' market betas from their time-series of returns
data = df.join(factors[['Mkt-RF']], how='left')
betas = least_squares(data, y=df.columns, x=['Mkt-RF'], stdres=True)
betas = betas.rename(columns={'Mkt-RF': 'BETA'})[['BETA', '_stdres']]
```

```
# collect test asset mean returns and betas
assets_df = betas[['BETA']].join(df.mean().rename('premiums')).sort_values('BETA')
```

```
# Orthogonalize polynomial (quadratic) beta^2 and residual-volatility features
betas['BETA2'] = smf.ols("I(BETA**2) ~ BETA",  data=betas).fit().resid
betas['RES'] = smf.ols("_stdres ~ BETA + BETA2", data=betas).fit().resid
r = rets.join(betas, on='port').sort_values(['port', 'Date'], ignore_index=True)
```

```
# run monthly Fama-MacBeth cross-sectional regressions
fm = r.groupby(by='Date')\
      .apply(least_squares, y=['ret'], x=['BETA', 'BETA2', 'RES'])
```

```
/tmp/ipykernel_1785309/3663017786.py:3: DeprecationWarning: DataFrameGroupBy.apply␣
↪operated on the grouping columns. This behavior is deprecated, and in a future␣
↪version of pandas the grouping columns will be excluded from the operation.␣
↪Either pass `include_groups=False` to exclude the groupings or explicitly select␣
↪the grouping columns after groupby to silence this warning.
  .apply(least_squares, y=['ret'], x=['BETA', 'BETA2', 'RES'])
```

```python
# compute time-series means and standard errors of the Fama-MacBeth coefficients
out = DataFrame(dict(mean=fm.mean(), stderr=fm.sem(), tstat=fm.mean()/fm.sem())).T
```

```python
print("Monthly Cross-sectional Regressions" +
      f" {min(rets['Date'])} to {max(rets['Date'])}")
out
```

```
Monthly Cross-sectional Regressions 1963-07 to 2024-12
```

```
        _intercept      BETA     BETA2       RES
mean      0.007817  0.000124 -0.008395  0.121528
stderr    0.001535  0.002255  0.002544  0.060507
tstat     5.091416  0.054945 -3.299643  2.008509
```

Clustered standard errors

Alternative corrections for standard errors account for both time-series and cross-sectional correlation, such as double clustering by firm and year.

```python
### Compare uncorrected to robust cov
ls = smf.ols("ret ~ BETA + BETA2 + RES", data=r).fit()
print(ls.summary())
# print(ls.get_robustcov_results('HC0').summary())
# print(ls.get_robustcov_results('HAC', maxlags=6).summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    ret   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                  0.000
Method:                 Least Squares   F-statistic:                     2.554
Date:                Mon, 03 Mar 2025   Prob (F-statistic):             0.0535
Time:                        07:02:21   Log-Likelihood:                 25722.
No. Observations:               18450   AIC:                        -5.144e+04
Df Residuals:                   18446   BIC:                        -5.141e+04
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.0078      0.002      4.174      0.000       0.004       0.011
BETA           0.0001      0.002      0.076      0.940      -0.003       0.003
BETA2         -0.0084      0.006     -1.390      0.165      -0.020       0.003
RES            0.1215      0.051      2.393      0.017       0.022       0.221
==============================================================================
Omnibus:                     1547.660   Durbin-Watson:                   1.778
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             9926.401
```

(continues on next page)

(continued from previous page)

```
Skew:                          -0.051   Prob(JB):                         0.00
Kurtosis:                       6.592   Cond. No.                         174.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly␣
↪specified.
```

```
print(ls.get_robustcov_results('hac-panel',
                               groups=r['port'],
                               maxlags=6).summary())
# print(ls.get_robustcov_results('cluster', groups=r['port']).summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    ret   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                  0.000
Method:                 Least Squares   F-statistic:                     1.636
Date:                Mon, 03 Mar 2025   Prob (F-statistic):              0.207
Time:                        07:02:21   Log-Likelihood:                 25722.
No. Observations:               18450   AIC:                        -5.144e+04
Df Residuals:                   18446   BIC:                        -5.141e+04
Df Model:                           3
Covariance Type:            hac-panel
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.0078      0.002      4.003      0.001       0.004       0.012
BETA           0.0001      0.002      0.066      0.948      -0.004       0.004
BETA2         -0.0084      0.006     -1.330      0.196      -0.021       0.005
RES            0.1215      0.066      1.847      0.077      -0.014       0.257
==============================================================================
Omnibus:                     1547.660   Durbin-Watson:                   1.778
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             9926.401
Skew:                          -0.051   Prob(JB):                         0.00
Kurtosis:                       6.592   Cond. No.                         174.
==============================================================================

Notes:
[1] Standard Errors are robust to cluster correlation (HAC-Panel)
```

4.3.2 Factor risk models

Beginning with Barra in the mid-1970's, industry practitioners have employed cross-sectional models to forecast risk premia and measure risk factors. Monthly cross-sectional regressions are run on standardized individual stock characteristics such as:

- **Size**: Market capitalization rank
- **Value**: Book-to-market ratio
- **Momentum**: 12-month return (excluding past month)
- **Reversal**: 1-month return reversal

The stock characteristic values are winsored at their 5% tails to reduce the influence of outliers. These risk premia are interpreted as returns on dollar-neutral portfolios with unit exposure to a characteristic and zero exposure to other characteristics.

Their estimated risk premiums are compared to Fama-French research factor returns (which are constructed as returns from long-short spread portfolios).

```
# Construct momentum and reversal signals

rebalbeg = 19640601
rebalend = LAST_DATE
# preload CRSP monthly into memory
monthly = CRSPBuffer(stocks=crsp, dataset='monthly',
                     fields=['ret', 'retx', 'prc'],
                     beg=bd.begmo(rebalbeg, -13),
                     end=bd.endmo(rebalend, 1))    # load with extra months around␣
↪rebaldates
```

```
intervals = {'mom': (2, 12, 1), 'strev': (1, 1, -1)}  # signal: (start, end, sign)
for label, past in intervals.items():
    out = []
    rebaldates = bd.date_range(bd.endmo(rebalbeg), rebalend, 'endmo')
    for rebaldate in tqdm(rebaldates, total=len(rebaldates)):
        start = bd.endmo(rebaldate, -past[1])
        beg1 = bd.offset(start, 1)
        end1 = bd.endmo(rebaldate, 1-past[0])
        df = monthly.get_universe(end1)
        # require data available at start month and at last month (universe)
        df['start'] = monthly.get_section(dataset='monthly',
                                          fields=['ret'],
                                          date_field='date',
                                          date=start).reindex(df.index)
        df[label] = past[2] * monthly.get_ret(beg1, end1).reindex(df.index)
        df['permno'] = df.index
        df['rebaldate'] = rebaldate
        df = df.dropna(subset=['start'])
        out.append(df[['rebaldate', 'permno', label]]) # append rows
    out = pd.concat(out, axis=0, ignore_index=True)
    n = signals.write(out, label, overwrite=True)
```

```
100%|██████████| 726/726 [00:43<00:00, 16.53it/s]
100%|██████████| 726/726 [00:41<00:00, 17.50it/s]
```

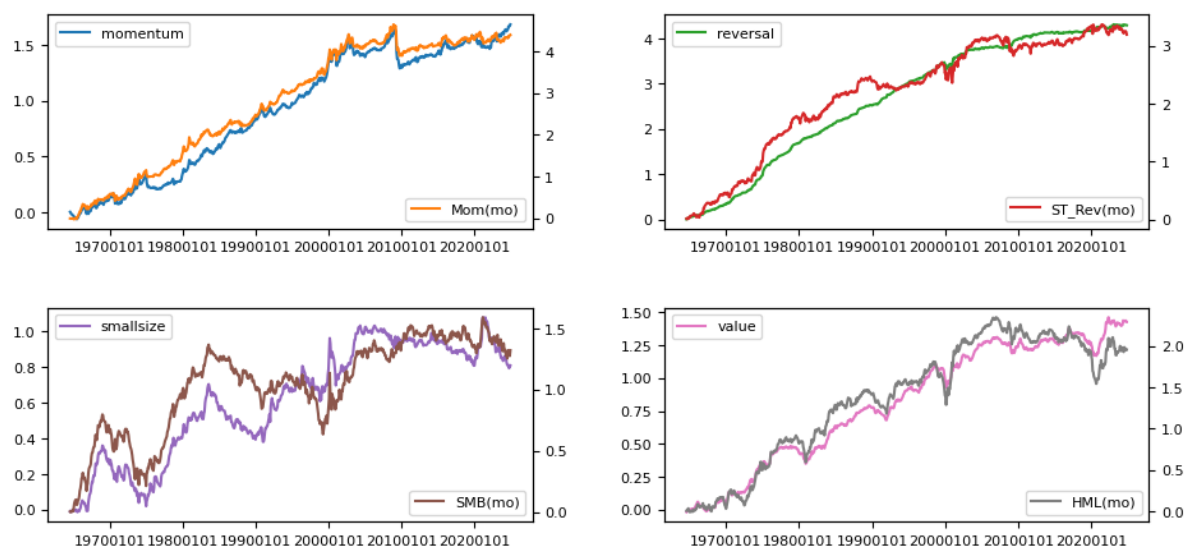
```
rebalbeg = 19640601
rebalend = LAST_DATE
rebaldates = crsp.bd.date_range(rebalbeg, rebalend, 'endmo')
loadings = dict()

# preload signal values
sf = {key: SignalsFrame(signals.read(key)) for key in ['hml', 'mom', 'strev']}

for pordate in tqdm(rebaldates):            # retrieve signal values every month
    date = bd.june_universe(pordate)
    univ = crsp.get_universe(date)
    smb = univ['capco'].rank(ascending=False).div(len(univ)).rename('smallsize')
    hml = sf['hml']('hml', date, bd.endmo(date, -12))['hml'].rename('value')
```

(continues on next page)

(continued from previous page)

```
    #beta = signals('beta', pordate, bd.begmo(pordate))['beta']*2/3 + 1/3 #shrink
    mom = sf['mom']('mom', pordate)['mom'].rename('momentum')
    strev = sf['strev']('strev', pordate)['strev'].rename('reversal')
    df = pd.concat((strev, hml, smb, mom), join='inner', axis=1)\
           .reindex(univ.index).dropna()
    loadings[pordate] = winsorize(df, quantiles=[0.05, 0.95])

# Compute coefficients from FM cross-sectional regressions
monthly = CRSPBuffer(stocks=crsp, dataset='monthly', fields=['ret'],
                     beg=bd.begmo(rebalbeg, -13), end=bd.endmo(rebalend, 1))

riskpremium = RiskPremium(sql=user, bench=bench, rf='RF', end=LAST_DATE)
out = riskpremium(stocks=monthly, loadings=loadings,
                  standardize=['value' ,'smallsize', 'momentum', 'reversal'])

# Compare time series of risk premiums to portfolio-sort benchmark eturns
benchnames = {'momentum': 'Mom(mo)',
              'reversal': 'ST_Rev(mo)',
              'smallsize':'SMB(mo)',
              'value': 'HML(mo)'}
out = riskpremium.fit(benchnames.values())  # to compare portfolio-sorts
riskpremium.plot(benchnames)
```

```
100%|██████████| 726/726 [00:31<00:00, 22.93it/s]
```




```
# Summarize time-series means of Fama-Macbeth risk premiums
df = out[0]
df['tvalue'] = df['mean']/df['stderr']
df['sharpe'] = np.sqrt(12) * df['mean']/df['std']
print("Fama-MacBeth Cross-sectional Regression Risk Premiums")
df.round(4)
```

```
Fama-MacBeth Cross-sectional Regression Risk Premiums
```

```
Factor Returns    mean  stderr     std  count   tvalue  sharpe
reversal        0.0059  0.0005  0.0143    725  11.1314  1.4321
value           0.0020  0.0004  0.0112    725   4.7378  0.6095
smallsize       0.0011  0.0007  0.0178    725   1.6812  0.2163
momentum        0.0023  0.0007  0.0175    725   3.5671  0.4589
```

```python
# Summarize time-series means of Fama-French portfolio-sort returns
df = out[2]
df['tvalue'] = df['mean']/df['stderr']
df['sharpe'] = np.sqrt(12) * df['mean']/df['std']
print("Fama-French Portfolio-Sorts")
df.round(4)
```

```
Fama-French Portfolio-Sorts
```

```
Benchmarks    mean  stderr     std  count  tvalue  sharpe
Mom(mo)     0.0061  0.0016  0.0423    725  3.8647  0.4972
ST_Rev(mo)  0.0044  0.0012  0.0315    725  3.7562  0.4833
SMB(mo)     0.0018  0.0011  0.0308    725  1.5941  0.2051
HML(mo)     0.0027  0.0011  0.0301    725  2.4116  0.3103
```

```python
# Show correlation of returns
print('Correlation of FM-Crossectional Risk Premiums and FF-Sorted Portfolio Returns')
pd.concat([out[1].join(out[4]), out[4].T.join(out[3])], axis=0).round(3)
```

```
Correlation of FM-Crossectional Risk Premiums and FF-Sorted Portfolio Returns
```

```
            reversal  value  smallsize  momentum  Mom(mo)  ST_Rev(mo)  \
reversal       1.000  0.016      0.077    -0.444   -0.388       0.793
value          0.016  1.000     -0.227    -0.194   -0.169      -0.019
smallsize      0.077 -0.227      1.000    -0.003    0.133       0.008
momentum      -0.444 -0.194     -0.003     1.000    0.884      -0.279
Mom(mo)       -0.388 -0.169      0.133     0.884    1.000      -0.307
ST_Rev(mo)     0.793 -0.019      0.008    -0.279   -0.307       1.000
SMB(mo)        0.145 -0.203      0.517    -0.066   -0.047       0.178
HML(mo)        0.051  0.810     -0.151    -0.207   -0.195       0.013

            SMB(mo)  HML(mo)
reversal      0.145    0.051
value        -0.203    0.810
smallsize     0.517   -0.151
momentum     -0.066   -0.207
Mom(mo)      -0.047   -0.195
ST_Rev(mo)    0.178    0.013
SMB(mo)       1.000   -0.150
HML(mo)      -0.150    1.000
```

4.4 Nonlinear regression

4.4.1 Feature transformations

A simple way to directly extend the linear model to accommodate non-linear relationships, using polynomial regression, is to include transformed versions of the predictors in the model, such as a quadratic term or several polynomial functions of the predictors, and use standard linear regression to estimate coefficients in order to produce a non-linear fit. The CAPM predicts that these coefficients on non-linear transformations of beta should be zero.

Raw polynomial terms may be highly correlated with each other: *Orthogonal polynomials* transform the raw data matrix of polynomial terms to another whose columns are a basis of orthogonal terms which span the same column space. For example, regress the second predictor on the first and replace its column with the residuals, then regress the third predictor on the first two and replace its column with the residuals, and so on.

Other feature transformation approaches include:

- dummy or binary indicator variable
- categorical variables with two or more levels
- binarization or turning a categorical variable into several binary variables (4)
- Legendre polynomals which are defined as a system of orthogonal polynomials over the interval $[-1, 1]$
- interaction term constructed by computing the product of the values of the two variables to capture the effect that response of one predictor is dependent on the value of another predictor.

4.4.2 Kernel regression

If there are already a large number of k features, then polynomial transformations, say up to degree d, may be computational expensive since we could be working in $O(k^d)$ dimensional space. Fortunately, many high-dimensional feature mappings, denoted $\phi(x)$, correspond to kernel functions K, where model fitting and prediction calculations only require inner products of these kernel matrices and we never need to explicitly represent vectors in the very high-dimensional feature space. For example, the kernel $K(x, y) = (x^T y + c)^d$, which requires only $O(k)$ to compute, expands to the feature space corresponding with all polynomial terms up to degree d of the features in x and y.

Kernels can be viewed as similarity metrics, that measure how close together the feature maps $\phi(x)$ and $\phi(y)$ are. The radial basis function (RBF), or Gaussian, kernel uses distance in Euclidean space which corresponds to an infinite-dimension feature mapping.

This application of Kernel functions that can be efficiently computed, where only their inner products are needed without ever explicitly computing their corresponding feature vectors in very high-dimensional space, has come to be known as the **kernel trick**.

```
factors
```

```
         Mkt-RF     SMB     HML      RF
Date
1926-07  0.0296 -0.0256 -0.0243  0.0022
1926-08  0.0264 -0.0117  0.0382  0.0025
1926-09  0.0036 -0.0140  0.0013  0.0023
1926-10 -0.0324 -0.0009  0.0070  0.0032
1926-11  0.0253 -0.0010 -0.0051  0.0031
...         ...     ...     ...     ...
2024-08  0.0161 -0.0355 -0.0113  0.0048
2024-09  0.0174 -0.0017 -0.0259  0.0040
```

(continues on next page)

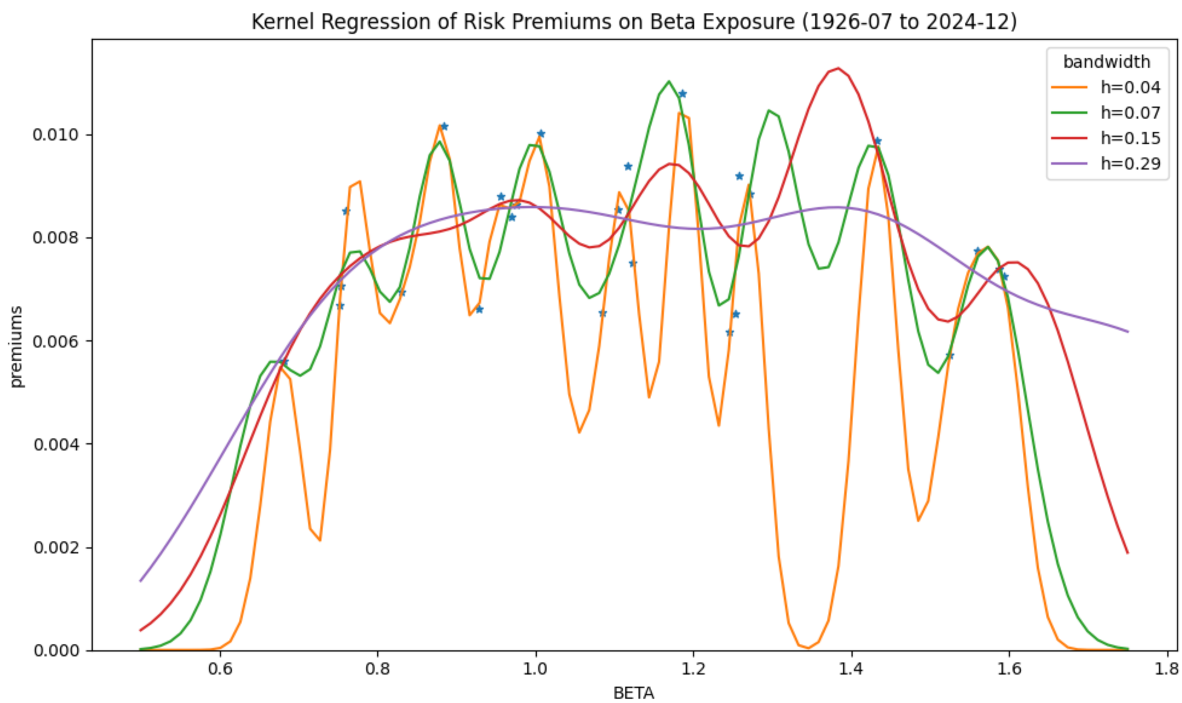
(continued from previous page)

```
2024-10 -0.0097 -0.0101  0.0089  0.0039
2024-11  0.0651  0.0463 -0.0005  0.0040
2024-12 -0.0317 -0.0273 -0.0295  0.0037

[1182 rows x 4 columns]
```

The concave shape of the fitted kernel regression curve is consistent with the negative average premiums observed for the squared-beta factor in earlier Fama-MacBeth tests.

```
y_train = assets_df[['premiums']].values
X_train = assets_df[['BETA']].values
X_test = np.linspace(0.5, 1.75, 100).reshape(-1, 1)
bandwidth = float((max(X_train) - min(X_train)) * 4 / len(X_train))

fig, ax = plt.subplots(figsize=(10,6))
legend = []
color = 1
for h in [0.25, 0.5, 1, 2]:
    for alpha in [0.01]:
        model = KernelRidge(alpha=alpha, kernel='rbf', gamma=1/(h*bandwidth)**2)
        model.fit(X=X_train, y=y_train)
        y_pred = model.predict(X_test)
        ax.plot(X_test, y_pred, ls='-', color=f"C{color}")
        legend.append(f"h={h*bandwidth:.2f}")
        color += 1
# scatter plot actual                                                         ␣
 ↪
assets_df.plot(x='BETA', y='premiums', kind='scatter', ax=ax, marker="*", color="C0")
ax.set_ylim(bottom=0)
plt.legend(legend, loc='best', title='bandwidth')
plt.title('Kernel Regression of Risk Premiums on Beta Exposure' +
          f" ({factors.index[0]} to {factors.index[-1]})")
plt.tight_layout()
```

```
/tmp/ipykernel_1785309/182542393.py:4: DeprecationWarning: Conversion of an array␣
 ↪with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you␣
 ↪extract a single element from your array before performing this operation.␣
 ↪(Deprecated NumPy 1.25.)
  bandwidth = float((max(X_train) - min(X_train)) * 4 / len(X_train))
```



References:

H. M. Markowitz, "Portfolio Selection," Journal of Finance 7, 1952, pp. 77–91.

W. F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," Journal of Finance 19, 1964, pp. 425–442.

J. Lintner, "Security Prices, Risk and Maximal Gains from Diversification," Journal of Finance 20, 1965, pp. 587–615, and J. Mossin, "Equilibrium in a Capital Asset Market," Econometrica 34, 1966, pp, 768–783.

Fama, Eugene F.; MacBeth, James D. (1973). "Risk, Return, and Equilibrium: Empirical Tests" . Journal of Political Economy. 81 (3): 607–636.

Black, F., and R. Litterman. 1992. "Global Portfolio Optimization." Financial Analysts Journal, vol. 48, no. 5 (September/October): 28-43

He, G., and R. Litterman. 1999. "The Intuition Behind Black-Litterman Model Portfolios." Goldman Sachs Investment Management Series.

R. C. Jones, T. Lim and P. J. Zangari. "The Black-Litterman Model for Structured Equity Portfolios," Journal of Portfolio Management, Vol. 33, No. 2, 2007, pp. 24-43. doi:10.3905/jpm.2007.674791

FRM Part I Exam Book Foundations of Risk Management Ch. 5.

CHAPTER

FIVE

CONTRARIAN TRADING

Fortune befriends the bold - Emily Dickinson

Contrarian trading strategies are based on the premise of mean reversion, which posits that asset prices tend to revert to their long-term average over time. This idea is integral to various investment strategies, as it suggests that prices that have deviated significantly from historical norms will eventually return to their equilibrium levels. In these strategies, mispricing can occur due to investor overreaction, leading to temporary opportunities for profitable trades. Common approaches, such as pairs trading and statistical arbitrage, capitalize on these price deviations by simultaneously taking opposing positions in correlated assets. This analysis examines the construction of a contrarian trading strategy, evaluates its performance using key risk-adjusted metrics like the Sharpe ratio, and assesses the effects of implementation shortfall and structural breaks in the strategy's effectiveness over time.

```
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
from tqdm import tqdm
import matplotlib.pyplot as plt
import rpy2.robjects as ro
from rpy2.robjects.packages import importr
from finds.database import SQL, RedisDB
from finds.structured import BusDay, CRSP, Benchmarks
from finds.recipes import fractile_split, least_squares
from finds.utils import PyR, row_formatted
from secret import credentials, paths, CRSP_DATE
VERBOSE = 0
#%matplotlib qt

importr('strucchange')    # R package to use
```

```
rpy2.robjects.packages.Package as a <module 'strucchange'>
```

```
# open connections
sql = SQL(**credentials['sql'], verbose=VERBOSE)
rdb = RedisDB(**credentials['redis'])
bd = BusDay(sql, verbose=VERBOSE)
crsp = CRSP(sql, bd, rdb=rdb, verbose=VERBOSE)
bench = Benchmarks(sql, bd)
```

5.1 Mean reversion

The concept of **mean reversion** forms the foundation of many trading strategies. According to the **law of one price**, similar assets should be priced similarly. When this does not hold, traders can exploit the mispricing through **arbitrage** by buying undervalued assets and short-selling overvalued ones, causing the prices to converge. Deviations from expected long-term values, such as yield spreads in fixed income assets, are typically not persistent. **Pairs trading** involves selecting two related securities and trading them to profit from their temporary price divergences. Meanwhile, **statistical arbitrage** (or **stat arb**) relies on complex algorithms to identify and exploit statistical relationships between securities. Although these relationships are not risk-free, stat arb strategies diversify across numerous positions to reduce exposure to risk. However, correlations between asset classes often increase during financial crises due to liquidity constraints and deleveraging, complicating risk management.

We evaluate a **contrarian strategy** based on weekly returns of US stocks, driven by the idea that stock mispricing arises from investor overreaction, as discussed by Lo and Mackinlay (1990). The **alpha** of this strategy can be measured by its **information coefficient** and **volatility** components, as outlined by Grinold (1994). To examine the strategy’s effectiveness over time, we apply tests for **structural breaks** with unknown change points. A key consideration in this analysis is the discrepancy between theoretical portfolio returns, based on assumed execution at decision prices, and actual returns, influenced by factors such as trading costs. The concept of **implementation shortfall** (introduced by Perold in 1988) captures the total cost of executing an investment decision, accounting for both explicit and implicit costs like market impact and opportunity costs.

Weekly returns reversals

Daily returns from CRSP are compounded in to weekly stock returns assuming Wednesday-close to Wednesday-close. This skips over weekend and holiday (which often occur over a long weekend) effects. The backtest starts in January 1974, after an expansion of the stocks universe the previous year, and excludes the smallest market cap quintile (based on NYSE breakpoints) comprising microcap stocks.

Let $\overline{r}_t$ and σ_t be the cross-sectional mean and standard deviation of stock returns in week t. Define $\tilde{r}_t = r_t - \overline{r}_t$ as the vector of demeaned stock returns, and $X_t = -\tilde{r}_t/\sigma_t$ as the vector of normalized scores. In other words, stocks’ “exposures” to (minus) their respective prior week’ s returns are standardized to have cross-sectional variance and standard deviation equal to 1.0.

Each week, a portfolio is rebalanced to hold an amount in each stock equal to their respective exposure values divided by the number of holdings $w_t = X_t/n$. hence the portfolio overall has unit exposure to prior week’ s returns $w_t^T X_t = 1$ and is dollar-neutral $\sum w_t = \sum X_t/n = 0$.

This portfolio construction approach can more generally incorporate additional signals where X may be a matrix with ones in the first column and standardized signal exposures in other columns. Then each row, except the first, of $W = (X'X)^{-1}X'$ contains stock weights of a long-short characteristic porfolio: a dollar-neutral, minimum-norm (in terms of squared weights) portfolio with unit exposure through long positions in stocks with positive exposure and short positions in stocks with negative exposure to the signal, and zero exposure to the other characteristics.

The portfolio’ s realized return in the following week $t+1$ is

$$W_t'\, r_{t+1} = \frac{X_t'\tilde{r}_t}{n} + \frac{X_t'\hat{r}_t}{n} = -\frac{\tilde{r}_t'\, \tilde{r}_{t+1}\, \sigma_{t+1}}{n\, \sigma_t\, \sigma_{t+1}} = -\rho_{t,t+1}\, \sigma_{t+1}$$

that is the product of the (negative) cross-sectional correlation of stock returns times the amount of cross-sectional stock volatility at week $t+1$.

```
weekday = 3           # wednesday close-to-close
bd = BusDay(sql, endweek=weekday)    # Generate weekly cal
begweek = 19740102    # increased stocks coverage in CRSP in Jan 1973
endweek = bd.endwk(CRSP_DATE, -1)
rebaldates = bd.date_range(begweek, endweek, freq='weekly')
retdates = bd.date_tuples(rebaldates)
```

(continues on next page)

(continued from previous page)

```
june_universe = 0  # to track date when reached a June end to update universe
year = 0           # to track new year to pre-load stocks datas in batch by year
results = []
lagged_weights = Series(dtype=float) # to track "turnover" of stock weights
for rebaldate, pastdates, nextdates in tqdm(zip(
        rebaldates[1:-1], retdates[:-1], retdates[1:]), total=len(rebaldates)-1):

    # screen universe each June: largest 5 size deciles
    d = bd.june_universe(rebaldate)
    if d != june_universe:  # need next June's universe
        june_universe = d                     # update universe every June
        univ = crsp.get_universe(june_universe)  # usual CRSP universe screen
        univ = univ[univ['decile'] <= 8]         # drop smallest quintile stocks

    # retrieve new annual batch of daily prices and returns when start new year
    if bd.begyr(rebaldate) != year:
        year = bd.begyr(rebaldate)
        prc = crsp.get_range(dataset='daily',
                             fields=['bidlo', 'askhi', 'prc', 'retx', 'ret'],
                             date_field='date',
                             beg=year,
                             end=bd.offset(bd.endyr(year), 10),
                             cache_mode="rw")

    # get past week's returns, require price at rebalance (decision) date
    past_week = prc[prc.index.get_level_values('date') == rebaldate]['prc']\
        .reset_index()\
        .set_index('permno')\
        .join(crsp.get_ret(*pastdates).reindex(univ.index))\
        .dropna()

    # convert past week's returns to desired standardized portfolio weights
    weights = ((past_week['ret'].mean() - past_week['ret']) /
               (past_week['ret'].std(ddof=0) * len(past_week)))

    # adjust past week's holdings by change stock price
    lagged_weights = lagged_weights.mul(crsp.get_ret(*pastdates, field='retx')\
                                        .reindex(lagged_weights.index)\
                                        .fillna(0) + 1)

    # compute how much to buy (or sell) to achieve desired new portfolio weights
    chg_weights = pd.concat([weights, -lagged_weights], axis=1)\
                    .fillna(0)\
                    .sum(axis=1)

    # calculate total abs weight as denominator for scaling turnover
    total_weight = weights.abs().sum() + lagged_weights.abs().sum()

    # get next week's gross returns
    next_week = crsp.get_ret(*nextdates).reindex(weights.index).fillna(0)

    # get next day's prices to compute one-day slippage cost
    next_day = prc[prc.index.get_level_values('date') ==
                   bd.offset(rebaldate, 1)]\
                   .reset_index()\
```

(continues on next page)

(continued from previous page)

```
                  .set_index('permno')\
                  .drop(columns='date')\
                  .reindex(chg_weights.index)

    # if no trade next day, then enter position at askhi (buy) or bidlo (sell)
    bidask = next_day['askhi'].where(chg_weights > 0, next_day['bidlo']).abs()

    # spread is relevant askhi or bidlo divided by recorded close, minus 1
    spread = next_day['prc'].where(next_day['prc'] > 0, bidask)\
                            .div(next_day['prc'].abs())\
                            .sub(1)\
                            .fillna(0)

    # finally, trade_prc is the next day's close, or the relevent askhi or bidlo
    trade_prc = next_day['prc'].where(next_day['prc'] > 0, bidask).fillna(0)

    # drift is next day's trade price with dividends over today's decision price
    # delay (positive is cost) will be chg_weights * drift
    drift = trade_prc.div(next_day['prc'].abs())\
                     .mul(1 + next_day['ret'])\
                     .sub(1)\
                     .fillna(0)


    # exit and enter delay should sum to chg_weights.dot(next_day['ret'])
    exit1 = -lagged_weights.dot(next_day['ret'].reindex(lagged_weights.index).
↪fillna(0))
    enter1 = weights.dot(next_day['ret'].reindex(weights.index).fillna(0))

    # accumulate weekly calculations
    results.append(DataFrame(
        {'ret': weights.dot(next_week),
         'exit1': exit1,
         'enter1': enter1,
         'delay': chg_weights.dot(next_day['ret'].fillna(0)),  # delay=enter+exit
         'spread': chg_weights.dot(spread),
         'slippage': chg_weights.dot(drift), # total slippage
         'ic': weights.corr(next_week),
         'n': len(next_week),
         'beg': nextdates[0],
         'end': nextdates[1],
         'absweight': np.sum(weights.abs()),
         'turnover': chg_weights.abs().sum()/total_weight,
         'vol': next_week.std(ddof=0)},
        index=[rebaldate]))

    # carry forward to next week as lagged portfolio weights
    lagged_weights = weights
```

```
100%|██████████| 2659/2660 [01:29<00:00, 29.74it/s]
```

```
# Combine accumulated computations and report
df = pd.concat(results, axis=0)
dates = df.index
df.index = pd.DatetimeIndex(df.index.astype(str))
```

(continues on next page)

(continued from previous page)

```
df['net'] = df['ret'].sub(df['slippage'])
# Show summary
cols = ['ic' ,'vol', 'ret', 'slippage', 'net', 'exit1', 'enter1', 'delay',
        'spread', 'turnover']
indexes = ['Information coefficient', 'Cross-sectional Volatility',
           'Gross return (alpha)', 'Slippage cost', 'Net (of slippage) return',
           '   Exit one day delay   ', '   Enter one day delay   ', 'Delay cost',
           'Spread cost', 'Portfolio turnover']
```

```
print(f'Summary of Weekly Mean Reversion Strategy {dates[0]}-{dates[-1]}')
pd.concat([df[cols].mean(axis=0).rename('mean'),
           df[cols].std(axis=0).rename('std')], axis=1)\
  .set_index(pd.Index(indexes)).round(4)
```

```
Summary of Weekly Mean Reversion Strategy 19740109-20241218
```

	mean	std
Information coefficient	0.0376	0.1022
Cross-sectional Volatility	0.0536	0.0191
Gross return (alpha)	0.0021	0.0080
Slippage cost	0.0017	0.0047
Net (of slippage) return	0.0005	0.0078
Exit one day delay	-0.0003	0.0029
Enter one day delay	0.0006	0.0036
Delay cost	0.0004	0.0041
Spread cost	0.0013	0.0023
Portfolio turnover	0.7363	0.0407

5.2 Implementation shortfall

Perold (1988) observed that the a paper portfolio based upon a well-known stock rankings system significantly outperformed the actual track record of funds that make use of the system. He defined implementation shortfall as the difference in return between a theoretical portfolio and the implemented portfolio, which captures explicit fees and commissions as well as market impact, delay and opportunity costs.

- **Decision price** is the price at the time the investment decision was made
- **Arrival price** is the midquote (mid-point of bid-ask prices) at the time the trader, broker or trading system received the order, or the trade decision is made.
- **Market drift** is the amount of buys (sells) multiplied by the increase (decrease) in execution price relative to the arrival price, due to execution delay.
- **Delay** is the adverse change in execution price relative to the decision price
- **Opportunity costs** are the profits lost due to trades that are cancelled or not executed.
- **Market impact costs** are bid-ask spreads as well as the amount that buying or selling moves the price against the buyer or seller

The **trader' s dilemma** refers to the trade-off between market drift and impact: one can trade faster with more impact to minimize market drift, or trade slower to minimize market impact but at the risk of the market drifting away.

In practice, stocks in the CRSP database are not available at finer than daily frequency, so we assume that trades are executed at the next day's closing prices.

If stocks do not trade, we estimate execution prices based on bid-ask spreads. This approach helps address slippage and delays in execution, although transaction costs cannot be directly observed in historical backtests.

Unfortunately, stock prices in CRSP are not available at a finer than daily frequency. We adjust estimated profits for slippage by waiting a full day after the decision price then setting the execution price at the next day's closing price, when stock market exchanges typically experience the most liquidity at the close of a trading dollar. For stocks that did not trade during that day, we assumed the desired buys are executed at the (higher) ask price and sells are executed at the (lower) bid price (in CRSP for such cases, closing bid and ask quotes are recorded and the closing price is set to the negative of the bid-ask average). This approach helps address slippage and delays in execution, although transaction costs cannot be directly observed in historical backtests.

Over the full period, much of the profitability of this version of the strategy appears to be dissipated after considering a one-day execution delay and bid-ask spreads.

5.3 Structural break with unknown changepoint

In a linear regression, the Chow test is commonly used to test for the presence of a structural break in the model at a period known *a priori*; it essentially constructs a test of whether the true coefficients on the independent variable split into two subsets are equal. Welch's test that two populations have equal means is a special case with no independent variables and only an intercept whose true values are tested in the two time periods. However, when the breakpoints are unknown, these standard tests are not applicable. Andrews (1993) and others have developed alternative tests, based on *supremum statistics*, for identifying changes in mean that occur at unknown points in the time series.

The **R library** `strucchange` provides tools for detecting structural breaks, and the **rpy2** Python package facilitates integration with R. The `PyR` wrapper class in the FinDS package facilitates converting Pandas DataFrames to and from R objects and function calls.

```
# Structural Break Test with Unknown Changepoint


# Set up data and formulas for R
Y = df['ret']
formula = ro.Formula('y ~ 1')
formula.environment['y'] = PyR(Y.values).ro

# Call R strucchange routines to compute breakpoint statistics
fstats_r = ro.r['Fstats'](formula, **{'from': 1})     # Fstats at every break
breakpoints_r = ro.r['breakpoints'](formula)          # candidate breakpoints
confint_r = ro.r['confint'](breakpoints_r, breaks=1) # conf interval for 1 break
sctest_r = ro.r['sctest'](fstats_r, **{'type': 'aveF'})

# Extract output from R results
confint = PyR(confint_r[0]).frame.iloc[0].astype(int) - 1  # R index starts at 1

output = dict(zip(confint.index, df.index[confint]))       # confidence interval

for k,v in zip(sctest_r.slots['names'][:3], sctest_r[:3]): # significance values
    output[k] = PyR(v).values[0]
output['mean(pre)'] = Y[df.index <= output['breakpoints']].mean()
output['mean(post)'] = Y[df.index > output['breakpoints']].mean()

fstat = [0] + list(PyR(fstats_r[0]).values) + [0, 0]  # pad before and after
```

(continues on next page)

(continued from previous page)

```
print("Structural break test with unknown changepoint")
DataFrame(output, index=['sctest'])
```

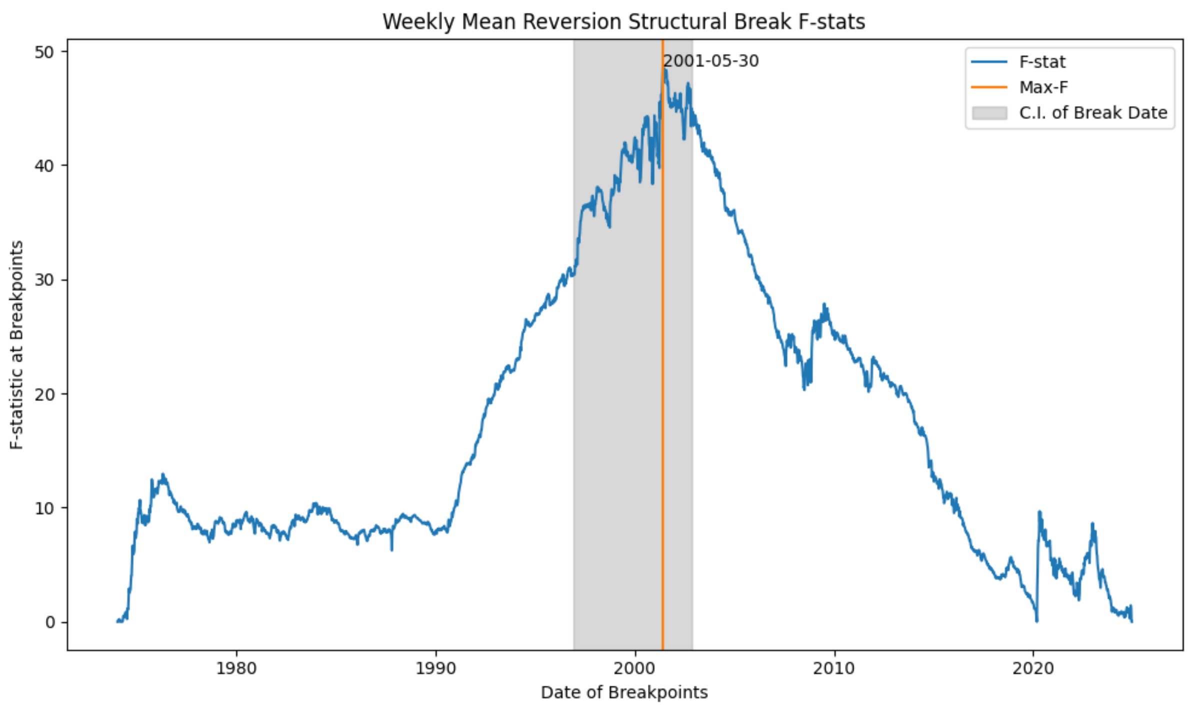
```
Structural break test with unknown changepoint
```

```
            2.5 % breakpoints     97.5 %  statistic  p.value     method  \
sctest 1996-12-04  2001-05-30 2002-11-20  17.588886      0.0  aveF test

        mean(pre)  mean(post)
sctest   0.003112    0.000969
```

Plot breakpoint F-stats

```
fig, ax = plt.subplots(num=2, clear=True, figsize=(10, 6))
ax.plot(df.index, fstat, color='C0')
argmax = np.nanargmax(fstat)             # where maximum fstat
ax.axvline(df.index[argmax], color='C1')
ax.axvspan(df.index[confint[0]], df.index[confint[2]], alpha=0.3, color='grey')
ax.legend(['F-stat', 'Max-F', 'C.I. of Break Date'])
ax.annotate(
    df.index[argmax].strftime('%Y-%m-%d'), xy=(df.index[argmax], fstat[argmax]))
ax.set_ylabel('F-statistic at Breakpoints')
ax.set_xlabel('Date of Breakpoints')
ax.set_title('Weekly Mean Reversion Structural Break F-stats')
plt.tight_layout()
```

```
/tmp/ipykernel_1657351/3930510136.py:5: FutureWarning: Series.__getitem__ treating
↪keys as positions is deprecated. In a future version, integer keys will always
↪be treated as labels (consistent with DataFrame behavior). To access a value by
↪position, use `ser.iloc[pos]`
  ax.axvspan(df.index[confint[0]], df.index[confint[2]], alpha=0.3, color='grey')
```



5.4 Performance evaluation

5.4.1 Information coefficient

Grinold (1994) linked the expected alpha of a signal to its information coefficient and an asset's signal score and idiosyncratic volatility: $\alpha = IC \times volatility \times score$.

- **IC**: The information coefficient can be understood as the correlation between a signal and residual returns. It tells how well forecasts align wih actual returns and is a measure of manager forecasting skill.
- **Volatility**: The volatility can be understood as an asset's residual risk. This component allows for forecast alpha to be expressed in units of returns.
- **Score**: The score is a standardized measure of an asset's raw signal exposure, and reflects relative expectations about an asset. Standardization, by subtracing the cross-sectional mean and dividing by the cross-sectional standard deviation, allows assets to be compared to one another and over time.

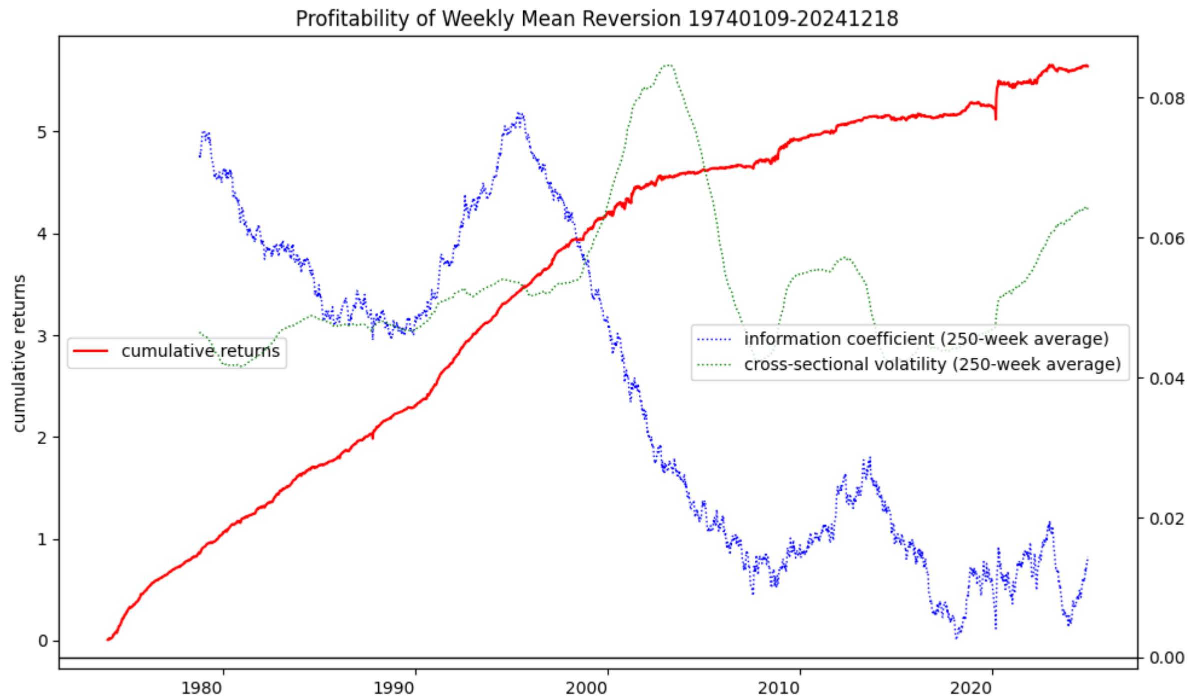
Recall that the weekly reversal portfolio W_t was constructed to be dollar-neutral with exposure equal to 1.0, with weekly profitability $W_t' \, r_{t+1} = -\rho_{t,t+1} \, \sigma_{t+1}$. Hence the **IC** of this strategy can be computed using the negative cross-sectional correlation of stock returns over time. The rolling average of alpha and its components—information coefficient and volatility—reveal trends over the strategy's life. The data show that while volatility fluctuated over time, the **IC** sharply declined after reaching a peak in the mid-1990s, continuing its downward trajectory through 2010.

```
## Plot returns, and rolling avg information coefficient and cross-sectional vol
fig, ax = plt.subplots(num=1, clear=True, figsize=(10, 6))
df['ret'].cumsum().plot(ax=ax, ls='-', color='r', rot=0)
ax.legend(['cumulative returns'], loc='center left')
ax.set_ylabel('cumulative returns')
bx = ax.twinx()
roll = 250  # 250 week rolling average ~ 5 years
```

(continues on next page)

(continued from previous page)

```
df['ic'].rolling(roll).mean().plot(ax=bx, ls=':', lw=1, rot=0, color='b')
df['vol'].rolling(roll).mean().plot(ax=bx, ls=':', lw=1, rot=0, color='g')
#bx.axhline(df['ic'].mean(), linestyle='-', color='C0', lw=2)
bx.axhline(0, linestyle='-', color='black', lw=1)
bx.legend([f"information coefficient ({roll}-week average)",
           f"cross-sectional volatility ({roll}-week average)"],
          loc='center right')
ax.set_title(f'Profitability of Weekly Mean Reversion {dates[0]}-{dates[-1]}')
plt.tight_layout()
```



5.4.2 Risk-adjusted performance measures

In a CAPM equilibrium, no investor can achieve an abnormal return, and each investment yields an identical risk-adjusted return. In the real world, assets may yield a return in excess of, or below, that which fairly compensates for their risk exposure. To assess the strategy's risk-adjusted performance, we employ several key metrics:

- Sharpe ratio - slope of the capital market line is the fair equilibrium compensation: $\dfrac{R_P - r_f}{\sigma_P}$
- Treynor ratio - uses beta which is an approporiate measure of risk for a well-diversified portfolio: $\dfrac{R_P - r_f}{\beta_P}$
- Jensen's alpha - the intercept of a CAPM regression should be zero in equilibrium: $R_P - r_f - \beta_P(R_M - r_f)$
- Appraisal ratio - Jensen's alpha scaled by the volatility of residual returns $\dfrac{\alpha}{\sigma_{P-M}}$
- Sortino ratio - focuses on downside risk relative a target required rate of return T: $\dfrac{R_P - T}{\sqrt{\sum_t \min(0, r_t - T)^2/N}}$

- Information ratio - adjusts performance relative to a benchmark, called the acive return, scaled by the volatility of active returns, called tracking error: $\dfrac{\hat{R_P} - \hat{R_B}}{\sigma_{R_P - R_B}}$
- M^2 (Modigliani-squared) - imagines that the given portfolio, P, is mixed with a position in T-bills so that the resulting portfolio P* matches the volatility of the market portfolio. Because the market index and portfolio P^* have the same standard deviation, their performance may be compared by simply subtracting returns: $R_{P^*} - R_M$

```
market = bench.get_series(permnos=['Mkt-RF'], field='ret').reset_index()
breakpoint = BusDay.to_date(output['breakpoints'])
out = dict()
for select, period in zip([dates > 0, dates <= breakpoint, dates > breakpoint],
                          ['Full', 'Pre-break', 'Post-break']):
    res = df[select].copy()
    res.index = dates[select]

    # align market returns and compute market regression beta
    #res['date'] = res.index
    res['mkt'] = [(1 + market[market['date'].between(*dt)]['Mkt-RF']).prod() - 1
                  for dt in res[['beg','end']].itertuples(index=False)]
    # model = lm(res['mkt'], res['ret'], flatten=True)
    model = least_squares(data=res, y=['ret'], x=['mkt'], stdres=True)

    # save df summary
    out[f"{period} Period"] = {
        'start date': min(res.index),
        'end date': max(res.index),
        'Sharpe Ratio': np.sqrt(52)*res['ret'].mean()/res['ret'].std(),
        'Average Gross Return': res['ret'].mean(),
        'Std Dev Returns': res['ret'].std(),
        'Market Beta': model.iloc[1],
        'Jensen Alpha (annualized)': model.iloc[0] * 52,
        'Appraisal Ratio': np.sqrt(52) * model.iloc[0] / model.iloc[2],
        'Information Coefficient': res['ic'].mean(),
        'Cross-sectional Vol': res['vol'].mean(),
        'Total Slippage Cost': res['slippage'].mean(),
        'Spread Cost': res['spread'].mean(),
        'Delay Cost': res['delay'].mean(),
        '  Exit Delay Cost': res['exit1'].mean(),
        '  Enter Delay Cost': res['enter1'].mean(),
        'Average Net Return': res['net'].mean(),
        'Portfolio Turnover': res['turnover'].mean(),
        #'Abs Weight': res['absweight'].mean(),
        'Average Num Stocks': int(res['n'].mean()),
    }
```

```
# Display as formatted DataFrame
fmts = dict.fromkeys(['start date', 'end date', 'Average Num Stocks'], '{:.0f}')
print("Subperiod Performance of Weekly Reversals")
row_formatted(DataFrame(out), formats=fmts, default='{:.4f}')
```

```
Subperiod Performance of Weekly Reversals
```

```
                        Full Period Pre-break Period Post-break Period
start date                 19740109         19740109          20010606
```

(continues on next page)

(continued from previous page)

```
end date                      20241218         20010530          20241218
Sharpe Ratio                    1.9185           3.6668            0.7300
Average Gross Return            0.0021           0.0031            0.0010
Std Dev Returns                 0.0080           0.0061            0.0096
Market Beta                     0.0898           0.0686            0.1109
Jensen Alpha (annualized)       0.1030           0.1566            0.0407
Appraisal Ratio                 1.8537           3.6607            0.6140
Information Coefficient         0.0376           0.0578            0.0140
Cross-sectional Vol             0.0536           0.0542            0.0529
Total Slippage Cost             0.0017           0.0028            0.0003
Spread Cost                     0.0013           0.0023            0.0002
Delay Cost                      0.0004           0.0005            0.0002
  Exit Delay Cost              -0.0003          -0.0004           -0.0001
  Enter Delay Cost              0.0006           0.0010            0.0002
Average Net Return              0.0005           0.0003            0.0006
Portfolio Turnover              0.7363           0.7393            0.7328
Average Num Stocks                1936             1981              1884
```

The structural break test identified a statistically significant change in the strategy's weekly returns around mid-2001. Comparing performance before and after this point reveals a clear decline in the annualized Sharpe ratio and average weekly returns, along with an increase in risk. This shift roughly coincides with the adoption of decimalization by the New York and American Stock Exchanges on January 29, 2001, which resulted in tighter bid-ask spreads. Prior to this change, U.S. markets quoted prices in fractions, with one-sixteenth (1/16) of a dollar being the smallest allowable price increment.

R usage notes:

```
/usr/bin/ld: cannot find -lgfortran
collect2: error: ld returned 1 exit status
```

- check versions are the same: `gfortran --version` and `gcc --version`
- select versions to be the same: `sudo update-alternatives --config gcc`

References:

Andrews, D.W.K., "Tests for parameter instability and structural change with unknown change point", Econometrica, 61 (1993), 821-856.

Grinold, Richard C., 1994, "Alpha is Volatility Times IC Times Score", The Journal of Portfolio Management, Summer 1994, 20(4), 9-16

Lo, Andrew W. and MacKinlay, A. Craig, 1990, "When Are Contrarian Profits Due to Stock Market Overreaction?", The Review of Financial Studies, 3(2), 175–205

Perold, Andre, 1988, "The Implementation Shortfall: Paper versus Reality", The Journal of Portfolio Management, 14(3), 4-9

CHAPTER

SIX

QUANT FACTORS

Quants do it with models - Anonymous

Factor investing is a systematic approach to asset pricing and portfolio management based on the premise that various risk factors drive asset returns. Factor-based investing recognizes that besides broad market exposure, additional systematic risks—such as value, momentum, and volatility—play a crucial role in determining returns. This framework has evolved over time, beginning with early models like the Capital Asset Pricing Model (CAPM) and expanding to multifactor models, behavioral theories, and adaptive market perspectives. This analysis explores the empirical performance of different factor strategies, and the methodologies used to evaluate them. We also examine historical backtests and employ clustering techniques to group similar investment strategies, seeking to identify style factors and construct effective benchmarks.

```
from pandas import DataFrame, Series
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.cm as cm
from sklearn.cluster import AgglomerativeClustering, KMeans
from scipy.cluster.hierarchy import dendrogram
from sklearn.metrics import silhouette_samples, silhouette_score
from tqdm import tqdm
import warnings
from datetime import datetime
from typing import List, Tuple
from finds.database import SQL, RedisDB
from finds.structured import (BusDay, Stocks, Benchmarks, Signals, SignalsFrame,
                              CRSP, PSTAT, IBES, CRSPBuffer)
from finds.backtesting import BackTest, univariate_sorts
from secret import credentials, paths, CRSP_DATE
VERBOSE = 0
if not VERBOSE:
    warnings.simplefilter(action='ignore', category=FutureWarning)


#%matplotlib qt
```

```
LAST_DATE = CRSP_DATE
# open connections
imgdir = paths['images']
sql = SQL(**credentials['sql'], verbose=VERBOSE)
user = SQL(**credentials['user'], verbose=VERBOSE)
rdb = RedisDB(**credentials['redis'])
bd = BusDay(sql, verbose=VERBOSE)
crsp = CRSP(sql, bd, rdb=rdb, verbose=VERBOSE)
pstat = PSTAT(sql, bd, verbose=VERBOSE)
```

(continues on next page)

(continued from previous page)

```
bench = Benchmarks(sql, bd, verbose=VERBOSE)
signals = Signals(user, verbose=VERBOSE)
ibes = IBES(sql, bd, verbose=VERBOSE)
backtest = BackTest(user, bench, 'RF', LAST_DATE, verbose=VERBOSE)
outdir = paths['scratch'] / 'output'
```

6.1 Factor investing

Factor investing posits that asset returns are driven by exposure to specific risk factors, which determine their risk premiums. The market itself is an investable factor, as described by the Capital Asset Pricing Model (CAPM), which asserts that market exposure is the sole driver of asset returns. However, additional factors such as interest rates, value-growth investing, low volatility strategies, and momentum portfolios have been widely recognized. Macroeconomic factors, including inflation and economic growth, also influence asset returns. Assets exhibit varying degrees of exposure to these risk factors, with greater exposure leading to higher risk premiums. Essentially, assets can be viewed as bundles of different factor exposures.

Early multifactor models include Stephen Ross’s (1976) Arbitrage Pricing Theory (APT), which argues that risk factors cannot be arbitraged or diversified away, and Robert Merton’s Intertemporal Capital Asset Pricing Model (ICAPM), which accounts for investors hedging risky positions over multiple time periods. Additionally, behavioral finance theories suggest that factor premiums arise due to investor biases, such as overreaction, underreaction, and bounded rationality.

6.1.1 Adaptive Markets Hypothesis

Andrew Lo’s (2004) *Adaptive Markets Hypothesis* proposes that financial markets are shaped by principles of evolutionary biology rather than fixed physical laws. This perspective suggests that investment performance fluctuates as the financial ecosystem and market conditions evolve. Lo advocates studying financial markets by analyzing different “species” of investors—individuals and institutions that share common traits—and tracking their size, growth, interactions, and behavioral tendencies.

6.1.2 Factor Zoo

John Cochrane (2011) coined the term *Factor Zoo* to highlight the rapid proliferation of newly identified factors in academic research. In response, Green, Hand, and Zhang (2017) systematically examined nearly 100 firm characteristic factors, addressing issues such as microcap stock overweighting and data snooping biases. Their study assessed the predictive power of these factors across different time periods.

```
# preload monthly stocks data
monthly = CRSPBuffer(stocks=crsp, dataset='monthly', fields=['ret', 'retx', 'prc'],
                     beg=19251201, end=CRSP_DATE)
```

```
# signals to flip signs when forming spread portfolios
leverage = {'mom1m':-1, 'mom36m':-1, 'pricedelay':-1, 'absacc':-1, 'acc':-1,
            'agr':-1, 'chcsho':-1, 'egr':-1, 'mve_ia':-1, 'pctacc':-1,
            'aeavol':-1, 'disp':-1, 'stdacc':-1, 'stdcf':-1, 'secured':-1,
            'maxret':-1, 'ill':-1, 'zerotrade':-1, 'cashpr':-1, 'chinv':-1,
            'invest':-1, 'cinvest':-1, 'idiovol':-1, 'retvol':-1}
```

Helper functions

```python
# to lag yearly characteristics
def as_lags(df, var, key, nlags):
    """Return dataframe with {nlags} of column {var}, same {key} value in row"""
    out = df[[var]].rename(columns={var: 0})      # first col: not shifted
    for i in range(1, nlags):
        prev = df[[key, var]].shift(i, fill_value=0) # next col: shifted i+1
        prev.loc[prev[key] != df[key], :] = np.nan   # require same {key} value
        out.insert(i, i, prev[var])
    return out
```

```python
# rolling window of returns
def as_rolling(df, other, width=0, dropna=True):
    """join next dataframe to a sliding window with fixed number of columns"""
    df = df.join(other, how='outer', sort=True, rsuffix='r')
    if width and len(df.columns) > width:          # if wider than width
        df = df.iloc[:, (len(df.columns)-width):]  #    then drop first cols
    if dropna:                                     # drop empty rows
        df = df[df.count(axis=1) > 0]
    df.columns = list(range(len(df.columns)))
    return df
```

```python
# pipeline to run backtest
def backtest_pipeline(backtest: BackTest,
                      stocks: Stocks,
                      holdings: DataFrame,
                      label: str,
                      benchnames: List[str],
                      suffix: str = '',
                      overlap: int = 0,
                      outdir: str ='',
                      num: int = None) -> DataFrame:
    """wrapper to run a backtest pipeline

    Args:
      backtest: To compute backtest results
      stocks: Where securities returns can be retrieved from (e.g. CRSP)
      holdings: dict (key int date) of Series holdings (key permno)
      label: Label of signal to backtest
      benchnames: Names of benchmarks to attribute portfolio performance
      overlap: Number of overlapping holdings to smooth
      num: Figure num to plot to

    Returns:
      DataFrame of performance returns in rows

    Notes:
      graph and summary statistics are output to jpg and (appended) html
      backtest object updated with performance and attribution data
    """
    summary = backtest(stocks, holdings, label, overlap=overlap)
    excess = backtest.fit(benchnames)
    backtest.write(label)
    backtest.plot(num=num, label=label + suffix)
    if VERBOSE:
        print(pd.Series(backtest.annualized, name=label + suffix)\
              .to_frame().T.round(3).to_string())
```

(continues on next page)

(continued from previous page)

```
    if outdir:
        # performance metrics from backtest to output
        sub = ['alpha', 'excess', 'appraisal', 'sharpe', 'welch-t', 'welch-p']
        with open(outdir / 'index.html', 'at') as f:
            f.write(f"<p><hr><h2>{label + suffix}</h2>\n<pre>\n")
            f.write("{}-{} {}\n".format(min(backtest.excess.index),
                                        max(backtest.excess.index),
                                        benchnames))
            f.write("{:12s} ".format("Annualized"))
            f.write("".join(f"{k:>10s}" for k in sub) + "\n")
            f.write("{:12s} ".format(label + ":"))
            f.write("".join(f"{backtest.annualized[k]:10.4f}" for k in sub))
            f.write(f"\n</pre><p>{datetime.now()}\n")
    return summary
```

6.1.3 Past prices

Momentum and dividend yield data are sourced from CRSP monthly records

```
beg, end = 19251231, LAST_DATE
intervals = {'mom12m': (2,12),
             'mom36m': (13,36),
             'mom6m': (2,6),
             'mom1m': (1,1)}
for label, past in tqdm(intervals.items(), total=len(intervals)):
    out = []
    rebaldates = bd.date_range(bd.endmo(beg, past[1]), end, 'endmo')
    for rebaldate in rebaldates:
        start = bd.endmo(rebaldate, -past[1])
        beg1 = bd.offset(start, 1)
        end1 = bd.endmo(rebaldate, 1-past[0])
        df = crsp.get_universe(end1)
        # require data available as of start month and end month (universe)
        df['start'] = monthly.get_section(dataset='monthly',
                                          fields=['ret'],
                                          date_field='date',
                                          date=start).reindex(df.index)
        df[label] = monthly.get_ret(beg1, end1).reindex(df.index)
        df['permno'] = df.index
        df['rebaldate'] = rebaldate
        df = df.dropna(subset=['start'])
        out.append(df[['rebaldate', 'permno', label]]) # append rows
    out = pd.concat(out, axis=0, ignore_index=True)
    n = signals.write(out, label, overwrite=True)

    beg, end = 19270101, LAST_DATE
    columns = ['chmom', 'divyld', 'indmom']
    out = []
    for rebaldate in bd.date_range(beg, end, 'endmo'):
        start = bd.endmo(rebaldate, -12)
        beg1 = bd.offset(start, 1)
        end1 = bd.endmo(rebaldate, -6)
        beg2 = bd.offset(end1, 1)
        end2 = bd.endmo(rebaldate)
```

(continues on next page)

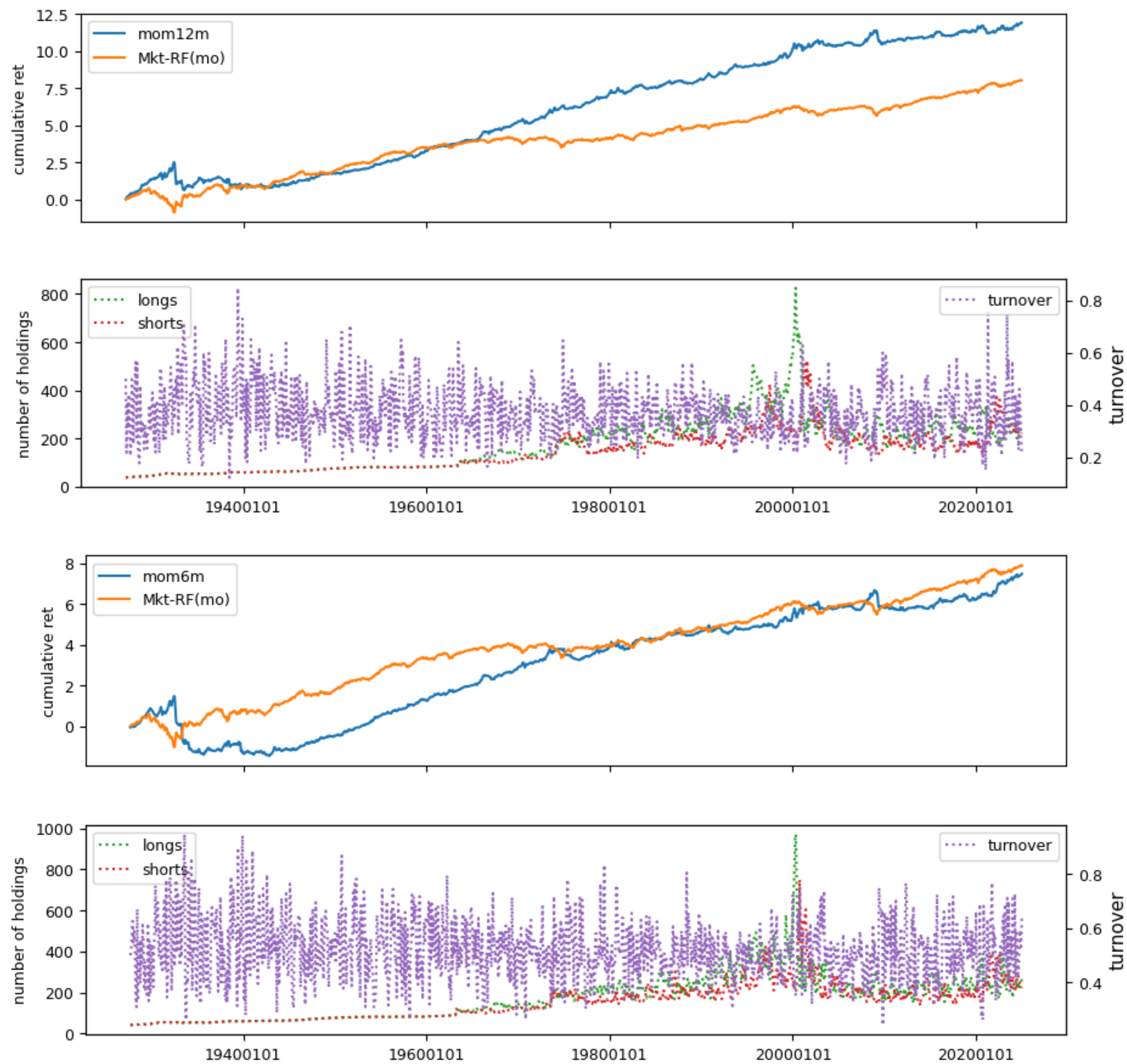
(continued from previous page)

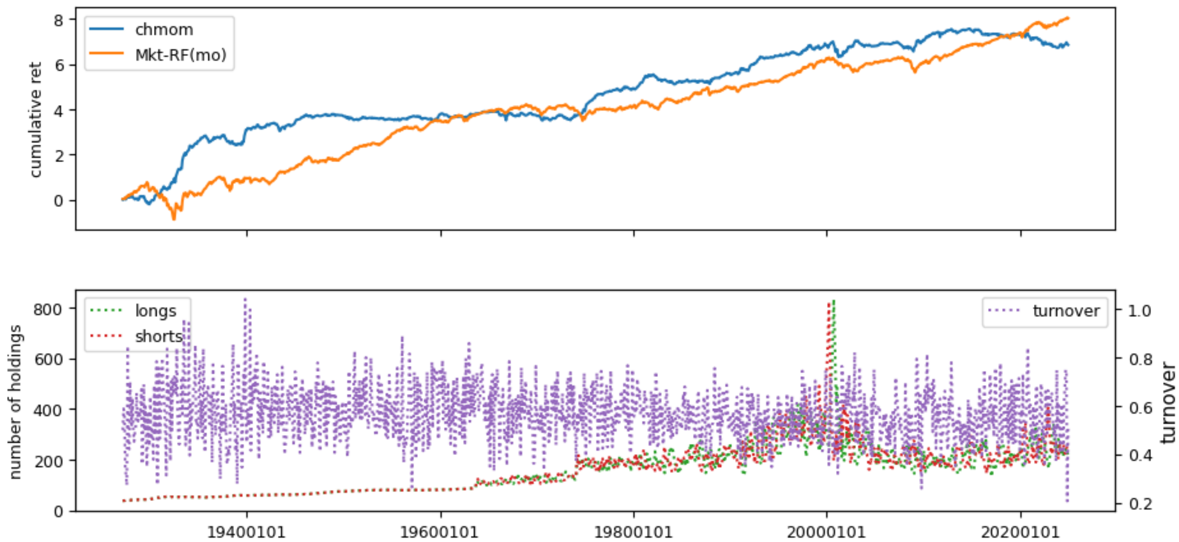
```
        df = crsp.get_universe(end1)
        df['start'] = monthly.get_section(dataset='monthly',
                                          fields=['ret'],
                                          date_field='date',
                                          date=start).reindex(df.index)
        df['end2'] = monthly.get_section(dataset='monthly',
                                         fields=['ret'],
                                         date_field='date',
                                         date=end2).reindex(df.index)
        df['mom2'] = monthly.get_ret(beg2, end2).reindex(df.index)
        df['mom1'] = monthly.get_ret(beg1, end1).reindex(df.index)
        df['divyld'] = crsp.get_divamt(beg1, end2)\
                           .reindex(df.index)['divamt']\
                           .div(df['cap'])\
                           .fillna(0)
        df['chmom'] = df['mom1'] - df['mom2']

        # 6-month two-digit sic industry momentum (group means of 'mom1')
        df['sic2'] = df['siccd'] // 100
        df = df.join(DataFrame(df.groupby(['sic2'])['mom1'].mean())\
                     .rename(columns={'mom1': 'indmom'}),
                     on='sic2', how='left')
        df['permno'] = df.index
        df['rebaldate'] = rebaldate
        out.append(df.dropna(subset=['start','end2'])\
                     [['rebaldate', 'permno'] + columns])
    out = pd.concat(out, axis=0, ignore_index=True)
    for label in columns:    # save signal values to sql
        n = signals.write(out, label, overwrite=True)
```

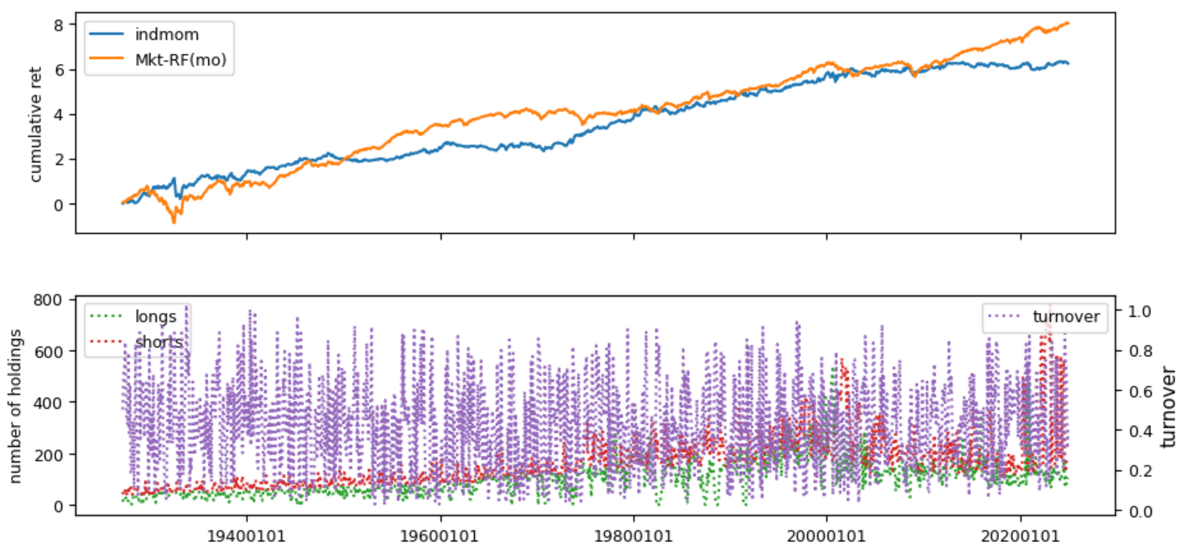
```
100%|██████████| 4/4 [47:03<00:00, 705.93s/it]
```

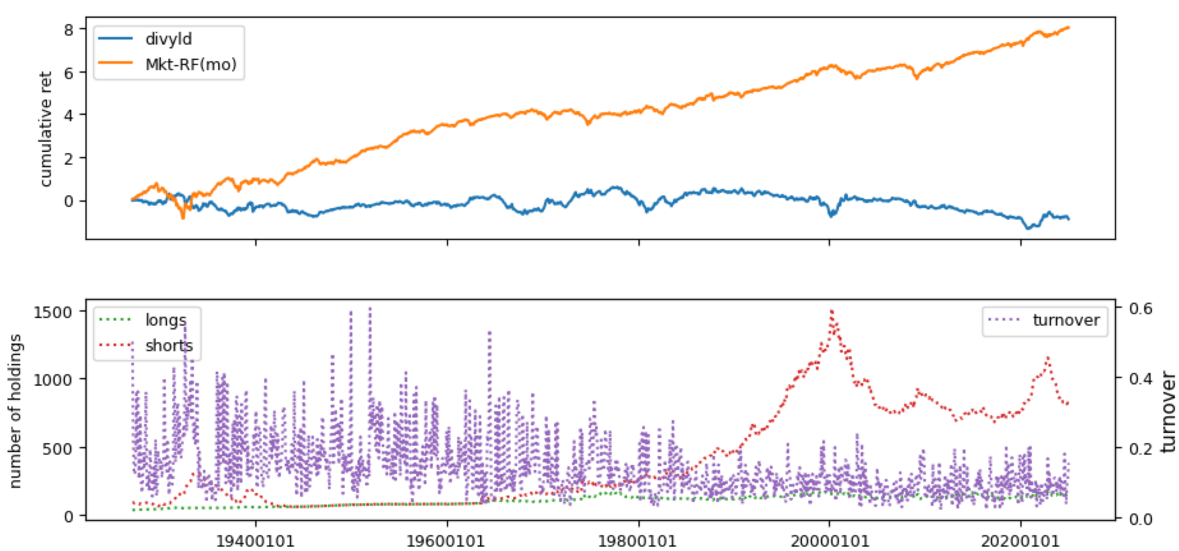
```
benchnames = ['Mkt-RF(mo)']
rebalbeg, rebalend = 19260101, LAST_DATE
columns = ['mom12m', 'mom6m', 'chmom', 'indmom', 'divyld', 'mom1m', 'mom36m']
for label in tqdm(columns):
    holdings = univariate_sorts(monthly,
                                label,
                                SignalsFrame(signals.read(label)),
                                rebalbeg,
                                rebalend,
                                window=1,
                                months=[],
                                maxdecile=8,
                                minprc=1.0,
                                pct=(10.0, 90.0),
                                leverage=leverage.get(label, 1))
    excess = backtest_pipeline(backtest,
                               monthly,
                               holdings,
                               label,
                               benchnames,
                               overlap=0,
                               outdir=outdir,
                               suffix=(leverage.get(label, 1) < 0)*'(-)')
```

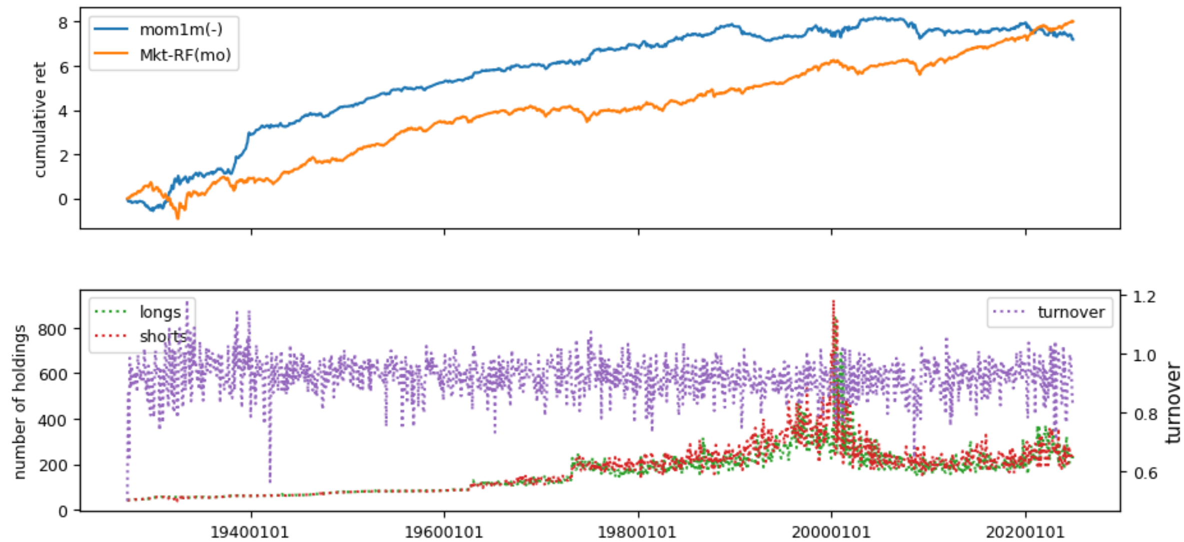
```
100%|██████████| 7/7 [22:01<00:00, 188.74s/it]
```

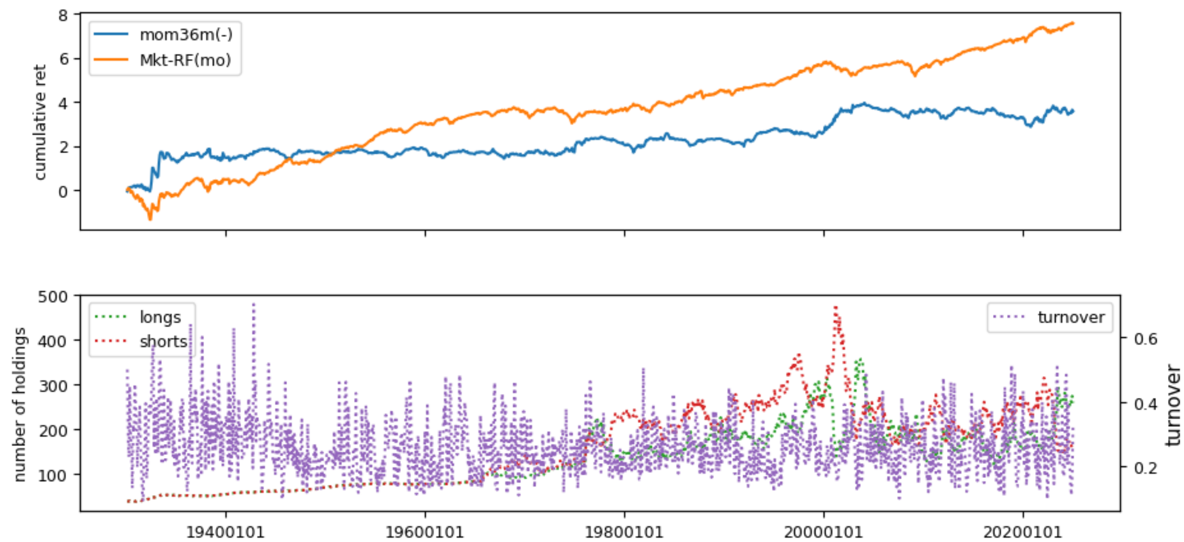



chmom
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
turnover
19400101
19600101
19800101
20000101
20200101


indmom
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
turnover
19400101
19600101
19800101
20000101
20200101


divyld
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
turnover
19400101
19600101
19800101
20000101
20200101






```python
# helper to calculate beta, idiovol and price delay from weekly returns
def regress(x: np.array, y: np.array) -> Tuple[float, float, float]:
    """helper method to calculate beta, idiovol and price delay

    Args:
      x: equal-weighted market returns (in ascending time order)
      y: stock returns (in ascending time order).  NaN's will be discarded.

    Returns:
      beta: slope from regression on market returns and intercept
      idiovol: mean squared error of residuals
      pricedelay: increase of adjusted Rsq with four market lags over without
    """
    v = np.logical_not(np.isnan(y))
    y = y[v]
    x = x[v]
    n0 = len(y)
    A0 = np.vstack([x, np.ones(len(y))]).T
```

(continues on next page)

(continued from previous page)

```
    b0 = np.linalg.inv(A0.T.dot(A0)).dot(A0.T.dot(y))    # univariate coeffs
    sse0 = np.mean((y - A0.dot(b0))**2)
    sst0 = np.mean((y - np.mean(y))**2)
    if (sst0>0 and sse0>0):
        R0 = (1 - ((sse0 / (n0 - 2)) / (sst0 / (n0 - 1))))
    else:
        R0 = 0
    y4 = y[4:]
    n4 = len(y4)
    A4 = np.vstack([x[0:-4], x[1:-3], x[2:-2], x[3:-1], x[4:],
                    np.ones(n4)]).T
    b4 = np.linalg.inv(A4.T.dot(A4)).dot(A4.T.dot(y4))   # four lagged coeffs
    sse4 = np.mean((y4 - A4.dot(b4))**2)
    sst4 = np.mean((y4 - np.mean(y4))**2)
    if sst4 > 0 and sse4 > 0:
        R4 = (1 - ((sse4 / (n4 - 6)) / (sst4 / (n4 - 1))))
    else:
        R4 = 0
    return [b0[0],
            sse0 or np.nan,
            (1 -(R0 / R4)) if R0>0 and R4>0 else np.nan]
```

Weekly price responses derived from CRSP daily records.

```
beg, end = 19260101, LAST_DATE
columns  = ['beta', 'idiovol', 'pricedelay']
wd = BusDay(sql, endweek='Wed')  # custom weekly trading day calendar
```

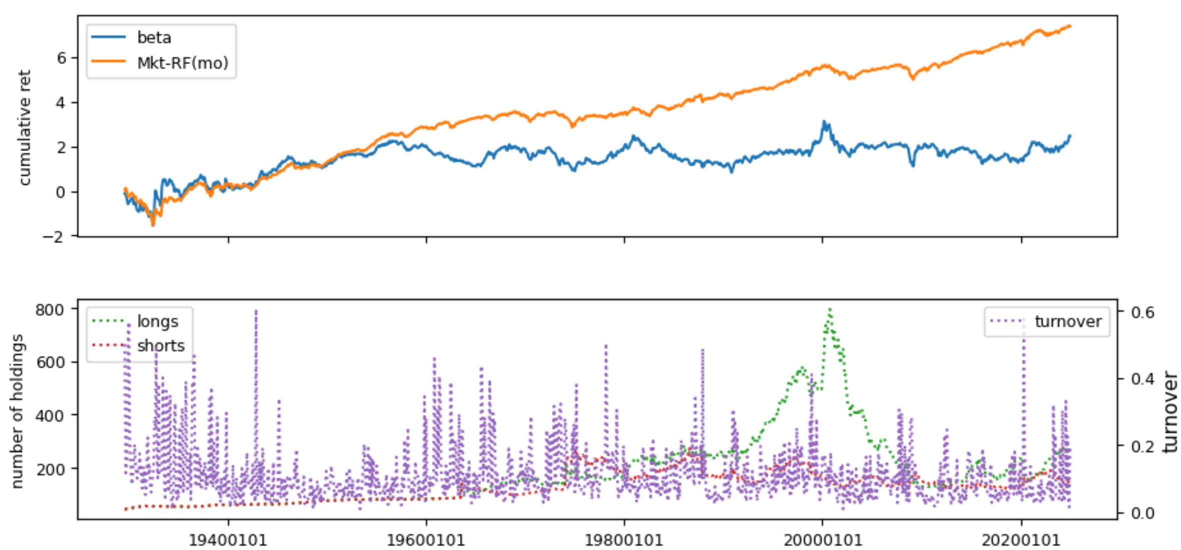
```
width    = 3*52+1            # up to 3 years of weekly returns
minvalid = 52                # at least 52 weeks required to compute beta
weekly   = DataFrame()       # rolling window of weekly stock returns
mkt      = DataFrame()       # to queue equal-weighted market returns
out      = []                # to accumulate final calculations
```

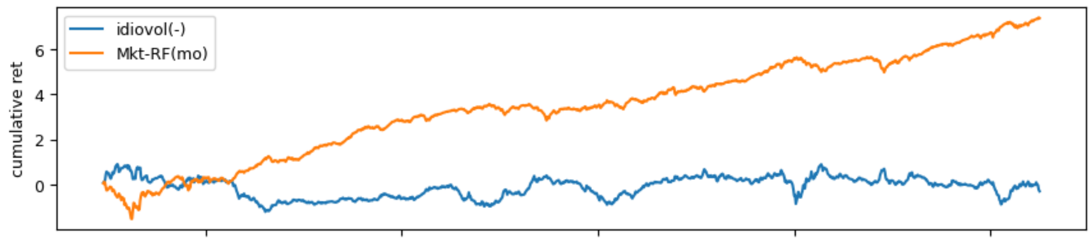
```
for date in tqdm(wd.date_range(beg, end, 'weekly')):
    df = crsp.get_ret(wd.begwk(date), date)
    mkt = as_rolling(mkt,       # rolling window of weekly mkt returns
                     DataFrame(data=[df.mean()], columns=[date]),
                     width=width)
    weekly = as_rolling(weekly, # rolling window of weekly stock returns
                        df.rename(date),
                        width=width)
    valid = weekly.count(axis=1) >= minvalid  # require min number weeks
    if valid.any():
        result = DataFrame([regress(mkt.values[0], y)
                            for y in weekly.loc[valid].values],
                           columns=columns)
        result['permno'] = weekly.index[valid].values
        result['rebaldate'] = date
        if wd.ismonthend(date): # signal value from last week of month
            out.append(result)
out = pd.concat(out, axis=0, ignore_index=True)
for label in columns:
    signals.write(out, label, overwrite=True)
```

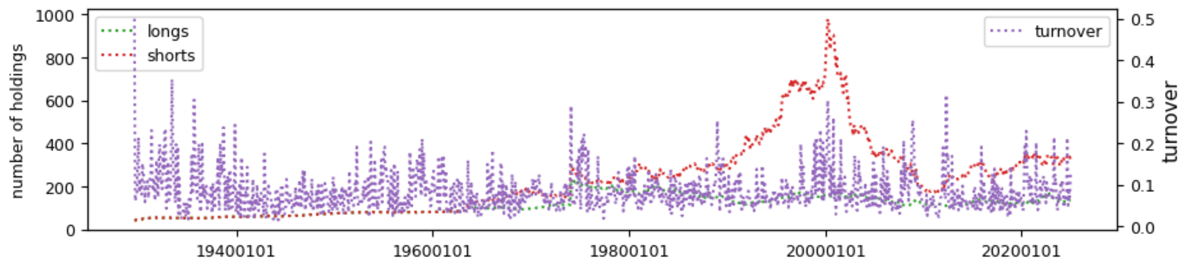
```
100%|██████████| 5165/5165 [31:49<00:00,  2.70it/s]
```

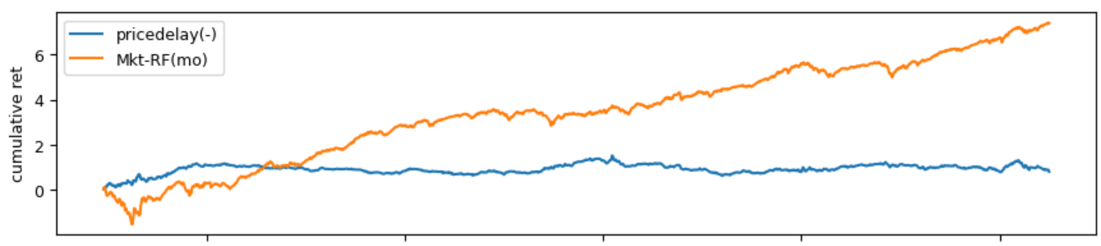
```
benchnames = ['Mkt-RF(mo)']
rebalbeg, rebalend = 19290601, LAST_DATE
for label in tqdm(columns):
    holdings = univariate_sorts(monthly,
                                label,
                                SignalsFrame(signals.read(label)),
                                rebalbeg,
                                rebalend,
                                window=1,
                                months=[],
                                maxdecile=8,
                                pct=(10., 90.),
                                leverage=leverage.get(label, 1))
    excess = backtest_pipeline(backtest,
                               monthly,
                               holdings,
                               label,
                               benchnames,
                               overlap=0,
                               outdir=outdir,
                               suffix=(leverage.get(label, 1) < 0)*'(-)')
```

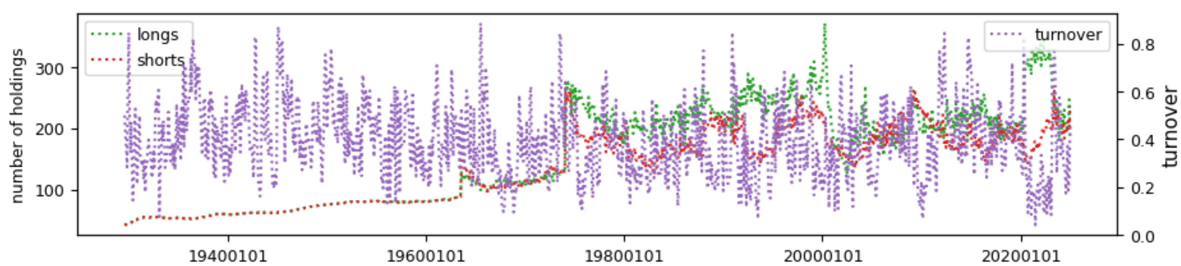
```
100%|██████████| 3/3 [07:48<00:00, 156.06s/it]
```











6.1.4 Liquidity

Liquidity signals are derived from daily stock return data.

```
beg, end = 19830601, LAST_DATE     # nasdaq/volume from after 1982
columns = ['ill', 'maxret', 'retvol', 'baspread', 'std_dolvol',
           'zerotrade', 'std_turn', 'turn']
```

```
out = []
dolvol = []
turn = DataFrame()     # to average turn signal over rolling 3-months
dt = bd.date_range(bd.begmo(beg,-3), end, 'endmo') # monthly rebalances
chunksize = 12         # each chunk is 12 months (1 year)
chunks = [dt[i:(i+chunksize)] for i in range(0, len(dt), chunksize)]
for chunk in tqdm(chunks):
    q = (f"SELECT permno, date, ret, askhi, bidlo, prc, vol, shrout "
```

(continues on next page)

(continued from previous page)

```
          f" FROM {crsp['daily'].key}"
          f" WHERE date>={bd.begmo(chunk[0])}"
          f"   AND date<={chunk[-1]}")         # retrieve a chunk
    f = crsp.sql.read_dataframe(q).sort_values(['permno', 'date'])
    f['baspread'] = ((f['askhi'] - f['bidlo']) / ((f['askhi'] + f['bidlo']) / 2))
    f['dolvol'] = f['prc'].abs() * f['vol']
    f['turn1'] = f['vol'] / f['shrout']
    f.loc[f['dolvol']>0, 'ldv'] = np.log(f.loc[f['dolvol']>0, 'dolvol'])
    f['ill'] = 1000000 * f['ret'].abs() / f['dolvol']

    for rebaldate in chunk:             # for each rebaldate in the chunk
        grouped = f[f['date'].ge(bd.begmo(rebaldate))
                    & f['date'].le(rebaldate)].groupby('permno')
        df = grouped[['ret']].max().rename(columns={'ret': 'maxret'})
        df['retvol'] = grouped['ret'].std()
        df['baspread'] = grouped['baspread'].mean()
        df['std_dolvol'] = grouped['ldv'].std()
        df['ill'] = grouped['ill'].mean()
        dv = grouped['dolvol'].sum()
        df.loc[dv > 0, 'dolvol'] = np.log(dv[dv > 0])
        df['turn1'] = grouped['turn1'].sum()
        df['std_turn'] = grouped['turn1'].std()
        df['countzero'] = grouped['vol'].apply(lambda v: sum(v==0))
        df['ndays'] = grouped['prc'].count()

        turn = as_rolling(turn, df[['turn1']], width=3)
        df['turn'] = turn.reindex(df.index).mean(axis=1, skipna=False)
        df.loc[df['turn1'].le(0), 'turn1'] = 0
        df.loc[df['ndays'].le(0), 'ndays'] = 0
        df['zerotrade'] = ((df['countzero'] + ((1/df['turn1'])/480000))
                           * 21/df['ndays'])

        df['rebaldate'] = rebaldate
        df = df.reset_index()
        out.append(df[['permno', 'rebaldate'] + columns])
        if rebaldate < bd.endmo(end):
            df['rebaldate'] = bd.endmo(rebaldate, 1)
            dolvol.append(df[['permno','rebaldate','dolvol']])
out = pd.concat(out, axis=0, ignore_index=True)
dolvol = pd.concat(dolvol, axis=0, ignore_index=True)
```

```
100%|██████████| 42/42 [25:56<00:00, 37.05s/it]
```

```
for label in columns:
    n = signals.write(out, label, overwrite=True)
n = signals.write(dolvol, 'dolvol', overwrite=True)
```

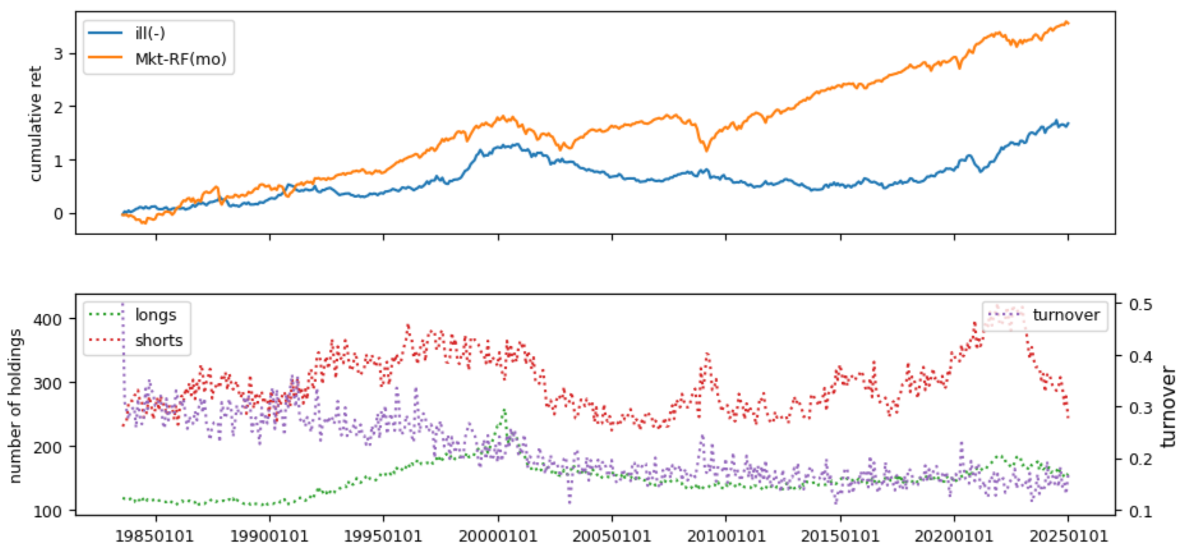
```
rebalbeg, rebalend = 19830601, LAST_DATE
benchnames = ['Mkt-RF(mo)']
for label in tqdm(columns + ['dolvol']):
    holdings = univariate_sorts(monthly,
                                label,
                                SignalsFrame(signals.read(label)),
                                rebalbeg,
```

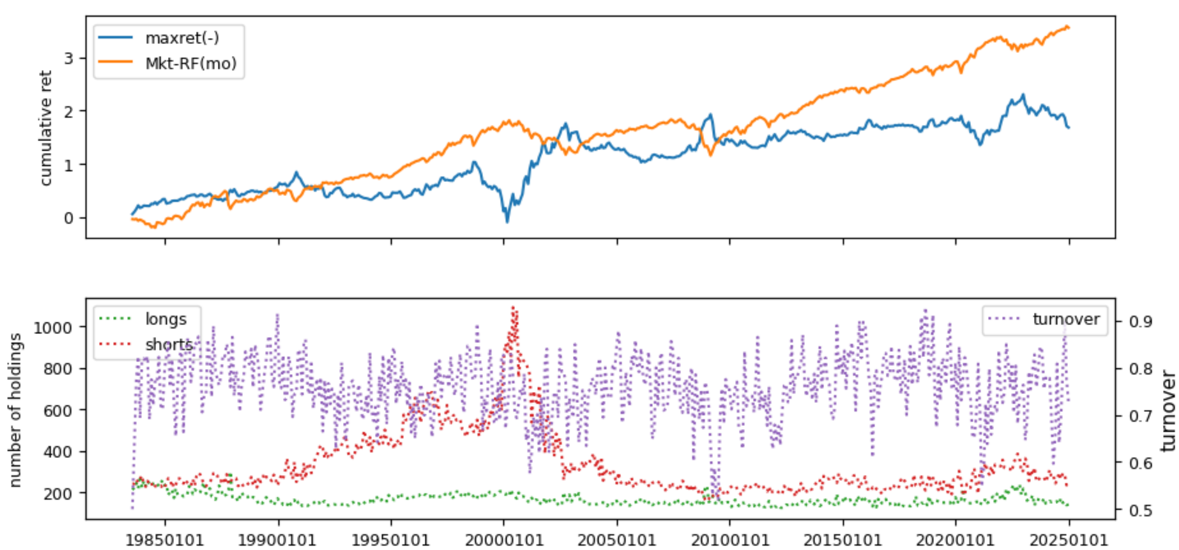
(continues on next page)

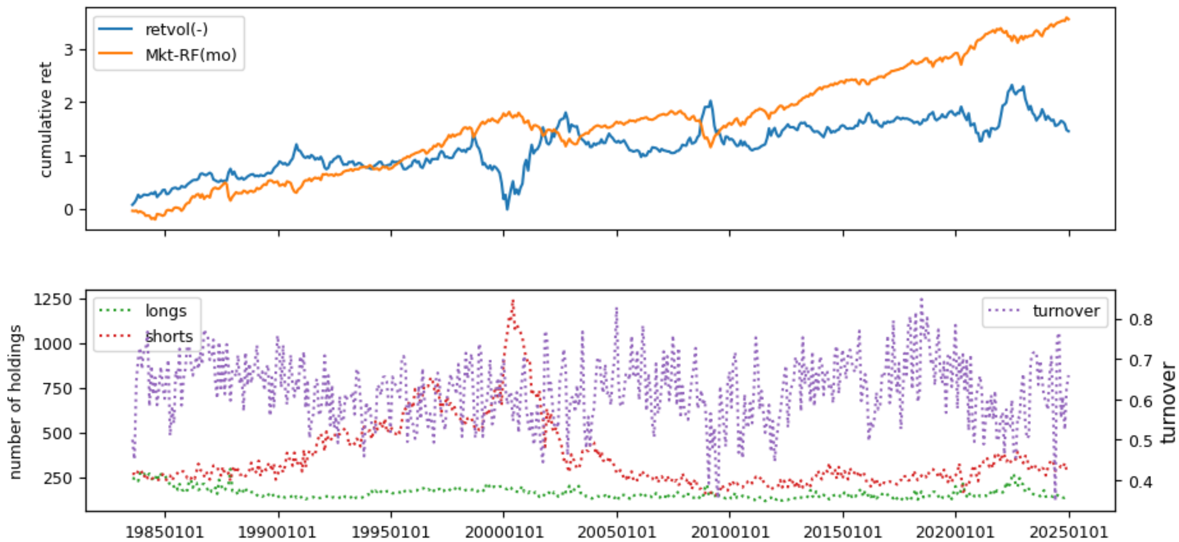
(continued from previous page)

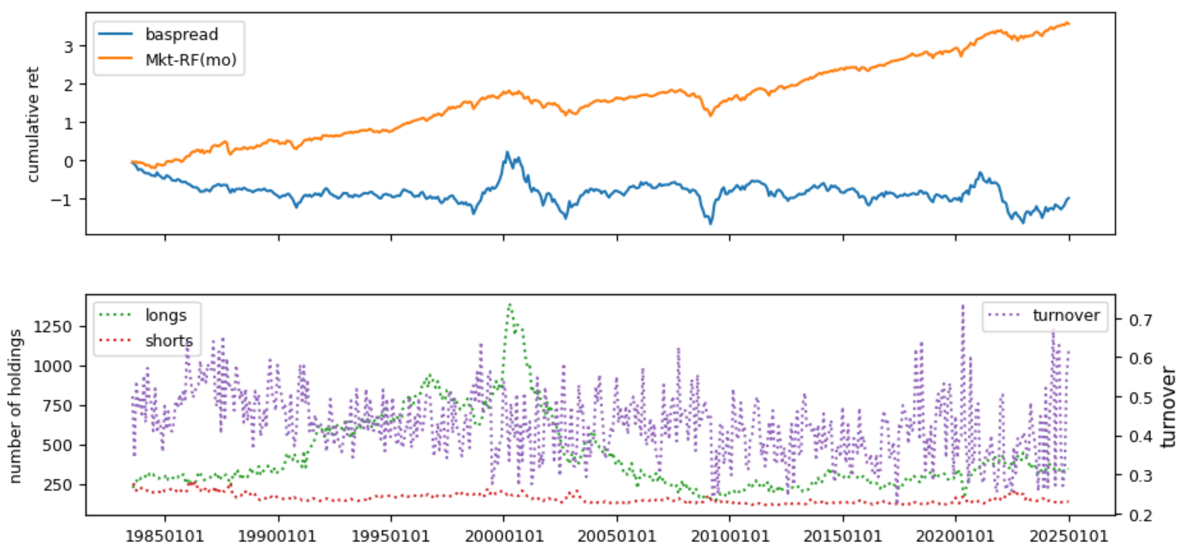
```
                                    rebalend,
                                    window=1,
                                    months=[],
                                    maxdecile=8,
                                    pct=(10., 90.),
                                    leverage=leverage.get(label, 1))
    excess = backtest_pipeline(backtest,
                                   monthly,
                                   holdings,
                                   label,
                                   benchnames,
                                   overlap=0,
                                   outdir=outdir,
                                   suffix=(leverage.get(label, 1) < 0)*'(-)')
```

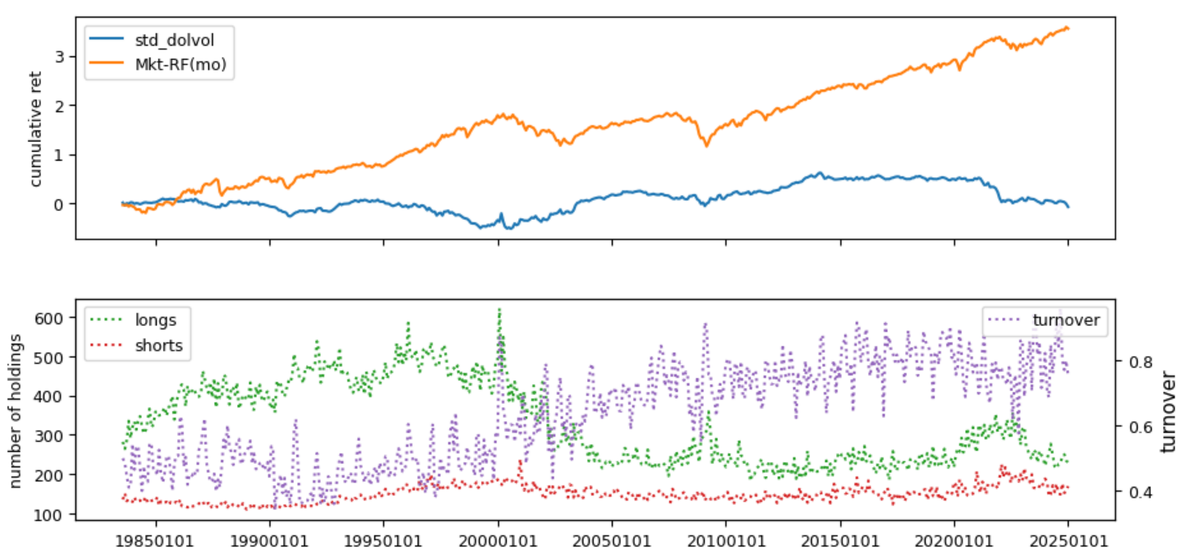
```
100%|██████████| 9/9 [09:46<00:00, 65.17s/it]
```

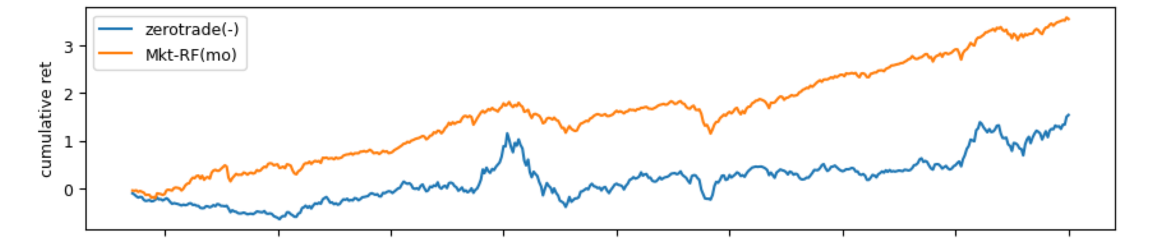


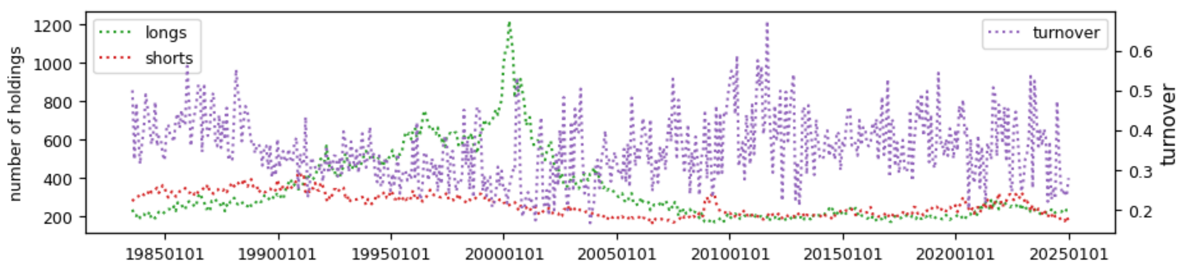


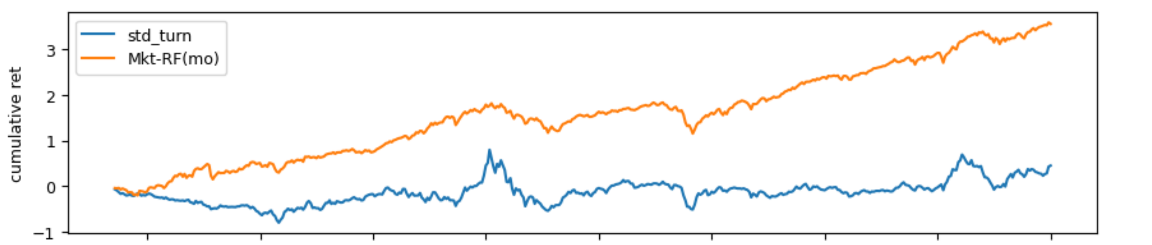

retvol(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19850101
19900101
19950101
20000101
20050101
20100101
20150101
20200101
20250101

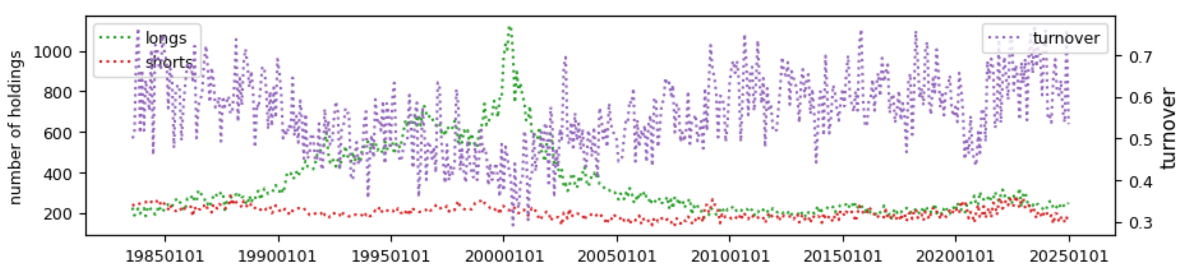

baspread
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19850101
19900101
19950101
20000101
20050101
20100101
20150101
20200101
20250101

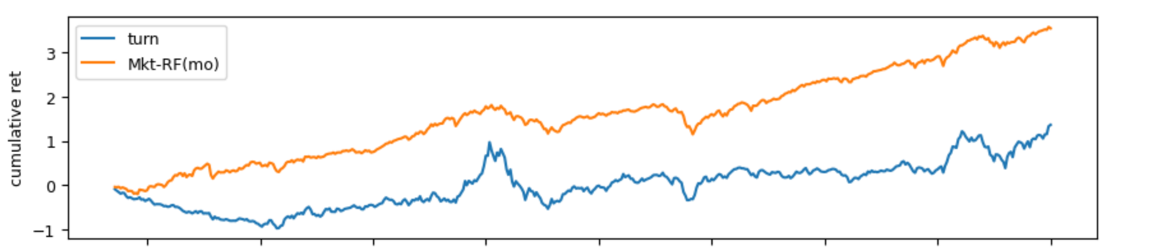

std_dolvol
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19850101
19900101
19950101
20000101
20050101
20100101
20150101
20200101
20250101

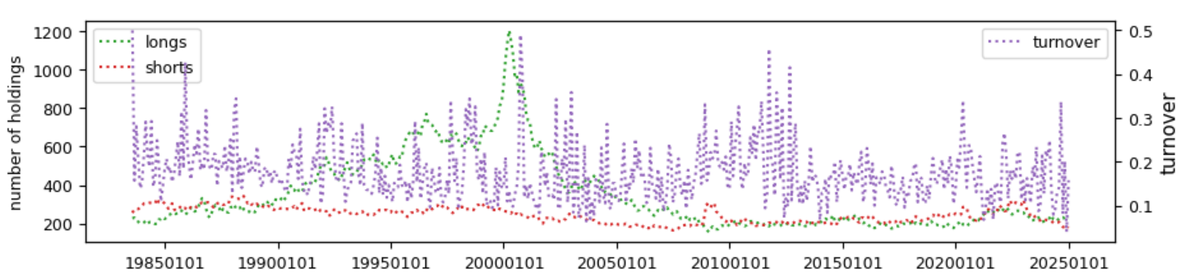

zerotrade(-)
Mkt-RF(mo)
cumulative ret

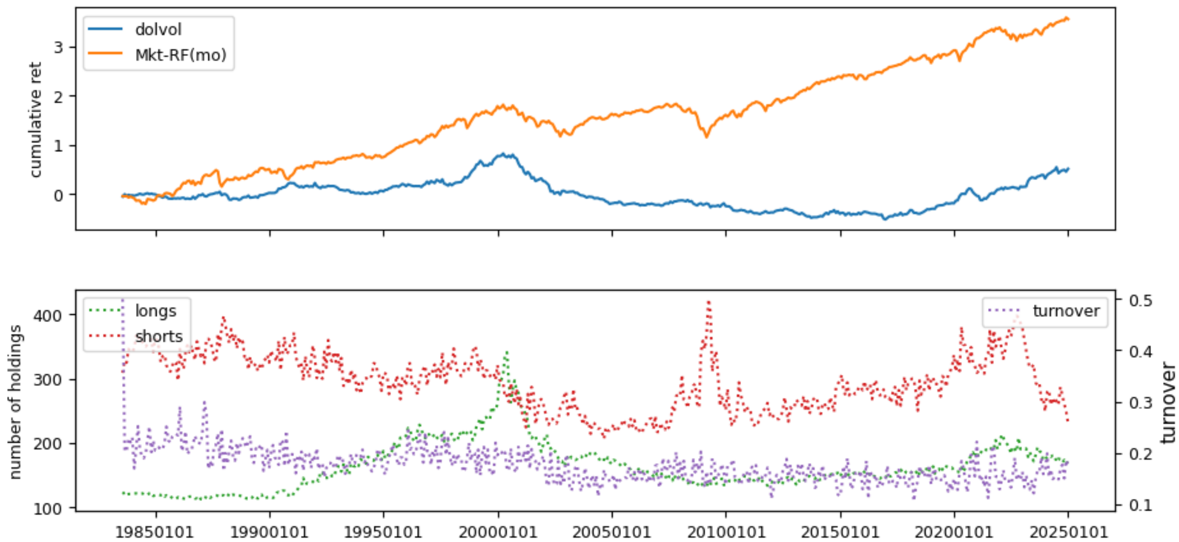

longs
shorts
turnover
number of holdings
turnover
19850101
19900101
19950101
20000101
20050101
20100101
20150101
20200101
20250101


std_turn
Mkt-RF(mo)
cumulative ret


longs
shorts
turnover
number of holdings
turnover
19850101
19900101
19950101
20000101
20050101
20100101
20150101
20200101
20250101


turn
Mkt-RF(mo)
cumulative ret


longs
shorts
turnover
number of holdings
turnover
19850101
19900101
19950101
20000101
20050101
20100101
20150101
20200101
20250101



6.1.5 Fundamentals

Fundamental signals are collected from Compustat, using both annual and quarterly datasets.

```
columns = ['absacc', 'acc', 'agr', 'bm', 'cashpr', 'cfp', 'chcsho',
           'chinv', 'depr', 'dy', 'egr', 'ep', 'gma', 'grcapx',
           'grltnoa', 'hire', 'invest', 'lev', 'lgr' ,
           'pchdepr', 'pchgm_pchsale', 'pchquick',
           'pchsale_pchinvt', 'pchsale_pchrect', 'pchsale_pchxsga',
           'pchsaleinv', 'pctacc', 'quick', 'rd_sale', 'rd_mve',
           'realestate', 'salecash', 'salerec', 'saleinv', 'secured',
           'sgr', 'sp', 'tang', 'bm_ia', 'cfp_ia', 'chatoia' , 'chpmia',
           'pchcapx_ia', 'chempia', 'mve_ia']
numlag = 6       # number of months to lag data for rebalance
end = LAST_DATE    # last data date
```

```
# retrieve annual, keep [permno, datadate] with non null prccq if any
fields = ['sic', 'fyear', 'ib', 'oancf', 'at', 'act', 'che', 'lct',
          'dlc', 'dltt', 'prcc_f', 'csho', 'invt', 'dp', 'ppent',
          'dvt', 'ceq', 'txp', 'revt', 'cogs', 'rect', 'aco', 'intan',
          'ao', 'ap', 'lco', 'lo', 'capx', 'emp', 'ppegt', 'lt',
          'sale', 'xsga', 'xrd', 'fatb', 'fatl', 'dm']
df = pstat.get_linked(dataset='annual',
                      fields=fields,
                      date_field='datadate',
                      where=(f"indfmt = 'INDL' "
                             f"  AND datafmt = 'STD'"
                             f"  AND curcd = 'USD' "
                             f"  AND popsrc = 'D'"
                             f"  AND consol = 'C'"
                             f"  AND datadate <= {end//100}31"))
fund = df.sort_values(['permno', 'datadate', 'ib'])\
         .drop_duplicates(['permno', 'datadate'])\
         .dropna(subset=['ib'])
fund.index = list(zip(fund['permno'], fund['datadate']))  # multi-index
fund['rebaldate'] = bd.endmo(fund.datadate, numlag)
```

```
# precompute, and lag common metrics: mve_f avg_at sic2
fund['sic2'] = np.where(fund['sic'].notna(), fund['sic'] // 100, 0)
fund['fyear'] = fund['datadate'] // 10000       # can delete this
fund['mve_f'] = fund['prcc_f'] * fund['csho']
```

```
lag = fund.shift(1, fill_value=0)
lag.loc[lag['permno'] != fund['permno'], fields] = np.nan
fund['avg_at'] = (fund['at'] + lag['at']) / 2
```

```
lag2 = fund.shift(2, fill_value=0)
lag2.loc[lag2['permno'] != fund['permno'], fields] = np.nan
lag['avg_at'] = (lag['at'] + lag2['at']) / 2
```

```
fund['bm'] = fund['ceq'] / fund['mve_f']
fund['cashpr'] = (fund['mve_f'] + fund['dltt'] - fund['at'])/fund['che']
fund['depr'] = fund['dp'] / fund['ppent']
fund['dy'] = fund['dvt'] / fund['mve_f']
fund['ep'] = fund['ib'] / fund['mve_f']
fund['lev'] = fund['lt'] / fund['mve_f']
fund['quick'] = (fund['act'] - fund['invt']) / fund['lct']
fund['rd_sale'] = fund['xrd'] / fund['sale']
fund['rd_mve'] = fund['xrd'] / fund['mve_f']
fund['realestate'] = ((fund['fatb'] + fund['fatl']) /
                      np.where(fund['ppegt'].notna(),
                               fund['ppegt'], fund['ppent']))
fund['salecash'] = fund['sale'] / fund['che']
fund['salerec'] = fund['sale'] / fund['rect']
fund['saleinv'] = fund['sale'] / fund['invt']
fund['secured'] = fund['dm'] / fund['dltt']
fund['sp'] = fund['sale'] / fund['mve_f']
fund['tang'] = (fund['che'] + fund['rect'] * 0.715 + fund['invt'] * 0.547
                + fund['ppent'] * 0.535) / fund['at']
```

```
# changes: agr chcsho chinv egr gma egr grcapx grltnoa emp invest lgr
fund['agr'] = (fund['at'] / lag['at'])
fund['chcsho'] = (fund['csho'] / lag['csho'])
fund['chinv'] = ((fund['invt'] - lag['invt']) / fund['avg_at'])
fund['egr'] = (fund['ceq'] / lag['ceq'])
fund['gma'] = ((fund['revt'] - fund['cogs']) / lag['at'])
fund['grcapx'] = (fund['capx'] / lag2['capx'])
fund['grltnoa'] =  (((fund['rect']
                      + fund['invt']
                      + fund['ppent']
                      + fund['aco']
                      + fund['intan']
                      + fund['ao']
                      - fund['ap']
                      - fund['lco']
                      - fund['lo'])
                     / (lag['rect']
                        + lag['invt']
                        + lag['ppent']
                        + lag['aco']
                        + lag['intan']
```

(continues on next page)

(continued from previous page)

```
                        + lag['ao']
                        - lag['ap']
                        - lag['lco']
                        - lag['lo']))
                    - ((fund['rect']
                        + fund['invt']
                        + fund['aco']
                        - fund['ap']
                        - fund['lco'])
                       - (lag['rect']
                          + lag['invt']
                          + lag['aco']
                          - lag['ap']
                          - lag['lco']))) / fund['avg_at']
fund['hire'] = ((fund['emp'] / lag['emp']) - 1).fillna(0)
fund['invest'] = (((fund['ppegt'] - lag['ppegt'])
                   + (fund['invt'] - lag['invt'])) / lag['at'])
fund['invest'] = fund['invest'].where(fund['invest'].notna(),
                                      ((fund['ppent'] - lag['ppent'])
                                       + (fund['invt'] - lag['invt'])) / lag['at'])
fund['lgr'] = (fund['lt'] / lag['lt'])
fund['pchdepr'] = ((fund['dp'] / fund['ppent']) / (lag['dp'] / lag['ppent']))
fund['pchgm_pchsale'] = (((fund['sale'] - fund['cogs']) / (lag['sale'] - lag['cogs']))
                         - (fund['sale'] / lag['sale']))
fund['pchquick'] = (((fund['act'] - fund['invt']) / fund['lct'])
                    / ((lag['act'] - lag['invt']) / lag['lct']))
fund['pchsale_pchinvt'] = ((fund['sale'] / lag['sale']) - (fund['invt'] / lag['invt
↪']))
fund['pchsale_pchrect'] = ((fund['sale'] / lag['sale']) - (fund['rect'] / lag['rect
↪']))
fund['pchsale_pchxsga'] = ((fund['sale'] / lag['sale']) - (fund['xsga'] / lag['xsga
↪']))
fund['pchsaleinv'] = ((fund['sale'] / fund['invt']) / (lag['sale'] / lag['invt']))
fund['sgr'] = (fund['sale'] / lag['sale'])
```

```
fund['chato'] = ((fund['sale'] / fund['avg_at']) - (lag['sale'] / lag['avg_at']))
fund['chpm'] = (fund['ib'] / fund['sale']) - (lag['ib'] / lag['sale'])
fund['pchcapx'] = fund['capx'] / lag['capx']
```

```
# compute signals with alternative definitions: acc absacc cfp
fund['_acc'] = (((fund['act'] - lag['act']) - (fund['che'] - lag['che']))
                - ((fund['lct'] - lag['lct']) - (fund['dlc'] - lag['dlc'])
                   - (fund['txp'] - lag['txp']) - fund['dp']))
fund['cfp'] = ((fund['ib'] - (((fund['act'] - lag['act']) - (fund['che'] - lag['che
↪']))
                              - ((fund['lct'] - lag['lct'])
                                 - (fund['dlc'] - lag['dlc'])
                                 - (fund['txp'] - lag['txp'])
                                 - fund['dp']))) / fund['mve_f'])
g = fund['oancf'].notnull()
fund.loc[g, 'cfp'] = fund.loc[g, 'oancf'] / fund.loc[g, 'mve_f']
fund.loc[g, '_acc'] = fund.loc[g, 'ib'] - fund.loc[g, 'oancf']
fund['acc'] = fund['_acc'] / fund['avg_at']
fund['absacc'] = abs(fund['_acc']) / fund['avg_at']
fund['pctacc'] = fund['_acc'] / abs(fund['ib'])
```

(continues on next page)

(continued from previous page)

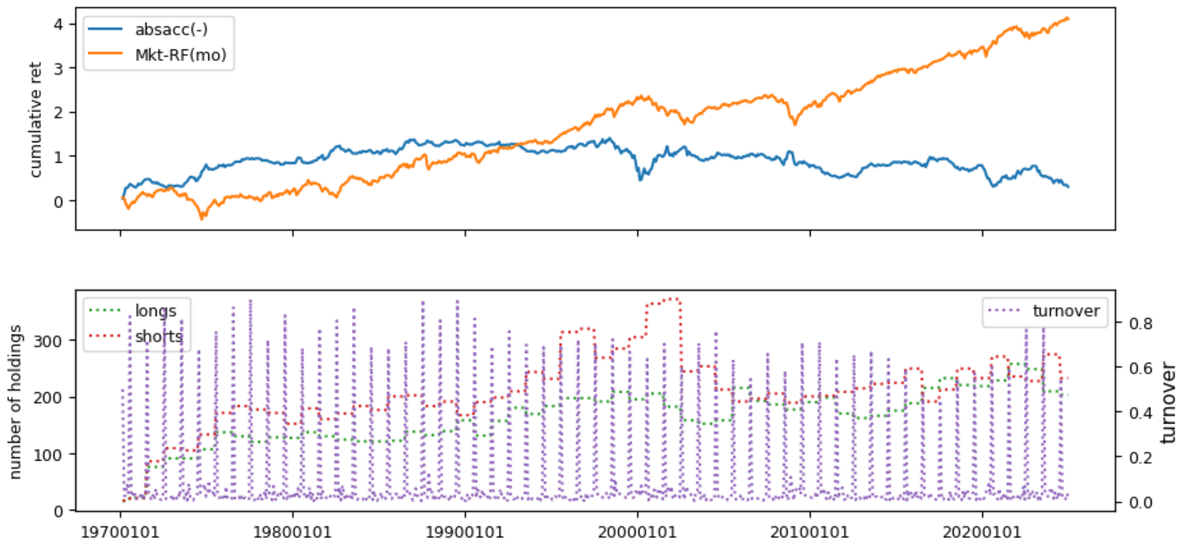
```
h = (fund['ib'].abs() <= 0.01)
fund.loc[h, 'pctacc'] = fund.loc[h, '_acc'] / 0.01
```

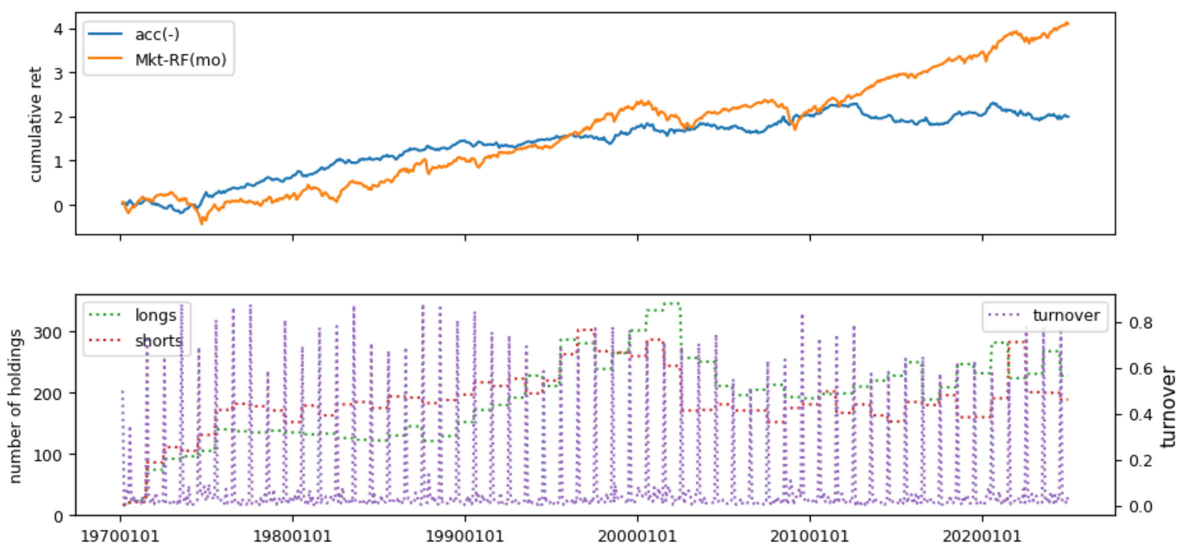
```
# industry-adjusted
cols = {'bm_ia': 'bm', 'cfp_ia': 'cfp', 'chatoia': 'chato',
        'chpmia': 'chpm', 'pchcapx_ia': 'pchcapx',
        'chempia': 'hire', 'mve_ia': 'mve_f'}
group = fund.groupby(['sic2', 'fyear'])
for k,v in cols.items():
    fund[k] = fund[v] - group[v].transform('mean')
```

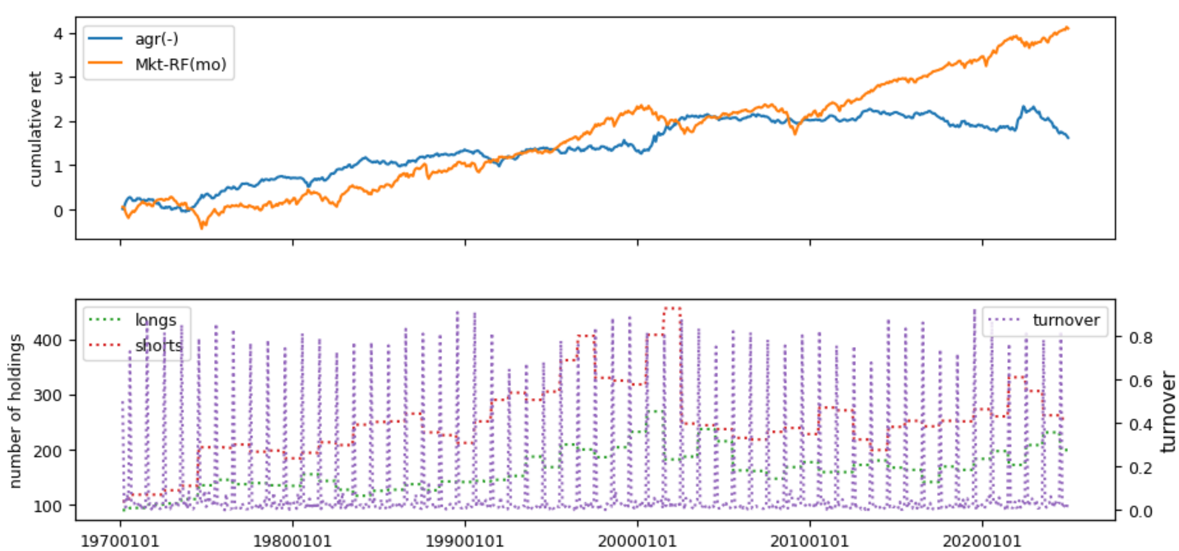
```
for label in columns:
    signals.write(fund, label, overwrite=True)
```

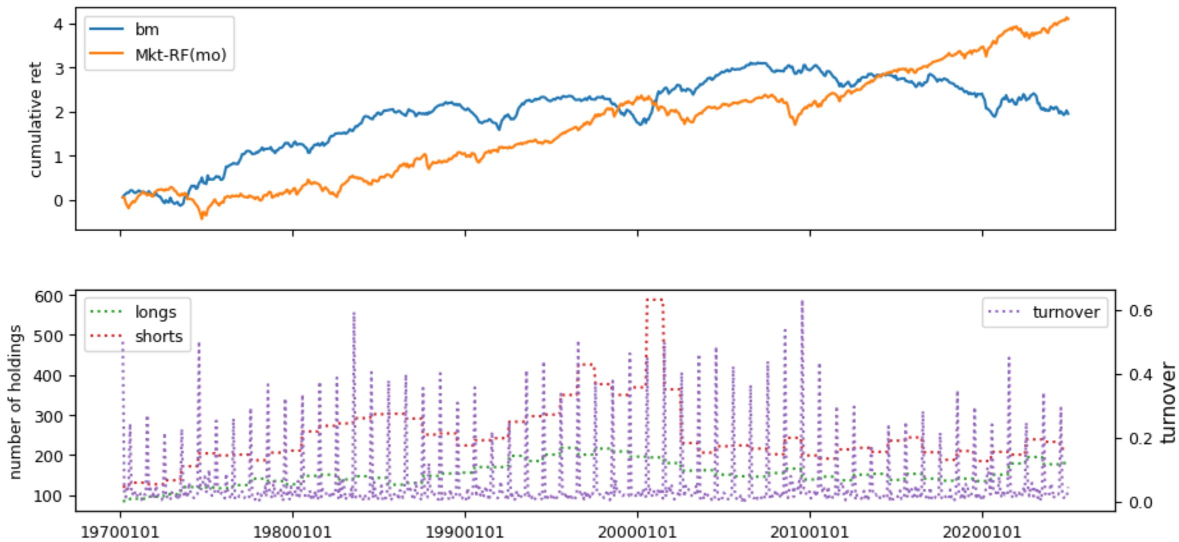
```
rebalbeg, rebalend = 19700101, LAST_DATE
benchnames = ['Mkt-RF(mo)'] #['Mom']  #['ST_Rev(mo)']   #
for label in tqdm(columns):
    holdings = univariate_sorts(monthly,
                                label,
                                SignalsFrame(signals.read(label)),
                                rebalbeg,
                                rebalend,
                                window=12,
                                months=[6],
                                maxdecile=8,
                                pct=(10., 90.),
                                leverage=leverage.get(label, 1))
    excess = backtest_pipeline(backtest,
                               monthly,
                               holdings,
                               label,
                               benchnames,
                               overlap=0,
                               outdir=outdir,
                               suffix=(leverage.get(label, 1) < 0)*'(-)')
```

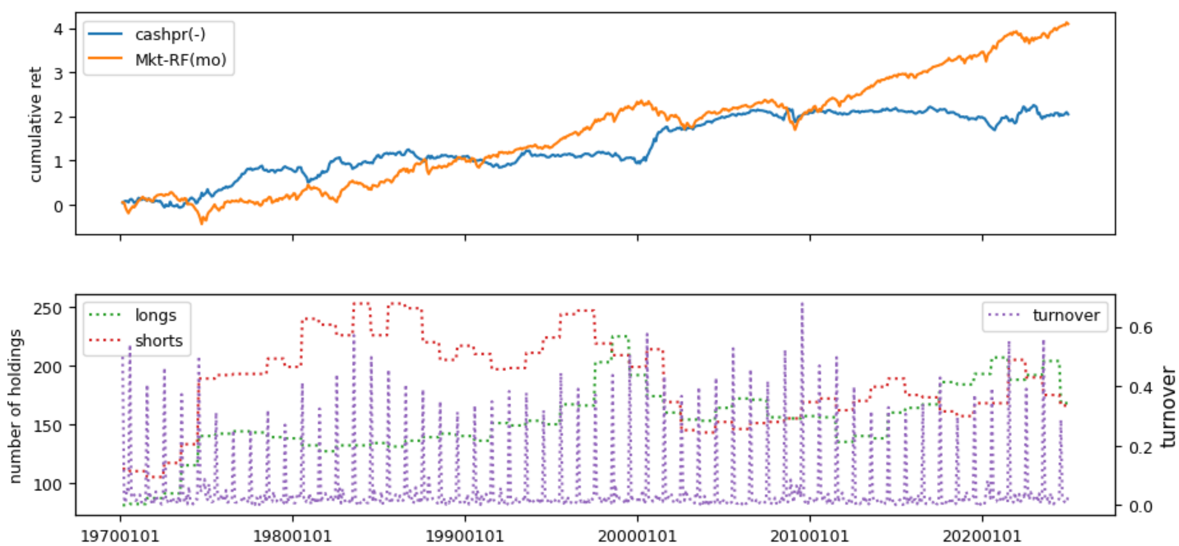
```
 44%|█████     | 20/45 [29:40<39:56, 95.88s/it] /home/terence/Dropbox/github/data-
↪science-notebooks/finds/backtesting/backtest.py:310: RuntimeWarning: More than␣
↪20 figures have been opened. Figures created through the pyplot interface␣
↪(`matplotlib.pyplot.figure`) are retained until explicitly closed and may␣
↪consume too much memory. (To control this warning, see the rcParam `figure.max_
↪open_warning`). Consider using `matplotlib.pyplot.close()`.
  fig, (ax1, ax2) = plt.subplots(2, 1, sharex=True, clear=True,
100%|███████████| 45/45 [1:09:08<00:00, 92.19s/it]
```

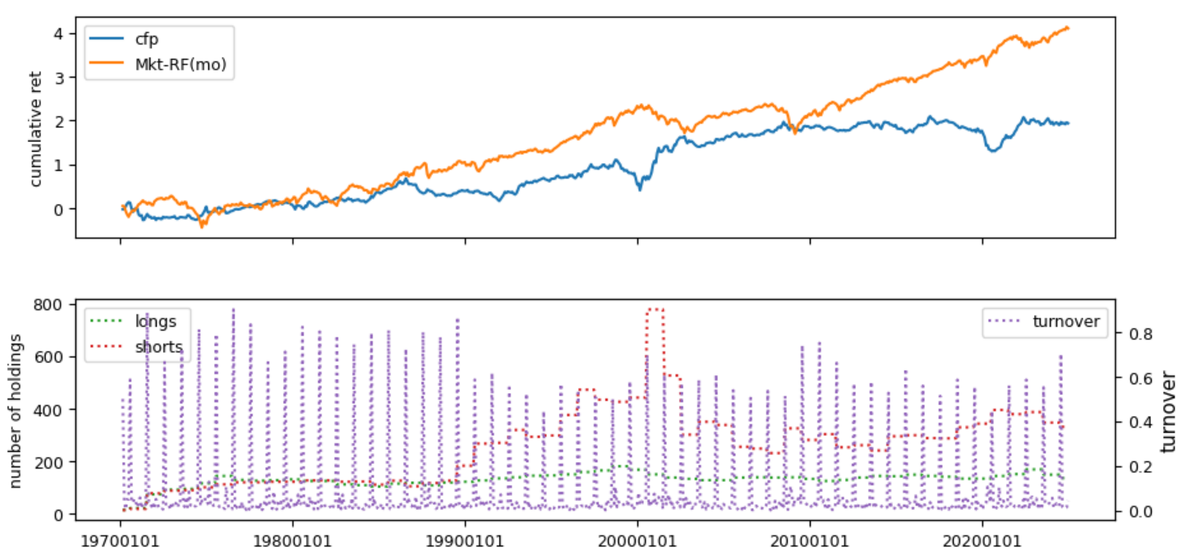

absacc(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

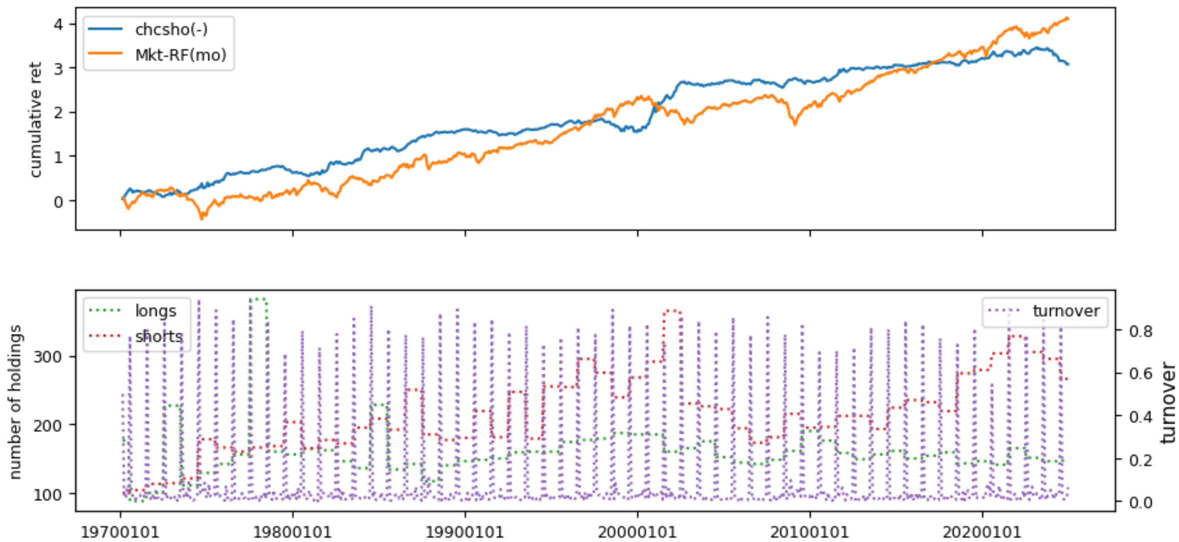

acc(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

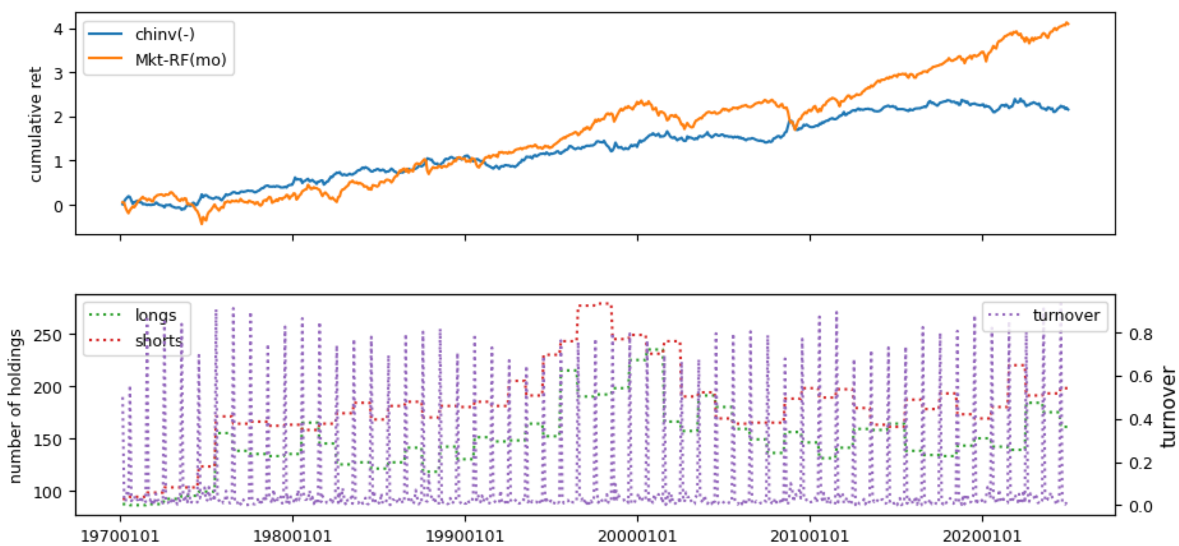

agr(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

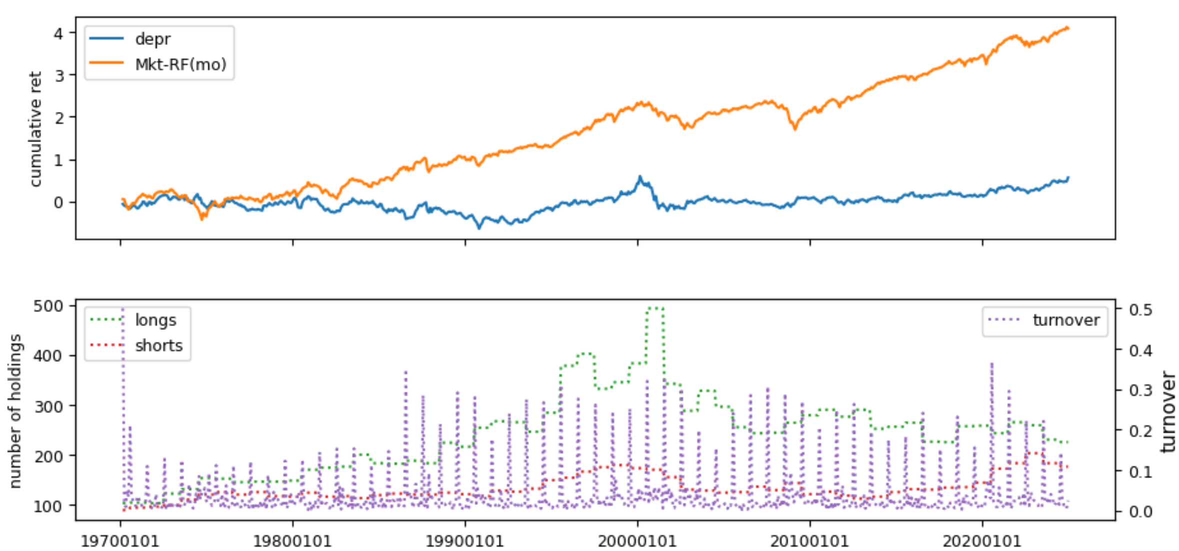

bm
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

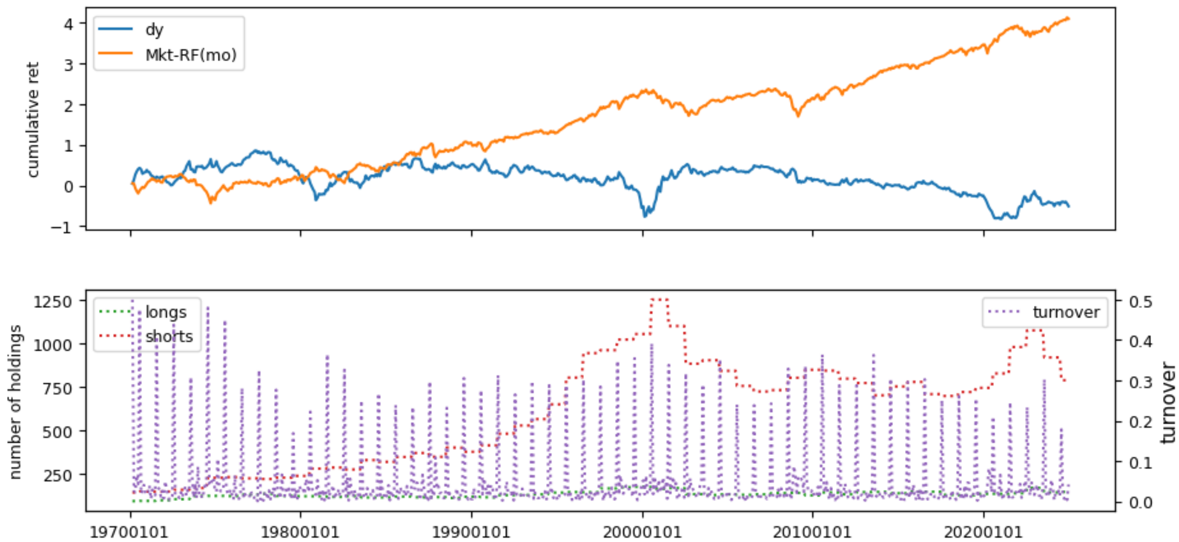

cashpr(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

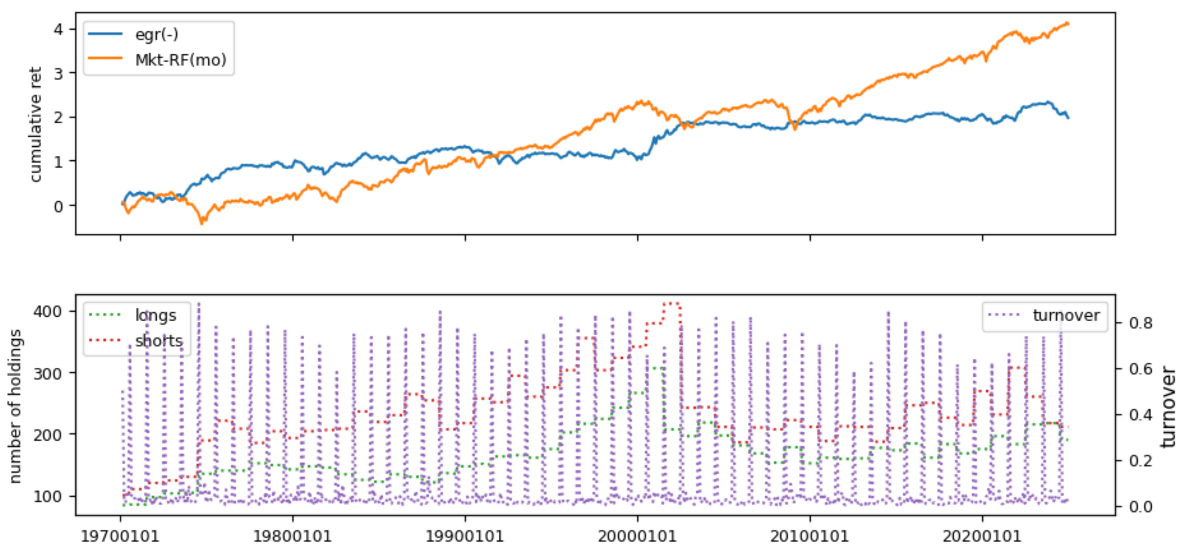

cfp
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

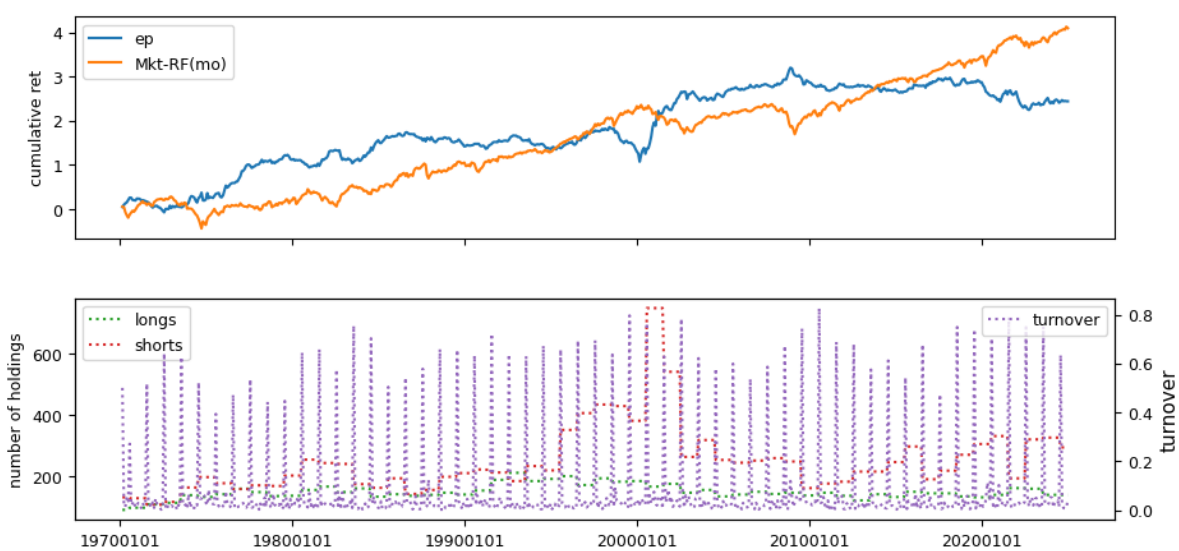

chcsho(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

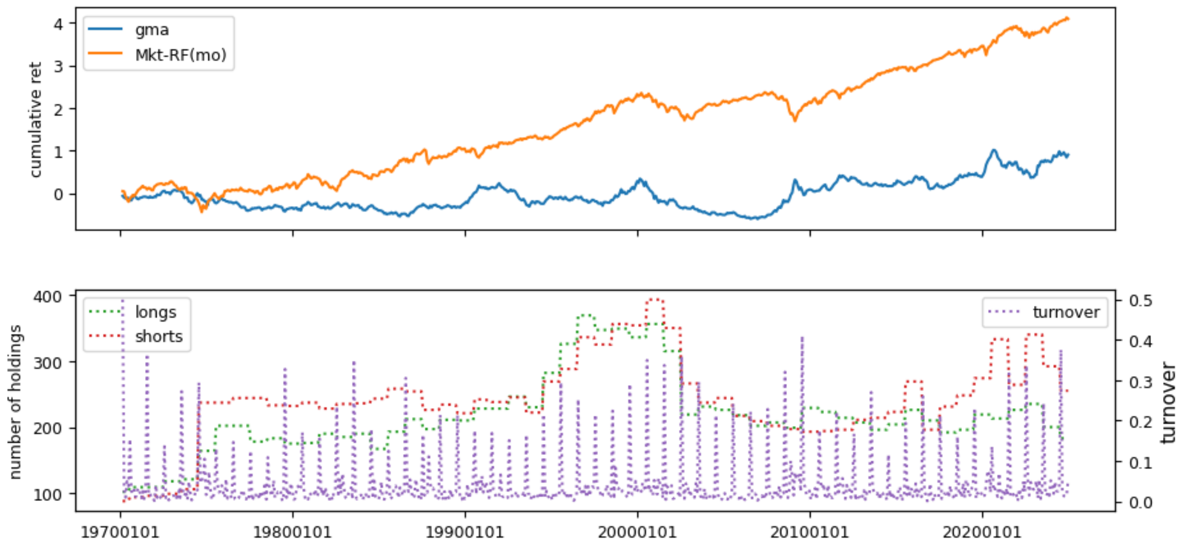

chinv(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

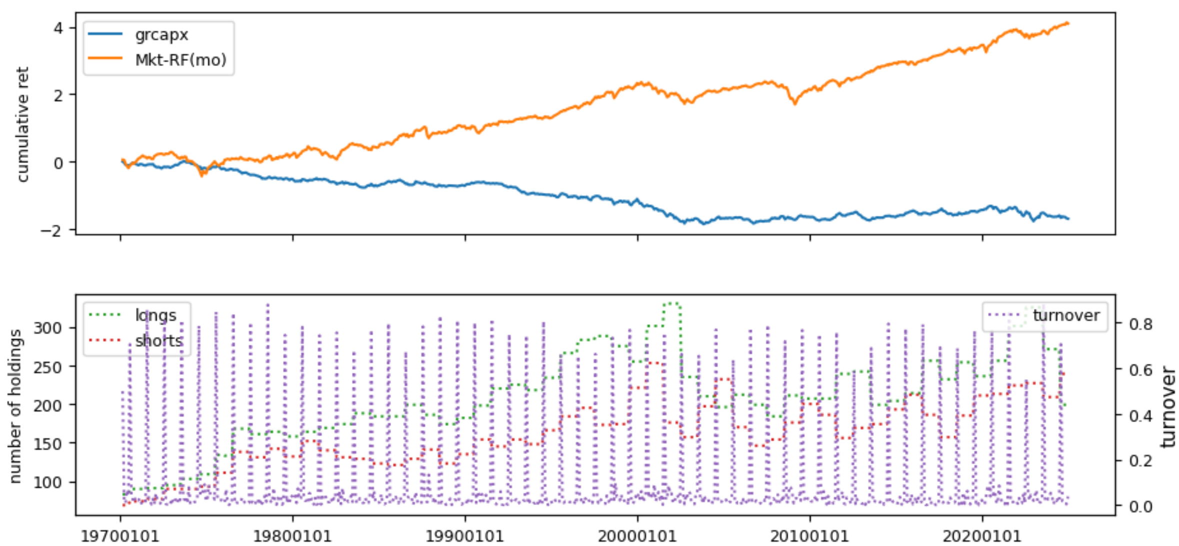

depr
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

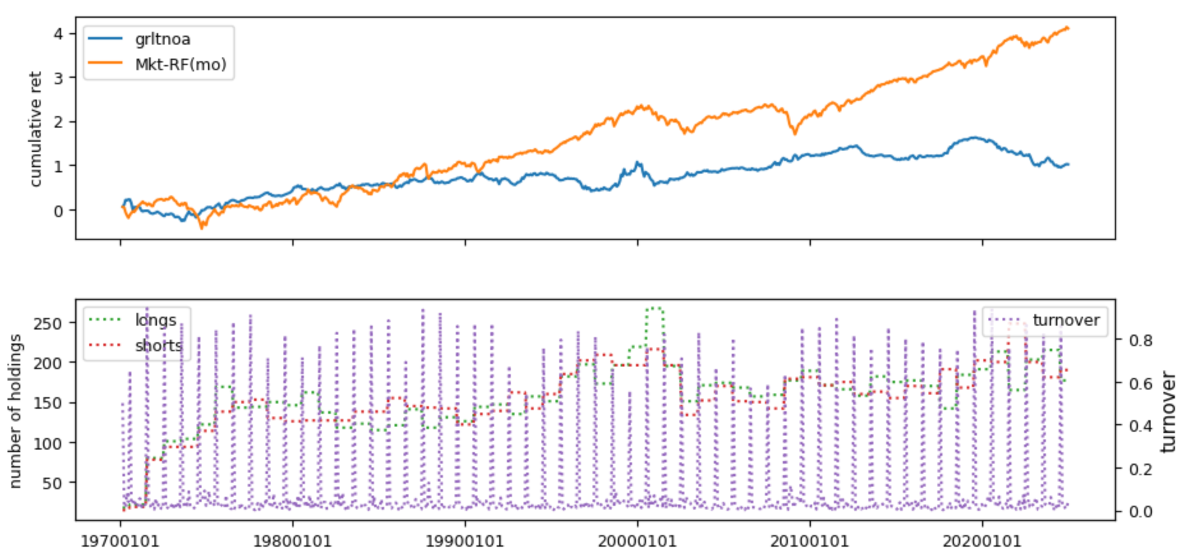

dy
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

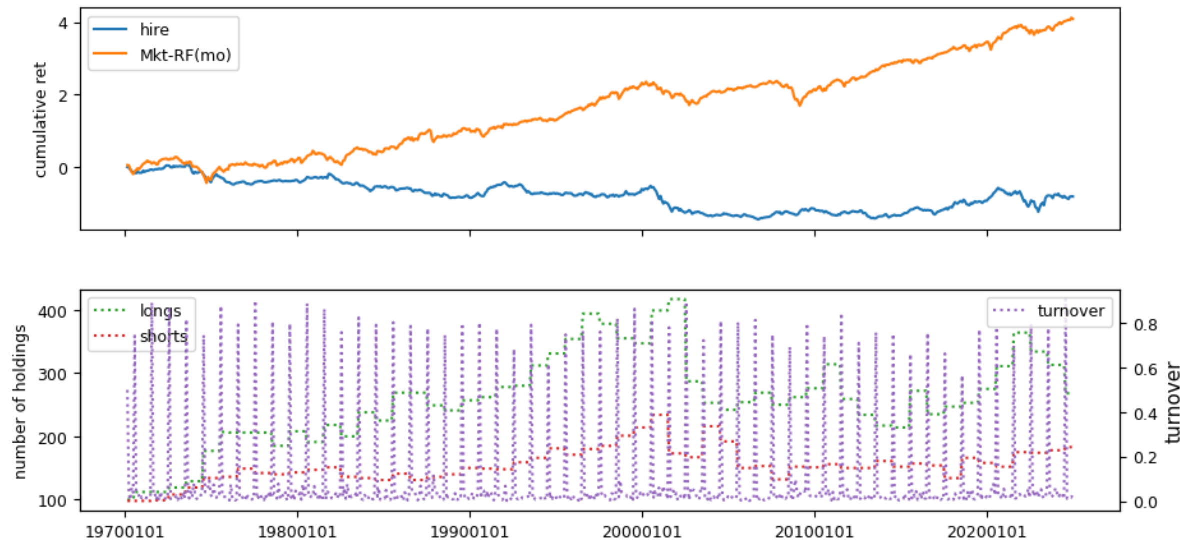

egr(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

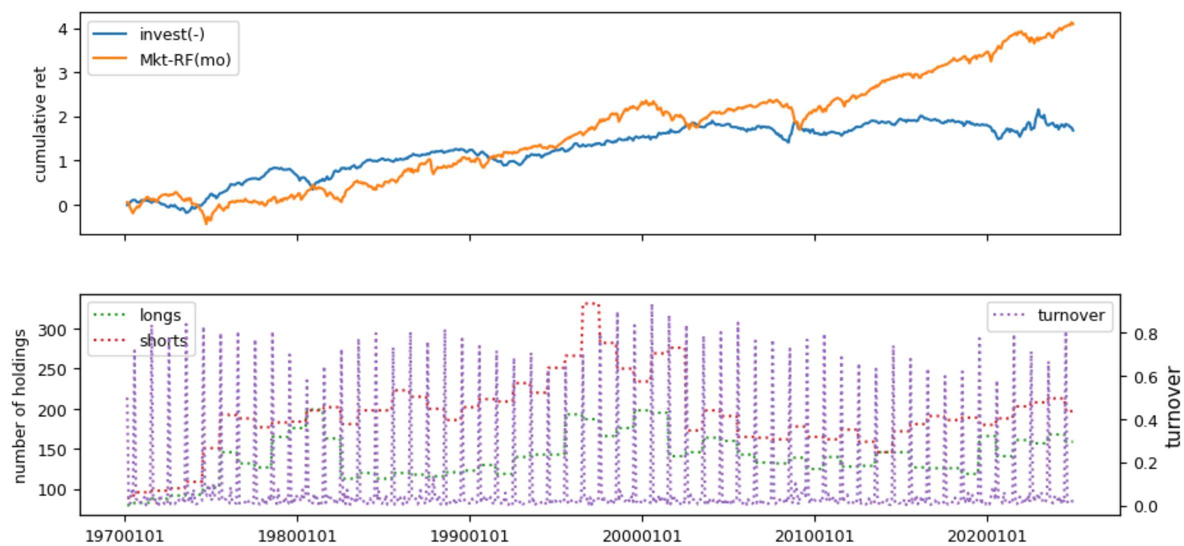

ep
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

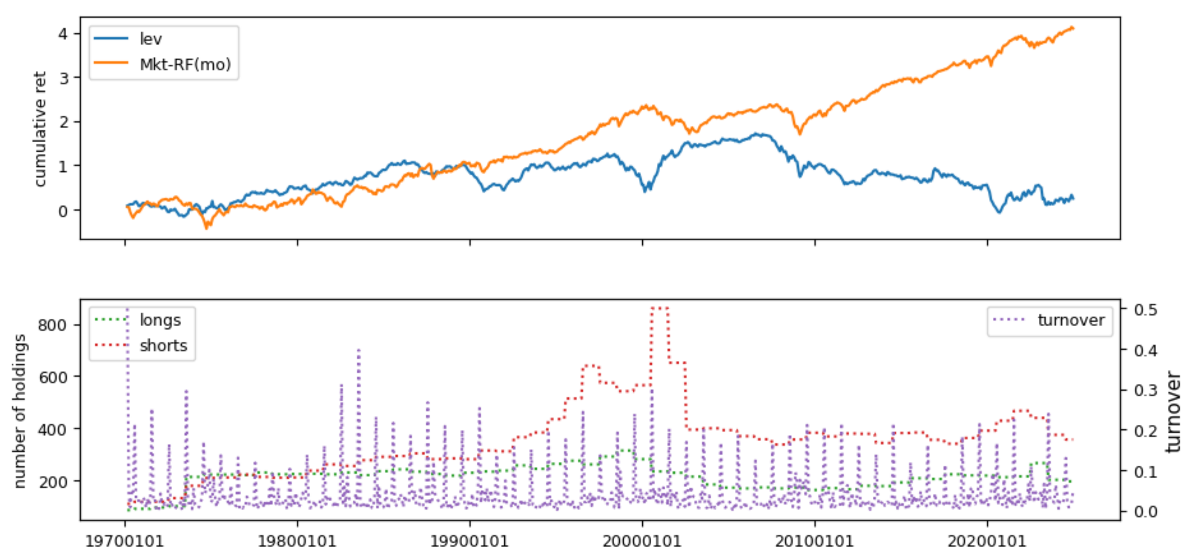

gma
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

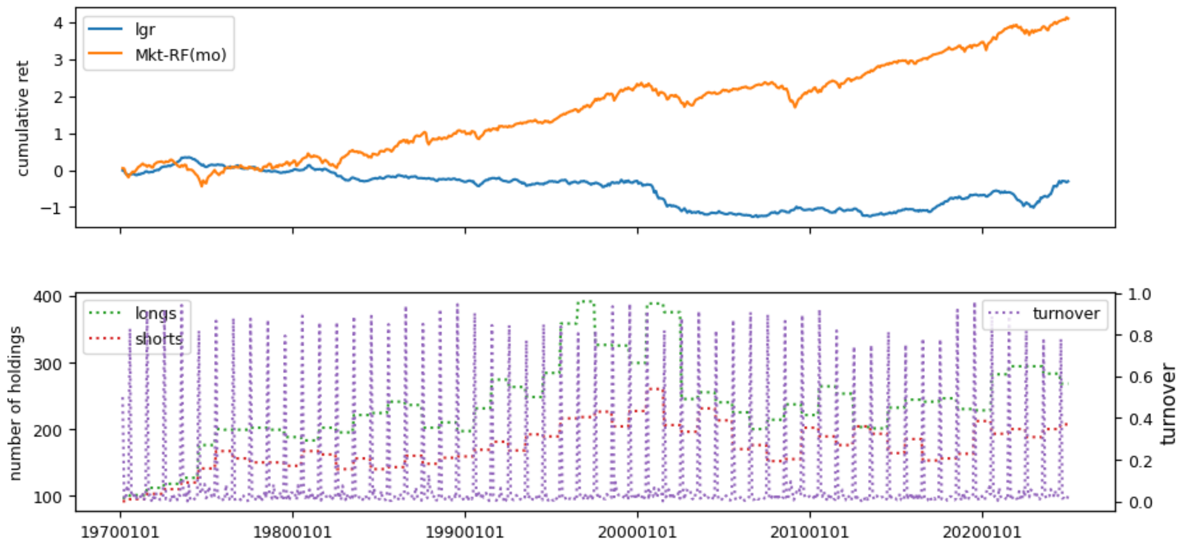

grcapx
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

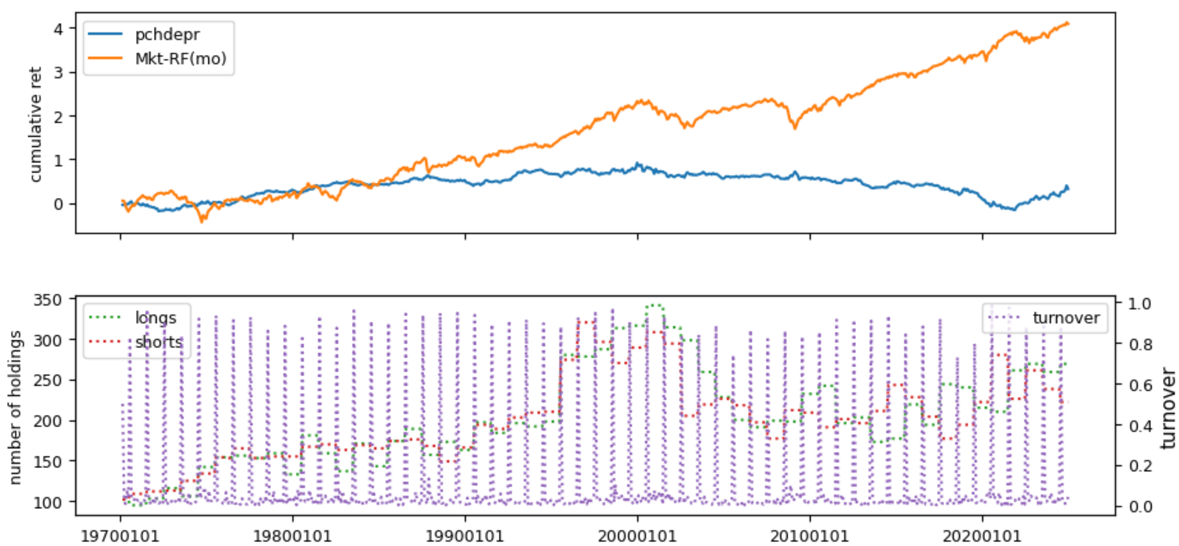

grltnoa
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

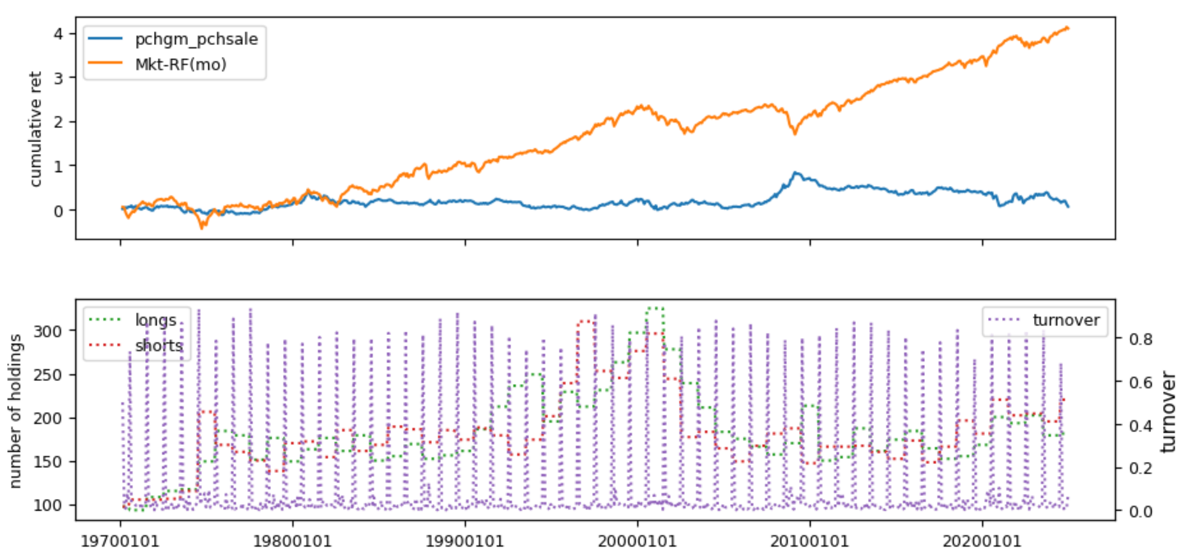

hire
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

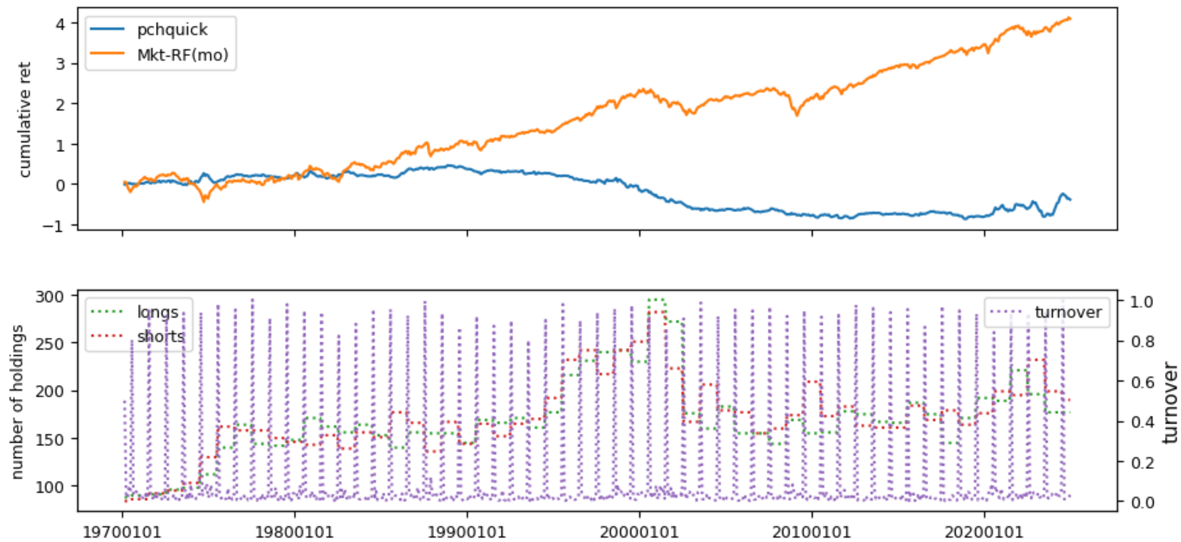

invest(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

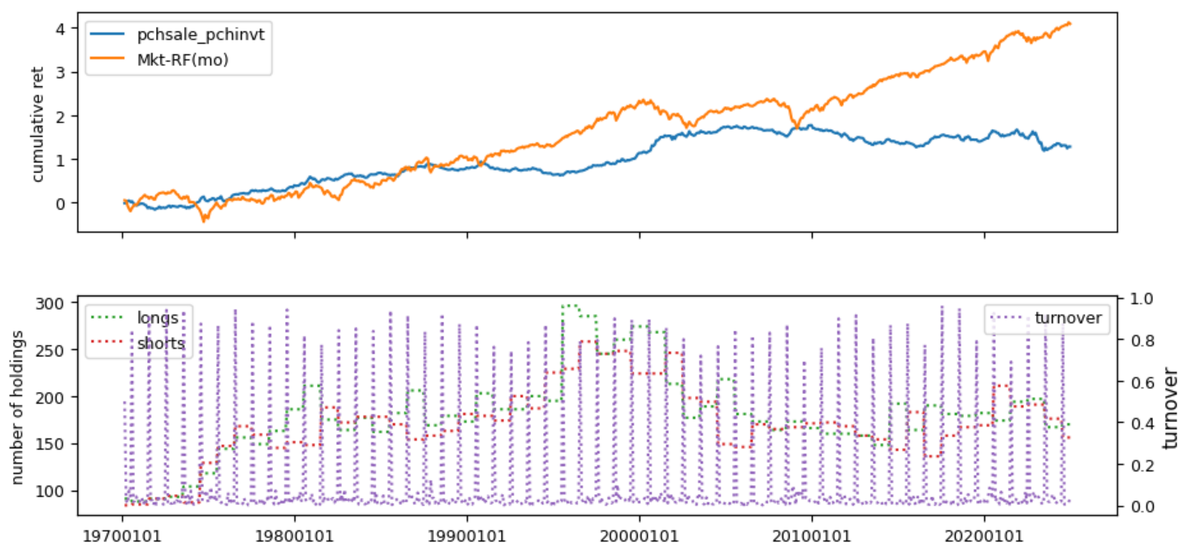

lev
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

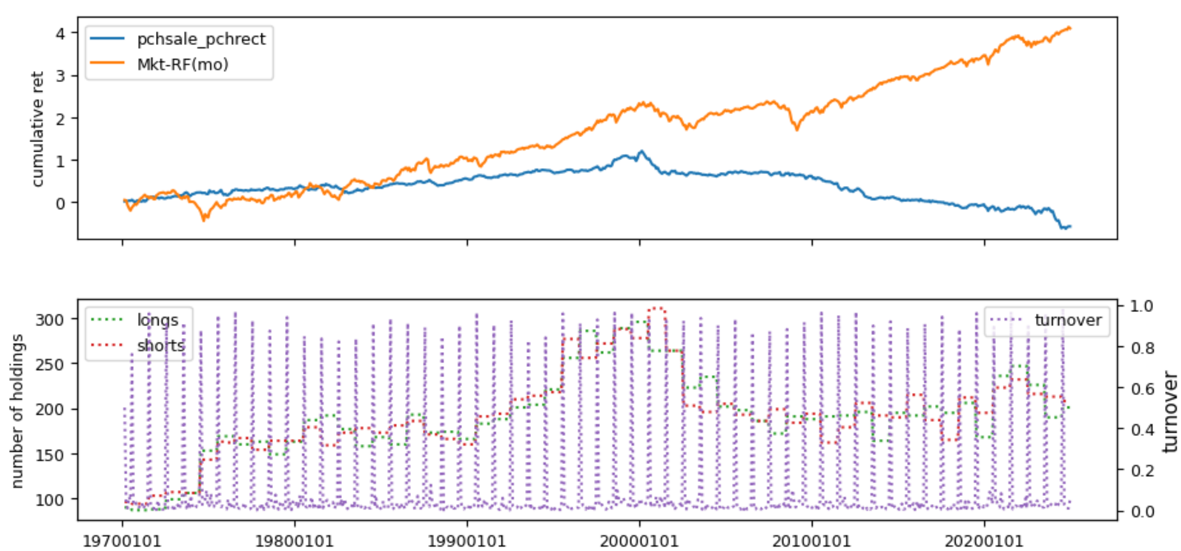

lgr
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

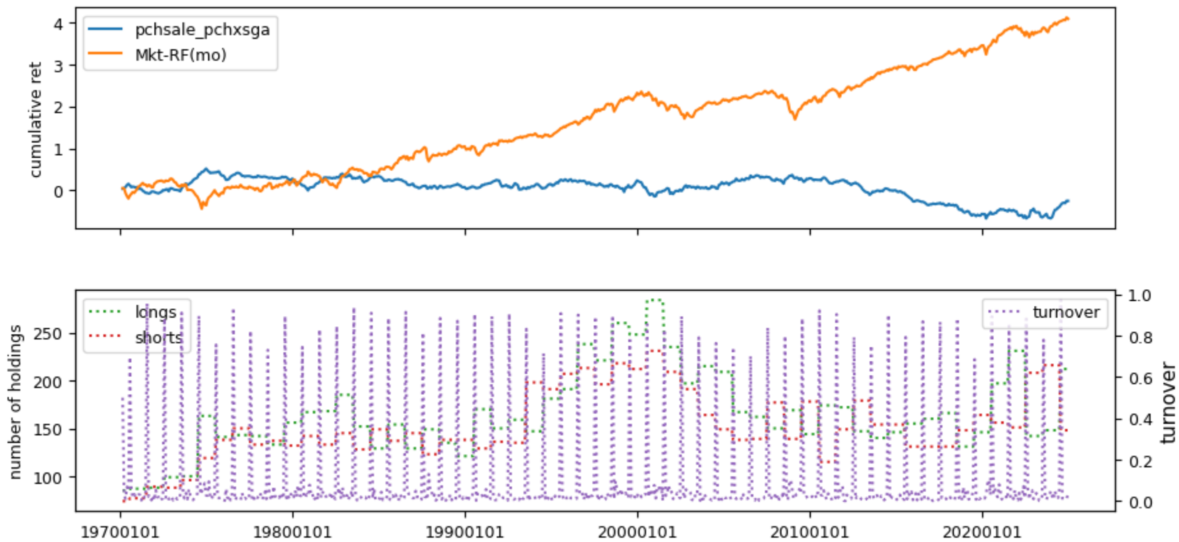

pchdepr
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101


pchgm_pchsale
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

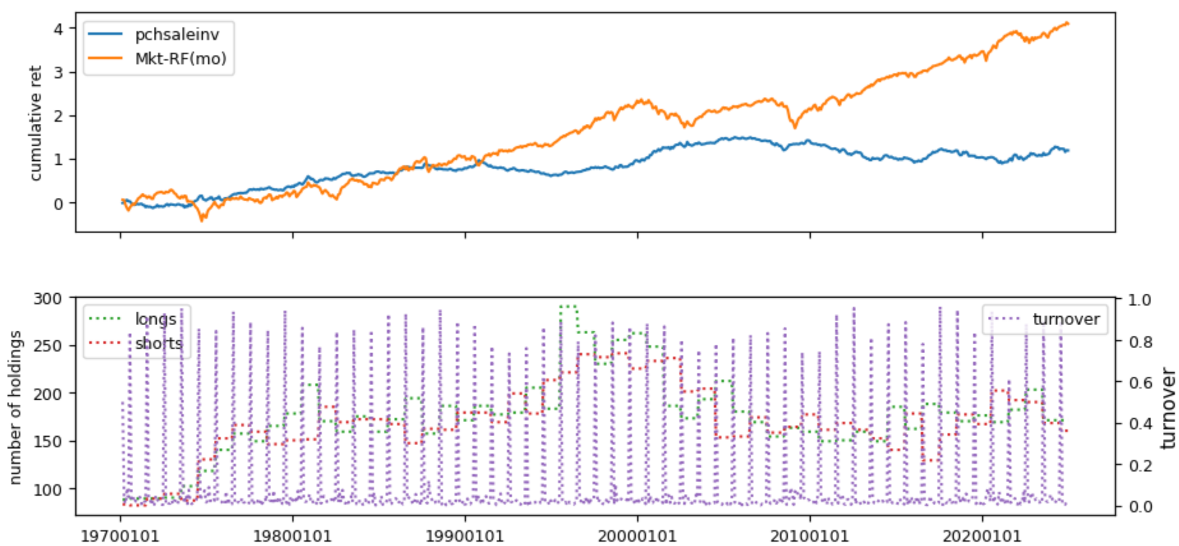

pchquick
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101


pchsale_pchinvt
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

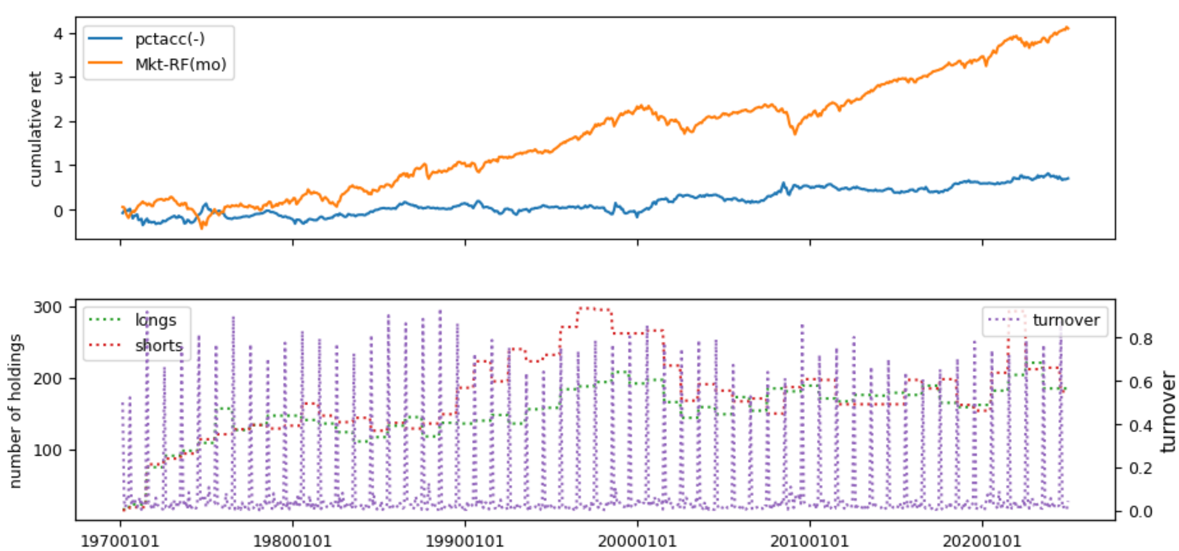

pchsale_pchrect
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101



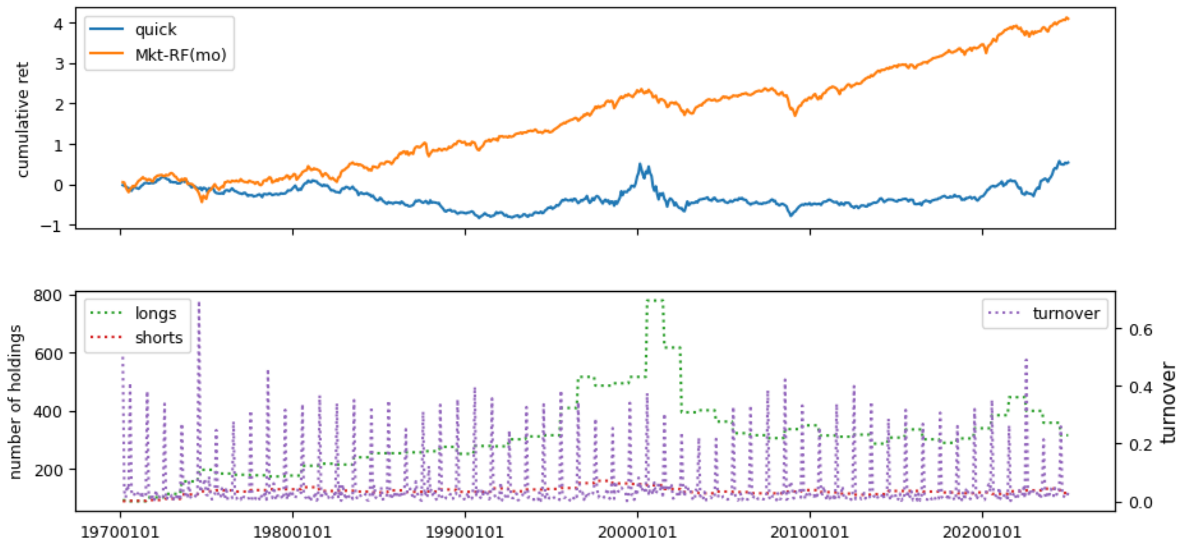
pchsale_pchxsga
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

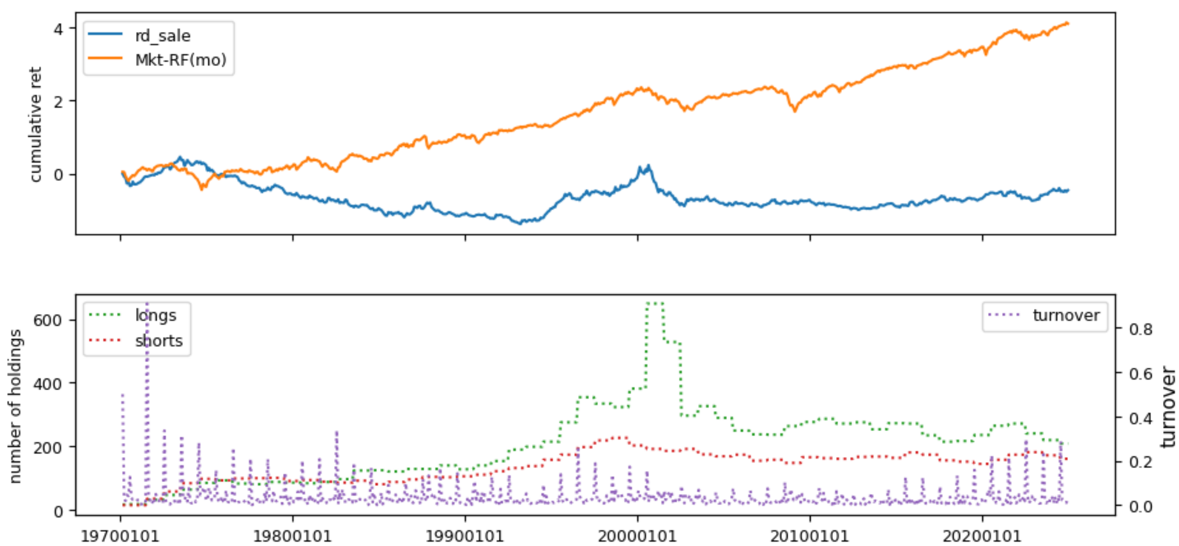


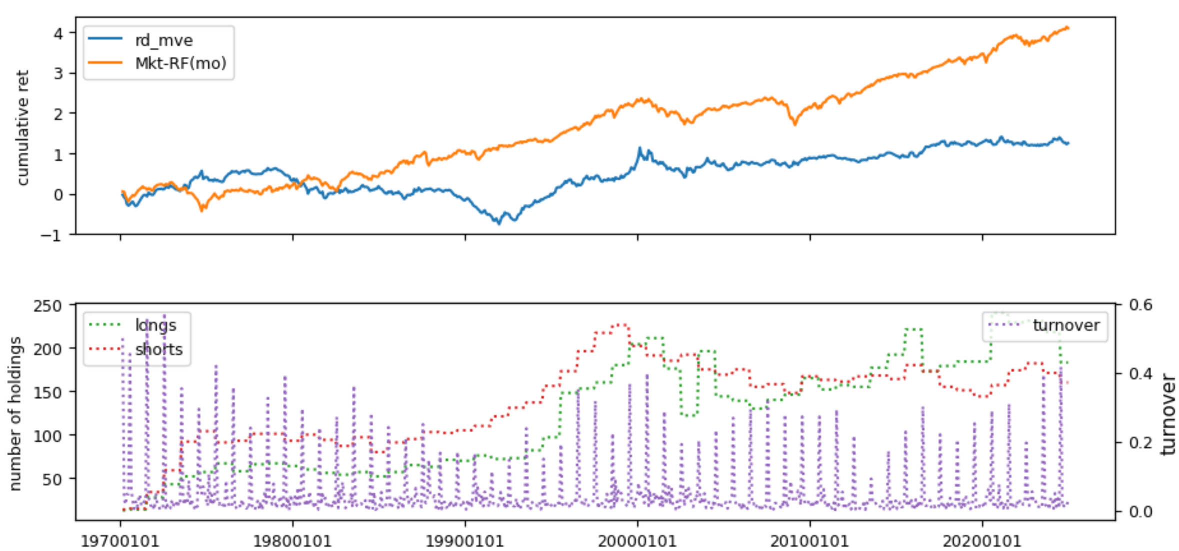
pchsaleinv
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

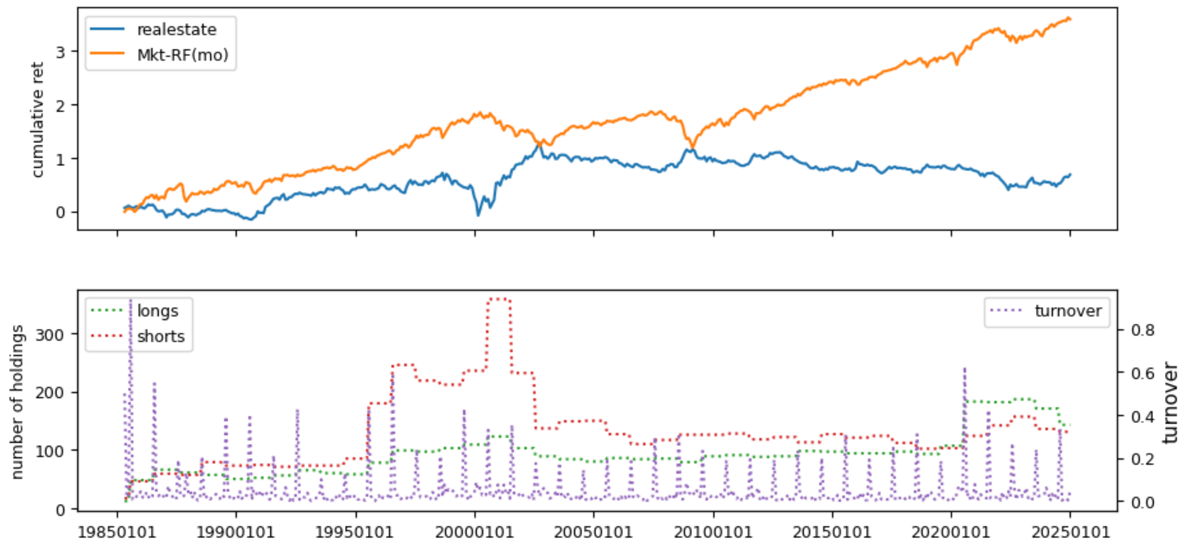


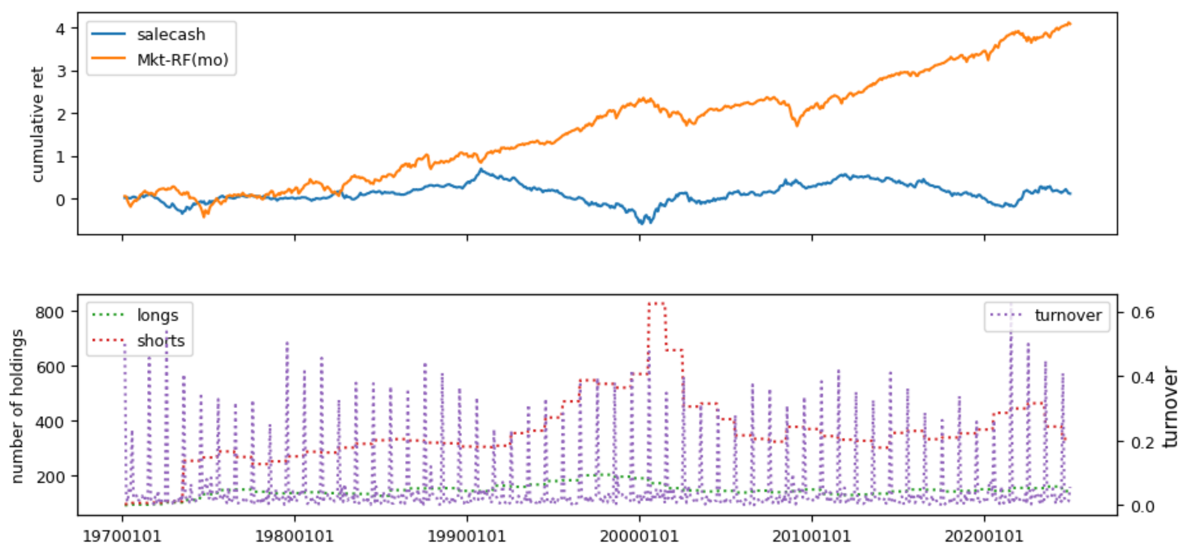
pctacc(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

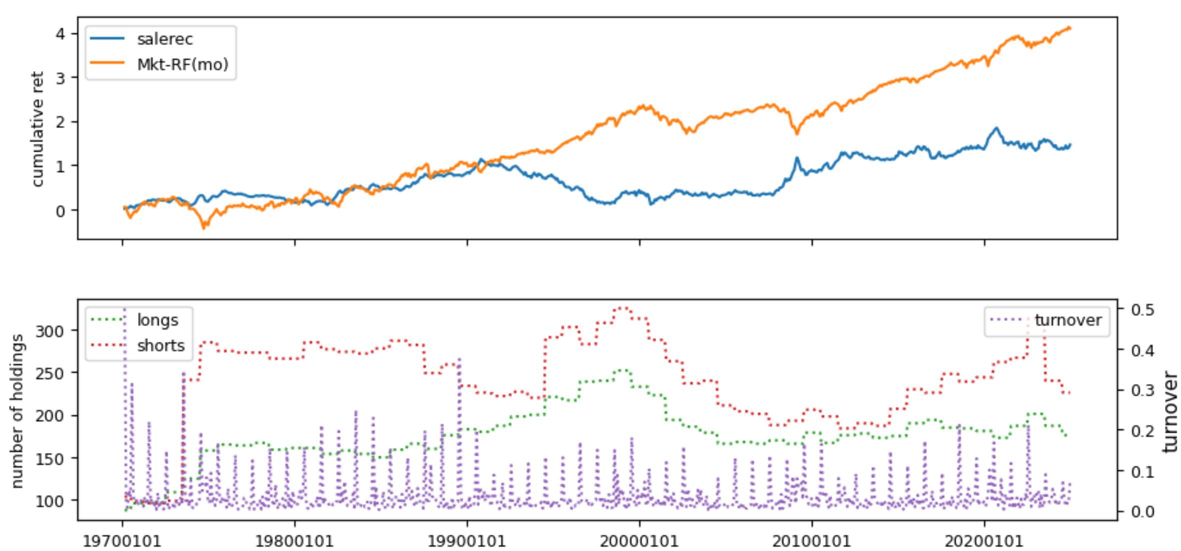

quick
Mkt-RF(mo)
cumulative ret
4
3
2
1
0
−1
longs
shorts
turnover
number of holdings
800
600
400
200
0.6
0.4
0.2
0.0
19700101
19800101
19900101
20000101
20100101
20200101

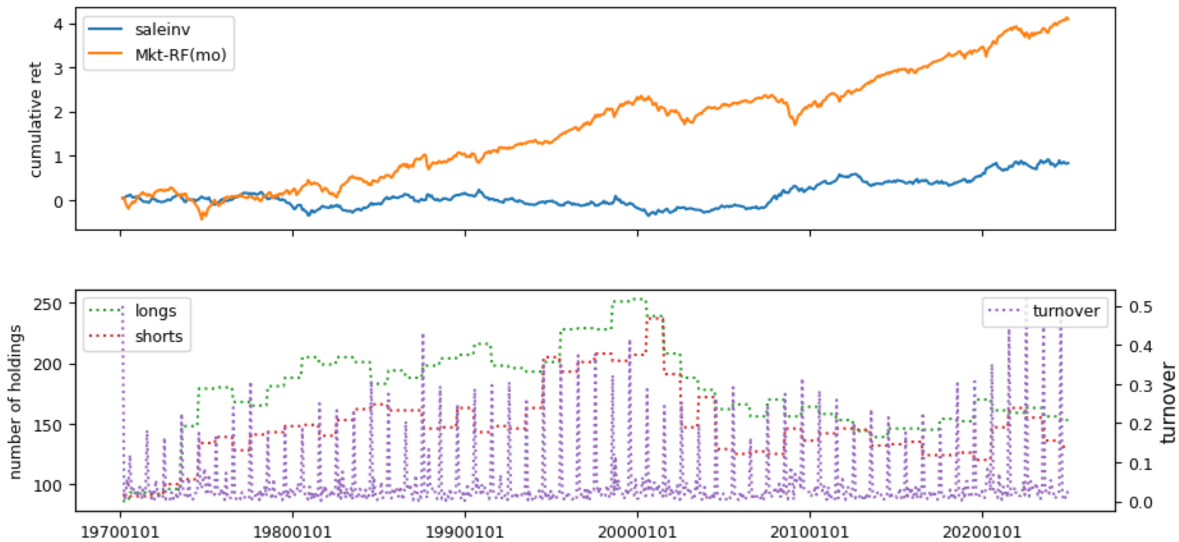

rd_sale
Mkt-RF(mo)
cumulative ret
4
2
0
longs
shorts
turnover
number of holdings
600
400
200
0
0.8
0.6
0.4
0.2
0.0
19700101
19800101
19900101
20000101
20100101
20200101

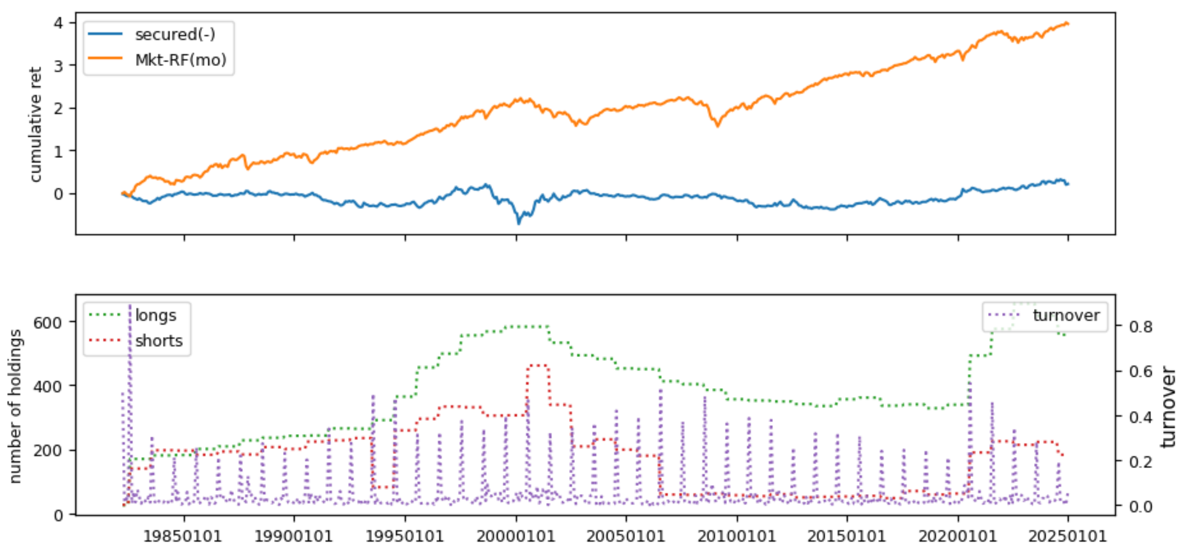

rd_mve
Mkt-RF(mo)
cumulative ret
4
3
2
1
0
−1
longs
shorts
turnover
number of holdings
250
200
150
100
50
0.6
0.4
0.2
0.0
19700101
19800101
19900101
20000101
20100101
20200101

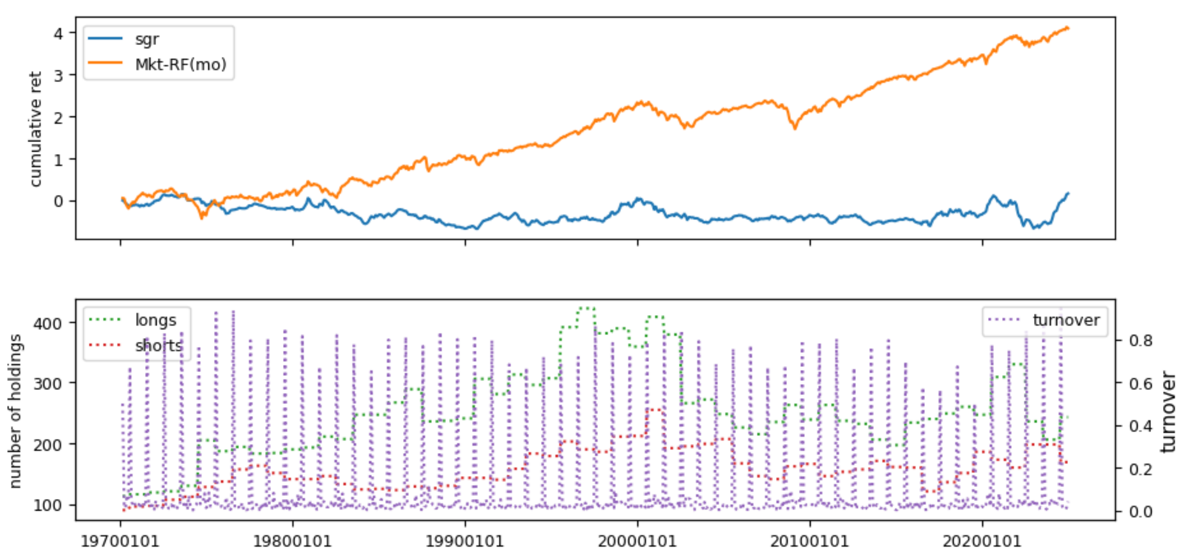

realestate
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19850101
19900101
19950101
20000101
20050101
20100101
20150101
20200101
20250101

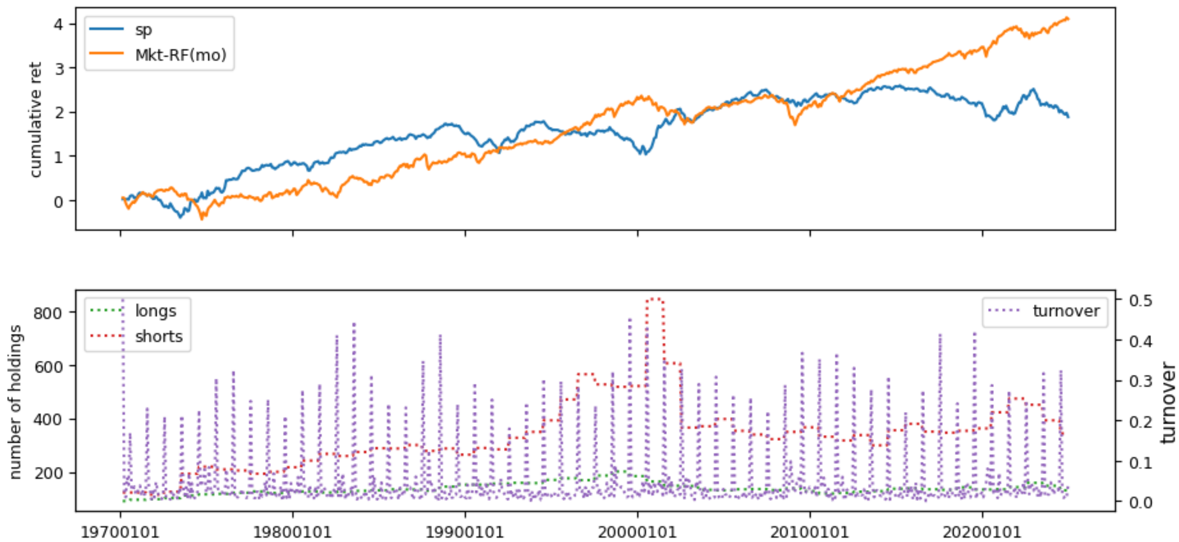

salecash
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

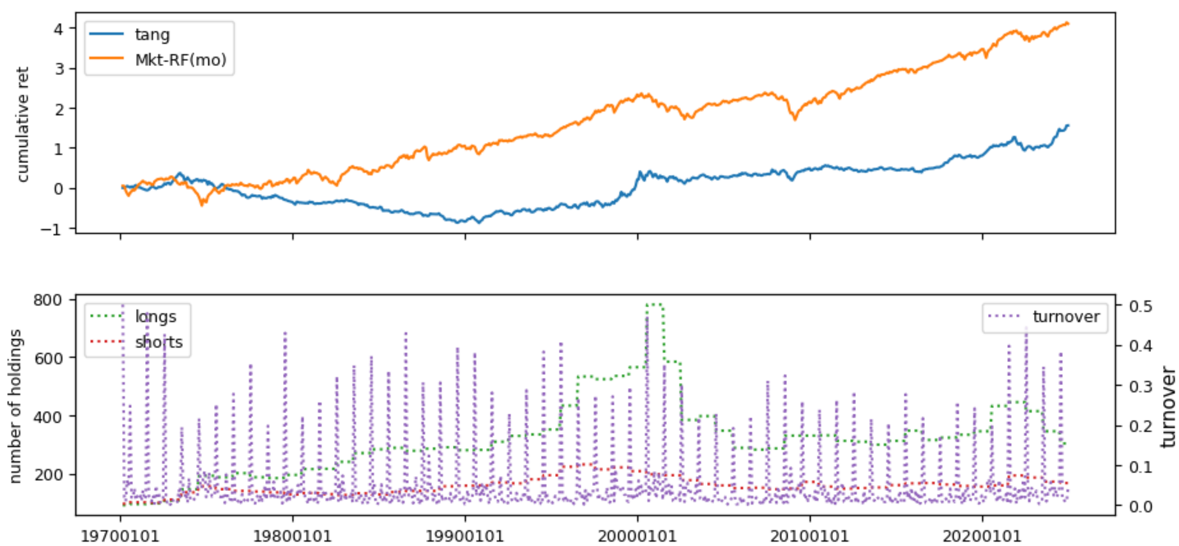

salerec
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

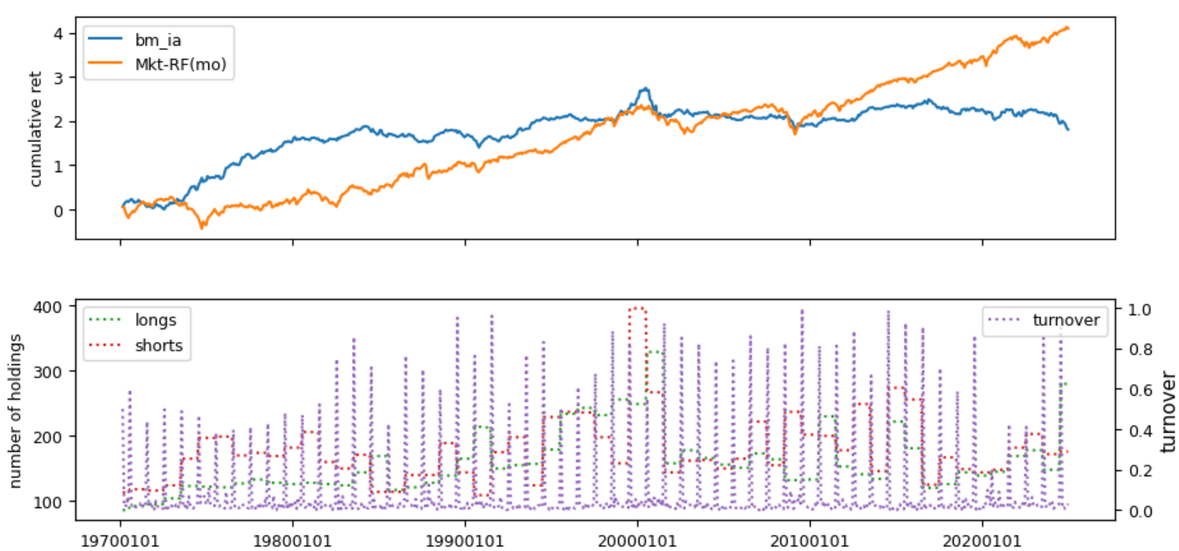

saleinv
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

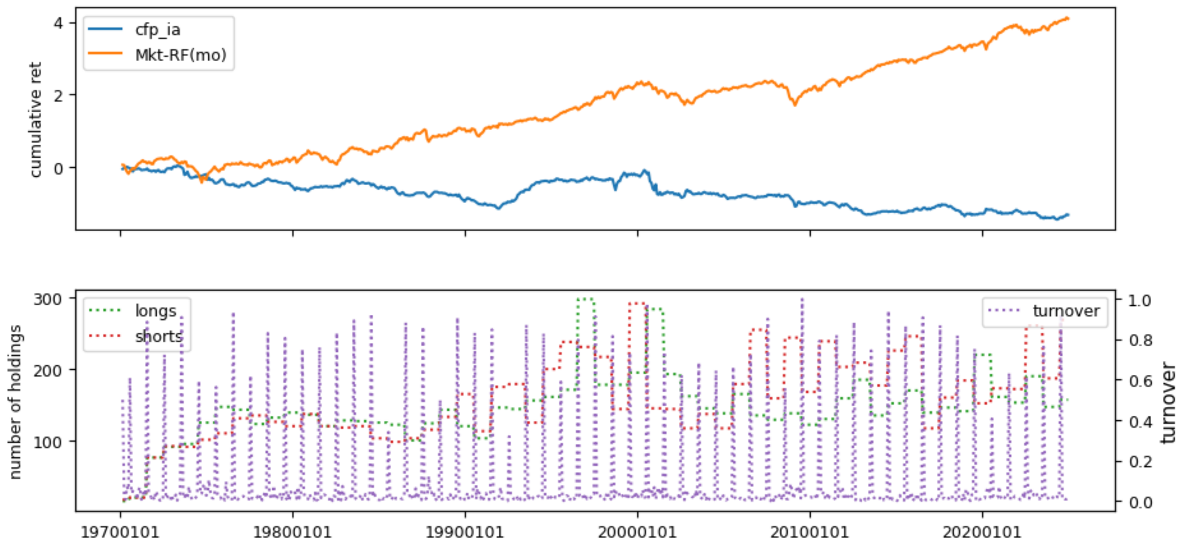

secured(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19850101
19900101
19950101
20000101
20050101
20100101
20150101
20200101
20250101

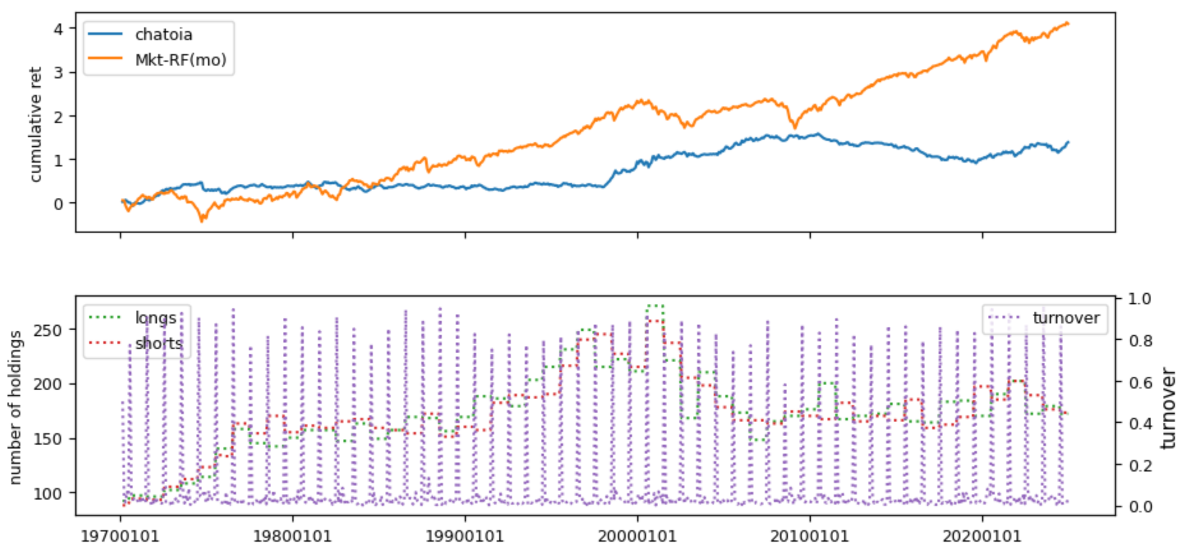

sgr
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

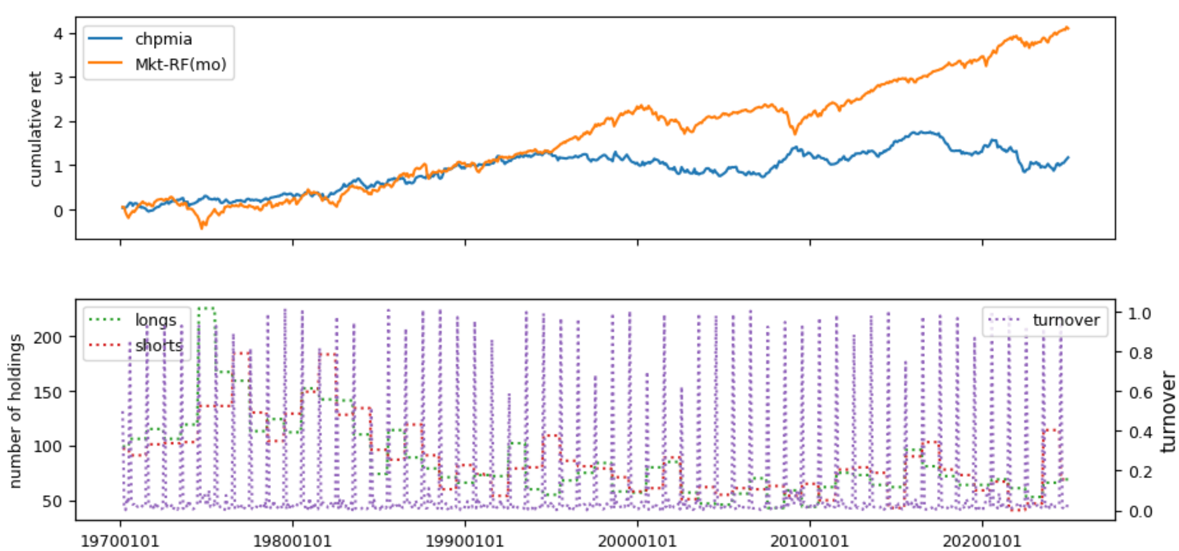

sp
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

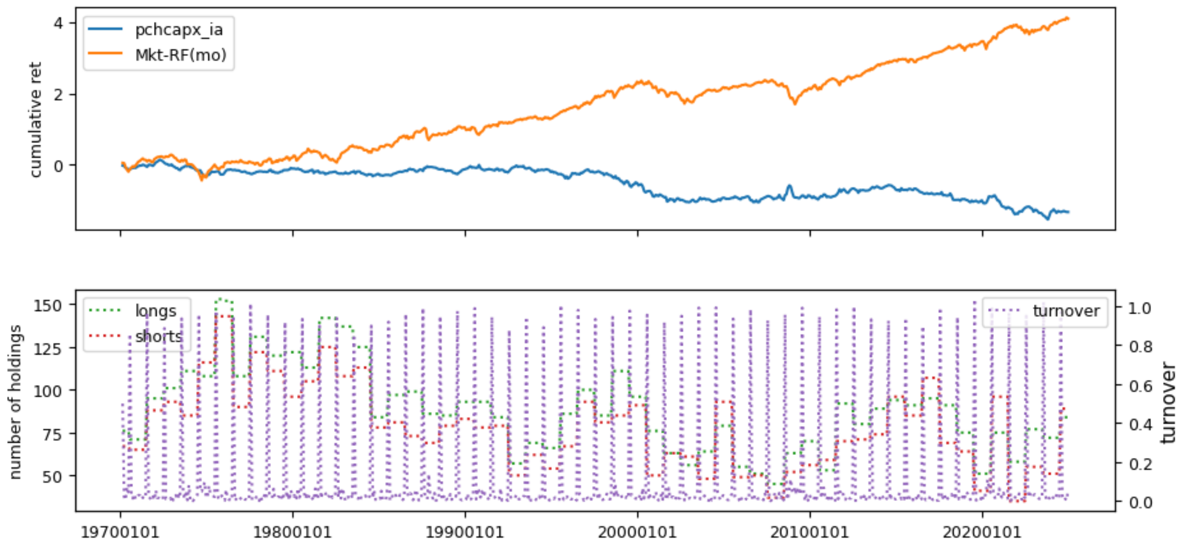

tang
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

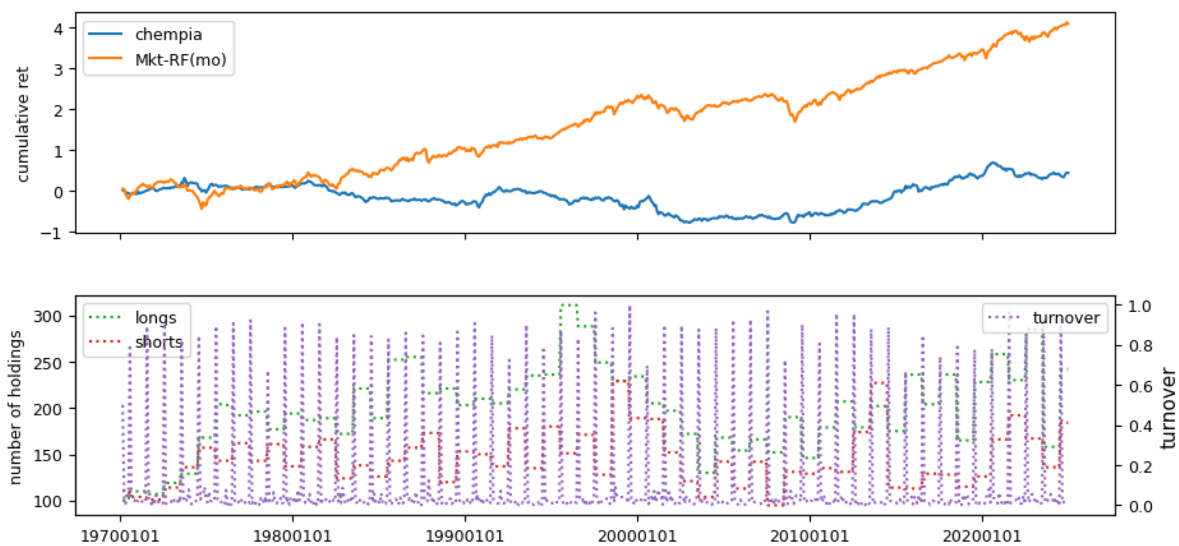

bm_ia
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

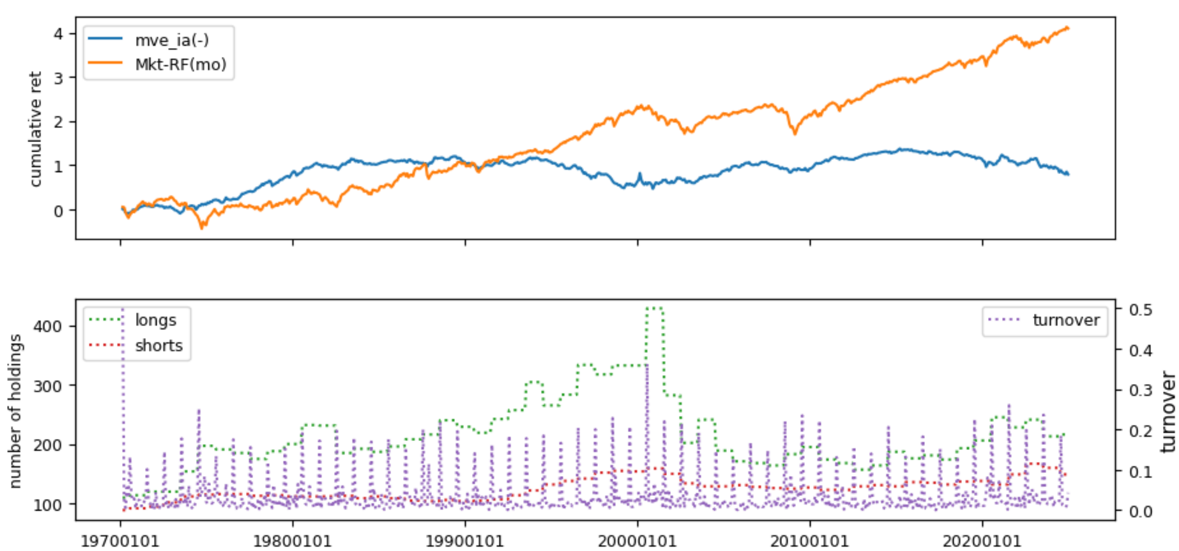

cfp_ia
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101


chatoia
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101


chpmia
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101


pchcapx_ia
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101


chempia
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101


mve_ia(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101

Fundamental signals from Compustat Quarterly

```
columns = ['stdacc', 'stdcf', 'roavol', 'sgrvol', 'cinvest', 'chtx',
           'rsup', 'roaq', 'cash', 'nincr']
numlag = 4        # require 4 month lag of fiscal data
end = LAST_DATE
```

```
# retrieve quarterly, keep [permno, datadate] key with non null prccq
fields = ['ibq', 'actq', 'cheq', 'lctq', 'dlcq', 'saleq', 'prccq',
          'cshoq', 'atq', 'txtq', 'ppentq']
df = pstat.get_linked(dataset='quarterly',
                      fields=fields,
                      date_field='datadate',
                      where=(f"datadate > 0 "
                             f"and datadate <= {end//100}31"))
fund = df.sort_values(['permno','datadate', 'ibq'])\
         .drop_duplicates(['permno', 'datadate'])\
         .dropna(subset=['ibq'])
fund.index = list(zip(fund['permno'], fund['datadate']))
rebaldate = bd.endmo(fund.datadate, numlag)
```

```
# compute current and lagged: scf sacc roaq nincr cinvest cash rsup chtx
lag = fund.shift(1, fill_value=0)
lag.loc[lag['permno'] != fund['permno'], fields] = np.nan
fund['_saleq'] = fund['saleq']
fund.loc[fund['_saleq'].lt(0.01), '_saleq'] = 0.01
```

```
fund['sacc'] = (((fund['actq'] - lag['actq']) - (fund['cheq'] - lag['cheq']))
                - ((fund['lctq'] - lag['lctq'])
                   - (fund['dlcq'] - lag['dlcq']))) / fund['_saleq']
fund['cinvest'] = (fund['ppentq'] - lag['ppentq']) / fund['_saleq']
fund['nincr'] = (fund['ibq'] > lag['ibq']).astype(int)
fund['scf']  = (fund['ibq'] / fund['_saleq']) - fund['sacc']
fund['roaq'] = (fund['ibq'] / lag['atq'])
fund['cash'] = (fund['cheq'] / fund['atq'])
```

```
lag4 = fund.shift(4, fill_value=0)
lag4.loc[lag4['permno'] != fund['permno'], fields] = np.nan
fund['rsup'] = ((fund['saleq'] - lag4['saleq'])
                / (fund['prccq'].abs() * fund['cshoq'].abs()))
fund['chtx'] = (fund['txtq'] - lag4['txtq']) / lag4['atq']
```

```
# for each var: make dataframe of 15 lags (column names=[0,...,15])
lags = {col : as_lags(fund, var=col, key='permno', nlags=16)
        for col in ['sacc', 'scf', 'roaq', 'rsup', 'cinvest', 'nincr']}
for i in range(1, 16):                      # lags[ninrc][i]=1 iff ibq
    lags['nincr'][i] *= lags['nincr'][i-1]  # increasing all prior qtrs

    # compute signals from the 15 lags
    fund['rebaldate'] = rebaldate
    fund['stdacc'] = lags['sacc'].std(axis=1, skipna=False)
    fund['stdcf'] = lags['scf'].std(axis=1, skipna=False)
    fund['roavol'] = lags['roaq'].std(axis=1, skipna=False)
    fund['sgrvol'] = lags['rsup'].std(axis=1, skipna=False)
```

(continues on next page)

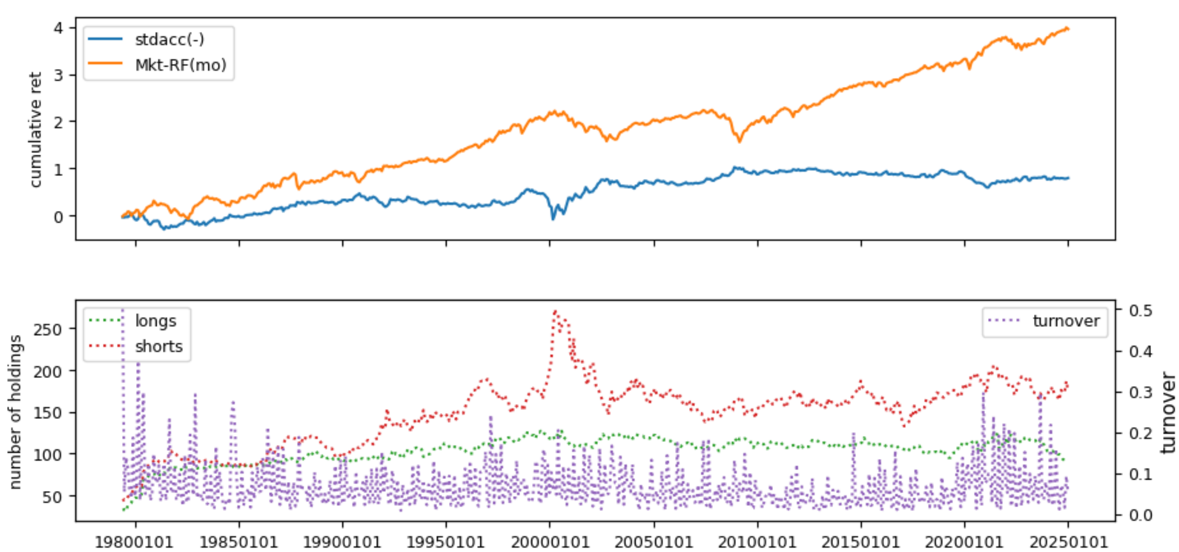
(continued from previous page)

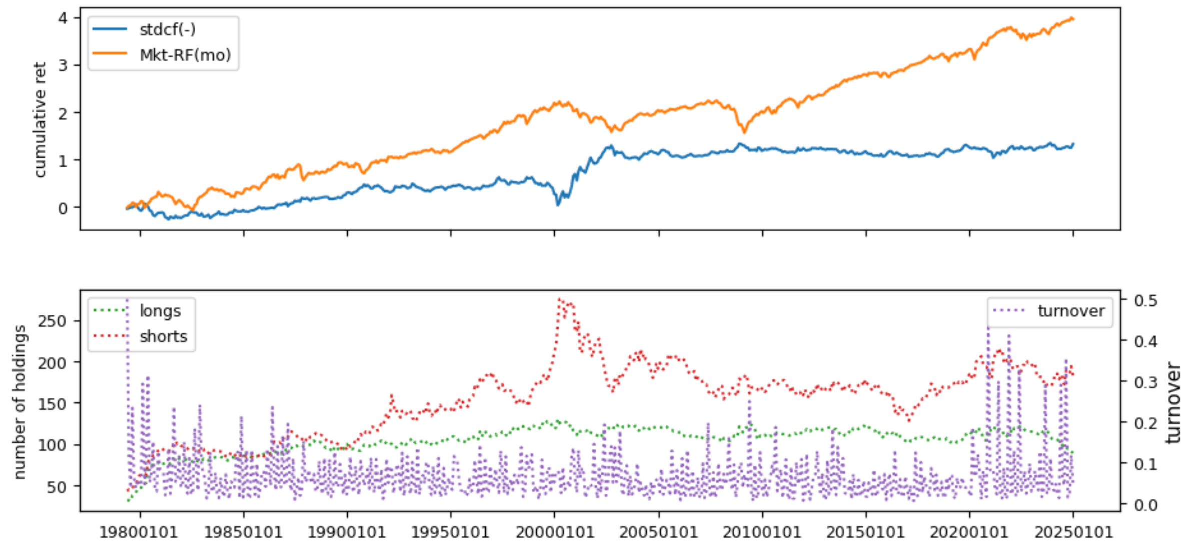
```
    fund['cinvest'] = (fund['cinvest'] -
                       lags['cinvest'][[1, 2, 3, 4]].mean(axis=1, skipna=False))

    # count number of consecutive increasing quarters
    fund['nincr'] = lags['nincr'][np.arange(8)].sum(axis=1)
```

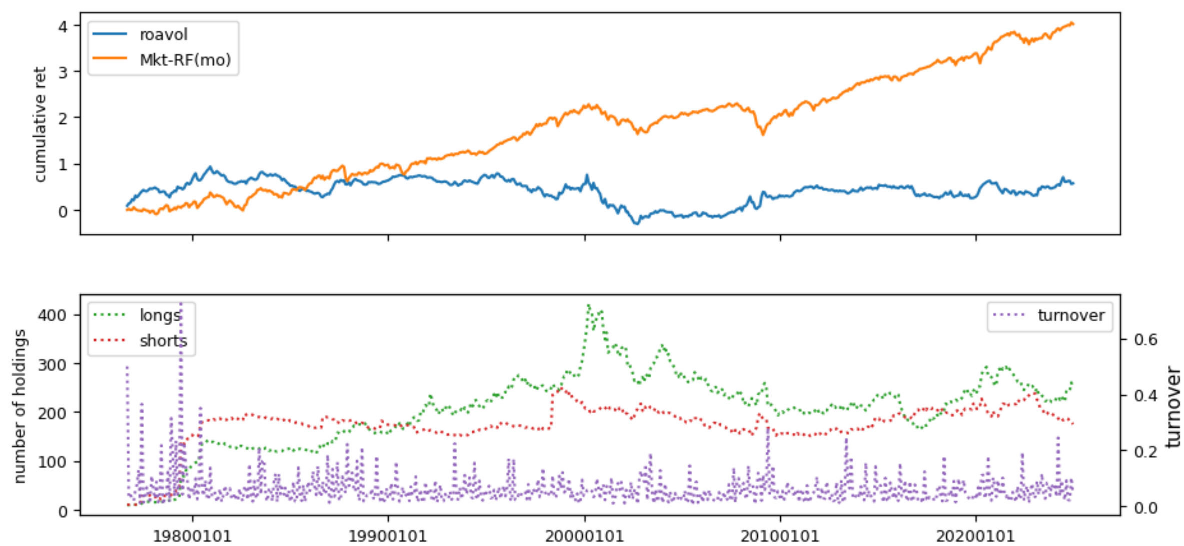
```
for label in columns:
    signals.write(fund, label, overwrite=True)
```

```
rebalbeg, rebalend = 19700101, LAST_DATE
benchnames = ['Mkt-RF(mo)']
for label in tqdm(columns):
    holdings = univariate_sorts(monthly,
                                label,
                                SignalsFrame(signals.read(label)),
                                rebalbeg,
                                rebalend,
                                window=3,
                                months=[],
                                maxdecile=8,
                                pct=(10., 90.),
                                leverage=leverage.get(label, 1))
    excess = backtest_pipeline(backtest,
                               monthly,
                               holdings,
                               label,
                               benchnames,
                               overlap=0,
                               outdir=outdir,
                               suffix='(-)'*(leverage.get(label, 1) < 0))
```

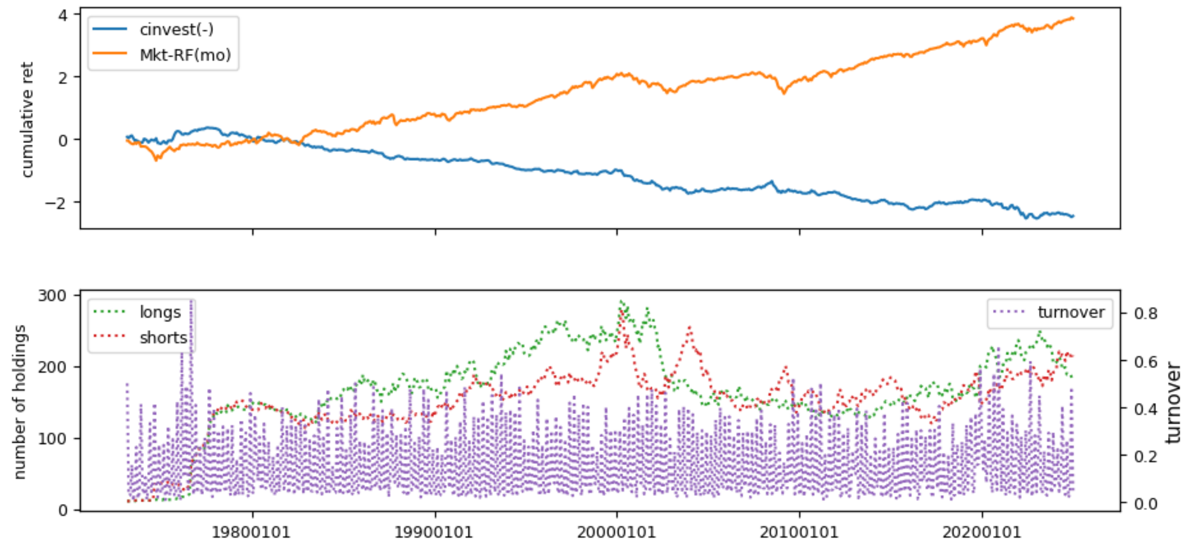
```
100%|██████████| 10/10 [11:53<00:00, 71.38s/it]
```

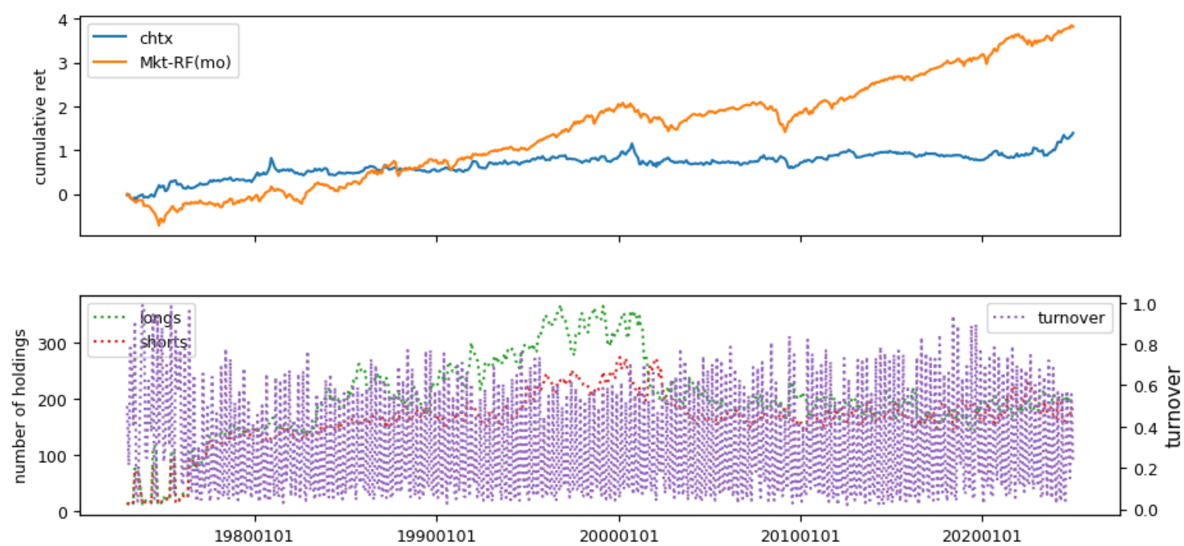


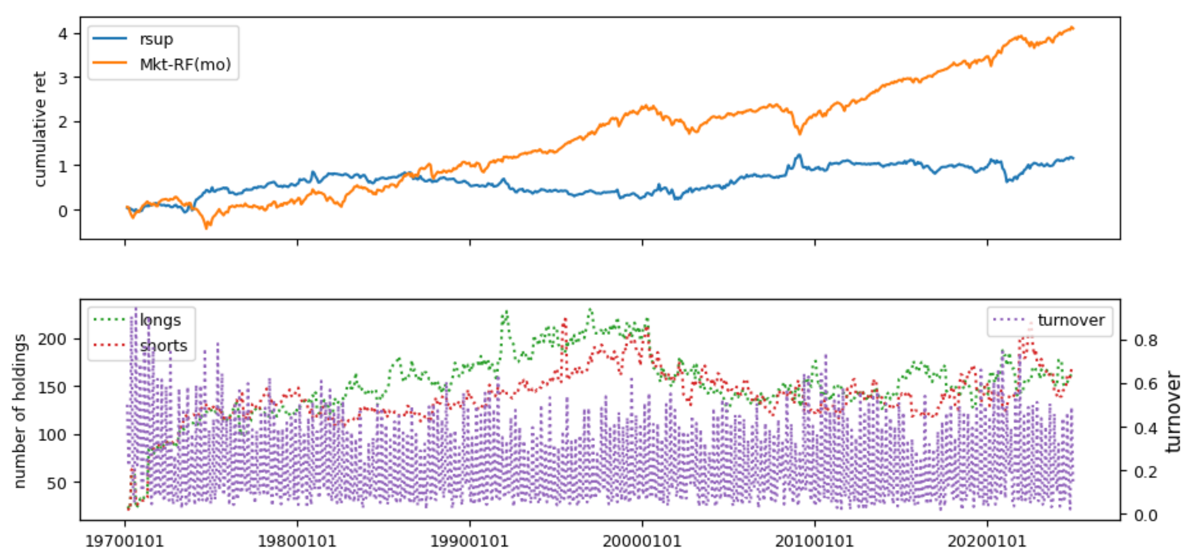

stdcf(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19800101
19850101
19900101
19950101
20000101
20050101
20100101
20150101
20200101
20250101


roavol
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19800101
19900101
20000101
20100101
20200101


sgrvol
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19800101
19900101
20000101
20100101
20200101


cinvest(-)
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19800101
19900101
20000101
20100101
20200101


chtx
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19800101
19900101
20000101
20100101
20200101


rsup
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
19700101
19800101
19900101
20000101
20100101
20200101


roaq
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
turnover
cash
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
turnover
nincr
Mkt-RF(mo)
cumulative ret
longs
shorts
turnover
number of holdings
turnover
19700101
19800101
19900101
20000101
20100101
20200101

6.1.6 Earnings Estimates

Earnings estimate signals are drawn from IBES data, including fiscal year 1 projections, long-term growth forecasts, and announcement dates linked to CRSP daily prices and Compustat quarterly.

```
columns = ['chfeps', 'chnanalyst', 'disp']
```

```
df = ibes.get_linked(dataset='statsum',
                     fields=['fpedats', 'meanest', 'medest',
                             'stdev', 'numest'],
                     date_field = 'statpers',
                     where=("meanest IS NOT NULL "
                            "  AND fpedats IS NOT NULL "
                            "  AND statpers IS NOT NULL"
                            "  AND fpi = '1'"))
out = df.sort_values(['permno', 'statpers', 'fpedats', 'meanest'])\
        .drop_duplicates(['permno', 'statpers', 'fpedats'])
out['rebaldate'] = bd.endmo(out['statpers'])
```

```
out['disp'] = out['stdev'] / abs(out['meanest'])
out.loc[abs(out['meanest']) < 0, 'disp'] = out['stdev'] / 0.01
```

```
lag1 = out.shift(1, fill_value=0)
f1 = (lag1['permno'] == out['permno'])
out.loc[f1, 'chfeps'] = out.loc[f1, 'meanest'] - lag1.loc[f1, 'meanest']
```

```
lag3 = out.shift(3, fill_value=0)
f3 = (lag3['permno'] == out['permno'])
out.loc[f3, 'chnanalyst'] = out.loc[f3, 'numest']-lag3.loc[f3, 'numest']
```

```
for label in columns:
    signals.write(out, label, overwrite=True)
```

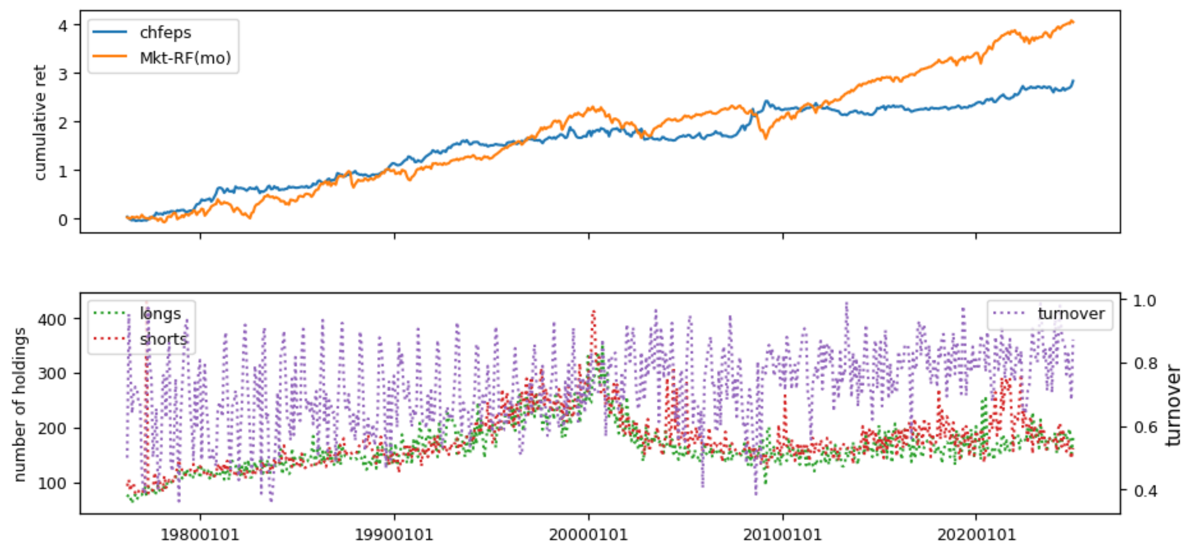
```
rebalbeg, rebalend = 19760101, LAST_DATE
benchnames = ['Mkt-RF(mo)']
for label in tqdm(columns):
    holdings = univariate_sorts(monthly,
                                label,
                                SignalsFrame(signals.read(label)),
                                rebalbeg,
                                rebalend,
                                window=3,
                                months=[],
                                maxdecile=8,
                                pct=(10., 90.),
                                leverage=leverage.get(label, 1))
    excess = backtest_pipeline(backtest,
                               monthly,
                               holdings,
                               label,
                               benchnames,
                               overlap=0,
```

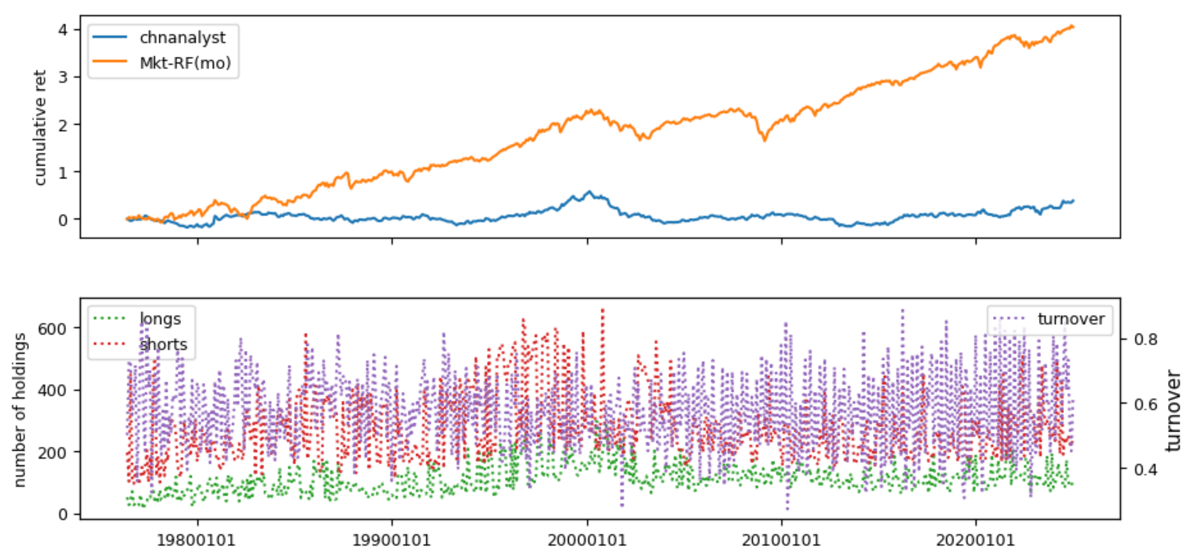
(continues on next page)

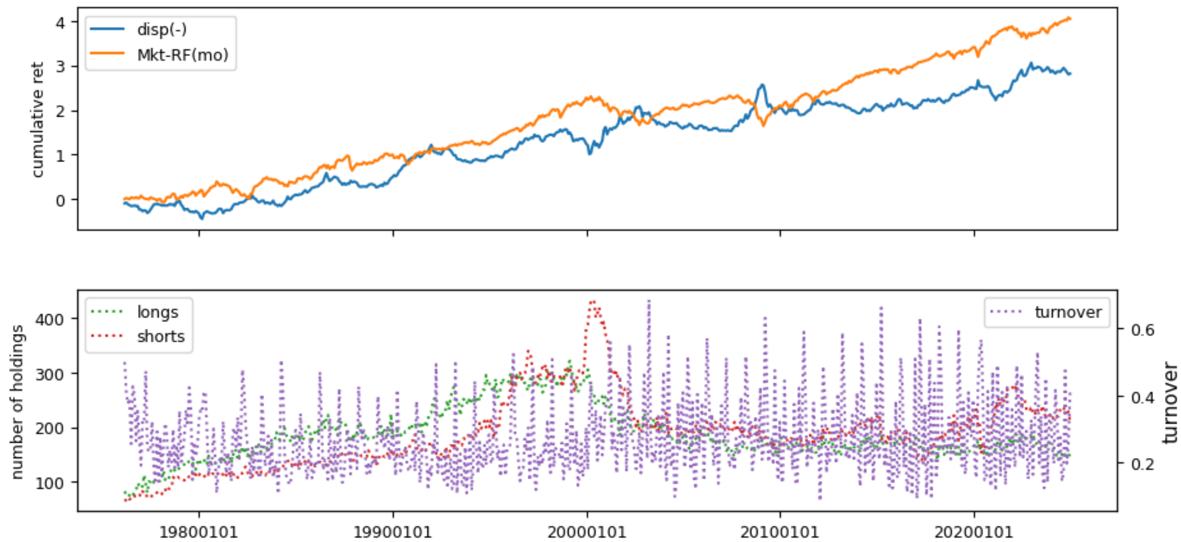
(continued from previous page)

```
                                  outdir=outdir,
                                  suffix=(leverage.get(label, 1) < 0)*'(-)')
```

```
100%|██████████| 3/3 [03:39<00:00, 73.25s/it]
```







IBES Long-term Growth signals

```
columns = ['fgr5yr']
```

```
df = ibes.get_linked(dataset='statsum',
                     fields = ['meanest'],
                     date_field = 'statpers',
                     where=("meanest IS NOT NULL "
                            "  AND fpi = '0'"
                            "  AND statpers IS NOT NULL"))
out = df.sort_values(['permno', 'statpers', 'meanest'])\
        .drop_duplicates(['permno', 'statpers'])\
        .dropna()
out['rebaldate'] = bd.endmo(out['statpers'])
out['fgr5yr'] = out['meanest']
signals.write(out, 'fgr5yr', overwrite=True)
```

```
1319938
```

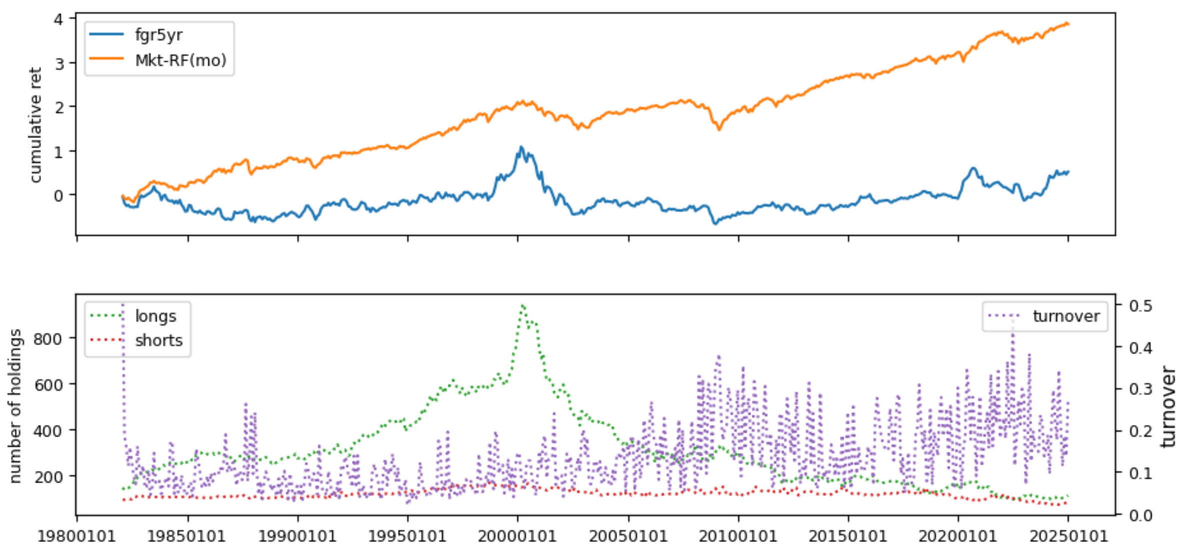
```
rebalbeg, rebalend = 19760101, LAST_DATE
benchnames = ['Mkt-RF(mo)']
for label in tqdm(columns):
    holdings = univariate_sorts(monthly,
                                label,
                                SignalsFrame(signals.read(label)),
                                rebalbeg,
                                rebalend,
                                window=3,
                                months=[],
                                maxdecile=8,
                                pct=(10., 90.),
                                leverage=leverage.get(label, 1))
    excess = backtest_pipeline(backtest,
                               monthly,
                               holdings,
                               label,
```

(continues on next page)

(continued from previous page)

```
                                    benchnames,
                                    overlap=0,
                                    outdir=outdir,
                                    suffix=(leverage.get(label, 1) < 0)*'(-)')
```

```
100%|███████████| 1/1 [01:06<00:00, 66.27s/it]
```



Announcement date in Quarterly, linked to CRSP daily

```
columns = ['ear', 'aeavol']
```

```
# retrieve rdq, and set rebalance date to at least one month delay
df = pstat.get_linked(dataset='quarterly',
                      fields=['rdq'],
                      date_field='datadate',
                      where=('rdq > 0'))
fund = df.sort_values(['permno', 'rdq', 'datadate'])\
         .drop_duplicates(['permno', 'rdq'])\
         .dropna()
fund['rebaldate'] = bd.offset(fund['rdq'], 2)
```

```
# ear is compounded return around 3-day window
out = crsp.get_window(dataset='daily',
                      field='ret',
                      date_field='date',
                      permnos=fund['permno'],
                      dates=fund['rdq'],
                      left=-1,
                      right=1)
fund['ear'] = (1 + out).prod(axis = 1).values
```

```
# aeavol is avg volume in 3-day window over 20-day average ten-days prior
actual = crsp.get_window(dataset='daily',
```

(continues on next page)

(continued from previous page)

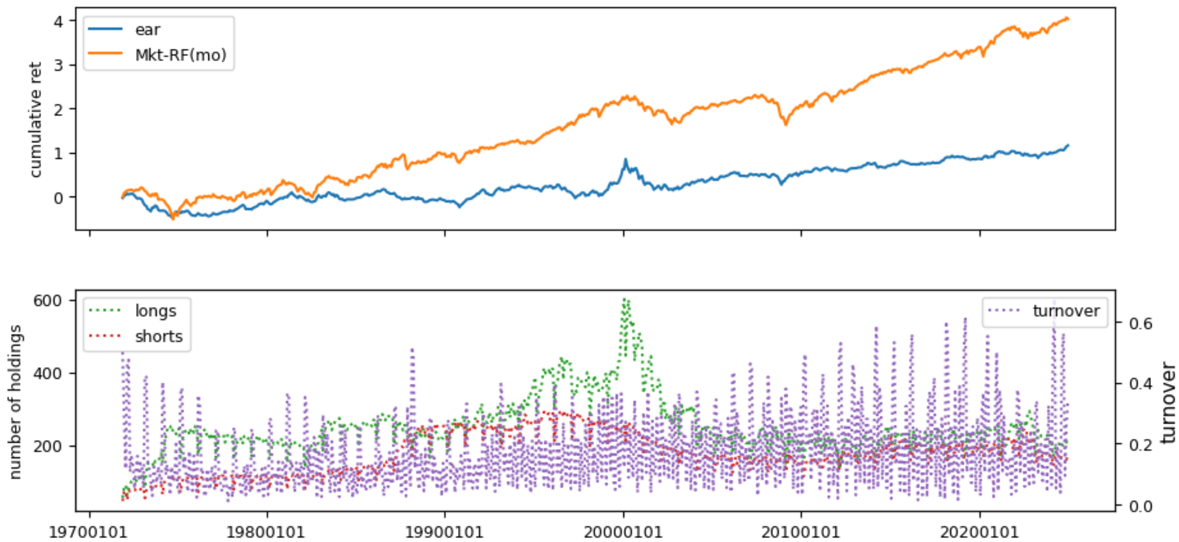
```
                        field='vol',
                        date_field='date',
                        permnos=fund['permno'],
                        dates=fund['rdq'],
                        left=-1,
                        right=1)
normal = crsp.get_window(dataset='daily',
                        field='vol',
                        date_field='date',
                        permnos=fund['permno'],
                        dates=fund['rdq'],
                        left=-30,
                        right=-11,
                        avg=True)
fund['aeavol'] = normal['vol'].values
```

```
signals.write(fund, 'ear', overwrite=True)
signals.write(fund, 'aeavol', overwrite=True)
```

```
968315
```

```
rebalbeg, rebalend = 19700101, LAST_DATE
benchnames = ['Mkt-RF(mo)']
for label in tqdm(columns):
    holdings = univariate_sorts(monthly,
                                label,
                                SignalsFrame(signals.read(label)),
                                rebalbeg,
                                rebalend,
                                window=3,
                                months=[],
                                maxdecile=8,
                                pct=(10., 90.),
                                leverage=leverage.get(label, 1))
    excess = backtest_pipeline(backtest,
                               monthly,
                               holdings,
                               label,
                               benchnames,
                               overlap=0,
                               outdir=outdir,
                               suffix=(leverage.get(label, 1) < 0)*'(-)')
```

```
100%|██████████| 2/2 [02:07<00:00, 63.88s/it]
```



IBES Fiscal Year 1 linked to Quarterly PSTAT

```
beg, end = 19760101, LAST_DATE
monthnum = lambda d: ((d//10000)-1900)*12 + ((d//100)%100) - 1
```

```
df = pstat.get_linked(dataset='quarterly',
                      fields=['prccq'],
                      date_field='datadate')
df = df.dropna()\
       .sort_values(['permno', 'datadate'])\
       .drop_duplicates(['permno', 'datadate'])
```

```
out = ibes.get_linked(dataset='statsum',
                      fields=['fpedats', 'meanest'],
                      date_field='statpers',
                      where="fpi='1'")
out = out.dropna()\
```

(continues on next page)

(continued from previous page)

```
         .sort_values(['permno', 'statpers', 'fpedats'])\
         .drop_duplicates(['permno', 'statpers'])
out['monthnum'] = monthnum(out['statpers'])
out = out.set_index(['permno', 'monthnum'], drop=False)
out['sfeq'] = np.nan
```

```
for num in range(4): # match ibes statpers to any datadate in last 4 mos
    df['monthnum'] = monthnum(df['datadate']) - num
    df = df.set_index(['permno', 'monthnum'], drop=False)
    out = out.join(df[['prccq']], how='left')
    out['sfeq'] = out['sfeq'].where(out['sfeq'].notna(),
                                    out['meanest'] / out['prccq'].abs())
    out = out.drop(columns=['prccq'])

    out['rebaldate'] = bd.endmo(out['statpers'])
    n = signals.write(out.reset_index(drop=True), 'sfeq', overwrite=True)
```

IBES Fiscal Year 1 linked to IBES price history

```
beg, end = 19760101, LAST_DATE
```

```
# retrieve monthly price history
df = ibes.get_linked(dataset='actpsum',
                     fields=['price'],
                     date_field='statpers')
hist = df.dropna()\
         .sort_values(['permno', 'statpers'])\
         .drop_duplicates(['permno', 'statpers'], keep='last')\
         .set_index(['permno', 'statpers'])
```

```
# retrieve monthly FY1 mean estimate
df = ibes.get_linked(dataset='statsum',
                     fields=['fpedats', 'meanest'],
                     date_field='statpers',
                     where="fpi='1' AND statpers <= fpedats")
out = df.dropna()\
        .sort_values(['permno', 'statpers', 'fpedats'])\
        .drop_duplicates(['permno', 'statpers'])\
        .set_index(['permno', 'statpers'])
```

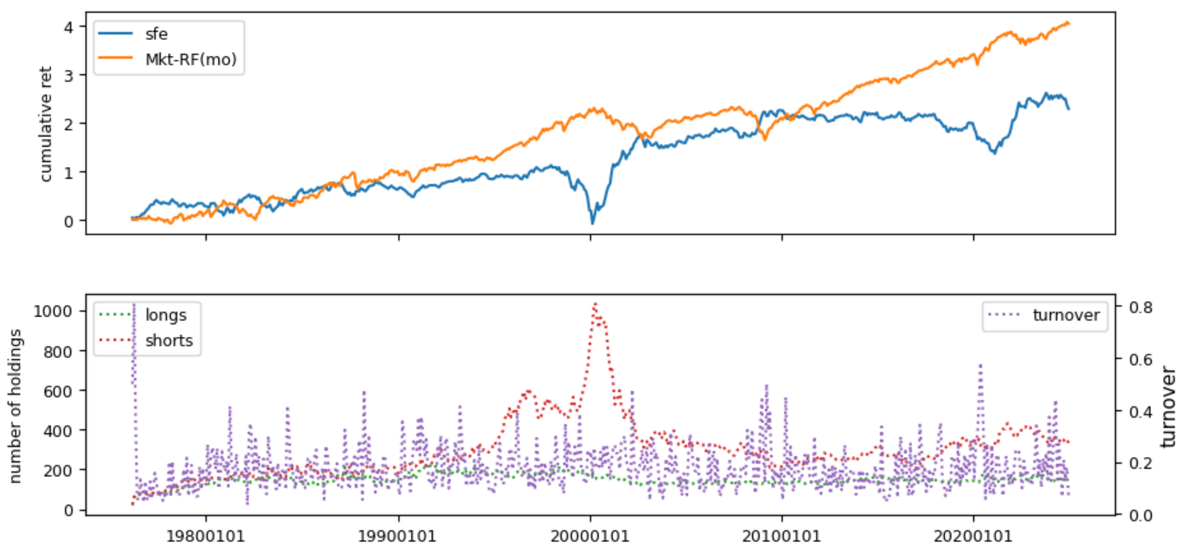
```
# join on [permno, statpers], and reindex on [permno, rebaldate]
out = out.join(hist[['price']], how='left').reset_index()
out['rebaldate'] = bd.endmo(out['statpers'])
out = out.set_index(['permno', 'rebaldate'])
out['sfe'] = out['meanest'].div(out['price'].abs())
n = signals.write(out.reset_index(), 'sfe', overwrite=True)
```

```
rebalbeg, rebalend = 19760101, LAST_DATE
benchnames = ['Mkt-RF(mo)']
label = 'sfe'
holdings = univariate_sorts(monthly,
                            label,
```

(continues on next page)

(continued from previous page)

```
                             SignalsFrame(signals.read(label)),
                             rebalbeg,
                             rebalend,
                             window=3,
                             months=[],
                             maxdecile=8,
                             pct=(10., 90.),
                             leverage=leverage.get(label, 1))
excess = backtest_pipeline(backtest,
                            monthly,
                            holdings,
                            label,
                            benchnames,
                            overlap=0,
                            outdir=outdir,
                            suffix=(leverage.get(label, 1) < 0)*'(-)')
```



IBES Fiscal Quarter 1, linked to Quarterly

```
columns = ['sue']
numlag = 4
end = LAST_DATE
```

```
# retrieve quarterly, keep [permno, datadate] key with non null prccq
df = pstat.get_linked(dataset='quarterly',
                      fields=['prccq', 'cshoq', 'ibq'],
                      date_field='datadate',
                      where=f"datadate <= {end//100}31")
fund = df.dropna(subset=['ibq'])\
         .sort_values(['permno', 'datadate', 'cshoq'])\
         .drop_duplicates(['permno', 'datadate'])
fund['rebaldate'] = bd.endmo(fund['datadate'], numlag)
fund = fund.set_index(['permno', 'rebaldate'], drop=False)
```

```
# retrieve ibes Q1 where forecast period <= fiscal date, keep latest
df = ibes.get_linked(dataset='statsum',
                     fields=['fpedats', 'medest', 'actual'],
                     date_field='statpers',
                     where=" fpi = '6' AND statpers <= fpedats")
summ = df.dropna()\
         .sort_values(['permno', 'fpedats', 'statpers'])\
         .drop_duplicates(['permno', 'fpedats'], keep='last')
summ['rebaldate'] = bd.endmo(summ['fpedats'], numlag)
summ = summ.set_index(['permno', 'statpers'])
```

```
# retrieve ibes price, then left join
df = ibes.get_linked(dataset='actpsum',
                     fields=['price'],
                     date_field='statpers')
hist = df.dropna()\
         .sort_values(['permno', 'statpers'])\
         .drop_duplicates(['permno', 'statpers'], keep='last')
hist = hist.set_index(['permno', 'statpers'])
summ = summ.join(hist[['price']], how='left')
summ = summ.reset_index()\
           .set_index(['permno', 'rebaldate'])\
           .reindex(fund.index)
```

```
# sue with ibes surprise and price
fund['sue'] = (summ['actual'] - summ['medest']) / summ['price'].abs()
```

```
# sue with ibes surprice and compustat quarterly price
fund['sue'] = fund['sue'].where(
    fund['sue'].notna(), (summ['actual'] - summ['medest']) / fund['prccq'].abs())
```

```
# sue with lag(4) difference in compustat quarterly and price
lag = fund.shift(4, fill_value=0)
fund['sue'] = fund['sue'].where(
    fund['sue'].notna() | (lag['permno'] != fund['permno']),
    ((fund['ibq'] - lag['ibq']) / (fund['prccq'] * fund['cshoq']).abs()))
```

```
signals.write(fund.reset_index(drop=True), 'sue', overwrite=True)
```

```
1089089
```

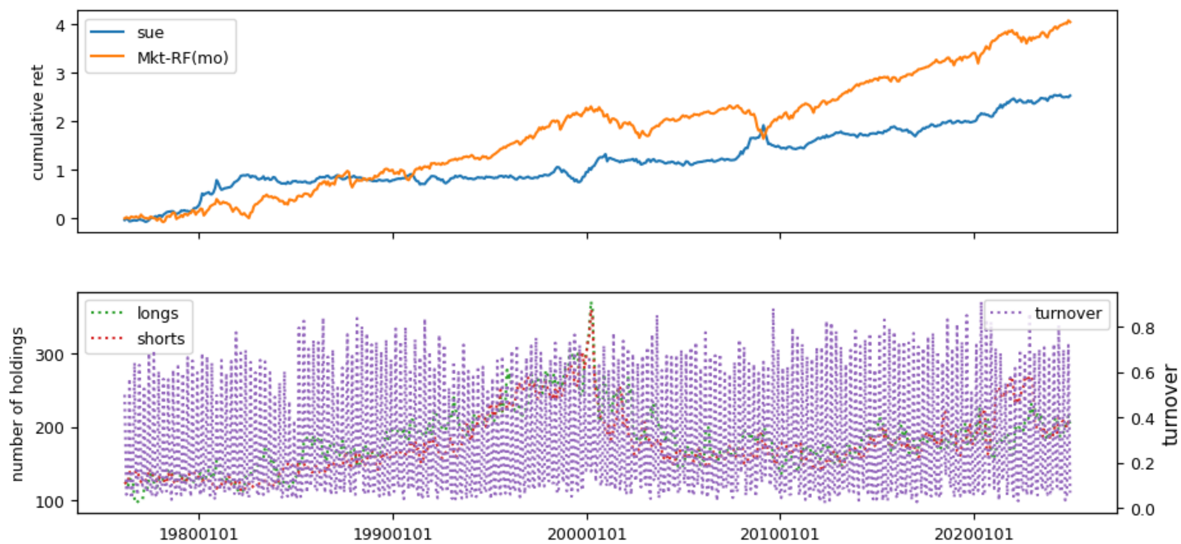
```
rebalbeg, rebalend = 19760101, LAST_DATE
benchnames = ['Mkt-RF(mo)']
for label in tqdm(columns):
    holdings = univariate_sorts(monthly,
                                label,
                                SignalsFrame(signals.read(label)),
                                rebalbeg,
                                rebalend,
                                window=3,
                                months=[],
                                maxdecile=8,
```

(continues on next page)

(continued from previous page)

```
                                    pct=(10., 90.),
                                    leverage=leverage.get(label, 1))
    excess = backtest_pipeline(backtest,
                               monthly,
                               holdings,
                               label,
                               benchnames,
                               overlap=0,
                               outdir=outdir,
                               suffix=(leverage.get(label, 1) < 0)*'(-)')
```

```
100%|██████████| 1/1 [01:14<00:00, 74.37s/it]
```



6.1.7 Backtests

All backtests are conducted on univariate, dollar-neutral spreads between the top and bottom deciles of a given characteristic. Stocks are value-weighted within each decile, excluding the smallest quintile of NYSE-listed firms by market capitalization. Spread portfolios are rebalanced monthly, with fundamental data lagged in the standard manner—six months for annual data and four months for quarterly data.

Each backtest generates a time series of cumulative spread portfolio and market index returns, along with metrics such as monthly turnover rates and the number of long and short positions. Results are ranked based on Welch's *t*-statistic, which tests for differences in mean returns before 2002 and after 2003. This aligns with findings from Green et al. (2017) and others, who observed a significant decline in the predictive power of many factors after this period.

Additionally, we compute the **maximum drawdown** for each strategy, defined as the largest peak-to-trough loss in cumulative returns before a new peak is reached. This measure provides insight into the worst-case historical performance of each strategy.

```
def maximum_drawdown(x: Series, is_price_level: bool = False) -> Series:
    """Compute max drawdown (max loss from previous high) period and returns"""
    cumsum = np.log(1 + x).cumsum()
    cummax = cumsum.cummax()
    end = (cummax - cumsum).idxmax()
```

(continues on next page)

(continued from previous page)

```
    beg = cumsum[cumsum.index <= end].idxmax()
    dd = cumsum.loc[[beg, end]]
    return np.exp(dd)
```

```
zoo = backtest.read().sort_values(['begret', 'permno'])
r = []
rets = []
for label in zoo.index:
    perf = backtest.read(label)
    rets.append(perf['ret'].rename(label))
    excess = {'ret': backtest.fit(['Mkt-RF(mo)'])}
    excess['annualized'] = backtest.annualized
    excess['dd'] = maximum_drawdown(backtest.perf['excess'])
    post = {'ret': backtest.fit(['Mkt-RF(mo)'],
                                beg=20020101).copy()}
    post['annualized'] = backtest.annualized.copy()
    s = label + ('(-)' if leverage.get(label, 1) < 0 else '')
    r.append(DataFrame({
        #             'Start': excess['ret'].index[0],
        'Sharpe Ratio': excess['annualized']['sharpe'],
        'Alpha': excess['annualized']['alpha'],
        'Appraisal Ratio': excess['annualized']['appraisal'],
        'Avg Ret': excess['ret']['excess'].mean(),
        'Vol': excess['ret']['excess'].std(ddof=0),
        'Welch-t': excess['annualized']['welch-t'],
        'Appraisal2002': post['annualized']['appraisal'],
        'Ret2002': post['ret']['excess'].mean(),
        'Drawdown': (excess['dd'].iloc[1]/excess['dd'].iloc[0]) - 1,
    }, index=[s]))
df = pd.concat(r, axis=0).round(4).sort_values('Welch-t')
```

```
pd.set_option("display.max_colwidth", None, 'display.max_rows', None)
df #.sort_values('Sharpe Ratio', ascending=False)
```

```
                  Sharpe Ratio   Alpha  Appraisal Ratio  Avg Ret     Vol  \
pchsale_pchrect        -0.1169 -0.0121          -0.1368  -0.0009  0.0256
mom1m(-)                0.3820  0.0568           0.3132   0.0061  0.0543
bm                      0.2233  0.0339           0.2141   0.0029  0.0457
bm_ia                   0.2413  0.0216           0.1611   0.0027  0.0393
agr(-)                  0.2493  0.0480           0.4320   0.0025  0.0341
pchsale_pchinvt         0.2434  0.0229           0.2392   0.0019  0.0276
chmom                   0.4007  0.0509           0.3114   0.0058  0.0494
pchsaleinv              0.2446  0.0197           0.2238   0.0018  0.0254
lev                     0.0261 -0.0002          -0.0012   0.0004  0.0488
absacc(-)               0.0380  0.0212           0.1520   0.0005  0.0414
ep                      0.2831  0.0639           0.4231   0.0037  0.0452
chcsho(-)               0.5415  0.0725           0.7490   0.0047  0.0298
invest(-)               0.2479  0.0450           0.3776   0.0025  0.0356
acc(-)                  0.3358  0.0377           0.3500   0.0030  0.0312
indmom                  0.3518  0.0643           0.3708   0.0053  0.0512
sp                      0.2303  0.0329           0.2220   0.0028  0.0428
pchdepr                 0.0617  0.0048           0.0506   0.0005  0.0273
egr(-)                  0.3157  0.0539           0.5072   0.0030  0.0327
sgrvol                  0.1918  0.0082           0.0562   0.0024  0.0440
```

(continues on next page)

(continued from previous page)

```
mom12m                 0.4699  0.1532            0.6550   0.0101  0.0732
realestate             0.1084  0.0508            0.3445   0.0014  0.0458
mom36m(-)              0.1778  0.0191            0.0953   0.0031  0.0601
cashpr(-)              0.2962  0.0475            0.3843   0.0031  0.0363
divyld                -0.0470  0.0377            0.2455  -0.0008  0.0552
aeavol(-)              0.2160  0.0381            0.3568   0.0020  0.0320
ill(-)                 0.3323  0.0504            0.4209   0.0034  0.0350
dy                    -0.0473  0.0382            0.2262  -0.0008  0.0569
chatoia                0.2567  0.0292            0.2994   0.0021  0.0282
chinv(-)               0.3601  0.0506            0.4775   0.0033  0.0314
retvol(-)              0.1306  0.1269            0.6067   0.0029  0.0772
maxret(-)              0.1807  0.1144            0.6383   0.0034  0.0646
pricedelay(-)          0.0819  0.0030            0.0302   0.0007  0.0292
chfeps                 0.4741  0.0673            0.5567   0.0048  0.0353
stdcf(-)               0.2132  0.0613            0.5020   0.0024  0.0391
idiovol(-)            -0.0126  0.0514            0.2623  -0.0003  0.0697
mve_ia(-)              0.1372  0.0025            0.0244   0.0012  0.0302
chpmia                 0.1598  0.0266            0.2003   0.0018  0.0385
disp(-)                0.3267  0.1025            0.6640   0.0048  0.0508
cfp                    0.2466  0.0511            0.3691   0.0029  0.0412
stdacc(-)              0.1414  0.0431            0.3801   0.0014  0.0354
pchsale_pchxsga       -0.0415 -0.0053           -0.0480  -0.0004  0.0320
grltnoa                0.1595  0.0208            0.1793   0.0015  0.0335
sfe                    0.2535  0.0763            0.4340   0.0039  0.0532
pchgm_pchsale          0.0106 -0.0035           -0.0337   0.0001  0.0297
cfp_ia                -0.1814 -0.0331           -0.2501  -0.0020  0.0386
mom6m                  0.3185  0.1085            0.5001   0.0064  0.0682
beta                   0.0807 -0.0733           -0.3806   0.0022  0.0906
dolvol                 0.1131  0.0017            0.0161   0.0010  0.0315
nincr                  0.3410  0.0242            0.2728   0.0026  0.0259
cinvest(-)            -0.4026 -0.0447           -0.3815  -0.0039  0.0339
cash                   0.2751  0.0143            0.1021   0.0035  0.0438
chtx                   0.2162  0.0211            0.1716   0.0022  0.0358
salecash               0.0170  0.0199            0.1773   0.0002  0.0342
rd_mve                 0.1412  0.0114            0.0716   0.0019  0.0463
chnanalyst             0.0875  0.0021            0.0240   0.0006  0.0255
fgr5yr                 0.0537 -0.0485           -0.2487   0.0010  0.0640
sue                    0.4709  0.0557            0.5087   0.0043  0.0317
pchcapx_ia            -0.1942 -0.0262           -0.2113  -0.0020  0.0359
roavol                 0.0768 -0.0063           -0.0425   0.0010  0.0440
baspread              -0.0851 -0.1231           -0.5790  -0.0020  0.0804
pchquick              -0.0755 -0.0085           -0.0918  -0.0006  0.0268
rd_sale               -0.0530 -0.0146           -0.0945  -0.0007  0.0446
pctacc(-)              0.1174  0.0248            0.2364   0.0011  0.0312
secured(-)             0.0364  0.0337            0.2793   0.0004  0.0376
std_turn               0.0539 -0.0490           -0.2884   0.0009  0.0582
std_dolvol            -0.0176  0.0007            0.0068  -0.0002  0.0315
rsup                   0.1644  0.0237            0.1859   0.0018  0.0369
depr                   0.0696 -0.0201           -0.1534   0.0008  0.0422
zerotrade(-)           0.1663 -0.0351           -0.1939   0.0031  0.0644
sgr                    0.0225 -0.0138           -0.1111   0.0002  0.0372
salerec                0.2028  0.0405            0.3172   0.0022  0.0378
roaq                   0.2858  0.0610            0.4358   0.0035  0.0422
turn                   0.1420 -0.0412           -0.2184   0.0027  0.0669
ear                    0.1909 -0.0019           -0.0185   0.0018  0.0329
quick                  0.0603 -0.0241           -0.1670   0.0008  0.0467
```

(continues on next page)

(continued from previous page)

```
gma                  0.1117  0.0201          0.1363   0.0014  0.0426
hire                -0.1215 -0.0324         -0.2790  -0.0012  0.0353
tang                 0.2328  0.0149          0.1259   0.0024  0.0351
grcapx              -0.2991 -0.0421         -0.4188  -0.0026  0.0298
saleinv              0.1401  0.0364          0.3711   0.0013  0.0312
chempia              0.0698 -0.0084         -0.0768   0.0007  0.0331
lgr                 -0.0573 -0.0162         -0.1720  -0.0005  0.0279

                 Welch-t  Appraisal2002  Ret2002  Drawdown
pchsale_pchrect  -3.1904        -0.6200  -0.0047   -0.8591
mom1m(-)         -2.7253        -0.3205  -0.0017   -0.7606
bm               -2.3167        -0.3007  -0.0019   -0.7651
bm_ia            -2.2583        -0.2118  -0.0012   -0.7011
agr(-)           -2.1174         0.0200  -0.0008   -0.5362
pchsale_pchinvt  -2.1010        -0.1343  -0.0008   -0.4944
chmom            -1.9158        -0.0634   0.0010   -0.6159
pchsaleinv       -1.7332        -0.1297  -0.0003   -0.4843
lev              -1.6920        -0.3338  -0.0035   -0.8698
absacc(-)        -1.6578        -0.1800  -0.0027   -0.7840
ep               -1.4698         0.2254   0.0007   -0.6637
chcsho(-)        -1.4605         0.5437   0.0027   -0.3253
invest(-)        -1.3936         0.1366   0.0001   -0.4268
acc(-)           -1.3486         0.0828   0.0011   -0.3966
indmom           -1.3305         0.2007   0.0020   -0.6892
sp               -1.2746        -0.0372   0.0004   -0.5726
pchdepr          -1.2651        -0.1436  -0.0011   -0.6979
egr(-)           -1.2025         0.3026   0.0013   -0.3384
sgrvol           -1.1672        -0.2804   0.0001   -0.6935
mom12m           -1.1488         0.5282   0.0058   -0.9670
realestate       -1.1485         0.2426  -0.0007   -0.6426
mom36m(-)        -1.0582        -0.0673  -0.0000   -0.7280
cashpr(-)        -1.0456         0.0969   0.0014   -0.4235
divyld           -1.0071        -0.1090  -0.0032   -0.9650
aeavol(-)        -0.9899         0.2019   0.0006   -0.5583
ill(-)           -0.8358         0.4068   0.0022   -0.6150
dy               -0.7118        -0.0399  -0.0025   -0.9190
chatoia          -0.6699         0.1768   0.0012   -0.5082
chinv(-)         -0.6658         0.2930   0.0023   -0.2747
retvol(-)        -0.6457         0.5100   0.0009   -0.7990
maxret(-)        -0.6188         0.5778   0.0017   -0.7173
pricedelay(-)    -0.5810        -0.1441  -0.0002   -0.6155
chfeps           -0.5669         0.5971   0.0040   -0.2969
stdcf(-)         -0.5562         0.4330   0.0015   -0.4787
idiovol(-)       -0.4946         0.3285  -0.0021   -0.9875
mve_ia(-)        -0.4777        -0.0749   0.0005   -0.5605
chpmia           -0.4676         0.1088   0.0009   -0.6303
disp(-)          -0.3850         0.7927   0.0039   -0.5300
cfp              -0.3045         0.2216   0.0024   -0.5708
stdacc(-)        -0.2576         0.3357   0.0011   -0.5056
pchsale_pchxsga  -0.2270        -0.1208  -0.0007   -0.7740
grltnoa          -0.2164         0.1620   0.0012   -0.5122
sfe              -0.1952         0.3435   0.0034   -0.7417
pchgm_pchsale    -0.1935         0.0227  -0.0002   -0.5895
cfp_ia           -0.1337        -0.3944  -0.0022   -0.8613
mom6m            -0.1285         0.5223   0.0060   -0.9885
beta             -0.1225        -0.4818   0.0016   -0.9586
```

(continues on next page)

(continued from previous page)

```
dolvol          -0.1030        0.1331   0.0009   -0.7624
nincr           -0.0929        0.3654   0.0024   -0.4061
cinvest(-)       0.0387       -0.3699  -0.0039   -0.9587
cash             0.0479        0.0796   0.0036   -0.7076
chtx             0.1783        0.3262   0.0025   -0.4779
salecash         0.2132        0.2326   0.0005   -0.7463
rd_mve           0.2541        0.0853   0.0024   -0.7868
chnanalyst       0.2806        0.1425   0.0010   -0.5443
fgr5yr           0.3620       -0.0030   0.0020   -0.8761
sue              0.3800        0.6849   0.0048   -0.4062
pchcapx_ia       0.4287       -0.0762  -0.0013   -0.8750
roavol           0.4640        0.0964   0.0019   -0.7812
baspread         0.4727       -0.5203  -0.0005   -0.9539
pchquick         0.5811       -0.0879   0.0001   -0.7586
rd_sale          0.6114       -0.0313   0.0005   -0.8706
pctacc(-)        0.6117        0.2568   0.0019   -0.3803
secured(-)       0.6222        0.3922   0.0014   -0.6406
std_turn         0.6391       -0.2285   0.0024   -0.8064
std_dolvol       0.7879        0.0782   0.0008   -0.5301
rsup             0.8444        0.4582   0.0032   -0.5353
depr             0.8697        0.0616   0.0024   -0.6522
zerotrade(-)     0.8812       -0.0779   0.0054   -0.8344
sgr              0.9168        0.0682   0.0018   -0.7062
salerec          0.9547        0.5194   0.0040   -0.6761
roaq             0.9856        0.7546   0.0054   -0.4532
turn             0.9945       -0.0831   0.0054   -0.8291
ear              1.0453        0.2661   0.0033   -0.5423
quick            1.2014        0.0018   0.0033   -0.7923
gma              1.2529        0.4330   0.0039   -0.6364
hire             1.4328        0.0264   0.0011   -0.8262
tang             1.4843        0.3043   0.0047   -0.7361
grcapx           1.5208       -0.1320  -0.0004   -0.8714
saleinv          1.9087        0.6251   0.0040   -0.4900
chempia          2.0683        0.3022   0.0038   -0.7319
lgr              2.2177        0.2295   0.0024   -0.8286
```

```python
X = pd.concat(rets, join='outer', axis=1).dropna()
X = X/X.std()   # standardize to unit variance
corr = X.corr()
dist = np.sqrt(1 - corr)
```

6.2 Cluster analysis

We use historical backtest returns as input features for cluster analysis, enabling the identification of peer benchmarks for different investment strategies. Strategies that exhibit similar return patterns tend to load on the same "style" factors and should be evaluated against one another.

The correlation between two return series is closely related to their Euclidean distance. When returns are standardized to have unit variance, the squared Euclidean norm between two series x and y can be expressed as:

$$d^2 = \sum_i (x_i - y_i)^2 = \sum_i x_i^2 + \sum_i y_i^2 - 2\sum_i x_i y_i = n + n - 2n\rho$$

since $\sum_i x_i^2 = \sum_i y_i^2 = n$, and $\rho = \sum_i x_i y_i / n$, where ρ represents the correlation between the two series. This relationship allows correlation to be rewritten as:

$$\rho = 1 - \frac{d^2}{2n}$$

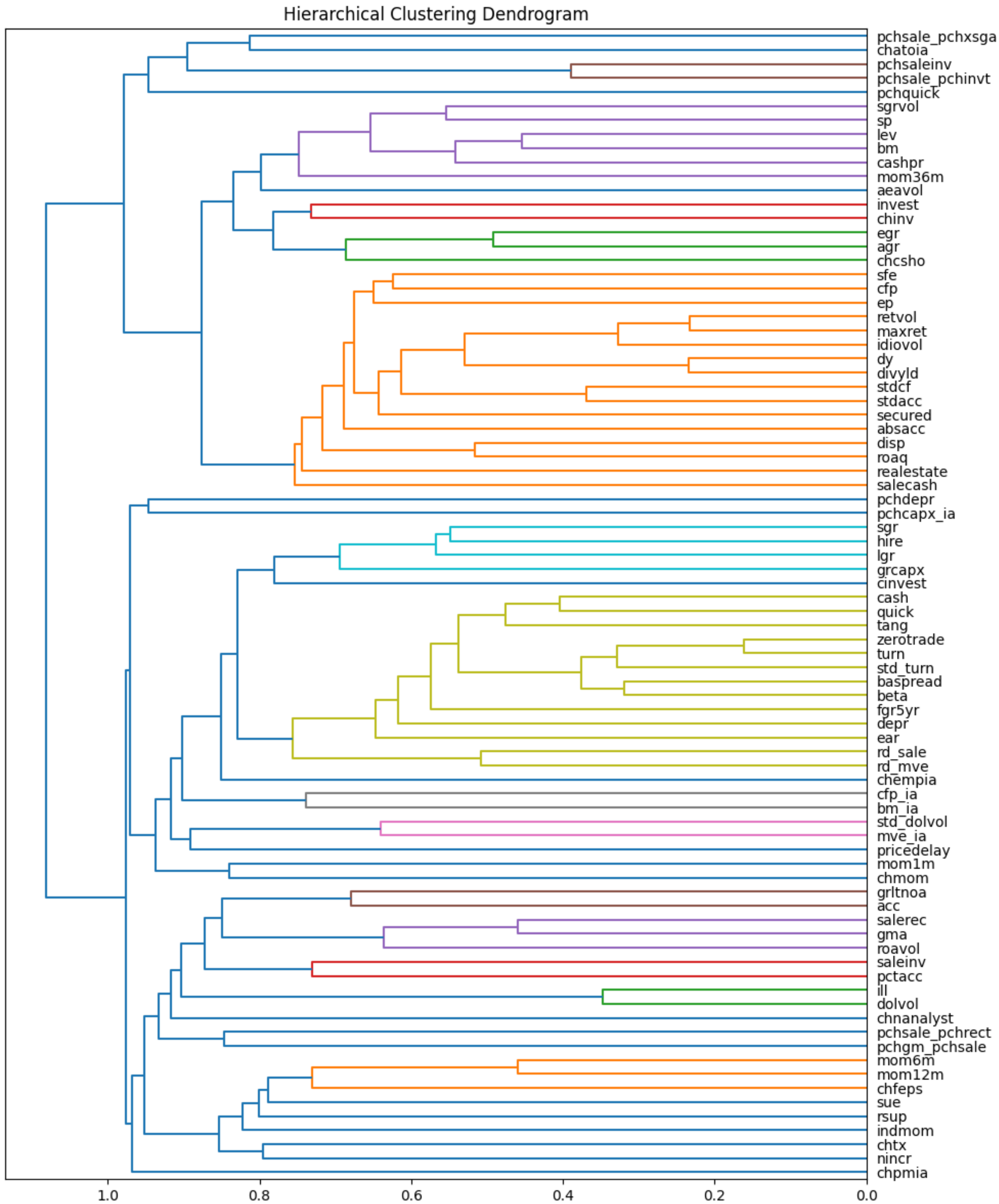
6.2.1 Hiearchical clustering

Hierarchical clustering builds a tree-like structure by iteratively merging clusters based on their similarity. Different linkage methods determine how clusters are merged:

- **Single Linkage**: Merges clusters based on the closest points, leading to elongated clusters.
- **Complete Linkage**: Uses the farthest points between clusters, producing compact, spherical clusters.
- **Average Linkage**: Merges clusters based on the average distance between all points, balancing between single and complete linkage.
- **Ward' s Method**: Minimizes within-cluster variance, resulting in clusters of similar size and shape.

```
model = AgglomerativeClustering(metric='precomputed', linkage='average',
                                distance_threshold=0, n_clusters=None).fit(dist)
```

```
fig, ax = plt.subplots(figsize=(10, 12))
plt.title("Hierarchical Clustering Dendrogram")
# Create linkage matrix and then plot the dendrogram
# scikit-learn: "Plot Hierarchical Clustering Dendrogram"
counts = np.zeros(model.children_.shape[0])
n_samples = len(model.labels_)
# create the counts of samples under each node
for i, merge in enumerate(model.children_):
    current_count = 0
    for child_idx in merge:
        if child_idx < n_samples:
            current_count += 1  # leaf node
        else:
            current_count += counts[child_idx - n_samples]
    counts[i] = current_count

linkage_matrix = np.column_stack([model.children_, model.distances_, counts])\
                   .astype(float)
# Plot the corresponding dendrogram
dendrogram(linkage_matrix, orientation='left', labels=dist.columns, leaf_font_size=10)
plt.tight_layout()
```


Hierarchical Clustering Dendrogram
pchsale_pchxsga
chatoia
pchsaleinv
pchsale_pchinvt
pchquick
sgrvol
sp
lev
bm
cashpr
mom36m
aeavol
invest
chinv
egr
agr
chcsho
sfe
cfp
ep
retvol
maxret
idiovol
dy
divyld
stdcf
stdacc
secured
absacc
disp
roaq
realestate
salecash
pchdepr
pchcapx_ia
sgr
hire
lgr
grcapx
cinvest
cash
quick
tang
zerotrade
turn
std_turn
baspread
beta
fgr5yr
depr
ear
rd_sale
rd_mve
chempia
cfp_ia
bm_ia
std_dolvol
mve_ia
pricedelay
mom1m
chmom
grltnoa
acc
salerec
gma
roavol
saleinv
pctacc
ill
dolvol
chnanalyst
pchsale_pchrect
pchgm_pchsale
mom6m
mom12m
chfeps
sue
rsup
indmom
chtx
nincr
chpmia
1.0
0.8
0.6
0.4
0.2
0.0

6.2.2 K-means clustering

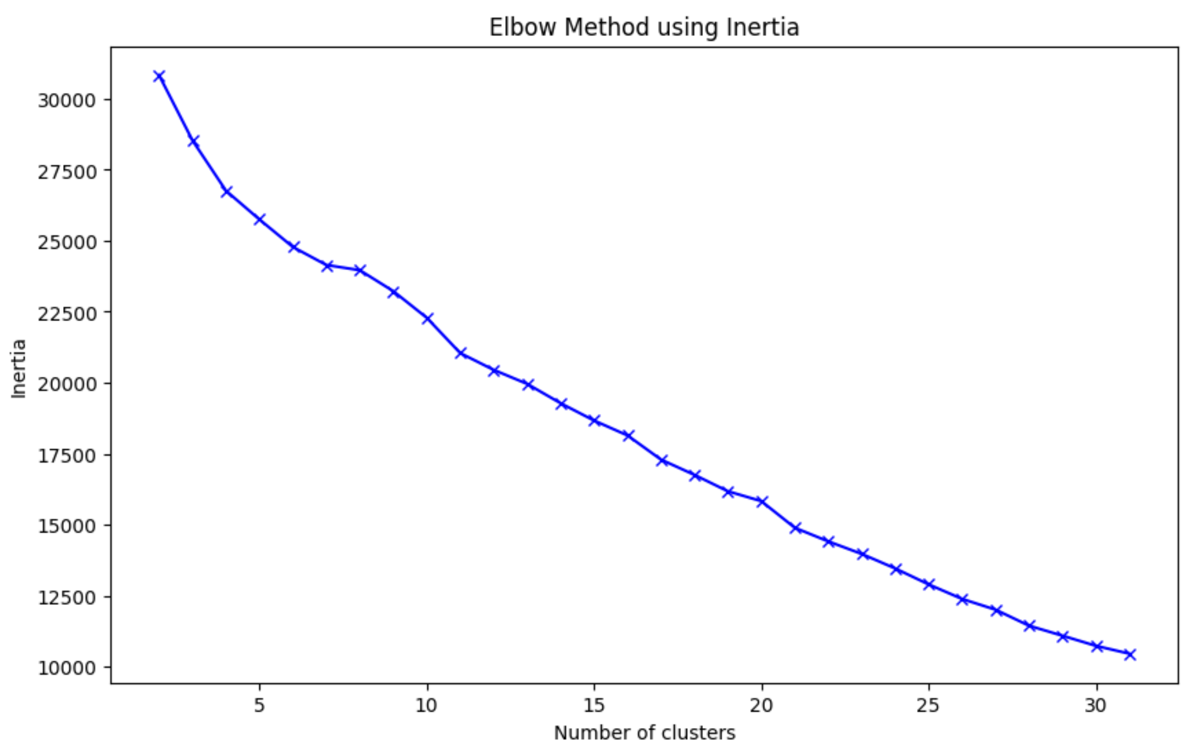
K-means is a widely used unsupervised learning algorithm that partitions data into K non-overlapping clusters. The process involves:

1. Selecting an initial number of clusters K.
2. Randomly assigning initial cluster centroids.
3. Iteratively assigning each data point to the nearest centroid.
4. Recalculating centroids based on the mean of points in each cluster.
5. Repeating until convergence.

K-means minimizes the within-cluster sum of squared distances, making it well-suited for datasets where clusters are roughly spherical and of similar size.

The Elbow Method is commonly used to determine the optimal number of clusters by plotting the within-cluster sum of squares for different values of K. The optimal K is where the graph forms an "elbow," indicating diminishing returns from adding more clusters.

```python
# Selecting number of centers with elbow method
inertias = []
n_clusters = list(range(2, 32))
for n_cluster in n_clusters:
    kmeans = KMeans(n_clusters=n_cluster, random_state=0, n_init="auto").fit(X.T)
    inertias.append(kmeans.inertia_)
plt.figure(figsize=(10, 6))
plt.plot(n_clusters, inertias, 'bx-')
plt.xlabel('Number of clusters')
plt.ylabel('Inertia')
plt.title('Elbow Method using Inertia')
plt.show()
```



Silhouette Analysis provides an alternative evaluation method by measuring how well-separated clusters are. The silhouette score compares cohesion (similarity within a cluster) to separation (difference from other clusters), with values ranging from -1 to 1. Higher scores indicate better-defined clusters, and the number of clusters maximizing the average silhouette score is considered optimal.

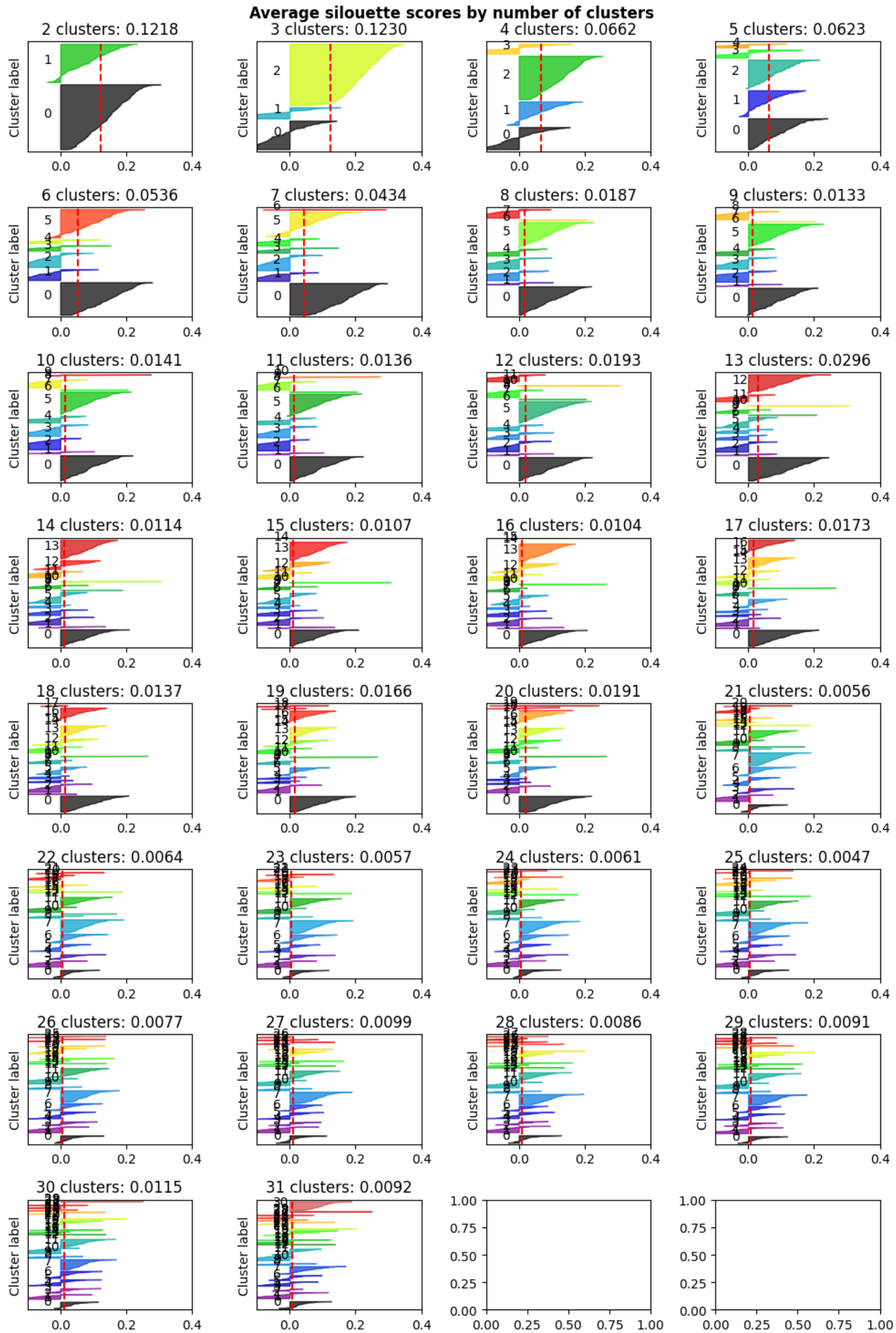
```
# scikit-learn: "Selecting the number of clusters with silhouette analysis on KMeans␣
↪clustering"
scores = {}
fig, ax = plt.subplots(ncols=4, nrows=int(np.round(len(n_clusters)/4)), figsize=(10,␣
↪15))
ax = ax.flatten()
for n_cluster, ax1 in zip(n_clusters, ax):
    clusterer = KMeans(n_clusters=n_cluster, n_init='auto', random_state=10)
    cluster_labels = clusterer.fit_predict(X)

    # The silhouette_score gives the average value for all the samples.
    # This gives a perspective into the density and separation of the formed
    # clusters
    silhouette_avg = silhouette_score(X, cluster_labels)
    print("For n_clusters =", n_cluster,
          "The average silhouette_score is :", silhouette_avg)
    scores[n_cluster] = silhouette_avg
    # Compute the silhouette scores for each sample
    sample_silhouette_values = silhouette_samples(X, cluster_labels)

    ax1.set_title(f"{n_cluster} clusters: {silhouette_avg:.4f}")
    ax1.set_xlim([-0.1, .4])
    ax1.set_ylim([0, len(X) + (n_cluster + 1) * 10])

    y_lower = 10
    for i in range(n_cluster):
        # Aggregate the silhouette scores for samples belonging to
        # cluster i, and sort them
        ith_cluster_silhouette_values = sample_silhouette_values[cluster_labels == i]

        ith_cluster_silhouette_values.sort()

        size_cluster_i = ith_cluster_silhouette_values.shape[0]
        y_upper = y_lower + size_cluster_i

        color = cm.nipy_spectral(float(i) / n_cluster)
        ax1.fill_betweenx(
            np.arange(y_lower, y_upper),
            0,
            ith_cluster_silhouette_values,
            facecolor=color,
            edgecolor=color,
            alpha=0.7,
        )

        # Label the silhouette plots with their cluster numbers at the middle
        ax1.text(-0.05, y_lower + 0.5 * size_cluster_i, str(i))

        # Compute the new y_lower for next plot
        y_lower = y_upper + 10  # 10 for the 0 samples
    ax1.set_ylabel("Cluster label")
```

(continues on next page)

(continued from previous page)

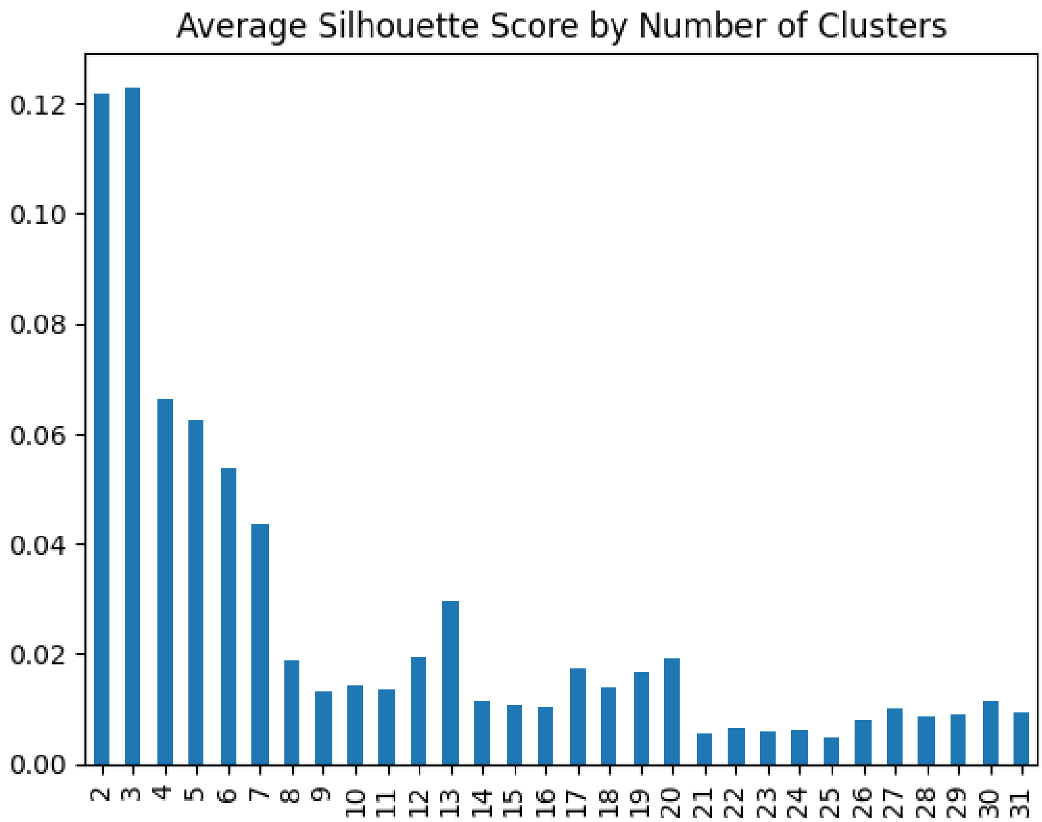
```
    # The vertical line for average silhouette score of all the values
    ax1.axvline(x=silhouette_avg, color="red", linestyle="--")

    ax1.set_yticks([])  # Clear the yaxis labels / ticks
#    ax1.set_xticks([-0.1, 0, 0.2, 0.4, 0.6, 0.8, 1])
    plt.suptitle("Average silouette scores by number of clusters", fontweight="bold")
    plt.tight_layout()
plt.show()
```

```
For n_clusters = 2 The average silhouette_score is : 0.12182520066378914
For n_clusters = 3 The average silhouette_score is : 0.12298455415373394
For n_clusters = 4 The average silhouette_score is : 0.06615920430551211
For n_clusters = 5 The average silhouette_score is : 0.062263203571522374
For n_clusters = 6 The average silhouette_score is : 0.05360073591339586
For n_clusters = 7 The average silhouette_score is : 0.043399891992264766
For n_clusters = 8 The average silhouette_score is : 0.01865524677198136
For n_clusters = 9 The average silhouette_score is : 0.013274893057161832
For n_clusters = 10 The average silhouette_score is : 0.014055548382794032
For n_clusters = 11 The average silhouette_score is : 0.013559353746117705
For n_clusters = 12 The average silhouette_score is : 0.019298192911994215
For n_clusters = 13 The average silhouette_score is : 0.029625503519344714
For n_clusters = 14 The average silhouette_score is : 0.011439750949169072
For n_clusters = 15 The average silhouette_score is : 0.010659851615184082
For n_clusters = 16 The average silhouette_score is : 0.010407061841623768
For n_clusters = 17 The average silhouette_score is : 0.017270693194767258
For n_clusters = 18 The average silhouette_score is : 0.01369810719820829
For n_clusters = 19 The average silhouette_score is : 0.016630208862778856
For n_clusters = 20 The average silhouette_score is : 0.019144464436508506
For n_clusters = 21 The average silhouette_score is : 0.005561006651677671
For n_clusters = 22 The average silhouette_score is : 0.006354452315559386
For n_clusters = 23 The average silhouette_score is : 0.005722873193904617
For n_clusters = 24 The average silhouette_score is : 0.006108458004688574
For n_clusters = 25 The average silhouette_score is : 0.004737653906771348
For n_clusters = 26 The average silhouette_score is : 0.007702186499322102
For n_clusters = 27 The average silhouette_score is : 0.009876913034750879
For n_clusters = 28 The average silhouette_score is : 0.008628670013359775
For n_clusters = 29 The average silhouette_score is : 0.00908060151674441
For n_clusters = 30 The average silhouette_score is : 0.01149725260958716
For n_clusters = 31 The average silhouette_score is : 0.009230744450581462
```


Average silouette scores by number of clusters
2 clusters: 0.1218
3 clusters: 0.1230
4 clusters: 0.0662
5 clusters: 0.0623
6 clusters: 0.0536
7 clusters: 0.0434
8 clusters: 0.0187
9 clusters: 0.0133
10 clusters: 0.0141
11 clusters: 0.0136
12 clusters: 0.0193
13 clusters: 0.0296
14 clusters: 0.0114
15 clusters: 0.0107
16 clusters: 0.0104
17 clusters: 0.0173
18 clusters: 0.0137
19 clusters: 0.0166
20 clusters: 0.0191
21 clusters: 0.0056
22 clusters: 0.0064
23 clusters: 0.0057
24 clusters: 0.0061
25 clusters: 0.0047
26 clusters: 0.0077
27 clusters: 0.0099
28 clusters: 0.0086
29 clusters: 0.0091
30 clusters: 0.0115
31 clusters: 0.0092
Cluster label

```
Series(scores).plot.bar(title='Average Silhouette Score by Number of Clusters')
```

```
<Axes: title={'center': 'Average Silhouette Score by Number of Clusters'}>
```

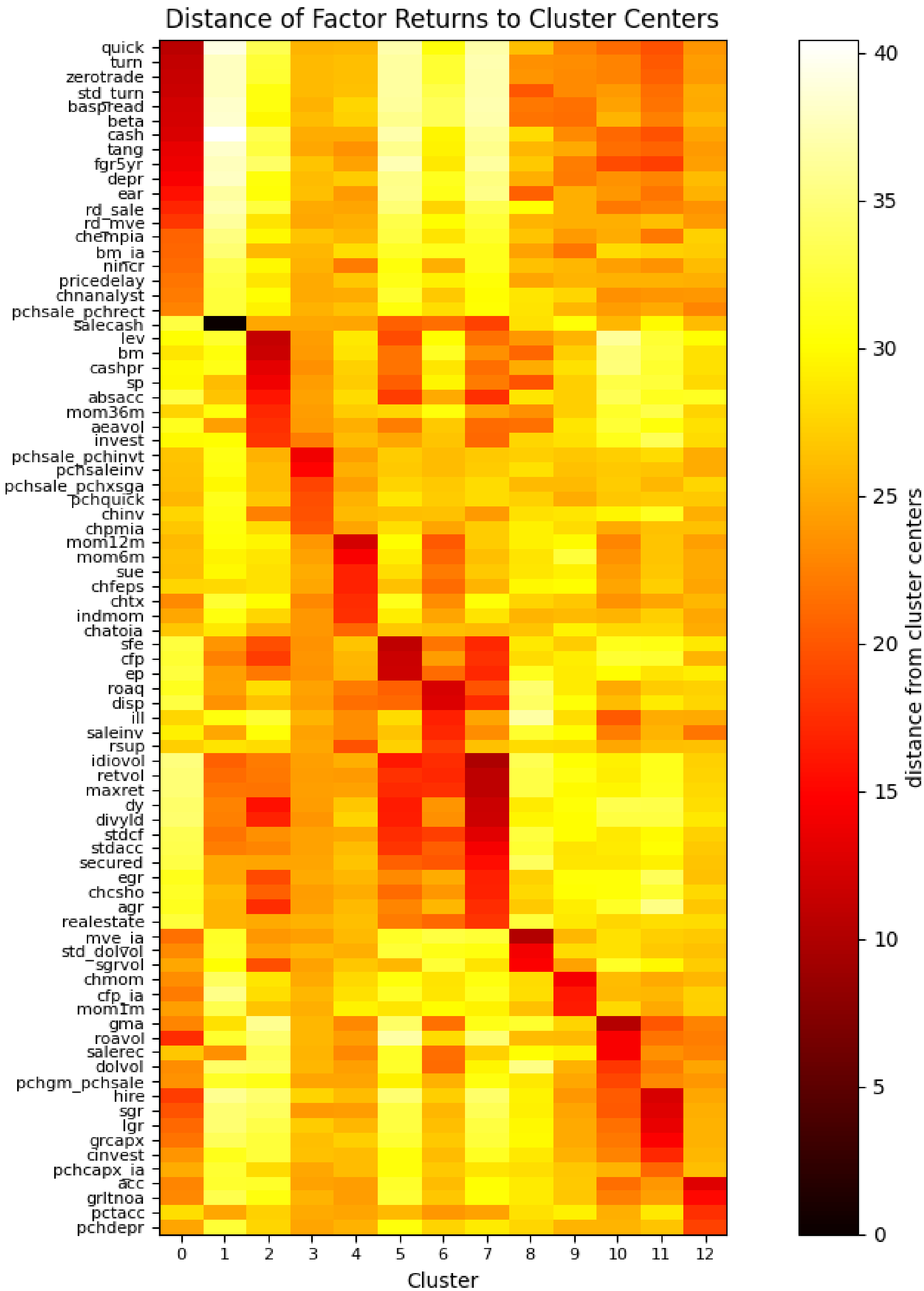


Heatmap of strategy correlations

A heatmap visualizes the distances between strategy returns (y-axis) and their respective cluster centers (x-axis), with clusters determined using the K-Means algorithm (K=20). The darkest-colored values appear along the jagged diagonal, indicating that strategy returns are closest to the centers of their assigned clusters. This visualization aids in identifying common style factors and risk exposures across different investment strategies.

```
kmeans = KMeans(n_clusters=13, random_state=0, n_init="auto").fit(X.T)
Z = DataFrame(kmeans.transform(X.T), index=X.columns)
Z['distance'] = np.min(Z.iloc[:, :kmeans.n_clusters], axis=1)
Z['cluster'] = np.argmin(Z.iloc[:, :kmeans.n_clusters], axis=1)
Z = Z.sort_values(['cluster', 'distance'])  # group by cluster and distance to center
```

```
fig, ax = plt.subplots(figsize=(10, 9))
plt.title('Distance of Factor Returns to Cluster Centers')
plt.imshow(Z.iloc[:, :kmeans.n_clusters], cmap='hot', aspect=1/3)
plt.yticks(range(len(Z)), labels=Z.index, fontsize=8)
plt.xticks(range(kmeans.n_clusters), labels=range(kmeans.n_clusters), fontsize=8)
plt.xlabel('Cluster')
plt.colorbar(label='distance from cluster centers')
plt.tight_layout()
plt.show()
```


Distance of Factor Returns to Cluster Centers
quick
turn
zerotrade
std_turn
baspread
beta
cash
tang
fgr5yr
depr
ear
rd_sale
rd_mve
chempia
bm_ia
nincr
pricedelay
chnanalyst
pchsale_pchrect
salecash
lev
bm
cashpr
sp
absacc
mom36m
aeavol
invest
pchsale_pchinvt
pchsaleinv
pchsale_pchxsga
pchquick
chinv
chpmia
mom12m
mom6m
sue
chfeps
chtx
indmom
chatoia
sfe
cfp
ep
roaq
disp
ill
saleinv
rsup
idiovol
retvol
maxret
dy
divyld
stdcf
stdacc
secured
egr
chcsho
agr
realestate
mve_ia
std_dolvol
sgrvol
chmom
cfp_ia
mom1m
gma
roavol
salerec
dolvol
pchgm_pchsale
hire
sgr
lgr
grcapx
cinvest
pchcapx_ia
acc
grltnoa
pctacc
pchdepr
0 1 2 3 4 5 6 7 8 9 10 11 12
Cluster
distance from cluster centers
0 5 10 15 20 25 30 35 40

References:

Stephen Ross, 1976, "The arbitrage theory of capital asset pricing", Journal of Economic Theory, Volume 13, Issue 3, December 1976, Pages 341-360

Robert C. Merton, 1973, "An Intertemporal Capital Asset Pricing Model", Econometrica, Vol. 41, No. 5 (Sep., 1973), pp. 867-887

Jeremiah Green, John R. M. Hand, X. Frank Zhang, 2017, "The Characteristics that Provide Independent Information about Average U.S. Monthly Stock Returns", The Review of Financial Studies, Volume 30, Issue 12, December 2017, Pages 4389–4436

John H. Cochrane, 2011, "Presidential Address: Discount Rates", The Journal of Finance, Volume 66, Issue 4, August 2011, Pages 1047-1108

Lo, Andrew W. (2004). "The Adaptive Markets Hypothesis: Market Efficiency from an Evolutionary Perspective". Journal of Portfolio Management. 5. 30: 15–29

Andrew Lo, 2017, "Adaptive Markets: Financial Evolution at the Speed of Thought".

Andrew Ang, 2014, "Asset Management: A Systematic Approach to Factor Investing"

FRM Part II Exam Book Investment Management Ch. 1-3.

Wharton Research Data Services.

CHAPTER SEVEN

EVENT STUDY

All strange and terrible events are welcome, but comforts we despise - Cleopatra

Event studies are a fundamental tool in financial research used to assess how specific events impact stock returns, see Mackinlay (1997). These events, such as mergers, earnings announcements, or regulatory changes, can cause deviations in stock prices known as abnormal returns. By comparing stock returns to a market benchmark, event studies help analysts determine whether an event had a statistically significant impact on a company's stock price.

This notebook explores key methodologies used in event studies, including the computation of abnormal returns, cumulative abnormal returns, and buy-and-hold abnormal returns. Additionally, it delves into the announcement effect, which examines how stock prices react immediately around an event, and investigates potential pre- and post-announcement drifts. To account for overlapping event windows and cross-sectional dependencies, we apply statistical corrections based on the work of Kolari et al. (2010, 2018).

We also introduce frequency domain methods, leveraging the Fourier Transform and Convolution Theorem to efficiently compute cross-correlations between time series. Finally, we address the multiple testing problem, ensuring that statistical inferences remain valid when analyzing a large number of events.

```python
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
from scipy.stats import norm
from statsmodels.stats.multitest import multipletests
from tqdm import tqdm
import warnings
from finds.database import SQL, RedisDB
from finds.structured import BusDay, Benchmarks, Stocks, CRSP, PSTAT
from secret import credentials, paths, CRSP_DATE
VERBOSE = 0
if not VERBOSE:
    warnings.simplefilter(action='ignore', category=FutureWarning)
#%matplotlib qt
```

```python
# open connections
sql = SQL(**credentials['sql'], verbose=VERBOSE)
user = SQL(**credentials['user'], verbose=VERBOSE)
bd = BusDay(sql)
rdb = RedisDB(**credentials['redis'])
keydev = PSTAT(sql, bd, verbose=VERBOSE)
crsp = CRSP(sql, bd, rdb=rdb, verbose=VERBOSE)
bench = Benchmarks(sql, bd, verbose=VERBOSE)
imgdir = paths['images'] / 'events'
```

7.1 Abnormal returns

Event studies analyze how stock returns respond to specific company events, such as mergers or public announcements. These studies examine deviations from expected returns, known as **abnormal returns**, in the days surrounding the news release.

Let the event date be $t = 0$, and define the stock return for firm i at time t as $R_{i,t}$. To provide a benchmark, we also compute the returns of a relevant market or sector index, denoted $R_{m,t}$. Then:

- **Abnormal returns (AR):** The deviation of a stock’ s return from the market return: $AR_{i,t} = R_{i,t} - R_{m,t}$
- **Average abnormal returns (AAR):** The mean abnormal return across all firms: $AAR_t = \frac{1}{N} \sum_i^N AR_{i,t}$
- **Cumulative average abnormal returns (CAR):** The sum of average abnormal returns over time: $CAR_T = \sum_t AAR_t$
- **Buy-and-hold average abnormal returns (BHAR):** The average buy-and-hold abnormal returns across firms: $BHAR_T = \frac{1}{N} \sum_i^N \left[\prod_t (1 + AR_{i,t}) - \prod_t (1 + R_{m,t})\right]$

7.1.1 Announcement effect

The **announcement effect** refers to the immediate stock price movement around a news release. The nature of this reaction varies:

- An **efficient reaction** occurs when the stock price stabilizes at its new level after the event.
- An **under-reaction** happens when the price continues trending in the same direction post-announcement.
- An **over-reaction** is when the price initially spikes but then reverses course.

In historical data, pinpointing the exact announcement time can be challenging. To account for this uncertainty, studies often consider an **announcement window**, typically covering one day before and after the event.

- A **pre-announcement drift** may indicate information leakage before the official announcement.
- A **post-announcement drift** may suggest that investors either under- or over-reacted to the news.

When analyzing multiple events, researchers often aggregate returns for stocks with the same announcement date. However, Kolari et al. (2010) caution that even small correlations among abnormal returns can lead to significant over-rejection of the null hypothesis of zero abnormal returns. To correct for this cross-sectional dependence, the unbiased estimate of abnormal return variance is:

$$\sigma^2 = \frac{s^2}{n}(1 + (n-1)\hat{r})$$

where:

- $\hat{r}$ is the average sample cross-sectional correlation of residuals,
- s^2 is the (biased) cross-sectional standard deviation of abnormal returns.

In the next subsection, we explore how cross-correlations can be be measured efficiently using the Fourier Transform technique from the field of signal processing.

7.1.2 FinDS `eventstudy` module

The `eventstudy` module in the FinDS package provides a comprehensive tool for analyzing announcement effects.

```
from finds.backtesting import EventStudy
eventstudy = EventStudy(user, bench=bench, stocks=crsp, max_date=CRSP_DATE)
```

```
# event window parameters
left, right, post = -1, 1, 21
end = bd.offset(CRSP_DATE, left - post)
beg = 20020101
mindays = 1000
```

- Retrieve event dates and company roles from CapitalIQ Key Developments

```
# sorted list of all eventids and roleids, provided in keydev class
events = sorted(keydev.event.index)
roles = sorted(keydev.role.index)
```

```
# str formatter to pretty print descriptions, provided in keydev class
eventformat = lambda e, r: "{event} ({eventid}) {role} [{roleid}]"\
    .format(event=keydev.event[e], eventid=e, role=keydev.role[r], roleid=r)
```

- Helper functions to retrieve size decile of universe stocks

```
class Universe:
    """Helper to lookup prevailing size decile of universe stocks"""
    def __init__(self, beg: int, end: int, stocks: Stocks, annual: bool = False):
        # whether to offset previous month or year end when lookup
        self.offset = stocks.bd.endyr if annual else stocks.bd.endmo

        # populate dictionary, keyed by permno and date, with size decile
        self.decile = dict()
        date_range = stocks.bd.date_range(start=self.offset(beg, -1), end=end,
                                          freq=12 if annual else 'e')
        for date in date_range:
            univ = crsp.get_universe(date)
            for permno in univ.index:
                self.decile[(permno, date)] = univ.loc[permno, 'decile']

    def __getitem__(self, item):
        """Returns size decile, else 0 if not universe stock"""
        permno, date = item
        return self.decile.get((permno, self.offset(date, -1)), 0)

univ = Universe(beg=beg, end=end, stocks=crsp)
```

- Define the event study pipeline:
  1. retrieve announcement dates for the desired event and test period
  2. filter the universe
  3. retrieve announcement window daily stock returns, and compute event study metrics

```
# run event study after screening stock universe
def event_pipeline(eventstudy: EventStudy,
                   beg: int,
                   end: int,
                   eventid: int,
                   roleid: int,
                   left: int,
                   right: int,
                   post: int) -> DataFrame:
    """helper to screen stock universe, and merge keydev events and crsp daily"""

    # Retrieve announcement dates for this event
    df = keydev.get_linked(
        dataset='keydev',
        date_field='announcedate',
        fields=['keydevid',
                'keydeveventtypeid',
                'keydevtoobjectroletypeid'],
        where=(f"announcedate >= {beg} "
               f"  and announcedate <= {end}"
               f"  and keydeveventtypeid = {eventid} "
               f"  and keydevtoobjectroletypeid = {roleid}"))\
               .drop_duplicates(['permno', 'announcedate'])\
               .set_index(['permno', 'announcedate'], drop=False)

    # Require universe size decile
    df['size'] = [univ[row.permno, row.announcedate] for row in df.itertuples()]

    # Call eventstudy to compute daily abnormal returns, with named label
    rows = eventstudy(label=f"{eventid}_{roleid}",
                      df=df[df['size'].gt(0)],
                      left=left,
                      right=right,
                      post=post,
                      date_field='announcedate')
    return df.loc[rows.to_records(index=False).tolist()] # restrict successful rows
```

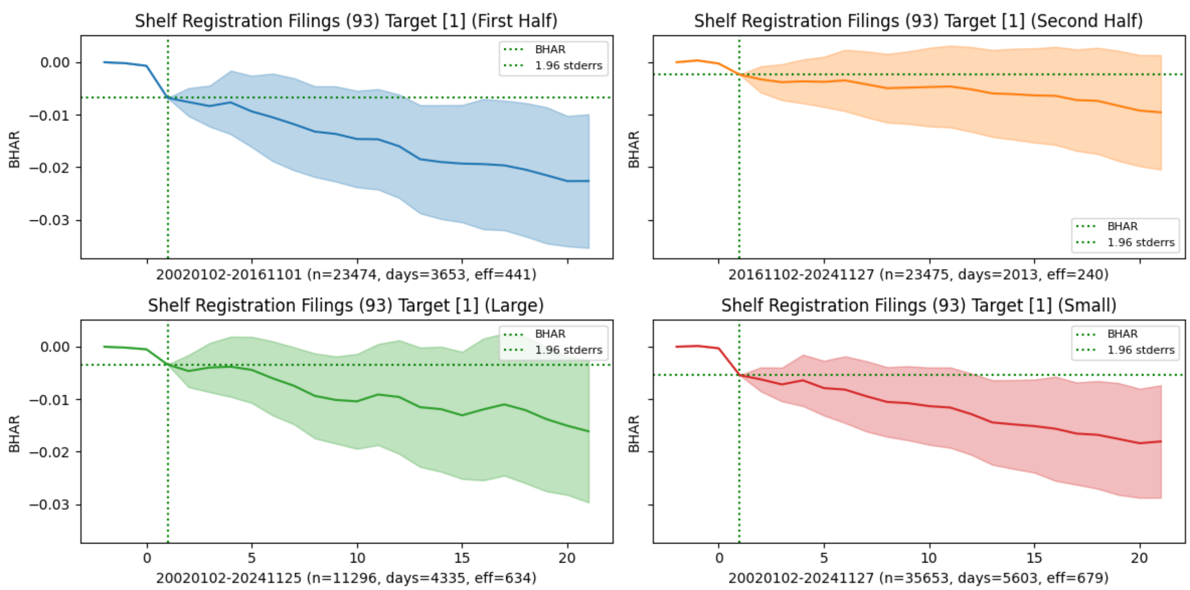
Examine and show subsample plots for events that exhibited large post-announcement drift

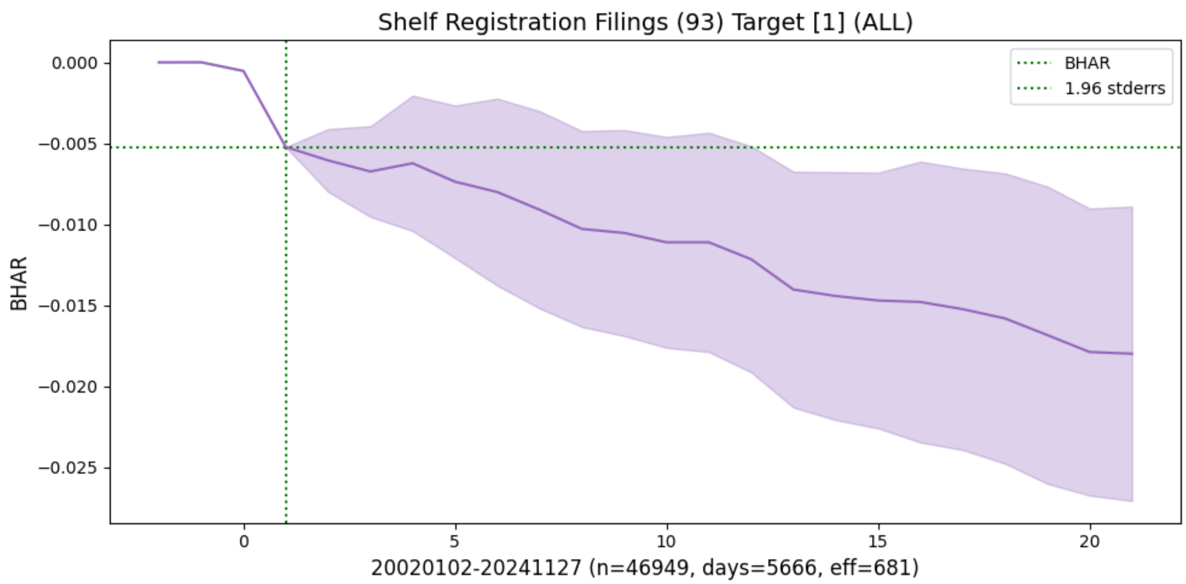
```
events_list = [[93, 1], [232, 1]]  # largest drift returns
for i, (eventid, roleid) in enumerate(events_list):
    df = event_pipeline(eventstudy,
                        eventid=eventid,
                        roleid=roleid,
                        beg=beg,
                        end=end,
                        left=left,
                        right=right,
                        post=post)
    halfperiod = np.median(df['announcedate'])
    sample = {'First Half': df['announcedate'].lt(halfperiod).values,
              'Second Half': df['announcedate'].ge(halfperiod).values,
              'Large': df['size'].le(5).values,
              'Small': df['size'].gt(5).values,
              'ALL': [],
             }
    fig, axes = plt.subplots(nrows=2, ncols=2, clear=True, figsize=(12, 6),
```

(continues on next page)

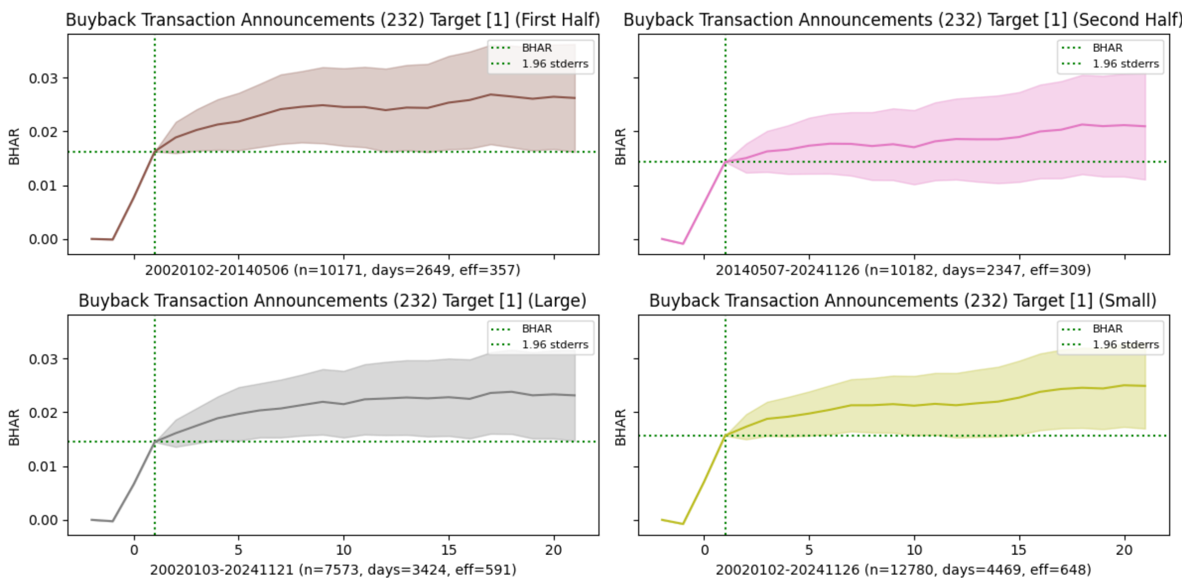
(continued from previous page)

```
                            sharex=True, sharey=True)
axes = axes.flatten()
for ifig, (label, rows) in enumerate(sample.items()):
    if ifig >= len(axes):
        plt.show()
        fig, ax = plt.subplots(clear = True, figsize=(10, 5))
    else:
        ax = axes[ifig]
    bhar = eventstudy.fit(model='sbhar', rows=rows)
    eventstudy.plot(model='sbhar',
                    title=eventformat(eventid, roleid) + f" ({label})",
                    drift=True,
                    ax=ax,
                    fontsize=(8 if ifig < len(axes) else 10),
                    c=f"C{i*5+ifig}")
    plt.tight_layout()
    plt.savefig(imgdir / (label + f"{eventid}_{roleid}.jpg"))
plt.show()
```

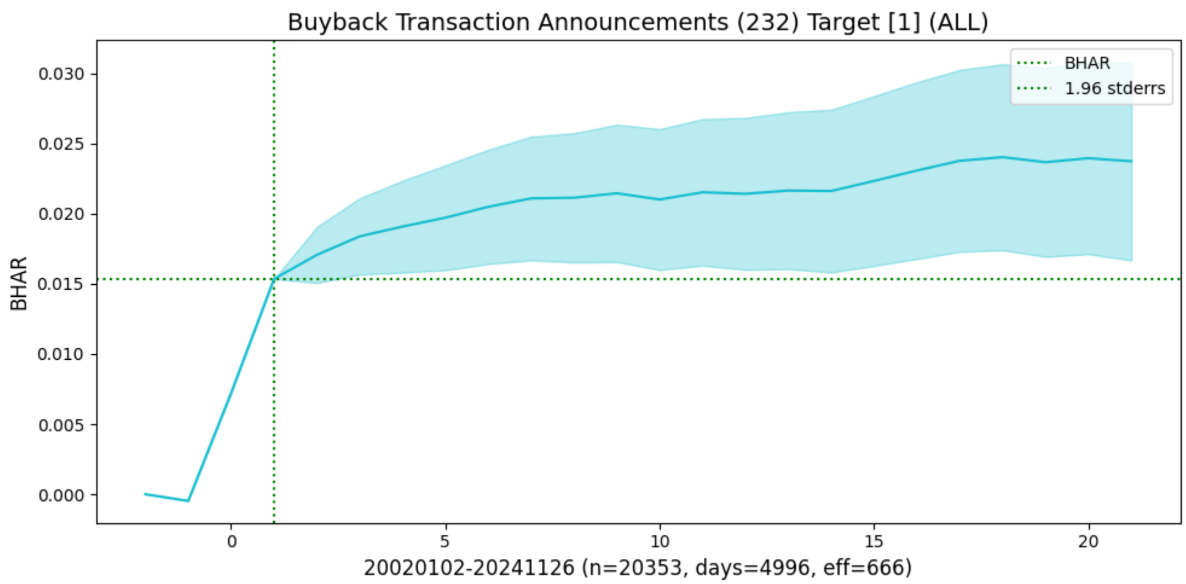




Shelf Registration Filings (93) Target [1] (ALL)
BHAR
1.96 stderrs
0.000
−0.005
−0.010
−0.015
−0.020
−0.025
BHAR
0
5
10
15
20
20020102-20241127 (n=46949, days=5666, eff=681)



Buyback Transaction Announcements (232) Target [1] (First Half)
20020102-20140506 (n=10171, days=2649, eff=357)
Buyback Transaction Announcements (232) Target [1] (Second Half)
20140507-20241126 (n=10182, days=2347, eff=309)
Buyback Transaction Announcements (232) Target [1] (Large)
20020103-20241121 (n=7573, days=3424, eff=591)
Buyback Transaction Announcements (232) Target [1] (Small)
20020102-20241126 (n=12780, days=4469, eff=648)
BHAR
1.96 stderrs
0.00
0.01
0.02
0.03
0
5
10
15
20



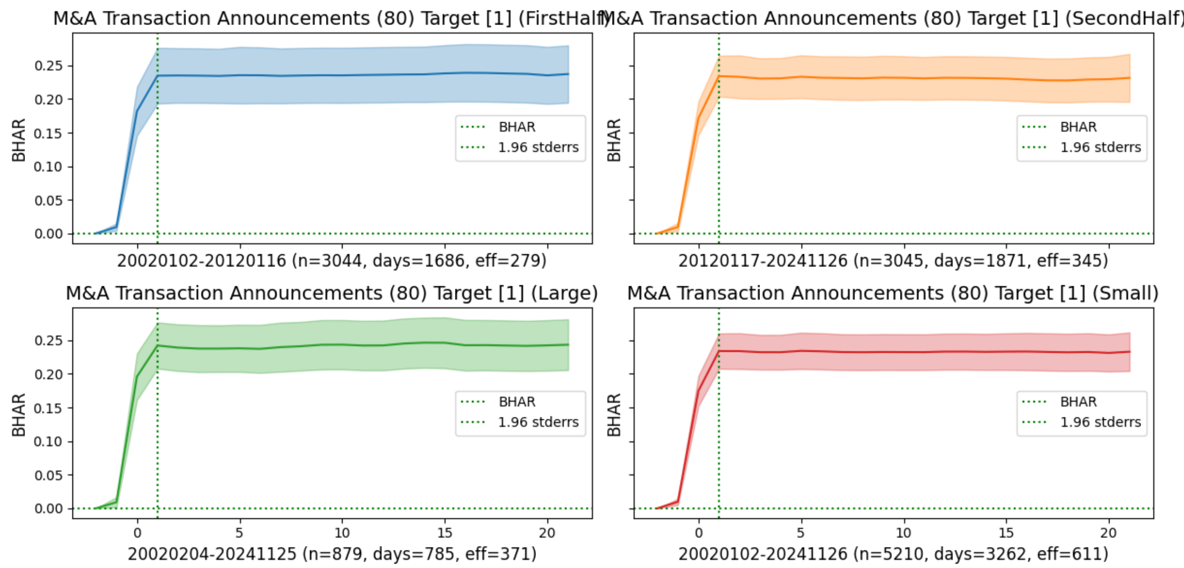
Examine and show subsample plots for events that exhibited large announcement window returns

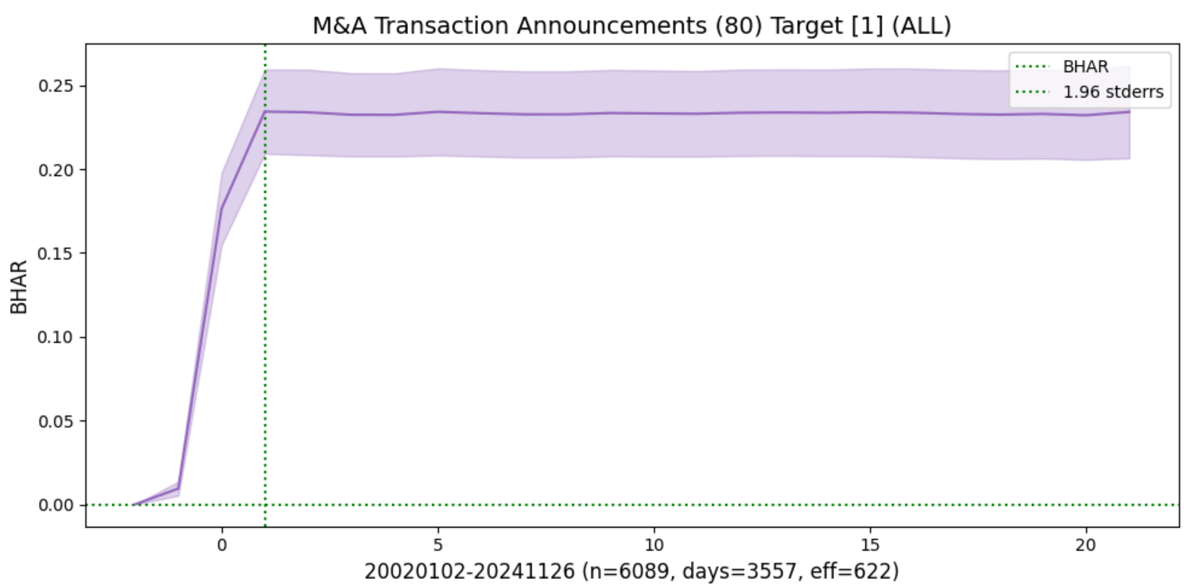
```
events_list = [[80,1], [26,1]]  # largest announcement window returns
for i, (eventid, roleid) in enumerate(events_list):
    #eventid, roleid = 50, 1
    #eventid, roleid = 83, 1
    df = event_pipeline(eventstudy,
                        eventid=eventid,
                        roleid=roleid,
                        beg=beg,
                        end=end,
                        left=left,
                        right=right,
                        post=post)
    halfperiod = np.median(df['announcedate'])
    sample = {'FirstHalf': df['announcedate'].lt(halfperiod).values,
              'SecondHalf': df['announcedate'].ge(halfperiod).values,
              'Large': df['size'].le(5).values,
              'Small': df['size'].gt(5).values,
              'ALL': []}
    fig, axes = plt.subplots(nrows=2, ncols=2, clear=True, figsize=(12, 6),
                             sharex=True, sharey=True)
    axes = axes.flatten()
    for ifig, (label, rows) in enumerate(sample.items()):
        if ifig >= len(axes):
            plt.show()
            fig, ax = plt.subplots(clear = True, figsize=(10, 5))
        else:
            ax = axes[ifig]
        bhar = eventstudy.fit(model='sbhar', rows=rows)
        eventstudy.plot(model='sbhar',
                        title=eventformat(eventid, roleid) + f" ({label})",
                        drift=False,
                        ax=ax,
                        c=f"C{i*5+ifig}")
        plt.tight_layout()
        plt.savefig(imgdir / (label + f"{eventid}_{roleid}.jpg"))
```

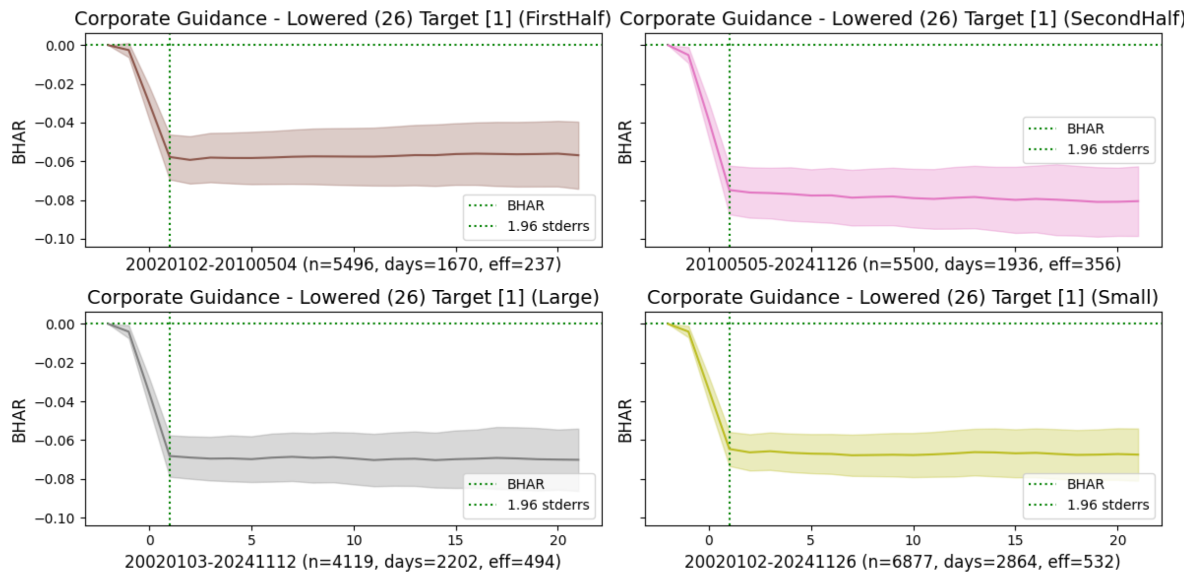
(continues on next page)

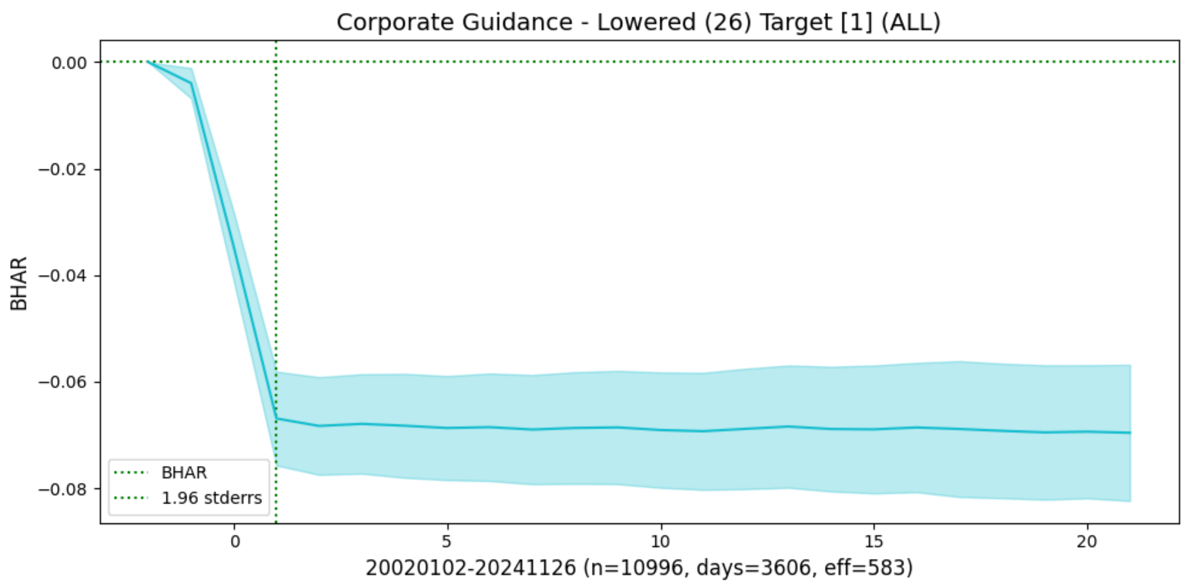
(continued from previous page)

```
    plt.show()
```









7.2 Fourier transforms

7.2.1 Cross-sectional Correlations

Kolari et al. (2018) address event windows that **partially overlap in calendar time**. To account for this overlap and cross-sectional correlation effects, they adjust the variance estimate as follows:

$$\sigma^2 = \frac{s^2}{n} m(1 + \tau(n-1)\rho)$$

where:

- m = length of the event window,
- τ = proportion of overlapping days between event windows,

- ρ = ratio of average covariance between abnormal returns to their variance.

If all events share the same calendar date, such that all event windows fully overlap, then ρ simplifies to the average cross-sectional correlation of residuals during the estimation period.

To estimate ρ, we assume cross-sectional correlations arise mainly from time misalignment. By shifting and aligning event window residuals for each stock pair to maximize their correlation, we approximate ρ as the average of these best-aligned correlations. We leverage the methodology of Fourier transforms and convolutions to effeciently compute all pairs of correlations.

7.2.2 Fast Fourier Transforms

Fourier transforms allow us to express a time series (or any function) as a sum of sinusoids. Given a function over an interval of length (L), we can decompose it into sinusoidal components with periods (L, L/2, L/3,) etc. Each component has:

- A magnitude (scale factor)
- A phase (shift)
- A frequency (inverse of the period)

This means that any function can be represented as a weighted sum of sinusoidal waves. By using complex numbers, we can efficiently store and manipulate these sinusoidal components in Fourier space rather than the original time domain. The **Fast Fourier Transform (FFT)** is an efficient algorithm for computing the **Discrete Fourier Transform (DFT)**. The DFT converts a sequence of time-domain data points into its sinusoidal coefficients, converting it from real space to frequency space. The **Inverse Fast Fourier Transform** reconstructs the original function.

7.2.3 Convolution Theorem

Convolution is a mathematical operation that combines two functions (f and g) to produce a third function ($f * g$), expressing how one function modifies the other:

$$(f * g)(t) = \int_{-\infty}^{\infty} f(s)g(t-s)ds$$

In **one dimension**, convolution acts like a **moving average**, smoothing a time series by weighting neighboring values. In **two dimensions**, it applies filters (e.g., blurring, edge detection) in image processing.

In signal processing, frequency-domain techniques efficiently compute cross-correlations across various time shifts. Instead of directly computing correlations for every possible lag (which is computationally expensive), we leverage the **Convolution Theorem**. A key property of Fourier transforms is that convolution in the time domain corresponds to multiplication in the frequency domain.

$$\mathcal{F}\{f(t) * g(t)\} = \mathcal{F}\{f(t)\}\mathcal{F}\{g(t)\}$$

Since direct convolution requires $O(N^2)$ operations, computing it directly is slow. However:

1. We can first apply the Fast Fourier Transform (FFT) to convert both functions into frequency space in $O(N \log N)$ time.
2. In frequency space, convolution is just element-wise multiplication of Fourier coefficients, which is very fast.
3. We then apply the Inverse FFT (IFFT) to obtain the final result in $O(N \log N)$ time.

```
# best alignment and cross-correlation using convolution theorem method
from scipy.fft import rfft, irfft
def fft_align(X):
    """Find best alignment, max cross-correlation and indices of all pairs of columns"
↪""

    def _normalize(X: np.ndarray) -> np.ndarray:
        """Helper to demean columns and divide by norm"""
        X = X - np.mean(X, axis=0)
        X = X / np.linalg.norm(X, axis=0)
        return X

    N, M = X.shape
    X = np.pad(_normalize(X), [(0, N), (0,0)])  # normalize and zero pad
    Y = rfft(np.flipud(X), axis=0)              # FFT of all series flipped
    X = rfft(X, axis=0)                         # FFT of all original series
    corr, disp, cols = [], [], []               # to accumulate results
    for col in range(M-1):  # at each iter: compute column col * all remaining columns
        conv = irfft(X[:, [col]] * Y[:, col+1:], axis=0)  # inverse of product of FFT
        corr.extend(np.max(conv, axis=0))
        shift = (N//2) + 1          # displacement location relative to center
        disp.extend(((np.argmax(conv, axis=0) + shift) % N) - shift + 1)
        cols.extend([(col, j) for j in range(col+1, M)])
    return corr, disp, cols
```

```
Z = np.random.uniform(size=(10000, 1))
fft_align(np.hstack((Z[:-1], Z[1:])))
```

```
([0.9998967076018761], [1], [(0, 1)])
```

```
#fft_align(np.hstack(np.hstack((Z[:-5], Z[5:])))[:,2:], Z[:-2])
fft_align(np.hstack((np.hstack((Z[:-5], Z[5:]))[:-2], Z[7:])))
```

```
([0.9997503757740265, 0.9993943885689067, 0.9996440440611477],
 [5, 7, 2],
 [(0, 1), (0, 2), (1, 2)])
```

7.3 Multiple testing

When testing a huge number of null hypotheses, we are bound to get some very testing small p-values by chance. **Data snooping** occurs when the analyst fits a great number of models to a dataset. When explanatory variables are selected using the data, t-ratios and F-ratios will be too large, thus overstating the importance of variables in the model. **Multiple testing** adjusts the hurdle for significance because some tests will appear significant by chance. The downside of doing this is that some truly significant strategies might be overlooked because they did not pass the more stringent hurdle. This is the classic tension between Type I errors and Type II errors. The Type I error is the false discovery (investing in an unprofitable trading strategy). The Type II error is missing a truly profitable trading strategy.

The **Family-Wise Error Rate (FWER)** controls the probability of making **at least one** false discovery. If all m null hypotheses are independent and true:

$$\text{FWER}(\alpha) = 1 - (1 - \alpha)^m$$

To maintain a stricter significance threshold, the **Bonferroni correction** divides the confidence level by the number of tests:

$$\alpha' = \frac{\alpha}{m}$$

This ensures that across multiple tests, the probability of making even **one** false discovery remains below α. However, it can be overly conservative.

The **false discovery rate (FDR)** controls the proportion of false positives among rejected hypotheses. The **Benjamini-Hochberg (BH) procedure** is a widely used method for FDR control. Instead of adjusting individual significance thresholds like Bonferroni, BH ranks p-values and sets a significance cutoff where false discoveries remain within acceptable limits.

Compute BHAR and CAR of all events

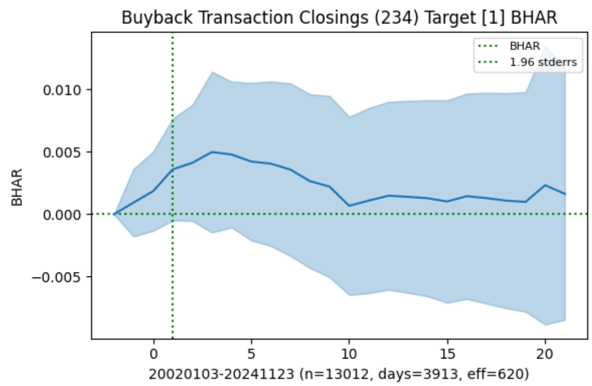
```
restart_event = 0 # 75,1
for i, eventid in tqdm(enumerate(events), total=len(events)):
    if eventid <= restart_event:  # kludge to resume loop
        continue
    for roleid in roles:
        # retrieve all returns observations of this eventid, roleid
        df = event_pipeline(eventstudy,
                            beg=beg,
                            end=end,
                            eventid=eventid,
                            roleid=roleid,
                            left=left,
                            right=right,
                            post=post)
        if df['announcedate'].nunique() < mindays:  # require min number of dates
            continue

        # compute both BHAR and CAR averages, then plot and save
        bhar = eventstudy.fit(model='sbhar')
        car = eventstudy.fit(model='scar')
        #eventstudy.write()
        eventstudy.write_summary()
        #print(eventstudy.label, eventid, roleid)

        fig, axes = plt.subplots(1, 2, clear=True, figsize=(12, 4), num=1)
        eventstudy.plot(model='sbhar', ax=axes[0],
                        title=eventformat(eventid, roleid) + ' BHAR',
                        fontsize=8, vline=[right])
        eventstudy.plot(model='scar', ax=axes[1],
                        title=eventformat(eventid, roleid) + ' CAR',
                        fontsize=8, vline=[right])
        plt.tight_layout()
        plt.savefig(imgdir / f"{eventid}_{roleid}.jpg")
```

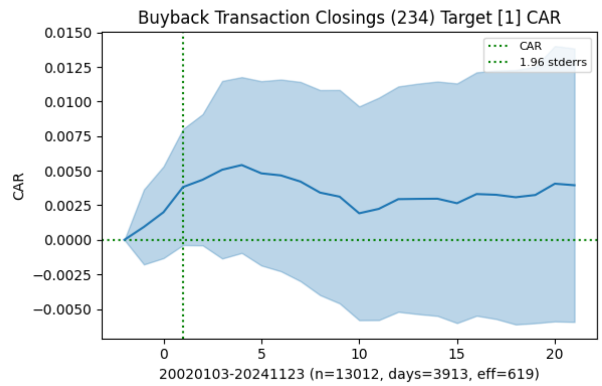
```
45%|█████     | 52/116 [20:04:50<22:46:35, 1281.17s/it]/home/terence/Dropbox/
↪github/data-science-notebooks/finds/recipes/filters.py:28: RuntimeWarning:␣
↪invalid value encountered in divide
 X = X / np.linalg.norm(X, axis=0)
53%|██████    | 62/116 [23:27:18<18:41:48, 1246.45s/it]/home/terence/Dropbox/
↪github/data-science-notebooks/finds/recipes/filters.py:28: RuntimeWarning:␣
↪invalid value encountered in divide
 X = X / np.linalg.norm(X, axis=0)
```

(continues on next page)

(continued from previous page)

```
 61%|██████     | 71/116 [26:38:56<16:18:29, 1304.66s/it]/home/terence/Dropbox/
↪github/data-science-notebooks/finds/recipes/filters.py:28: RuntimeWarning:␣
↪invalid value encountered in divide
  X = X / np.linalg.norm(X, axis=0)
100%|██████████| 116/116 [42:41:25<00:00, 1324.87s/it]
```





Summarize BHAR’ s of all events

```
# Create (eventid, roleid) multiindex for table of all BHAR's
df = eventstudy.read_summary(model='sbhar')\
               .set_index('label')\
               .drop(columns=['rho', 'tau', 'created'])
df = df[df['effective'] > 400].sort_values('post_t')  # sorted by post drift t-stat
multiIndex = DataFrame(df.index.str.split('_').to_list()).astype(int)
df.index = pd.MultiIndex.from_frame(multiIndex, names=['eventid', 'roleid'])
df['event'] = keydev.event[df.index.get_level_values(0)].values
df['role'] = keydev.role[df.index.get_level_values(1)].values
print("Post-Announcement Drift")
pd.set_option('display.max_rows', None)
df.set_index(['event', 'role']).drop(columns=['model'])
```

```
Post-Announcement Drift
```

```
                                                              beg       end  \
event                                        role
Shelf Registration Filings                   Target      20020102  20241127
Special/Extraordinary Shareholders Meeting   Target      20040106  20241127
Name Changes                                 Target      20020430  20241121
Changes in Company Bylaws/Rules              Target      20020430  20241127
Product-Related Announcements                Target      20020102  20241127
M&A Transaction Closings                     Target      20020108  20241125
Auditor Changes                              Target      20020103  20241127
Executive/Board Changes - Other              Target      20020101  20241127
Executive Changes - CFO                      Target      20020102  20241127
Business Expansions                          Target      20020102  20241127
Special Calls                                Target      20030211  20241127
Executive Changes - CEO                      Target      20020102  20241127
M&A Transaction Cancellations                Buyer       20020107  20241122
Client Announcements                         Target      20020102  20241127
Follow-on Equity Offerings                   Target      20020102  20241127
```

(continues on next page)

(continued from previous page)

```
Investor Activism - Target Communication    Target       20020402  20241127
Earnings Calls                              Target       20020108  20241127
Strategic Alliances                         Target       20020102  20241127
Lawsuits & Legal Issues                     Target       20020101  20241127
Investor Activism - Activist Communication  Target       20020102  20241126
Corporate Guidance - Lowered                Target       20020102  20241126
Private Placements                          Buyer        20020103  20241127
M&A Transaction Announcements               Seller       20020101  20241127
M&A Transaction Closings                    Buyer        20020101  20241127
Company Conference Presentations            Target       20041101  20241127
M&A Calls                                   Target       20031027  20241125
Index Constituent Adds                      Target       20020102  20241127
Buyback Transaction Closings                Target       20020103  20241123
M&A Rumors and Discussions                  Target       20020918  20241127
Shareholder/Analyst Calls                   Target       20020108  20241127
Labor-related Announcements                 Target       20020111  20241121
Business Reorganizations                    Target       20020102  20241122
Conferences                                 Participant  20070125  20241127
M&A Transaction Announcements               Buyer        20020101  20241127
Delayed SEC Filings                         Target       20020102  20241127
Earnings Release Date                       Target       20020411  20241127
M&A Transaction Closings                    Seller       20020101  20241127
M&A Transaction Announcements               Target       20020102  20241126
Discontinued Operations/Downsizings         Target       20020102  20241127
Fixed Income Offerings                      Target       20020101  20241127
Corporate Guidance - New/Confirmed          Target       20020101  20241127
Private Placements                          Target       20020103  20241127
Seeking Acquisitions/Investments            Target       20020103  20241125
Board Meeting                               Target       20020204  20241120
Considering Multiple Strategic Alternatives Target       20040324  20241122
Announcements of Sales/Trading Statement    Target       20020102  20241119
Debt Financing Related                      Target       20020102  20241127
Annual General Meeting                      Target       20020122  20241127
Seeking to Sell/Divest                      Target       20020110  20241125
End of Lock-Up Period                       Target       20020115  20241126
Buyback Tranche Update                      Target       20020107  20241127
Announcements of Earnings                   Target       20020101  20241127
Analyst/Investor Day                        Target       20040204  20241126
Investor Activism - Proxy/Voting Related    Target       20020107  20241126
Delistings                                  Target       20020102  20241127
Impairments/Write Offs                      Target       20020408  20241126
Corporate Guidance - Raised                 Target       20020103  20241126
Index Constituent Drops                     Target       20020107  20241126
Buyback - Change in Plan Terms              Target       20020102  20241126
Buyback Transaction Announcements           Target       20020102  20241126

                                                           rows  days  \
event                                       role
Shelf Registration Filings                  Target        46949  5666
Special/Extraordinary Shareholders Meeting  Target         6183  3353
Name Changes                                Target         1563  1329
Changes in Company Bylaws/Rules             Target        27834  4825
Product-Related Announcements               Target       209897  5767
M&A Transaction Closings                    Target         1508  1234
Auditor Changes                             Target         9563  3888
Executive/Board Changes - Other             Target       222258  5766
```

(continues on next page)

(continued from previous page)

```
Executive Changes - CFO                      Target        22715  5544
Business Expansions                          Target        58224  5709
Special Calls                                Target        16738  4501
Executive Changes - CEO                      Target        17079  5269
M&A Transaction Cancellations                Buyer          1508  1258
Client Announcements                         Target       212795  5761
Follow-on Equity Offerings                   Target        21917  5219
Investor Activism - Target Communication     Target         5248  3059
Earnings Calls                               Target       236084  5442
Strategic Alliances                          Target        29517  5463
Lawsuits & Legal Issues                      Target        41158  5599
Investor Activism - Activist Communication   Target         8556  3875
Corporate Guidance - Lowered                 Target        10996  3606
Private Placements                           Buyer         11631  4362
M&A Transaction Announcements                Seller        12325  4804
M&A Transaction Closings                     Buyer         43177  5699
Company Conference Presentations             Target       339716  4975
M&A Calls                                    Target         7218  3305
Index Constituent Adds                       Target        29372  2248
Buyback Transaction Closings                 Target        13012  3913
M&A Rumors and Discussions                   Target        21747  4605
Shareholder/Analyst Calls                    Target        24420  4417
Labor-related Announcements                  Target         3975  2485
Business Reorganizations                     Target         4398  2766
Conferences                                  Participant  296636  3676
M&A Transaction Announcements                Buyer         25352  5501
Delayed SEC Filings                          Target        13297  2078
Earnings Release Date                        Target       226653  5294
M&A Transaction Closings                     Seller        18467  5138
M&A Transaction Announcements                Target         6089  3557
Discontinued Operations/Downsizings          Target        18782  4447
Fixed Income Offerings                       Target        28237  5403
Corporate Guidance - New/Confirmed           Target       148835  5649
Private Placements                           Target        15216  5057
Seeking Acquisitions/Investments             Target        37787  5239
Board Meeting                                Target         6563  2705
Considering Multiple Strategic Alternatives Target         4017  2491
Announcements of Sales/Trading Statement     Target        12367  3306
Debt Financing Related                       Target        41158  5708
Annual General Meeting                       Target        71528  5094
Seeking to Sell/Divest                       Target         5250  2922
End of Lock-Up Period                        Target         7107  3293
Buyback Tranche Update                       Target       113233  5073
Announcements of Earnings                    Target       343641  5712
Analyst/Investor Day                         Target         6664  3353
Investor Activism - Proxy/Voting Related     Target         7449  3165
Delistings                                   Target        17906  4928
Impairments/Write Offs                       Target        23529  3527
Corporate Guidance - Raised                  Target        20044  4162
Index Constituent Drops                      Target        18202  3039
Buyback - Change in Plan Terms               Target         6897  3412
Buyback Transaction Announcements            Target        20353  4996

                                                          effective    window  \
event                                        role
Shelf Registration Filings                   Target          681.0 -0.005224
```

(continues on next page)

(continued from previous page)

```
Special/Extraordinary Shareholders Meeting  Target          534.0 -0.002029
Name Changes                                Target          454.0  0.012626
Changes in Company Bylaws/Rules             Target          592.0 -0.000070
Product-Related Announcements               Target          686.0  0.005712
M&A Transaction Closings                    Target          430.0  0.010726
Auditor Changes                             Target          609.0 -0.003402
Executive/Board Changes - Other             Target          685.0 -0.000533
Executive Changes - CFO                     Target          677.0 -0.007618
Business Expansions                         Target          688.0  0.002006
Special Calls                               Target          597.0  0.014764
Executive Changes - CEO                     Target          671.0 -0.005253
M&A Transaction Cancellations               Buyer           455.0 -0.000106
Client Announcements                        Target          687.0  0.008237
Follow-on Equity Offerings                  Target          667.0 -0.030227
Investor Activism - Target Communication    Target          525.0  0.008690
Earnings Calls                              Target          654.0  0.000499
Strategic Alliances                         Target          679.0  0.010685
Lawsuits & Legal Issues                     Target          682.0 -0.000897
Investor Activism - Activist Communication  Target          576.0  0.014757
Corporate Guidance - Lowered                Target          583.0 -0.066909
Private Placements                          Buyer           645.0  0.000440
M&A Transaction Announcements               Seller          662.0  0.009878
M&A Transaction Closings                    Buyer           687.0  0.004541
Company Conference Presentations            Target          592.0  0.001272
M&A Calls                                   Target          566.0  0.059589
Index Constituent Adds                      Target          517.0 -0.004618
Buyback Transaction Closings                Target          620.0  0.003593
M&A Rumors and Discussions                  Target          584.0  0.013315
Shareholder/Analyst Calls                   Target          585.0  0.001265
Labor-related Announcements                 Target          485.0  0.001723
Business Reorganizations                    Target          519.0 -0.001773
Conferences                                 Participant     451.0 -0.000775
M&A Transaction Announcements               Buyer           682.0  0.010533
Delayed SEC Filings                         Target          519.0 -0.017189
Earnings Release Date                       Target          632.0 -0.000074
M&A Transaction Closings                    Seller          672.0  0.003516
M&A Transaction Announcements               Target          622.0  0.234380
Discontinued Operations/Downsizings         Target          594.0 -0.009840
Fixed Income Offerings                      Target          677.0 -0.004932
Corporate Guidance - New/Confirmed          Target          687.0 -0.001445
Private Placements                          Target          657.0  0.023461
Seeking Acquisitions/Investments            Target          668.0 -0.000697
Board Meeting                               Target          473.0  0.002824
Considering Multiple Strategic Alternatives Target          479.0 -0.008435
Announcements of Sales/Trading Statement    Target          588.0  0.004235
Debt Financing Related                      Target          684.0  0.004347
Annual General Meeting                      Target          620.0  0.001460
Seeking to Sell/Divest                      Target          504.0 -0.002623
End of Lock-Up Period                       Target          565.0 -0.060075
Buyback Tranche Update                      Target          646.0  0.000906
Announcements of Earnings                   Target          685.0  0.000446
Analyst/Investor Day                        Target          548.0  0.008374
Investor Activism - Proxy/Voting Related    Target          530.0 -0.001100
Delistings                                  Target          625.0 -0.020833
Impairments/Write Offs                      Target          521.0 -0.003683
Corporate Guidance - Raised                 Target          631.0  0.033832
```

(continues on next page)

(continued from previous page)

```
Index Constituent Drops                      Target          507.0 -0.003266
Buyback - Change in Plan Terms               Target          605.0  0.009150
Buyback Transaction Announcements            Target          666.0  0.015343

                                                          window_t      post  \
event                                        role
Shelf Registration Filings                   Target      -2.542320 -0.012755
Special/Extraordinary Shareholders Meeting   Target      -0.537886 -0.018210
Name Changes                                 Target       1.944510 -0.023076
Changes in Company Bylaws/Rules              Target      -0.024033 -0.010242
Product-Related Announcements                Target       4.608120 -0.004112
M&A Transaction Closings                     Target       1.184190 -0.019140
Auditor Changes                              Target      -1.173250 -0.009535
Executive/Board Changes - Other              Target      -0.537347 -0.003171
Executive Changes - CFO                      Target      -3.312440 -0.006563
Business Expansions                          Target       1.663570 -0.003755
Special Calls                                Target       1.100980 -0.014743
Executive Changes - CEO                      Target      -1.572760 -0.007032
M&A Transaction Cancellations                Buyer       -0.019428 -0.010588
Client Announcements                         Target       7.380600 -0.002300
Follow-on Equity Offerings                   Target      -5.005380 -0.007275
Investor Activism - Target Communication     Target       2.362150 -0.004881
Earnings Calls                               Target       0.481335 -0.002042
Strategic Alliances                          Target       2.626260 -0.003538
Lawsuits & Legal Issues                      Target      -0.413061 -0.002723
Investor Activism - Activist Communication   Target       4.288130 -0.004063
Corporate Guidance - Lowered                 Target     -14.887900 -0.002680
Private Placements                           Buyer        0.272817 -0.002208
M&A Transaction Announcements                Seller       2.630630 -0.002869
M&A Transaction Closings                     Buyer        3.186290 -0.001394
Company Conference Presentations             Target       1.292770 -0.001116
M&A Calls                                    Target       8.955770 -0.002523
Index Constituent Adds                       Target      -1.557500 -0.002675
Buyback Transaction Closings                 Target       1.725140 -0.001955
M&A Rumors and Discussions                   Target       4.876590 -0.001790
Shareholder/Analyst Calls                    Target       0.428922 -0.002059
Labor-related Announcements                  Target       0.546605 -0.002090
Business Reorganizations                     Target      -0.381681 -0.002317
Conferences                                  Participant -0.643617 -0.000928
M&A Transaction Announcements                Buyer        3.253760 -0.001108
Delayed SEC Filings                          Target      -3.797660 -0.001306
Earnings Release Date                        Target      -0.071140 -0.000190
M&A Transaction Closings                     Seller       1.461850 -0.000303
M&A Transaction Announcements                Target      18.265800 -0.000214
Discontinued Operations/Downsizings          Target      -2.844410 -0.000069
Fixed Income Offerings                       Target      -2.922310  0.000168
Corporate Guidance - New/Confirmed           Target      -0.883251  0.000366
Private Placements                           Target       4.328510  0.002351
Seeking Acquisitions/Investments             Target      -0.305334  0.001316
Board Meeting                                Target       0.980749  0.002164
Considering Multiple Strategic Alternatives Target      -1.135750  0.012629
Announcements of Sales/Trading Statement     Target       1.329410  0.005084
Debt Financing Related                       Target       2.013270  0.002883
Annual General Meeting                       Target       0.950287  0.004452
Seeking to Sell/Divest                       Target      -0.531814  0.006912
End of Lock-Up Period                        Target      -9.088620  0.007541
```

(continues on next page)

(continued from previous page)

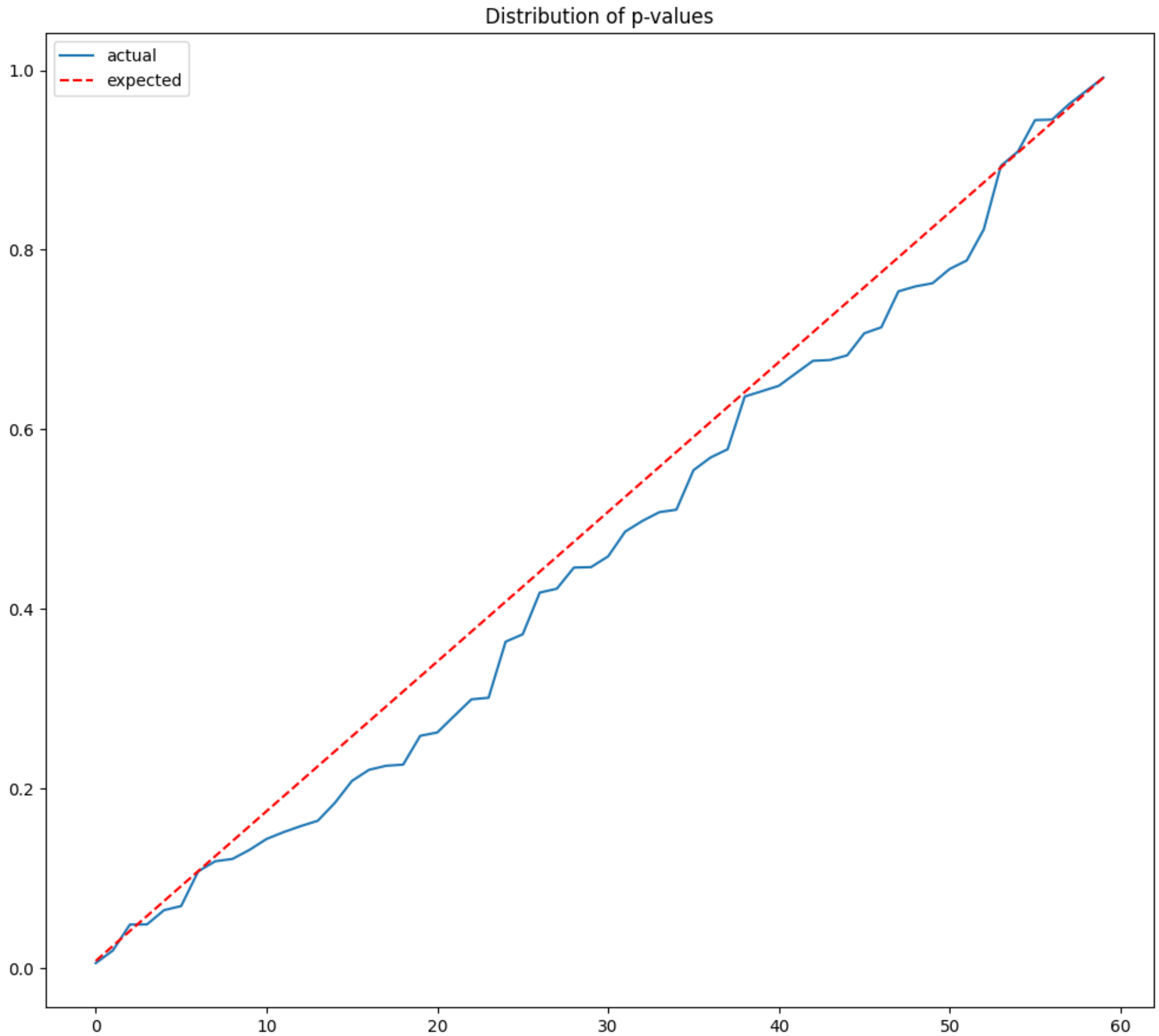
```
Buyback Tranche Update                      Target        0.559217  0.002991
Announcements of Earnings                   Target        0.305990  0.002854
Analyst/Investor Day                        Target        3.238440  0.006292
Investor Activism - Proxy/Voting Related    Target       -0.472323  0.011292
Delistings                                  Target       -3.871270  0.016593
Impairments/Write Offs                      Target       -1.122100  0.008273
Corporate Guidance - Raised                 Target       11.737600  0.006226
Index Constituent Drops                     Target       -0.751127  0.017488
Buyback - Change in Plan Terms              Target        3.837660  0.008018
Buyback Transaction Announcements           Target        7.037260  0.008389

                                                           post_t
event                                       role
Shelf Registration Filings                  Target      -2.753130
Special/Extraordinary Shareholders Meeting  Target      -1.970180
Name Changes                                Target      -1.968600
Changes in Company Bylaws/Rules             Target      -1.815650
Product-Related Announcements               Target      -1.607030
M&A Transaction Closings                    Target      -1.506510
Auditor Changes                             Target      -1.433420
Executive/Board Changes - Other             Target      -1.410500
Executive Changes - CFO                     Target      -1.390520
Business Expansions                         Target      -1.327890
Special Calls                               Target      -1.257180
Executive Changes - CEO                     Target      -1.120140
M&A Transaction Cancellations               Buyer       -1.037350
Client Announcements                        Target      -0.908251
Follow-on Equity Offerings                  Target      -0.892820
Investor Activism - Target Communication    Target      -0.801679
Earnings Calls                              Target      -0.761636
Strategic Alliances                         Target      -0.741016
Lawsuits & Legal Issues                     Target      -0.677482
Investor Activism - Activist Communication  Target      -0.657731
Corporate Guidance - Lowered                Target      -0.590799
Private Placements                          Buyer       -0.569800
M&A Transaction Announcements               Seller      -0.472392
M&A Transaction Closings                    Buyer       -0.464066
Company Conference Presentations            Target      -0.455673
M&A Calls                                   Target      -0.436285
Index Constituent Adds                      Target      -0.417274
Buyback Transaction Closings                Target      -0.416104
M&A Rumors and Discussions                  Target      -0.409002
Shareholder/Analyst Calls                   Target      -0.313663
Labor-related Announcements                 Target      -0.306394
Business Reorganizations                    Target      -0.301770
Conferences                                 Participant -0.281036
M&A Transaction Announcements               Buyer       -0.268724
Delayed SEC Filings                         Target      -0.113672
Earnings Release Date                       Target      -0.069660
M&A Transaction Closings                    Seller      -0.069015
M&A Transaction Announcements               Target      -0.028792
Discontinued Operations/Downsizings         Target      -0.010337
Fixed Income Offerings                      Target       0.047205
Corporate Guidance - New/Confirmed          Target       0.133957
Private Placements                          Target       0.223887
Seeking Acquisitions/Investments            Target       0.366793
```

(continues on next page)

(continued from previous page)

```
Board Meeting                                  Target        0.375795
Considering Multiple Strategic Alternatives Target        0.556299
Announcements of Sales/Trading Statement       Target        0.662073
Debt Financing Related                         Target        0.696258
Annual General Meeting                         Target        0.760826
Seeking to Sell/Divest                         Target        0.809188
End of Lock-Up Period                          Target        1.033670
Buyback Tranche Update                         Target        1.077510
Announcements of Earnings                      Target        1.128810
Analyst/Investor Day                           Target        1.208400
Investor Activism - Proxy/Voting Related       Target        1.211770
Delistings                                     Target        1.223670
Impairments/Write Offs                         Target        1.460700
Corporate Guidance - Raised                    Target        1.547110
Index Constituent Drops                        Target        1.557680
Buyback - Change in Plan Terms                 Target        1.846240
Buyback Transaction Announcements              Target        2.327460
```

```
# Expected p-values (with continuity correction)
pvals = norm.cdf(-df['post_t'].abs()) * 2
fig, ax = plt.subplots(1, 1, clear=True, figsize=(10, 9))
ax.plot(sorted(pvals))
ax.plot([0, len(pvals)-1], [0.5/len(pvals), (len(pvals)-0.5)/len(pvals)], 'r--')
ax.set_title('Distribution of p-values')
ax.legend(['actual', 'expected'])
plt.tight_layout()
```




```
# Number of rejections with uncorrected pvals
#argmin = np.argmin(pvals)
#header = df.iloc[argmin][['event', 'role', 'days', 'effective', 'post_t']]
alpha = 0.05
uncorrected = DataFrame({'rejections': sum(pvals < alpha),
                         'min p-value': min(pvals)},
                        index=['uncorrected'])
uncorrected.round(4)
```

```
             rejections  min p-value
uncorrected           4       0.0059
```

7.3.1 Bonferroni correction

The best-known FWER test is called the Bonferroni test which adjusts for the multiple tests. Given the chance that one test could randomly show up as significant, the Bonferroni requires the confidence level to increase. Instead of 5%, you take the 5% and divide by the number of tests, that is, 5%/10 = 0.5%. Again equivalently, you need to be 99.5% confident with 10 tests that you are not making a single false discovery. In terms of the t-statistic, the Bonferroni requires a statistic of at least 2.8 for 10 tests. For 1,000 tests, the statistic must exceed 4.1.

It sets the threshold for rejecting each hypothesis to α/m, by applying the union bound inequality:

$$\text{FWER}(\alpha) = \Pr(\cup_{j=1}^{m} A_j) \leq \sum_{j=1}^{m} \Pr(A_j) \leq m \times \frac{\alpha}{m} = \alpha$$

where A_j denotes the probability of rejecting the j-th hypothesis. The Bonferroni correction can be quite conservative, in the sense that the true FWER is often quite a bit lower than the nominal (or target) FWER;

```
# The Bonferroni p-values bonf are simply the uncorrected pvalues multiplied by
# the number of samples and truncated to be less than or equal to 1.
reject, bonf_corrected, _, _ =  multipletests(pvals, alpha=alpha, method='bonferroni')
bonf = DataFrame({'rejections': np.sum(bonf_corrected < alpha),
                  'min p-value': np.min(bonf_corrected)},
                 index=['bonferroni'])
pd.concat([uncorrected, bonf], axis=0).round(4)
```

```
             rejections  min p-value
uncorrected           4       0.0059
bonferroni            0       0.3542
```

7.3.2 Holm's step-down procedure

Holm's method, also known as Holm's step-down procedure or the Holm-Bonferroni method, is an alternative to the Bonferroni procedure. Holm's method method controls the FWER, but it is less conservative than Bonferroni, in the sense that it will reject more null hypotheses, typically resulting in fewer Type II errors and hence greater power. Holm's method makes no independence assumptions about the hypothesis tests, and is uniformly more powerful than the Bonferroni method –it will always reject at least as many null hypotheses as Bonferroni–and so it should always be preferred. It is worth noting that in Holm's procedure, the threshold that we use to reject each null hypothesis actually depends on the values of all the p-values.

The Holm method begins by sorting the tests from the lowest p-value (most significant) to the highest (least significant), and comparing a threshold computed with the Holm function. In contrast to the Bonferroni, which has a single threshold for all tests, the other tests will have a different hurdle under Holm. Starting from the first test, we sequentially compare the p-values with their hurdles. When we first come across the test such that its p-value fails to meet the hurdle, we reject this test and all others with higher p-values.

```
# Holm method
reject, holm_corrected, _, _ =  multipletests(pvals, alpha=alpha, method='holm')
holm = DataFrame({'rejections': np.sum(holm_corrected < alpha),
                  'min p-value': np.min(holm_corrected)},
                 index=['holm'])
pd.concat([uncorrected, bonf, holm], axis=0).round(4)
```

```
             rejections  min p-value
uncorrected           4       0.0059
```

(continues on next page)

(continued from previous page)

```
bonferroni           0       0.3542
holm                 0       0.3542
```

7.3.3 Benjamin-Hochberg procedure

The false discovery rate approach allows an expected proportional error rate. As such, it is less stringent than both the Bonferroni and the Holm test. FDR control is much milder –and more powerful –than FWER control, in the sense that it allows us to reject many more null hypotheses, with a cost of substantially more false positives.

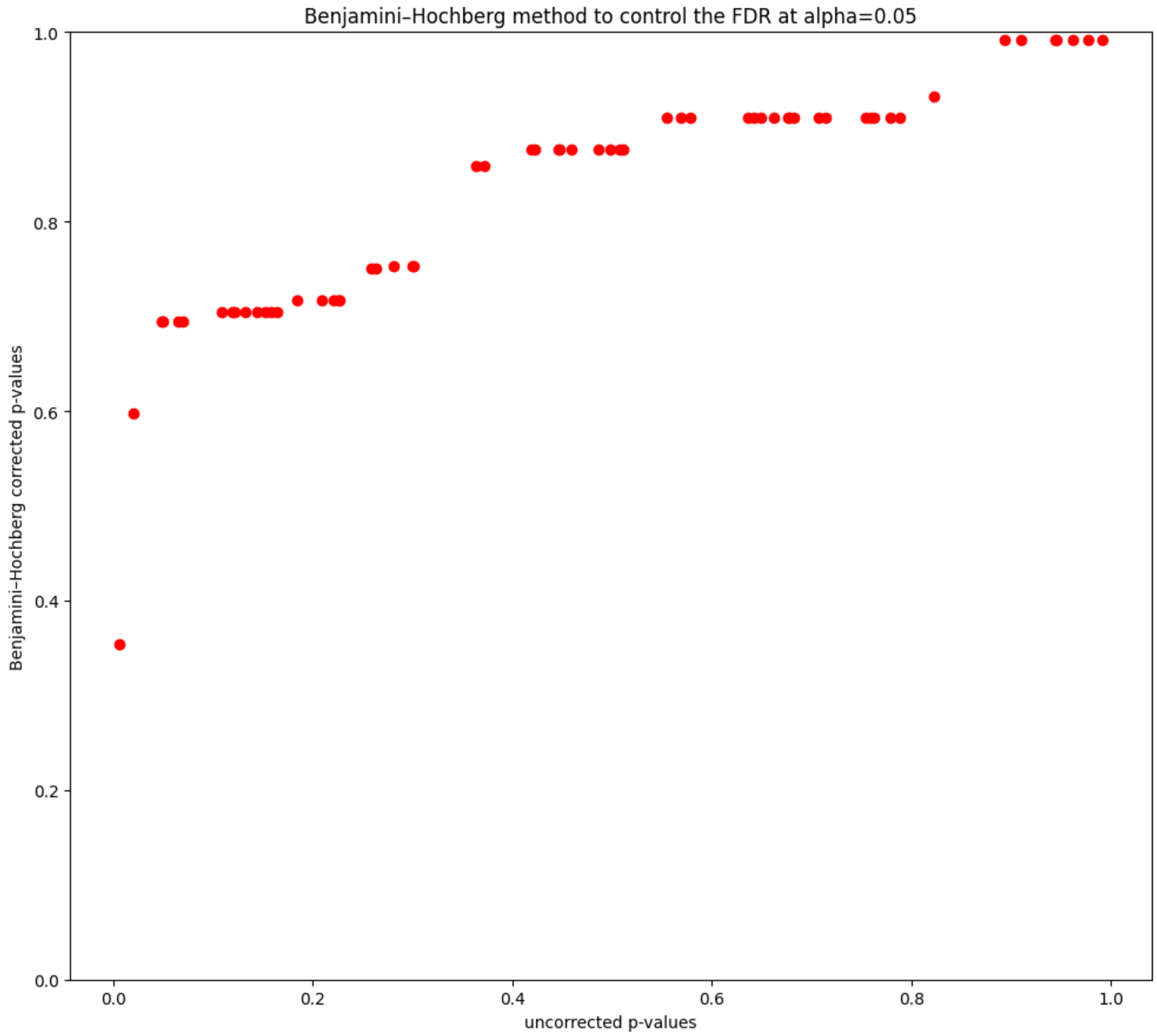
Similar to the Holm test, Benjamin-Hochberg also relies on the distribution of test statistics. However, in contrast to the Holm test that begins with the most significant test, the Benjamin-Hochberg approach starts with the least significant. We sort the tests from the lowest p-value (most significant) to the highest (least significant). Starting from the last test, we sequentially compare the p-values with their Benjamin-Hochberg thresholds. When we first come across the test such that its p-value falls below its threshold, we declare this test significant and all tests that have a lower p-value.

The `multipletests` function can be used to carry out the Benjamini–Hochberg procedure. The q-values output by the Benjamini–Hochberg procedure can be interpreted as the smallest FDR threshold at which we would reject a particular null hypothesis. For instance, a q-value of 0.1 indicates that we can reject the corresponding null hypothesis at an FDR of 10% or greater, but that we cannot reject the null hypothesis at an FDR below 10%.

```python
# Benjamini-Hochberg method
reject, fdr_bh_corrected, _, _ =  multipletests(pvals, alpha=alpha, method='fdr_bh')
fdr_bh = DataFrame({'rejections': np.sum(fdr_bh_corrected < alpha),
                    'min p-value': np.min(fdr_bh_corrected)},
                   index=['fdr_bh'])
pd.concat([uncorrected, bonf, holm, fdr_bh], axis=0).round(4)
```

```
             rejections  min p-value
uncorrected           4       0.0059
bonferroni            0       0.3542
holm                  0       0.3542
fdr_bh                0       0.3542
```

```python
# Plot uncorrected and corrected p-values from Benjamini-Hochberg method
fig, ax = plt.subplots(figsize=(10, 9))
ax.scatter(pvals, fdr_bh_corrected, color='red')
ax.set_ylim(bottom=0, top=1)
ax.set_title(f"Benjamini-Hochberg method to control the FDR at alpha={alpha}")
ax.set_xlabel('uncorrected p-values')
ax.set_ylabel('Benjamini-Hochberg corrected p-values')
plt.tight_layout()
```



References

Kolari, James W. and Pynnonen, Seppo, 2010, "Event Study Testing with Cross-sectional Correlation of Abnormal Returns". The Review of Financial Studies, 23(11), 3996-4025

Kolari, James W., Paper, Bernd, and Pynnonen, Seppo, 2018, "Event Study Testing with Cross-sectional Correlation Due to Partially Overlapping Event Windows", working paper

MacKinlay, A. Craig "Event Studies in Economics and Finance", Journal of Economic Literature, March 1997.

CHAPTER

EIGHT

ECONOMIC INDICATORS

What we learn from history is that people don' t learn from history - Warren Buffett

Economic data is fundamental to financial analysis, policymaking, and investment strategies. However, many economic indicators are subject to revisions, meaning initial estimates may change over time as more accurate data becomes available. Understanding these revisions is crucial for interpreting past economic conditions, refining forecasting models, and making informed decisions. We explore retrieving data from online sources such as the Federal Reserve Economic Data (FRED), its archival counterpart (ALFRED), and key derived datasets such as FRED-MD and FRED-QD. Additionally, we examine the impact of data revisions on critical economic indicators like Total Nonfarm Payrolls (PAYEMS), and methods for detecting outliers in historical data.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import textwrap
from finds.readers import Alfred, fred_md, fred_qd
from finds.utils import plot_date, plot_groupbar
from finds.recipes import is_outlier
from datetime import datetime
from pprint import pprint
from secret import credentials
VERBOSE = 0
# %matplotlib qt
```

8.1 FRED

Federal Reserve Economic Data (FRED) is a widely used online database maintained by the Federal Reserve Bank of St. Louis, providing access to hundreds of thousands of economic data series from national and international sources. Users can retrieve data via the website, an Excel add-in, or API calls.

8.2 Retrieving data from websites

Economic data can be retrieved from the web through several methods:

1. **Downloading structured files** –Many websites provide data in formats like CSV, Excel, or JSON, making it easy to import into analytical tools.
2. **Web scraping** –Extracting information directly from web pages by identifying specific HTML tags or text patterns.
3. **Using APIs** –Some platforms, including FRED, offer APIs that allow developers to automate data retrieval via structured queries.

8.2.1 Download structured files

Many economic data providers allow users to download pre-structured files containing historical and current data. These files often include metadata, timestamps, and adjustment information.

```
# This URL is the location of the FRED-MD csv file from the St Louis FRED
url = 'https://www.stlouisfed.org/-/media/project/frbstl/stlouisfed/research/fred-md/
↪monthly/current.csv'
```

```
# Pandas has several built-in readers for csv, xml, json, excel and even html files ␣
↪
df = pd.read_csv(url, header=0)
df
```

```
        sasdate        RPI  W875RX1  DPCERA3M086SBEA     CMRMTSPLx  \
0    Transform:      5.000      5.0            5.000  5.000000e+00
1      1/1/1959   2583.560   2426.0           15.188  2.766768e+05
2      2/1/1959   2593.596   2434.8           15.346  2.787140e+05
3      3/1/1959   2610.396   2452.7           15.491  2.777753e+05
4      4/1/1959   2627.446   2470.0           15.435  2.833627e+05
..          ...        ...      ...              ...           ...
788    8/1/2024  20007.209  16322.1          121.052  1.530317e+06
789    9/1/2024  20044.142  16333.7          121.690  1.541305e+06
790   10/1/2024  20128.752  16397.9          121.948  1.539382e+06
791   11/1/2024  20161.687  16432.8          122.519  1.544190e+06
792   12/1/2024  20184.060  16457.8          123.013           NaN

          RETAILx    INDPRO   IPFPNSS   IPFINAL   IPCONGD  ...  \
0         5.00000    5.0000    5.0000    5.0000    5.0000  ...
1     18235.77392   21.9616   23.3868   22.2620   31.6664  ...
2     18369.56308   22.3917   23.7024   22.4549   31.8987  ...
3     18523.05762   22.7142   23.8459   22.5651   31.8987  ...
4     18534.46600   23.1981   24.1903   22.8957   32.4019  ...
..            ...       ...       ...       ...       ...  ...
```

(continues on next page)

(continued from previous page)

```
788  710038.00000  103.0135  100.9825  100.9803  102.2118  ...
789  716388.00000  102.5969  100.3826  100.0630  101.9696  ...
790  720393.00000  102.0854   99.5434   98.9267  101.3127  ...
791  725925.00000  102.2549   99.8216   99.4970  101.7893  ...
792  729191.00000  103.1942  100.5351  100.1302  102.2582  ...

     DNDGRG3M086SBEA  DSERRG3M086SBEA  CES0600000008  CES2000000008  \
0              6.000            6.000           6.00           6.00   
1             18.294           10.152           2.13           2.45   
2             18.302           10.167           2.14           2.46   
3             18.289           10.185           2.15           2.45   
4             18.300           10.221           2.16           2.47   
..               ...              ...            ...            ...   
788          119.653          128.291          31.26          35.81   
789          119.220          128.682          31.44          36.00   
790          119.064          129.169          31.55          36.22   
791          119.112          129.375          31.61          36.21   
792          119.689          129.760          31.73          36.44   

     CES3000000008  UMCSENTx  DTCOLNVHFNM   DTCTHFNM     INVEST  VIXCLSx  
0             6.00       2.0         6.00       6.00     6.0000   1.0000  
1             2.04       NaN      6476.00   12298.00    84.2043      NaN  
2             2.05       NaN      6476.00   12298.00    83.5280      NaN  
3             2.07       NaN      6508.00   12349.00    81.6405      NaN  
4             2.08       NaN      6620.00   12484.00    81.8099      NaN  
..             ...       ...          ...        ...        ...      ...  
788          27.97      67.9    551667.22  933066.90  5327.6461  19.6750  
789          28.11      70.1    553347.06  934283.59  5368.5818  17.6597  
790          28.14      70.5    554377.25  937299.96  5407.3304  19.9478  
791          28.29      71.8    555000.61  938899.31  5382.4019  15.9822  
792          28.34      74.0          NaN        NaN  5370.6184  15.6997  

[793 rows x 127 columns]
```

8.2.2 Web scraping

Web scraping involves extracting data from unstructured web pages by identifying patterns in the HTML structure. This method is useful when structured data files are unavailable, but it requires compliance with website policies.

```
# URL that displays the most popular series in the FRED economic data web site
url = f"https://fred.stlouisfed.org/tags/series?ob=pv&pageID=1"
```

```
# use requests package to retrieve the web page
import requests
data = requests.get(url)
data    # a response code of 200 indicates the request has succeeded
```

```
<Response [200]>
```

```
# the content is just a byte-string that you can parse with Python string (or other)␣
↪methods
data.content[:200]
```

```
b'<!DOCTYPE html>\n<html lang="en">\n<head>\n    <meta http-equiv="X-UA-Compatible
↪" content="IE=edge">\n    <meta charset="utf-8">\n                <title>
↪Economic Data Series by Tag | FRED | St. Louis Fed</'
```

```python
# use the BeautifulSoup package to parse html formats
from bs4 import BeautifulSoup
soup = BeautifulSoup(data.content, 'lxml')

# based on this snippet, we want to extract the href property of the series-title␣
↪class tag
print(soup.decode()[39000:40000])
```

```
s="series-title pager-series-title-gtm" href="/series/T10Y2Y" id="titleLink" style=
↪"font-size:1.2em; padding-bottom: 2px">10-Year Treasury Constant Maturity Minus␣
↪2-Year Treasury Constant Maturity</a></h3>
</div>
<div class="display-results-popularity-bar d-none d-sm-block col-sm-2">
<span aria-label="popularity 100% popular" class="popularity-bar-span-parent" data-
↪target="popularity-bar-span-T10Y2Y" tabindex="0" title="100% popular">
<span aria-hidden="true" class="popularity_bar" style="padding-top: 3px; padding-
↪left:60px;"> </span> <span aria-hidden="true" class="popularity_bar_background"␣
↪id="popularity-bar-span-T10Y2Y"> </span></span>
</div>
</td>
</tr>
<tr class="series-pager-attr">
<td colspan="2">
<div class="series-meta series-group-meta">
<span class="attributes">Percent, Not Seasonally Adjusted</span>
<br class="clear"/>
</div>
<div class="series-meta">
<input aria-labelledby="unitLinkT10Y2Y" class="pager-item-checkbox pager-check-
↪series-gtm" name="sids[0]" type="checkbox" v
```

```python
# identify all the tags whose class starts with 'series-title'
tags = soup.findAll(name='a', attrs={'class': 'series-title'})
tags[0]    # show first tag found
```

```
<a class="series-title pager-series-title-gtm" href="/series/T10Y2Y" id="titleLink
↪" style="font-size:1.2em; padding-bottom: 2px">10-Year Treasury Constant␣
↪Maturity Minus 2-Year Treasury Constant Maturity</a>
```

```python
# extract desired substring (which is a data series mnemonic) from the href property
details = [tag.get('href').split('/')[-1] for tag in tags]  # only want substring␣
↪after last '/'
details[0]  # show first mnemonic string found
```

```
'T10Y2Y'
```

8.2.3 Using APIs

APIs (Application Programming Interfaces) enable direct communication with data servers, allowing for real-time data retrieval. Many economic research institutions, including the St Louis Fed, offer APIs to access macroeconomic data programmatically.

```
# an API call is simply a URL string containing your parameters for the request
url = "{root}?series_id={series_id}&file_type={file_type}&api_key={api_key}".format(
    root="https://api.stlouisfed.org/fred/series", # base url of the API call
    series_id=details[0],                          # mnemonic of the data series to␣
↪retrieve
    file_type='json',                              # request data be returned in json␣
↪format
    api_key=credentials['fred']['api_key'])        # private api key (obtain from␣
↪FRED for free)
```

```
# make the API call to retrieve the data
data = requests.get(url)
data.content
```


```
b'{"realtime_start":"2025-02-28","realtime_end":"2025-02-28","seriess":[{"id":
↪"T10Y2Y","realtime_start":"2025-02-28","realtime_end":"2025-02-28","title":"10-
↪Year Treasury Constant Maturity Minus 2-Year Treasury Constant Maturity",
↪"observation_start":"1976-06-01","observation_end":"2025-02-28","frequency":
↪"Daily","frequency_short":"D","units":"Percent","units_short":"%","seasonal_
↪adjustment":"Not Seasonally Adjusted","seasonal_adjustment_short":"NSA","last_
↪updated":"2025-02-28 16:02:07-06","popularity":100,"notes":"Starting with the␣
↪update on June 21, 2019, the Treasury bond data used in calculating interest␣
↪rate spreads is obtained directly from the U.S. Treasury Department (https:\\/\\/
↪www.treasury.gov\\/resource-center\\/data-chart-center\\/interest-rates\\/Pages\\
↪/TextView.aspx?data=yield).\\r\\nSeries is calculated as the spread between 10-
↪Year Treasury Constant Maturity (BC_10YEAR) and 2-Year Treasury Constant␣
↪Maturity (BC_2YEAR). Both underlying series are published at the U.S. Treasury␣
↪Department (https:\\/\\/www.treasury.gov\\/resource-center\\/data-chart-center\\/
↪interest-rates\\/Pages\\/TextView.aspx?data=yield)."}]}'
```


```
# use the json package to convert byte-string data content
import json
v = json.loads(data.content)
v
```


```
{'realtime_start': '2025-02-28',
 'realtime_end': '2025-02-28',
 'seriess': [{'id': 'T10Y2Y',
   'realtime_start': '2025-02-28',
   'realtime_end': '2025-02-28',
   'title': '10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant␣
↪Maturity',
   'observation_start': '1976-06-01',
   'observation_end': '2025-02-28',
   'frequency': 'Daily',
   'frequency_short': 'D',
   'units': 'Percent',
   'units_short': '%',
```


(continues on next page)

(continued from previous page)

```
   'seasonal_adjustment': 'Not Seasonally Adjusted',
   'seasonal_adjustment_short': 'NSA',
   'last_updated': '2025-02-28 16:02:07-06',
   'popularity': 100,
   'notes': 'Starting with the update on June 21, 2019, the Treasury bond data␣
↪used in calculating interest rate spreads is obtained directly from the U.S.␣
↪Treasury Department (https://www.treasury.gov/resource-center/data-chart-center/
↪interest-rates/Pages/TextView.aspx?data=yield).\r\nSeries is calculated as the␣
↪spread between 10-Year Treasury Constant Maturity (BC_10YEAR) and 2-Year␣
↪Treasury Constant Maturity (BC_2YEAR). Both underlying series are published at␣
↪the U.S. Treasury Department (https://www.treasury.gov/resource-center/data-
↪chart-center/interest-rates/Pages/TextView.aspx?data=yield).'}]}
```

```python
# Pandas can create a DataFrame directly from a dict data structure
df = DataFrame(v['seriess'])
df
```

```
       id realtime_start realtime_end  \
0  T10Y2Y     2025-02-28   2025-02-28

                                              title observation_start  \
0  10-Year Treasury Constant Maturity Minus 2-Yea...        1976-06-01

  observation_end frequency frequency_short    units units_short  \
0      2025-02-28     Daily               D  Percent           %

       seasonal_adjustment seasonal_adjustment_short           last_updated  \
0  Not Seasonally Adjusted                       NSA  2025-02-28 16:02:07-06

   popularity                                              notes
0         100  Starting with the update on June 21, 2019, the...
```

8.2.4 ALFRED (Archival FRED)

ALFRED extends FRED's functionality by preserving historical versions of economic data. This allows researchers to track how data revisions impact economic narratives over time.

```python
today = int(datetime.today().strftime('%Y%m%d'))
alf = Alfred(api_key=credentials['fred']['api_key'], verbose=VERBOSE)
```

8.2.5 Popular FRED series

FRED organizes its data into **categories, frequencies, and seasonal adjustments**. Some of the most frequently accessed series include employment figures, inflation metrics, and GDP growth rates. A current list of the most popular FRED series can be found here.

```python
# scrape FRED most popular page
popular = {}
titles = Alfred.popular(1)
for title in titles:
```

(continues on next page)

(continued from previous page)

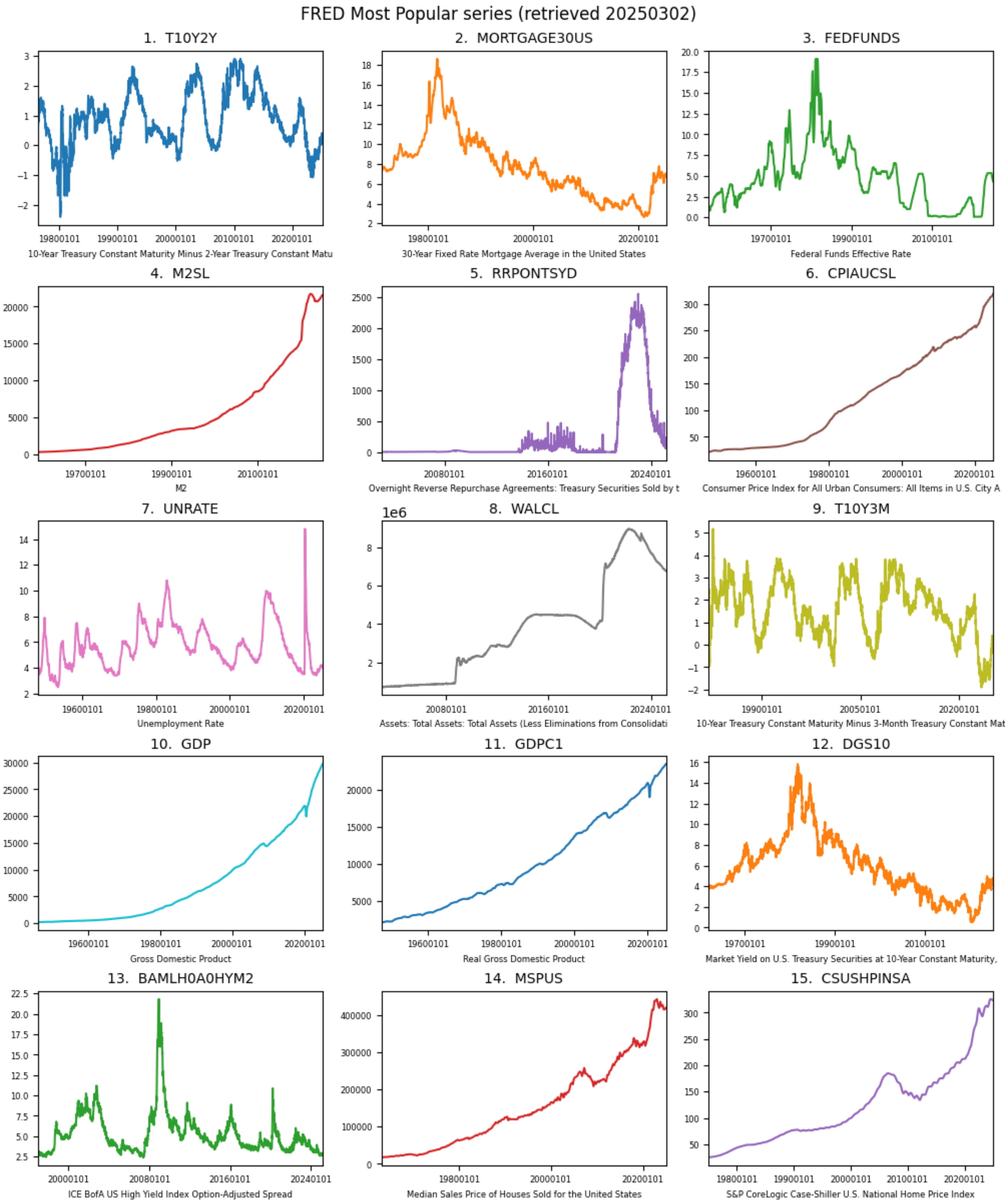
```
    series = alf.request_series(title)   # requests 'series' FRED api
    if not series.empty:
        popular[title] = series.iloc[-1][['title', 'popularity']]
print(f"Most Popular Series in FRED, retrieved {today}")
DataFrame.from_dict(popular, orient='index')
```

```
Most Popular Series in FRED, retrieved 20250302
```

	title	popularity
T10Y2Y	10-Year Treasury Constant Maturity Minus 2-Yea...	100
MORTGAGE30US	30-Year Fixed Rate Mortgage Average in the Uni...	99
FEDFUNDS	Federal Funds Effective Rate	98
M2SL	M2	93
RRPONTSYD	Overnight Reverse Repurchase Agreements: Treas...	95
CPIAUCSL	Consumer Price Index for All Urban Consumers: ...	95
UNRATE	Unemployment Rate	95
WALCL	Assets: Total Assets: Total Assets (Less Elimi...	94
T10Y3M	10-Year Treasury Constant Maturity Minus 3-Mon...	94
GDP	Gross Domestic Product	93
GDPC1	Real Gross Domestic Product	92
DGS10	Market Yield on U.S. Treasury Securities at 10...	92
BAMLH0A0HYM2	ICE BofA US High Yield Index Option-Adjusted S...	92
MSPUS	Median Sales Price of Houses Sold for the Unit...	90
CSUSHPINSA	S&P CoreLogic Case-Shiller U.S. National Home ...	88
T10YIE	10-Year Breakeven Inflation Rate	89
FPCPITOTLZGUSA	Inflation, consumer prices for the United States	85
M1SL	M1	84

```
# plot popular series
fig, axes = plt.subplots(ncols=3, nrows=5, figsize=(10, 12), layout='constrained')
for cn, (ax, title) in enumerate(zip(np.ravel(axes), titles[:15])):
    series = alf(title)
    plot_date(series, ax=ax, title=f"{cn+1}.  {title}", xlabel=alf.header(title)[:70],
              fontsize=6, ls='-', cn=cn, nbins=4)
plt.suptitle(f"FRED Most Popular series (retrieved {today})")
```

```
Text(0.5, 0.98, 'FRED Most Popular series (retrieved 20250302)')
```


FRED Most Popular series (retrieved 20250302)
1. T10Y2Y
10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant Matu
2. MORTGAGE30US
30-Year Fixed Rate Mortgage Average in the United States
3. FEDFUNDS
Federal Funds Effective Rate
4. M2SL
M2
5. RRPONTSYD
Overnight Reverse Repurchase Agreements: Treasury Securities Sold by t
6. CPIAUCSL
Consumer Price Index for All Urban Consumers: All Items in U.S. City A
7. UNRATE
Unemployment Rate
8. WALCL
1e6
Assets: Total Assets: Total Assets (Less Eliminations from Consolidati
9. T10Y3M
10-Year Treasury Constant Maturity Minus 3-Month Treasury Constant Mat
10. GDP
Gross Domestic Product
11. GDPC1
Real Gross Domestic Product
12. DGS10
Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity,
13. BAMLH0A0HYM2
ICE BofA US High Yield Index Option-Adjusted Spread
14. MSPUS
Median Sales Price of Houses Sold for the United States
15. CSUSHPINSA
S&P CoreLogic Case-Shiller U.S. National Home Price Index

8.2.6 FRED series categories

One of the most closely watched FRED series is `Total Nonfarm Payroll Employment (PAYEMS)`, a key labor market indicator. This series belongs to broader employment-related categories.

```
# Retrieve grandparent,  parent and siblings of series
series_id, freq = 'PAYEMS', 'M'
category = alf.categories(series_id).iloc[0]
grand_category = alf.get_category(category['parent_id'])
parent_category = alf.get_category(category['id'])
category.to_frame().T
```

```
           id           name parent_id
PAYEMS  32305  Total Nonfarm        11
```

```
print(f"Super category {grand_category['id']}: {grand_category['name']}")
if 'notes' in grand_category:
    print(textwrap.fill(grand_category['notes']))
```

```
Super category 11: Current Employment Statistics (Establishment Survey)
The establishment survey provides data on employment, hours, and
earnings by industry.  Numerous conceptual and methodological
differences between the current population (household) and
establishment surveys result in important distinctions in the
employment estimates derived from the surveys. Among these are:   The
household survey includes agricultural workers, the self- employed,
unpaid family workers, and private household workers among the
employed. These groups are excluded from the establishment survey.
The household survey includes people on unpaid leave among the
employed. The establishment survey does not.   The household survey is
limited to workers 16 years of age and older. The establishment survey
is not limited by age.   The household survey has no duplication of
individuals, because individuals are counted only once, even if they
hold more than one job. In the establishment survey, employees working
at more than one job and thus appearing on more than one payroll are
counted separately for each appearance.   For more information, visit
http://www.bls.gov/news.release/empsit.tn.htm.
```

```
print("Parent categories:")
for child in grand_category['children']:
    node = alf.get_category(child['id'])
    if node:
        print(f" {node['id']}: {node['name']} "
              f" (children={len(node['children'])}, series={len(node['series'])})")
```

```
Parent categories:
 32305: Total Nonfarm  (children=0, series=5)
 32306: Total Private  (children=0, series=27)
 32307: Goods-Producing  (children=0, series=27)
 32326: Service-Providing  (children=0, series=1)
 32308: Private Service-Providing  (children=0, series=27)
 32309: Mining and Logging  (children=0, series=39)
 32310: Construction  (children=0, series=41)
 32311: Manufacturing  (children=0, series=31)
```

(continues on next page)

(continued from previous page)

```
32312: Durable Goods  (children=0, series=63)
32313: Nondurable Goods  (children=0, series=55)
32314: Trade, Transportation, and Utilities  (children=0, series=27)
32315: Wholesale Trade  (children=0, series=33)
32316: Retail Trade  (children=0, series=55)
32317: Transportation and Warehousing  (children=0, series=47)
32318: Utilities  (children=0, series=27)
32319: Information  (children=0, series=39)
32320: Financial Activities  (children=0, series=51)
32321: Professional and Business Services  (children=0, series=55)
32322: Education and Health Services  (children=0, series=51)
32323: Leisure and Hospitality  (children=0, series=41)
32324: Other Services  (children=0, series=33)
32325: Government  (children=0, series=23)
```

```
print("Sibling series:")
for child in parent_category['series']:
    if child['id'] == series_id:
        node = child
    print(f"  {child['id']}: {child['title']} {child['seasonal_adjustment']}"
          f" (popularity={child['popularity']})")
```

```
Sibling series:
  CES0000000010: Women Employees, Total Nonfarm Seasonally Adjusted (popularity=4)
  CES0000000039: Women Employees-To-All Employees Ratio: Total Nonfarm Seasonally
↪Adjusted (popularity=16)
  CEU0000000010: Women Employees, Total Nonfarm Not Seasonally Adjusted
↪(popularity=1)
  PAYEMS: All Employees, Total Nonfarm Seasonally Adjusted (popularity=83)
  PAYNSA: All Employees, Total Nonfarm Not Seasonally Adjusted (popularity=47)
```

```
print(f"{node['id']}: {node['title']} {node['seasonal_adjustment']}",
      f" ({node['observation_start']}-{node['observation_end']})")
print()
print(textwrap.fill(node['notes']))
```

```
PAYEMS: All Employees, Total Nonfarm Seasonally Adjusted  (1939-01-01-2025-01-01)

All Employees: Total Nonfarm, commonly known as Total Nonfarm Payroll,
is a measure of the number of U.S. workers in the economy that
excludes proprietors, private household employees, unpaid volunteers,
farm employees, and the unincorporated self-employed. This measure
accounts for approximately 80 percent of the workers who contribute to
Gross Domestic Product (GDP).  This measure provides useful insights
into the current economic situation because it can represent the
number of jobs added or lost in an economy. Increases in employment
might indicate that businesses are hiring which might also suggest
that businesses are growing. Additionally, those who are newly
employed have increased their personal incomes, which means (all else
constant) their disposable incomes have also increased, thus fostering
further economic expansion.  Generally, the U.S. labor force and
levels of employment and unemployment are subject to fluctuations due
to seasonal changes in weather, major holidays, and the opening and
```

(continues on next page)

(continued from previous page)

```
closing of schools. The Bureau of Labor Statistics (BLS) adjusts the
data to offset the seasonal effects to show non-seasonal changes: for
example, women's participation in the labor force; or a general
decline in the number of employees, a possible indication of a
downturn in the economy. To closely examine seasonal and non-seasonal
changes, the BLS releases two monthly statistical measures: the
seasonally adjusted All Employees: Total Nonfarm (PAYEMS) and All
Employees: Total Nonfarm (PAYNSA), which is not seasonally adjusted.
The series comes from the 'Current Employment Statistics
(Establishment Survey).'  The source code is: CES0000000001
```

8.3 Revisions and vintage dates

Economic data revisions occur as new information becomes available, improving the accuracy of initial estimates. The Bureau of Labor Statistics (BLS), for instance, releases an initial estimate of Total Nonfarm Payroll Employment (PAYEMS) on the first Friday of each month. However, this figure is a very rough estimate, which is then revised in subsequent months as more firm-level data is collected.

These revisions can be significant, sometimes altering economic assessments. ALFRED, the archival FRED tool, allows users to compare initial estimates with later revisions. For the monthly values of PAYEMS in 2023, we examine the total amount of changes at each subsequent revision.

```python
start, end = 20230101, 20231231
data = {}
print(f"{alf.header(series_id)} (retrieved {today}):")
latest = alf(series_id, start=start, end=end, freq=freq, realtime=True)
latest
```

```
All Employees, Total Nonfarm (retrieved 20250302):
```

	PAYEMS	realtime_start	realtime_end
date			
20230131	154780	20250207	99991231
20230228	155086	20250207	99991231
20230331	155171	20250207	99991231
20230430	155387	20250207	99991231
20230531	155614	20250207	99991231
20230630	155871	20250207	99991231
20230731	156019	20250207	99991231
20230831	156176	20250207	99991231
20230930	156334	20250207	99991231
20231031	156520	20250207	99991231
20231130	156661	20250207	99991231
20231231	156930	20250207	99991231

```python
print("First Release:")
data[0] = alf(series_id, release=1, start=start, end=end, freq=freq, realtime=True)
data[0]
```

```
First Release:
```

```
          PAYEMS  realtime_start  realtime_end
date
20230131  155073        20230203      20230309
20230228  155350        20230310      20230406
20230331  155569        20230407      20230504
20230430  155673        20230505      20230601
20230531  156105        20230602      20230706
20230630  156204        20230707      20230803
20230731  156342        20230804      20230831
20230831  156419        20230901      20231005
20230930  156874        20231006      20231102
20231031  156923        20231103      20231207
20231130  157087        20231208      20240104
20231231  157232        20240105      20240201
```

```
print("Second Release:")
data[1] = alf(series_id, release=2, start=start, end=end, freq=freq, realtime=True)
data[1]
```

```
Second Release:
```

```
          PAYEMS  realtime_start  realtime_end
date
20230131  155039        20230310      20230406
20230228  155333        20230407      20230504
20230331  155420        20230505      20230601
20230430  155766        20230602      20230706
20230531  155995        20230707      20230803
20230630  156155        20230804      20230831
20230731  156232        20230901      20231005
20230831  156538        20231006      20231102
20230930  156773        20231103      20231207
20231031  156888        20231208      20240104
20231130  157016        20240105      20240201
20231231  157347        20240202      20240307
```

```
print("Third Release:")
data[2] = alf(series_id, release=3, start=start, end=end, freq=freq, realtime=True)
data[2]
```

```
Third Release:
```

```
          PAYEMS  realtime_start  realtime_end
date
20230131  155007        20230407      20240201
20230228  155255        20230505      20240201
20230331  155472        20230602      20240201
20230430  155689        20230707      20240201
20230531  155970        20230804      20240201
20230630  156075        20230901      20240201
20230731  156311        20231006      20240201
20230831  156476        20231103      20240201
20230930  156738        20231208      20240201
```

(continues on next page)

(continued from previous page)

```
20231031  156843        20240105      20240201
20231130  157014        20240202      20250206
20231231  157304        20240308      20250206
```

```
print("Fourth Release:")
data[3] = alf(series_id, release=4, start=start, end=end, freq=freq, realtime=True)
data[3]
```

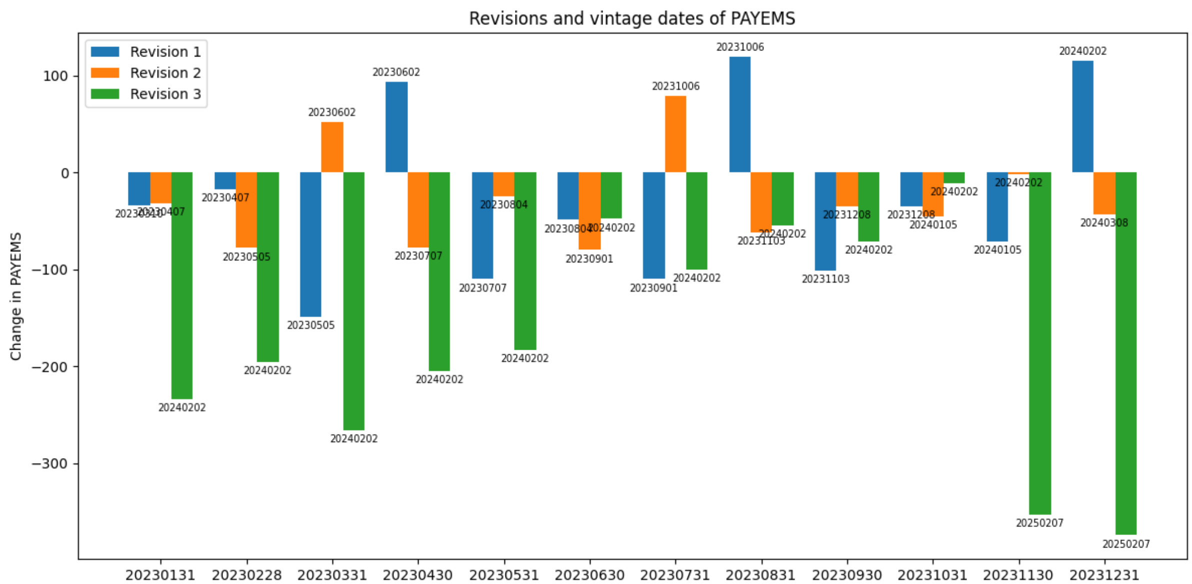
```
Fourth Release:
```

```
          PAYEMS  realtime_start  realtime_end
date
20230131  154773        20240202      20250206
20230228  155060        20240202      20250206
20230331  155206        20240202      20250206
20230430  155484        20240202      20250206
20230531  155787        20240202      20250206
20230630  156027        20240202      20250206
20230731  156211        20240202      20250206
20230831  156421        20240202      20250206
20230930  156667        20240202      20250206
20231031  156832        20240202      20250206
20231130  156661        20250207      99991231
20231231  156930        20250207      99991231
```

```
df = pd.concat([(data[i][series_id] - data[i-1][series_id]).rename(f"Revision {i}")
                for i in range(1, len(data))], axis=1)
labels = pd.concat([data[i]['realtime_start'].rename(f"Revision {i}")
                    for i in range(1, len(data))], axis=1).fillna(0).astype(int)
DataFrame(df.sum(axis=0).rename("Total revisions ('000)"))
```

```
            Total revisions ('000)
Revision 1                    -349
Revision 2                    -348
Revision 3                   -2095
```

```
#df = pd.concat([data[i][series_id].rename(f"Revision {i}")
#                 for i in range(1, len(data))], axis=1)
#labels = pd.concat([data[i]['realtime_start'].rename(f"Revision {i}")
#                     for i in range(1, len(data))], axis=1).fillna(0).astype(int)
fig, ax = plt.subplots(figsize=(12, 6))
plot_groupbar(df, labels=labels, ax=ax)
plt.legend()
plt.ylabel(f'Change in {series_id}')
plt.title(f'Revisions and vintage dates of {series_id}')
plt.tight_layout()
plt.show()
```



8.3.1 FRED-MD and FRED-QD

FRED-MD (Monthly Database) and FRED-QD (Quarterly Database) are curated datasets that streamline access to macroeconomic indicators. These datasets mimic the coverage of macroeconomic datasets used in the research literature and are updated in real-time, relieving users from the task of incorporating data changes and revisions. Historical monthly snap-shots of the datasets are also available.

8.3.2 Release dates

The timing of data releases is crucial for market participants and policymakers.

```
md_df, md_transform = fred_md()
end = md_df.index[-1]
out = {}
for i, title in enumerate(md_df.columns):
    out[title] = alf(series_id=title,
                     release=1,
                     start=end, # within 4 days of monthend
                     end=end,
                     realtime=True)
    if title.startswith('S&P'):  # stock market data available same day close
        out[title] = Series({end: end}, name='realtime_start').to_frame()
    elif title in alf.splice_:   # these series were renamed or spliced
        if isinstance(Alfred.splice_[title], str):  # if renamed
            out[title] = alf(series_id=Alfred.splice_[title],
                             release=1,
                             start=end-4,  # within 4 days of monthend
                             end=end,
                             realtime=True)
        else:  # if FRED-MD series was spliced
            out[title] = pd.concat([alf(series_id=sub,
                                        reglease=1,
                                        start=end-4,  # within 4 days of monthend
```

(continues on next page)

(continued from previous page)

```
                                                 end=end,
                                                 realtime=True)
                                         for sub in Alfred.splice_[title][1:]])
```

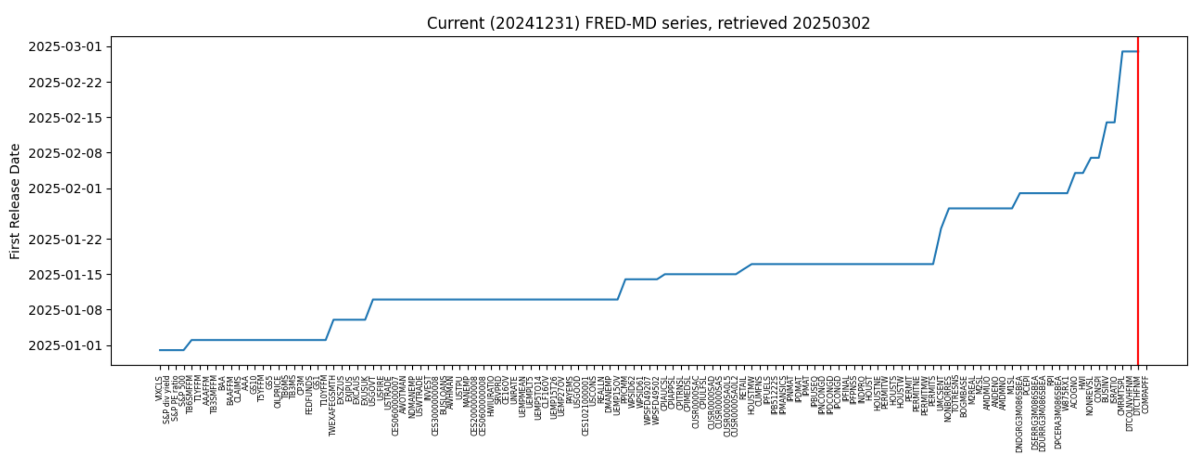
```
FRED-MD vintage: monthly/current.csv
```

```
# date convention of Consumer Sentiment
df = alf('UMCSENT', release=1, realtime=True)
out['UMCSENT'] = df[df['realtime_start'] > end - 4].iloc[:1]
```

```
# weekly averages of Claims
df = alf('ICNSA', release=1, realtime=True)
out['CLAIMS'] = df[df['realtime_start'] > end - 4].iloc[:1]
```

```
# Plot release dates of series in FRED-MD
release = Series({k: str(min(v['realtime_start'])) if v is not None and len(v)
                  else None  for k,v in out.items()}).sort_values()
```

```
fig, ax = plt.subplots(clear=True, num=1, figsize=(13, 5))
ax.plot(pd.to_datetime(release, errors='coerce'))
ax.axvline(release[~release.isnull()].index[-1], c='r')
ax.set_title(f"Current ({end}) FRED-MD series, retrieved {today}")
ax.set_ylabel('First Release Date')
ax.set_xticks(np.arange(len(release)))
ax.set_xticklabels(release.index, rotation=90, fontsize='xx-small')
plt.tight_layout()
```




```
# Check if recently released data available to update latest FRED-MD
↪
md_missing = md_df.iloc[-1]
md_missing = md_missing[md_missing.isnull()]
print("Recent values available to update missing in current FRED-MD")
for series_id in md_missing.index:
    print(alf.splice(series_id).iloc[-3:])
```

```
Recent values available to update missing in current FRED-MD
date
20241031    1538666.0
20241130    1544822.0
20241231    1555153.0
Name: CMRMTSPL, dtype: float64
date
20241031    7839
20241130    8156
20241231    7600
Name: HWI, dtype: int64
date
20241130    1.145345
20241231    1.103689
20250131         NaN
Name: HWIURATIO, dtype: float64
date
20241031    248120.0
20241130    248160.0
20241231    248851.0
Name: ACOGNO, dtype: float64
date
20241031    2585582.0
20241130    2588757.0
20241231    2584314.0
Name: BUSINV, dtype: float64
date
20241031    1.37
20241130    1.37
20241231    1.35
Name: ISRATIO, dtype: float64
date
20241031    3736897.53
20241130    3745366.76
20241231    3763355.59
Name: NONREVSL, dtype: float64
date
20241130    149.697308
20241231    149.793644
20250131           NaN
Name: CONSPI, dtype: float64
date
20241231    37.90
20250131    37.66
20250228    37.53
Name: S&P PE ratio, dtype: float64
date
20241031    554951.25
20241130    556075.09
20241231    558854.68
Name: DTCOLNVHFNM, dtype: float64
date
20241031    938525.34
20241130    941204.79
20241231    946489.00
Name: DTCTHFNM, dtype: float64
```

```
# Find any missing series observations, if any, now available to update current FRED-
↪MD
Series(release.values, index=[(s, alf.header(s)) for s in release.index])\
    .tail(len(md_missing))
```

```
(W875RX1, Real personal income excluding current transfer receipts)                ␣
↪                20250131
(ACOGNO, Manufacturers' New Orders: Consumer Goods)                                ␣
↪                20250204
(HWI, Help Wanted Index for United States)                                         ␣
↪                20250204
(NONREVSL, Nonrevolving Consumer Credit Owned and Securitized)                     ␣
↪                20250207
(CONSPI, Nonrevolving consumer credit to Personal Income)                          ␣
↪                20250207
(BUSINV, Total Business Inventories)                                               ␣
↪                20250214
(ISRATIO, Total Business: Inventories to Sales Ratio)                              ␣
↪                20250214
(CMRMTSPL, Real Manufacturing and Trade Industries Sales)                          ␣
↪                20250228
(DTCOLNVHFNM, Consumer Motor Vehicle Loans Owned by Finance Companies, Level)      ␣
↪                20250228
(DTCTHFNM, Total Consumer Loans and Leases Owned and Securitized by Finance␣
↪Companies, Level)    20250228
(COMPAPFF, 3-Month Commercial Paper Minus FEDFUNDS)                                ␣
↪                    None
dtype: object
```

8.4 Outliers

1. **Interquartile Range (IQR) Approach** –Filters data within median ± 10 times the interquartile range to minimize extreme values.
2. **Tukey's Rule** –Proposed by John Tukey, this method classifies data points as "outliers" if they fall beyond 1.5 times the interquartile range (IQR) of the first or third quartile, that is outside of [Q1 - 1.5(Q3-Q1), Q3 + 1.5(Q3-Q1)], and as **"far out"** if beyond 3 times the IQR.

```
payems = alf('PAYEMS', freq=freq, realtime=True, diff=1, log=1).dropna().iloc[:,0]
payems
```

```
date
19390228    0.005898
19390331    0.005962
19390430   -0.006162
19390531    0.006789
19390630    0.006678
              ...
20240930    0.001517
20241031    0.000278
20241130    0.001647
20241231    0.001934
```

(continues on next page)

(continued from previous page)

```
20250131    0.000899
Name: PAYEMS, Length: 1032, dtype: float64
```

```
for method in ['tukey', 'farout', 'iq10']:
    print(f"Outliers fraction detected by {method}:", np.mean(is_outlier(payems,␣
↪method=method)).round(4))
payems.iloc[is_outlier(payems, method='iq10')]
```

```
Outliers fraction detected by tukey: 0.0969
Outliers fraction detected by farout: 0.0329
Outliers fraction detected by iq10: 0.0029
```

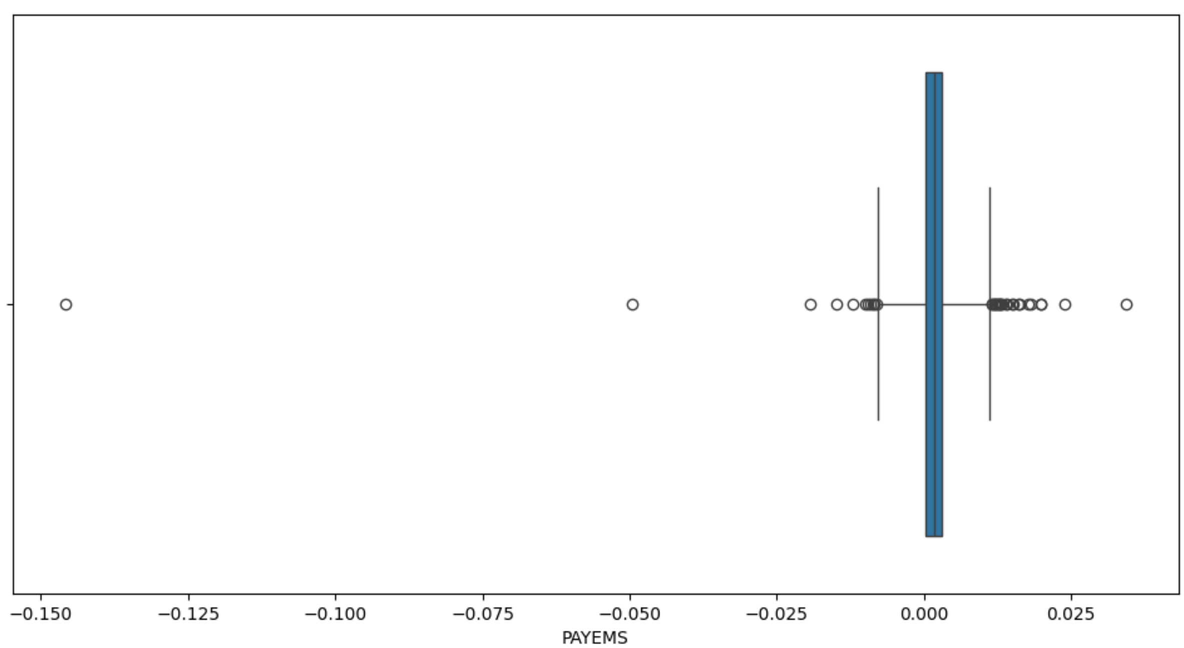
```
date
19450930   -0.049622
20200430   -0.145794
20200630    0.034217
Name: PAYEMS, dtype: float64
```

Box-and-whiskers plot

A box plot shows the quartiles of the data while the whiskers extend to show the rest of the distribution, except for points that are determined to be "outliers", which are more than some multiple of the inter-quartile range (IQR) beyond the first and third quartiles.

```
import seaborn as sns
fig, ax = plt.subplots(figsize=(12, 6))
sns.boxplot(payems, ax=ax, orient='h', whis=3) # whiskers at 3xIQR
```

```
<Axes: xlabel='PAYEMS'>
```



Referenes:

https://fred.stlouisfed.org/

https://www.stlouisfed.org/research/economists/mccracken/fred-databases

McCracken, M. W., & Ng, S. (2016). FRED-MD: A Monthly Database for Macroeconomic Research. Journal of Business & Economic Statistics, 34(4), 574–589.

McCracken, M.W., Ng, S., 2020. FRED-QD: A Quarterly Database for Macroeconomic Research, Federal Reserve Bank of St. Louis Working Paper 2020- 005

Katrina Stierholz, 2018, Economic Data Revisions: What They Are and Where to Find Them https://journals.ala.org/index.php/dttp/article/view/6383/8404

CHAPTER

NINE

LINEAR REGRESSION DIAGONOSTICS

In economics, the majority is always wrong - John Kenneth Galbraith

For a linear regression model to produce reliable and interpretable results, it must satisfy certain assumptions. Diagnosing potential issues such as heteroskedasticity, multicollinearity, omitted variables, and influential data points is important for model validity. We explore key diagnostic tests for linear regression, including methods for detecting violations of assumptions, evaluating the impact of outliers, and assessing model fit through residual plots. Additionally, we discuss techniques for robust standard errors when assumptions are violated, such as heteroskedasticity- and autocorrelation-consistent (HAC) estimators.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import patsy
import statsmodels.formula.api as smf
from statsmodels.stats.outliers_influence import variance_inflation_factor
from finds.readers import Alfred
from finds.utils import plot_fitted, plot_leverage, plot_scale, plot_qq
from secret import credentials
VERBOSE = 0
# matplotlib qt
```

```
alf = Alfred(api_key=credentials['fred']['api_key'])
```

For this analysis, we retrieve monthly Consumer Price Index (CPI) data as the dependent (endogenous) variable and Producer Price Index (PPI) data as the independent (exogenous) variable. The model uses the monthly differences of the logarithms of both series to account for changes over time.

```
# difference of logs of CPI and PPI monthly series from FRED
series_id, freq, start = 'CPIAUCSL', 'M', 0   #19740101
exog_id = 'WPSFD4131'

data = pd.concat([alf(s, start=start) for s in [series_id, exog_id]], axis=1)
data.index = pd.DatetimeIndex(data.index.astype(str))
data = np.log(data).diff().dropna()  # model the changes in logs of the series
DataFrame.from_dict({s: alf.header(s) for s in [series_id, exog_id]},
                    orient='index', columns=['Description'])
```

```
                                                   Description
CPIAUCSL   Consumer Price Index for All Urban Consumers: ...
WPSFD4131  Producer Price Index by Commodity: Final Deman...
```

9.1 Model assumptions

A valid linear regression model must satisfy the following assumptions:

1. **Linearity**: The expected value of y_i follows a linear function of the independent variables: $E[y_i] = b_0 + b_1 x_{i1} + ... + b_k x_{ik}$
2. **Exogeneity**: The explanatory variables $\{x_{i1}, ..., x_{ik}\}$ are non-stochastic and not correlated with the error term.
3. **Homoscedasticity**: The variance of the dependent variable remains constant: $Var(y_i) = \sigma^2$
4. **Independence**: The observations $\{y_i\}$ are independent of each other.
5. **Normality**: The error terms follow a normal distribution.

When assumptions 1-4 hold, the least squares estimator:

- Provides an **unbiased** estimate of the regression coefficients: $b = (X'X)^{-1}X'y$
- Has a variance-covariance matrix: $Var(b) = \sigma^2(X'X)^{-1}$
- The standard error for each coefficient b_j is: $se(b_j) = \sigma\sqrt{(X'X)^{-1}_{[j+1,j+1]}}$

When all five assumptions hold, the least squares estimator follows a normal distribution, enabling valid statistical inference.

```python
# Run Linear Regression (with exog and 2 lags)
dmf = (f'{series_id} ~ {series_id}.shift(1) + {series_id}.shift(2) + {exog_id}.
↪shift(1)')
model = smf.ols(formula=dmf, data=data).fit()
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:               CPIAUCSL   R-squared:                       0.453
Model:                            OLS   Adj. R-squared:                  0.451
Method:                 Least Squares   F-statistic:                     167.5
Date:                Sun, 02 Mar 2025   Prob (F-statistic):           4.43e-79
Time:                        22:25:01   Log-Likelihood:                 2815.2
No. Observations:                 610   AIC:                            -5622.
Df Residuals:                     606   BIC:                            -5605.
Df Model:                           3
Covariance Type:            nonrobust
==================================================================================
                         coef    std err          t      P>|t|      [0.025      0.
↪975]
------------------------------------------------------------------------------
↪---
```

(continues on next page)

(continued from previous page)

```
Intercept                0.0009      0.000      6.276      0.000       0.001       0.
↪001
CPIAUCSL.shift(1)        0.5821      0.041     14.199      0.000       0.502       0.
↪663
CPIAUCSL.shift(2)       -0.0479      0.041     -1.174      0.241      -0.128       0.
↪032
WPSFD4131.shift(1)       0.2011      0.036      5.609      0.000       0.131       0.
↪271
==============================================================================
Omnibus:                      116.912   Durbin-Watson:                   2.051
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              584.706
Skew:                          -0.752   Prob(JB):                    1.08e-127
Kurtosis:                       7.555   Cond. No.                         522.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly␣
↪specified.
```

9.1.1 Heteroskedasity and HAC robust errors

If the variance of residuals is not constant (**heteroskedasticity**), the usual **Ordinary Least Squares (OLS)** standard error formula, $\sigma^2(X'X)^{-1}$, no longer holds. While OLS coefficient estimates remain **consistent**, their standard errors may be misestimated. A more general form of the variance-covariance matrix is:

$$(X'X)^{-1}(X'\Omega X)(X'X)^{-1}$$

where different choices of Ω provide **robust standard error estimators**.

- **White's (1980) heteroskedasticity-consistent estimator** (also known as the sandwich estimator) uses the diagonal of squared residuals.
- Alternative estimators account for leverage effects in the design matrix.

If error terms exhibit **serial correlation**, standard heteroskedasticity-robust errors may still be misleading. **Newey and West (1987)** introduced the **Heteroskedasticity and Autocorrelation Consistent (HAC)** estimator, which applies a weighting scheme to correct for autocorrelation. The truncation parameter for lag selection is often chosen as:

$$m = 0.75T^{1/3}$$

where autocorrelation coefficients are weighted as follows:

$$1 + 2\sum_{j=1}^{m-1} \frac{m-j}{m}\hat{\rho}_j$$

```
robust = model.get_robustcov_results(cov_type='HAC', use_t=None, maxlags=0)
print(robust.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:               CPIAUCSL   R-squared:                       0.453
Model:                            OLS   Adj. R-squared:                  0.451
Method:                 Least Squares   F-statistic:                     108.8
```

(continues on next page)

(continued from previous page)

```
Date:                  Sun, 02 Mar 2025   Prob (F-statistic):          2.25e-56
Time:                          22:25:01   Log-Likelihood:                2815.2
No. Observations:                   610   AIC:                          -5622.
Df Residuals:                       606   BIC:                          -5605.
Df Model:                             3
Covariance Type:                    HAC
=====================================================================================
                         coef    std err          t      P>|t|      [0.025      0.
↪975]
-------------------------------------------------------------------------------------
↪---
Intercept              0.0009      0.000      5.779      0.000       0.001       0.
↪001
CPIAUCSL.shift(1)      0.5821      0.074      7.903      0.000       0.437       0.
↪727
CPIAUCSL.shift(2)     -0.0479      0.053     -0.902      0.367      -0.152       0.
↪056
WPSFD4131.shift(1)     0.2011      0.055      3.652      0.000       0.093       0.
↪309
==============================================================================
Omnibus:                      116.912   Durbin-Watson:                   2.051
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              584.706
Skew:                          -0.752   Prob(JB):                    1.08e-127
Kurtosis:                       7.555   Cond. No.                         522.
==============================================================================

Notes:
[1] Standard Errors are heteroscedasticity and autocorrelation robust (HAC) using␣
↪0 lags and without small sample correction
```

9.1.2 Multicollinearity and variance inflation factors

Multicollinearity arises when explanatory variables are highly correlated, leading to unstable coefficient estimates. The **Variance Inflation Factor (VIF)** quantifies the degree of multicollinearity by measuring how much a predictor ' s variance is inflated due to correlation with other predictors:

$$\mathrm{VIF_i} = \frac{1}{1 - R_i^2}$$

where R_i^2 is obtained by regressing X_i on all other predictors.

A VIF > 5 or 10 suggests that the variable is highly collinear with other explanatory variables, potentially leading to large standard errors and unreliable estimates.

```python
Y, X = patsy.dmatrices(dmf + ' - 1', data=data)  # exclude intercept term
print("Variance Inflation Factors")
Series({X.design_info.column_names[i]: variance_inflation_factor(X, i)
        for i in range(X.shape[1])}, name='VIF').to_frame()
```

```
Variance Inflation Factors
```

```
                         VIF
CPIAUCSL.shift(1)   3.414717
```

(continues on next page)

(continued from previous page)

```
CPIAUCSL.shift(2)   3.404137
WPSFD4131.shift(1)  2.285974
```

9.1.3 Omitted variables

Leaving out an important variable from the regression model can lead to biased estimates. The consequences are:

1. **Bias in Included Variables**: If the omitted variable is correlated with included variables, their regression coefficients will capture some of its effect, leading to inconsistent estimates.
2. **Inflated Residual Variance**: The estimated residuals will include both true shocks and the effects of the omitted variable, reducing model accuracy.

Conversely, including an extraneous (irrelevant) variable does not introduce bias but increases standard errors, making it harder to detect significant effects.

9.2 Residual diagnostics

Residual plots help evaluate model fit, identify outliers and detect potential specification issues.
An ideal model would have residuals that are not systematically related to any of the included explanatory variables. Standardized residuals may alternatively be used so that the magnitude of deviation is more apparent.

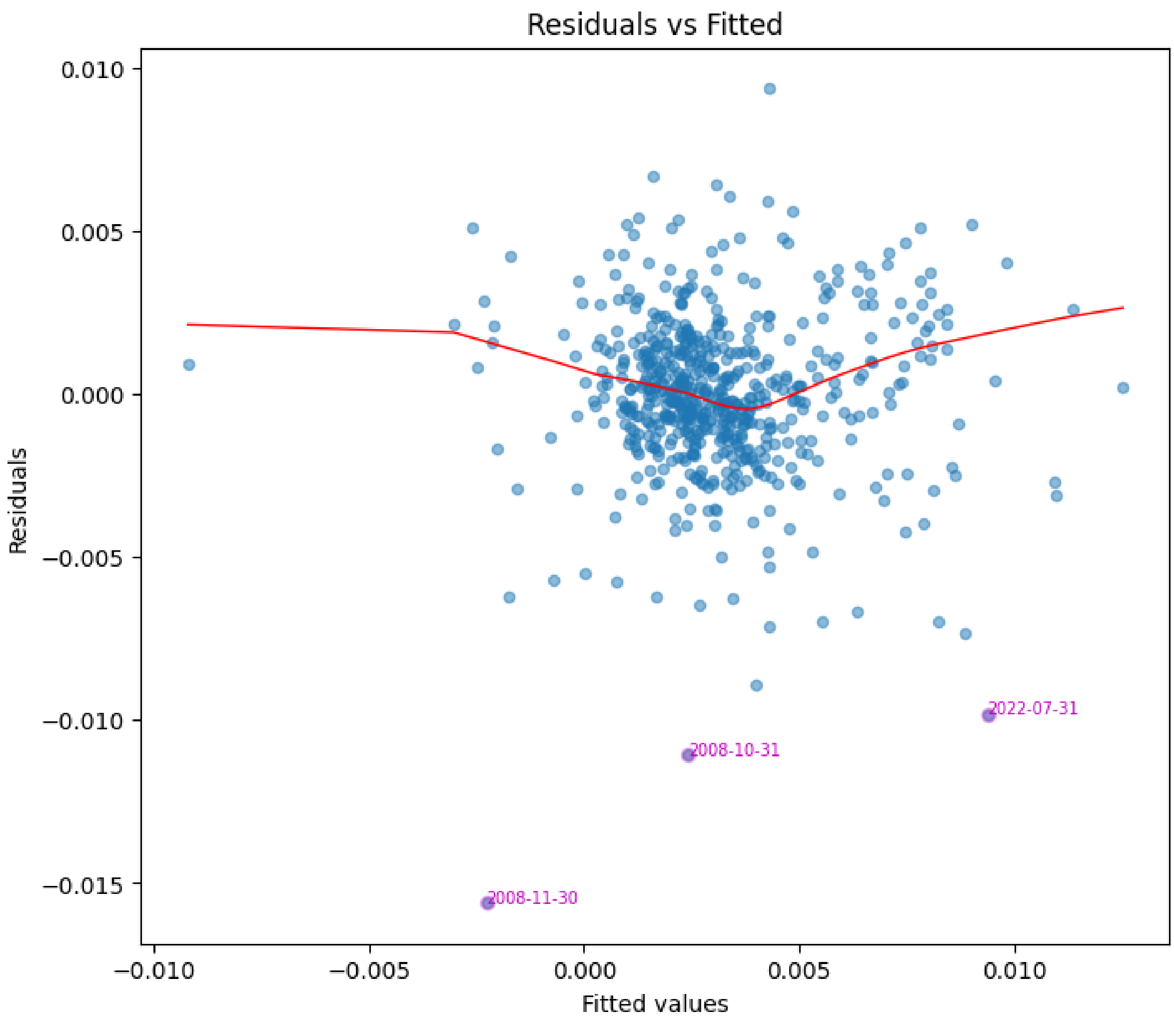
9.2.1 Residuals vs fitted plot

This plot assesses whether residuals exhibit nonlinear patterns. Ideally, residuals should be randomly scattered around zero, with no discernible trend. A systematic pattern suggests model misspecification or omitted variables.

```python
# Plot residuals and identify outliers
fig, ax = plt.subplots(clear=True, figsize=(8,7))
z = plot_fitted(fitted=model.fittedvalues,
                resid=model.resid,
                ax=ax)
print("Residual Outliers")
z.to_frame().T
```

```
Residual Outliers
```

```
date        2022-07-31  2008-10-31  2008-11-30
outliers     -0.009815   -0.011066   -0.015599
```



9.2.2 Normal QQ plot

A **quantile-quantile (Q-Q) plot** compares residuals to a normal distribution. If residuals are normally distributed, data points should align along a 45-degree reference line. **Outliers** may appear as deviations from this line, indicating potential issues such as a heavy-tailed distribution.

```
fig, ax = plt.subplots(clear=True, figsize=(8,7))
plot_qq(model.resid, ax=ax)
```

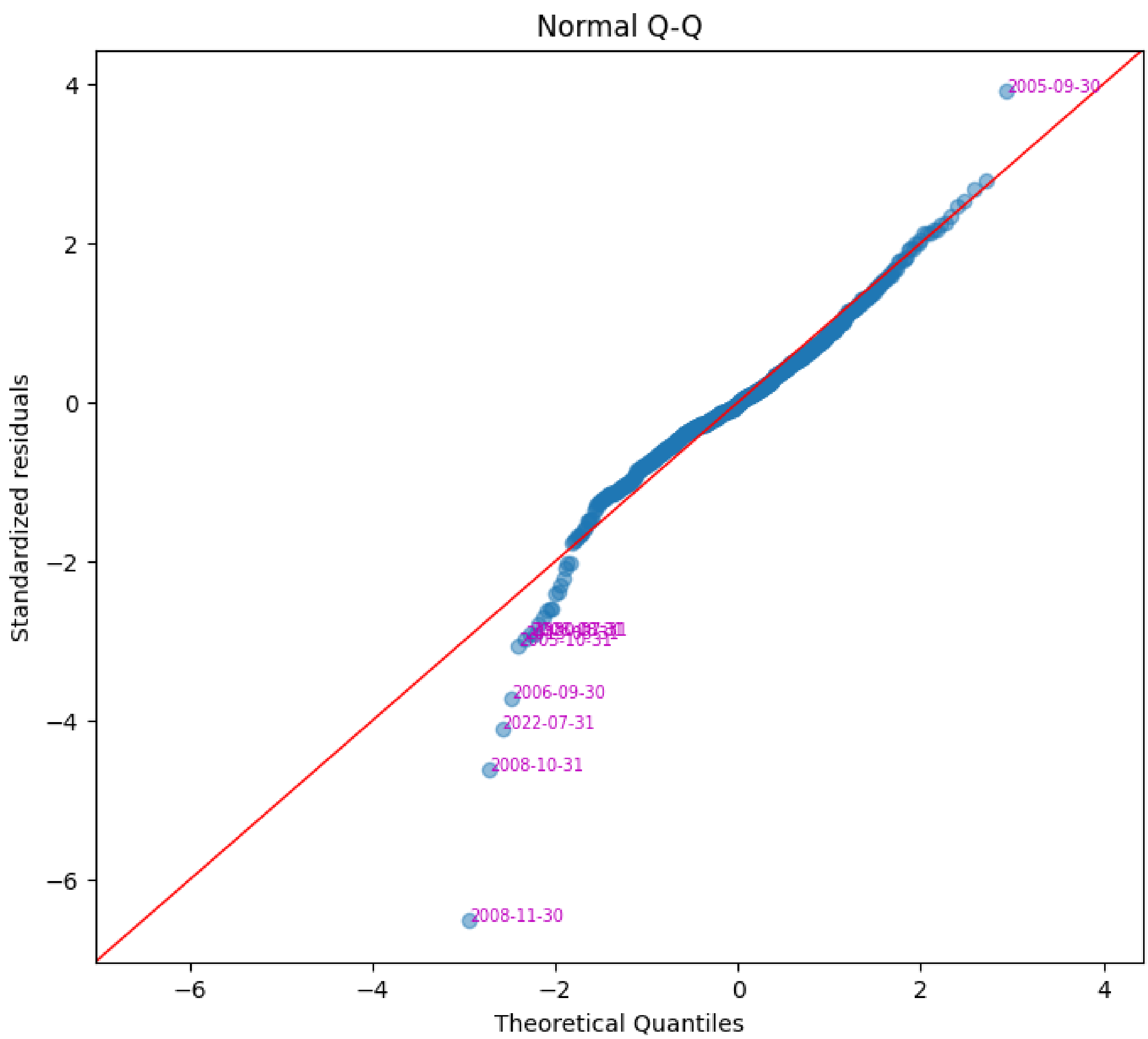
```
/home/terence/env3.11/lib/python3.11/site-packages/statsmodels/graphics/gofplots.
↪py:1043: UserWarning: color is redundantly defined by the 'color' keyword
↪argument and the fmt string "b" (-> color=(0.0, 0.0, 1.0, 1)). The keyword
↪argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```

```
             residuals  standardized
date
2008-11-30  -0.015599     -6.511279
```

(continues on next page)

(continued from previous page)

```
2008-10-31  -0.011066     -4.619178
2022-07-31  -0.009815     -4.097059
2006-09-30  -0.008903     -3.716483
2005-10-31  -0.007321     -3.056102
2013-03-31  -0.007108     -2.967065
2008-08-31  -0.007003     -2.923170
1980-07-31  -0.006997     -2.920733
2005-09-30   0.009362      3.907884
```

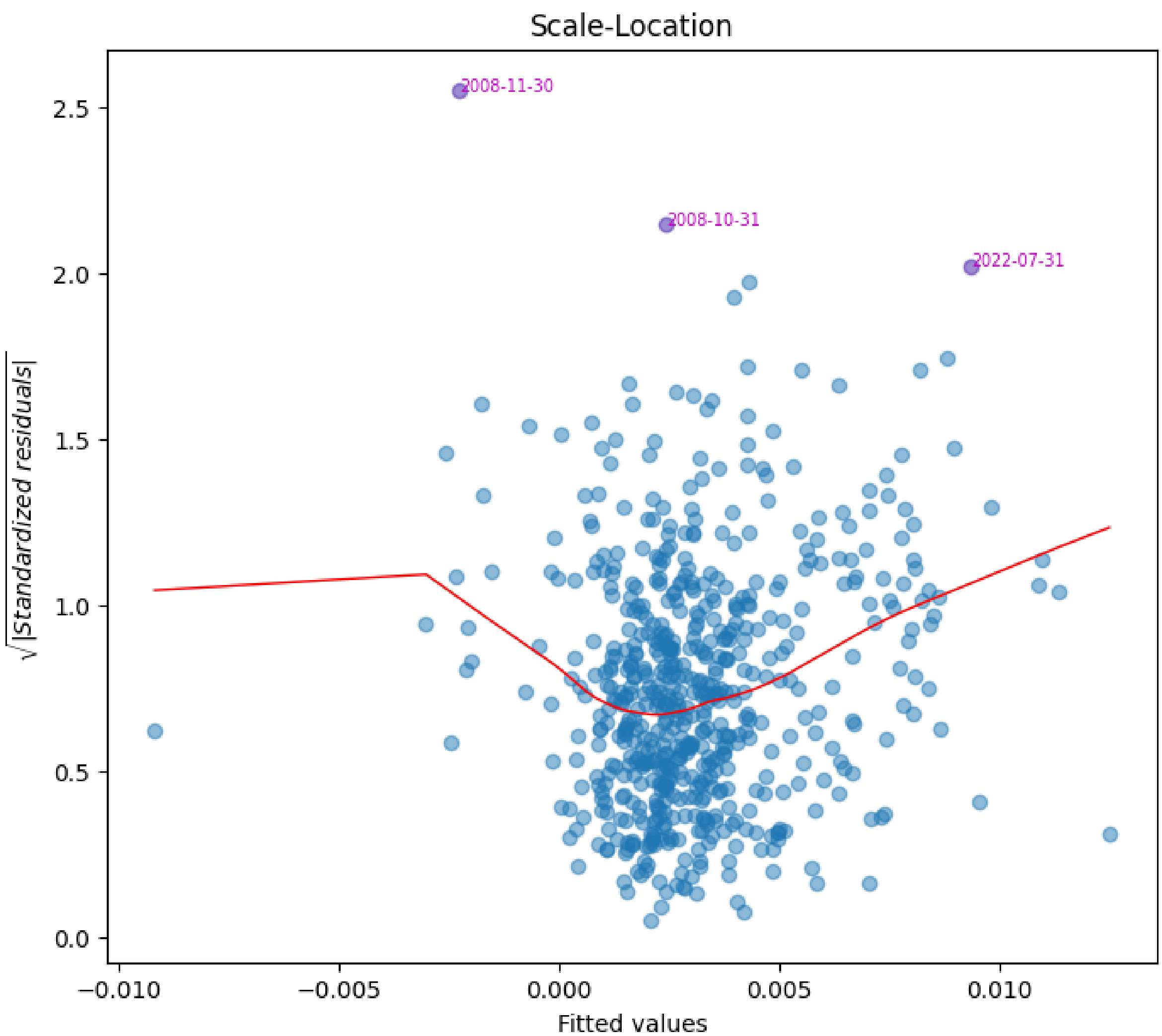


9.2.3 Scale-location plot

This plot checks for homoscedasticity (constant variance). Residuals should be evenly spread across predictor values. A funnel-shaped pattern suggests heteroskedasticity, requiring robust standard errors.

```
fig, ax = plt.subplots(clear=True, figsize=(8,7))
plot_scale(model.fittedvalues, model.resid, ax=ax)
```

```
array([579, 414, 415])
```



9.2.4 Leverage and influential points

Certain data points can disproportionately affect regression estimates. The projection matrix from the least squares estimator, also called the **hat matrix**, $H = X(X^T X)^{-1} X^T$ identifies **leverage points**, where the diagonal element h_{ii} measures the influence of the i-th observation.

A point may have **high leverage** but not necessarily influence the regression results significantly. **Cook' s Distance** measures influence based on both residual magnitude and leverage:

$$D_i = \frac{1}{p} t_i^2 \frac{h_{ii}}{1 - h_{ii}}$$

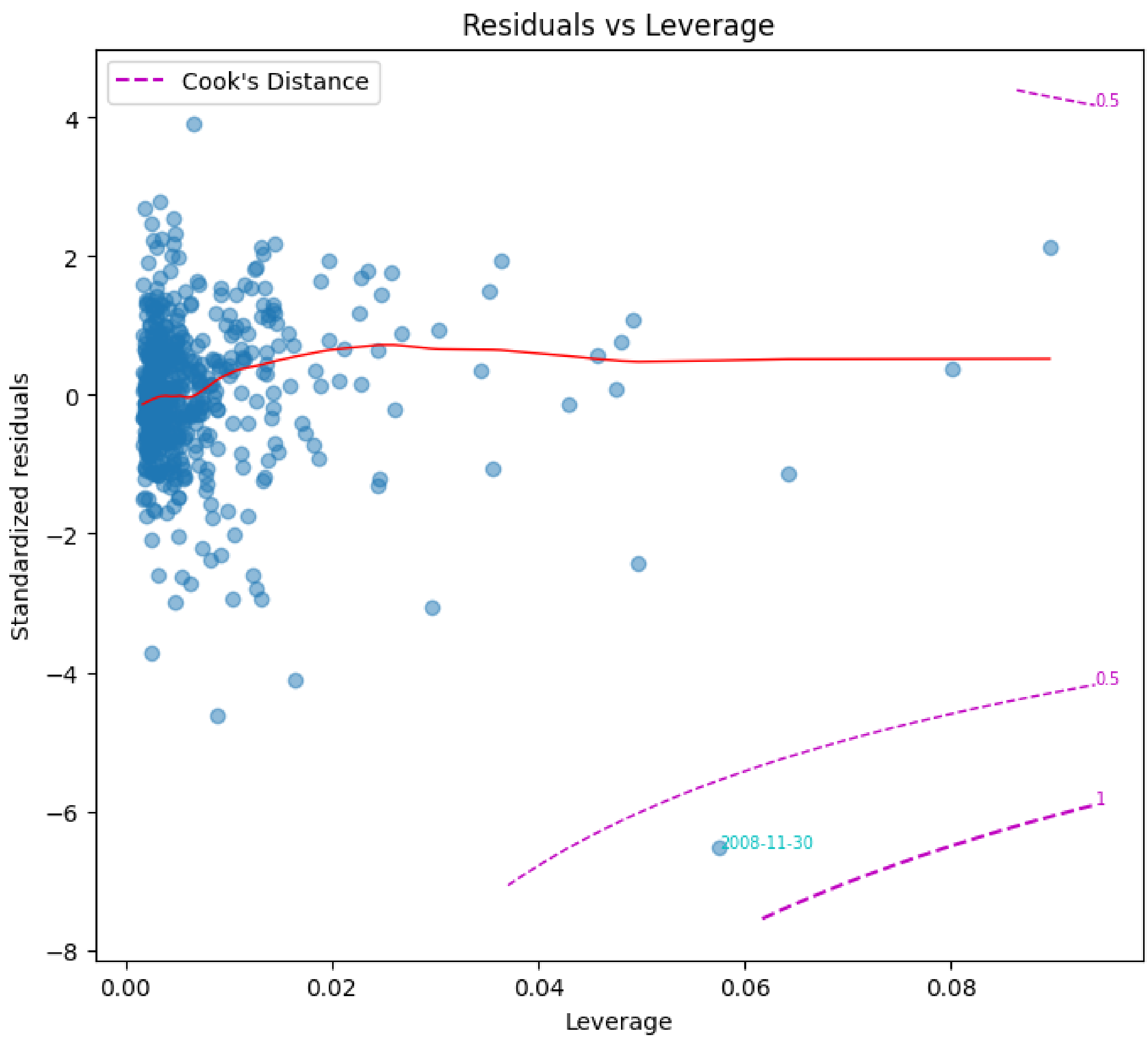
where:

- p is the number of regression parameters.
- $t_i = \frac{\hat{\epsilon}_i}{\hat{\sigma}(1 - h_{ii})}$ is the *studentized residual*, which accounts for non-constant variance.
- $\hat{\sigma} = \sqrt{\sum_{j=1}^{n} \hat{\epsilon}_j^2 / n}$.

D_i > 1 $ suggests an influential point that may need further investigation. The **Residuals vs Leverage plot** helps visualize these influential points.

```
fig, ax = plt.subplots(clear=True, figsize=(8,7))
plot_leverage(model.resid, model.get_influence().hat_matrix_diag,
              model.get_influence().cooks_distance[0],
              ddof=len(model.params), ax=ax)
```

```
Empty DataFrame
Columns: [influential, cook's D, leverage]
Index: []
```



References:

White, Halbert (1980). "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity" . Econometrica. 48 (4): 817–838.

Newey, Whitney K., and Kenneth D. West. 1987. "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." Econometrica 55: 703–8.

https://library.virginia.edu/data/articles/diagnostic-plots

FRM Part I Exam Book Quantitative Analysis Ch 9

CHAPTER TEN

TIME SERIES ANALYSIS

The only thing we know about the future is that it will be different - Peter Drucker

Time series analysis is a fundamental statistical technique used to analyze data points collected over time. A time series can exhibit multiple components, such as long-term trends, cyclical fluctuations, seasonal variations, and irregular random movements. Understanding these components enables analysts to build accurate predictive models. We explore key concepts in time series analysis, including trend patterns, seasonality, stationarity, autocorrelation, and various statistical models such as autoregressive (AR), moving average (MA), and autoregressive moving average (ARMA) models. We also examine forecasting methods and statistical tests used to assess time series properties.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import seaborn as sns
from statsmodels.tsa.ar_model import AutoReg, ar_select_order
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf, acf
from statsmodels.tsa.seasonal import seasonal_decompose
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.tsa.stattools import grangercausalitytests, adfuller
from statsmodels.tsa.api import VAR
import statsmodels.formula.api as smf
from sklearn.metrics import mean_squared_error
from finds.readers import Alfred
from secret import credentials
VERBOSE = 0
# %matplotlib qt
```

We retrieve the Industrial Production (IP) total index monthly time series from FRED.

```
series_id, freq, start = 'IPB50001N', 'ME', 0  # not seasonally adjusted
alf = Alfred(api_key=credentials['fred']['api_key'])
df = alf(series_id, log=1, freq=freq, start=start).dropna()
date_title = f" ({df.index[0]//100}-{df.index[-1]//100})"
df.index = pd.to_datetime(df.index, format='%Y%m%d')
df.index.freq = freq                 # set index to datetime type with 'M frequency
alf.header(series_id)
```

```
'Industrial Production: Total Index'
```

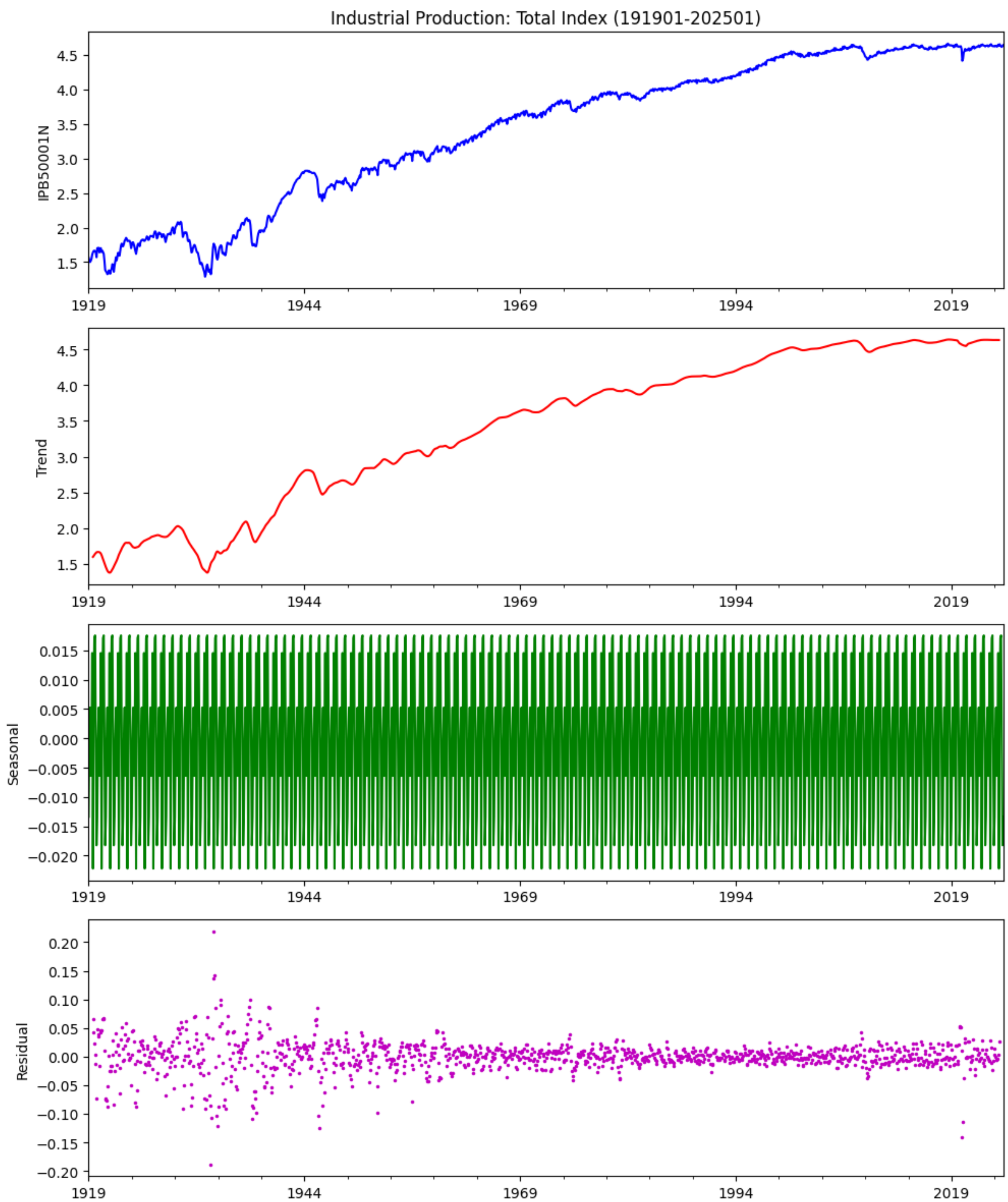
10.1 Seasonality

A time series can be broken down into three key components:

- **Trend** –Represents long-term movements in the data.
- **Seasonality** –Captures predictable and recurring changes within a specific time frame.
- **Cyclical** –Encompasses longer-term cycles that are influenced by broader economic or structural factors.

If a time series Y_t exhibits seasonality, its mean can differ across periods, repeating every s periods (e.g., quarterly with $s = 4$ or monthly with $s = 12$). Deterministic seasonal effects can be effectively modeled using dummy variables.

```
## Seasonality Decomposition Plot
result = seasonal_decompose(df, model='add')
fig, ax = plt.subplots(nrows=4, ncols=1, clear=True, figsize=(10, 12))
result.observed.plot(ax=ax[0], title=alf.header(result.observed.name) + date_title,
                     ylabel=result.observed.name, xlabel='', c='b')
result.trend.plot(ax=ax[1], ylabel='Trend', xlabel='', c='r')
result.seasonal.plot(ax=ax[2], ylabel='Seasonal', xlabel='', c='g')
result.resid.plot(ax=ax[3], ls='', ms=3, marker='.', c='m',
                  ylabel='Residual', xlabel='')
plt.tight_layout()
```


Industrial Production: Total Index (191901-202501)
IPB50001N
Trend
Seasonal
Residual
1919
1944
1969
1994
2019

10.2 Stationarity

A time series is considered **covariance-stationary** if it meets three key conditions:

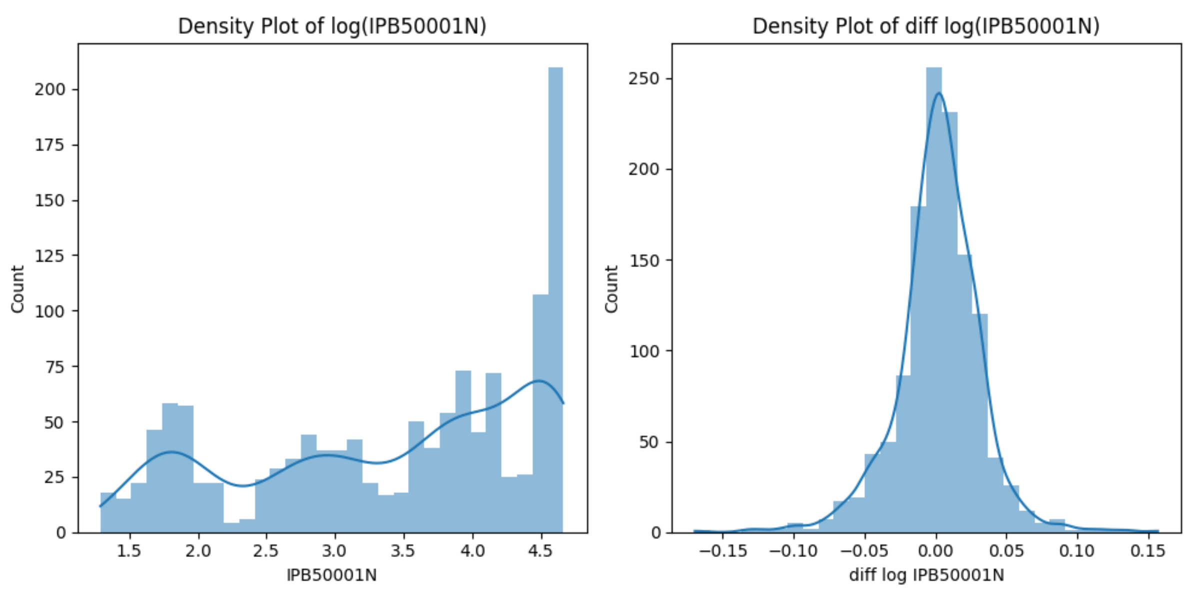
1. The mean remains constant over time, i.e., $E[Y_t] = m$ for all t.
2. The variance is finite and does not change over time, i.e., $V[Y_t] = \gamma_0 < \infty$.
3. The autocovariance depends only on the time lag h and not on t, i.e., $Cov[Y_t, Y_{t-h}] = \gamma_h$.

Covariance stationarity depends on the first two moments of a time series. Stationarity is crucial for modeling and forecasting because stationary series exhibit stable statistical properties. The **Augmented Dickey-Fuller (ADF) test** helps determine whether a time series has a unit root, with the null hypothesis stating that a unit root is present. If the p-value is below a critical threshold, the null hypothesis can be rejected, indicating stationarity.

```
values = df.diff().dropna().values.squeeze()
adf = adfuller(values)
DataFrame.from_dict({"I(1)": list(adf[:4]) + list(adf[4].values())},
                    orient='index',
                    columns=(['Test Statistic', 'p-value', 'Lags Used', 'Obs Used'] +
                             [f"critical({k})" for k in adf[4].keys()])).round(3)
```

```
      Test Statistic  p-value  Lags Used  Obs Used  critical(1%)  \
I(1)          -7.266      0.0         23      1248        -3.436

      critical(5%)  critical(10%)
I(1)        -2.864         -2.568
```

```
# Histogram Plot and Kernel Density Estimate
fig, axes = plt.subplots(1, 2, clear=True, figsize=(10,5))
sns.histplot(df.dropna(), bins=30, lw=0, kde=True, ax=axes[0])
axes[0].set_title(f"Density Plot of log({series_id})")
sns.histplot(df.diff().dropna().rename(f"diff log {series_id}"),
             bins=30, lw=0, kde=True, ax=axes[1]) #line_kws={"color": "r"}
axes[1].set_title(f"Density Plot of diff log({series_id})")
plt.tight_layout()
```

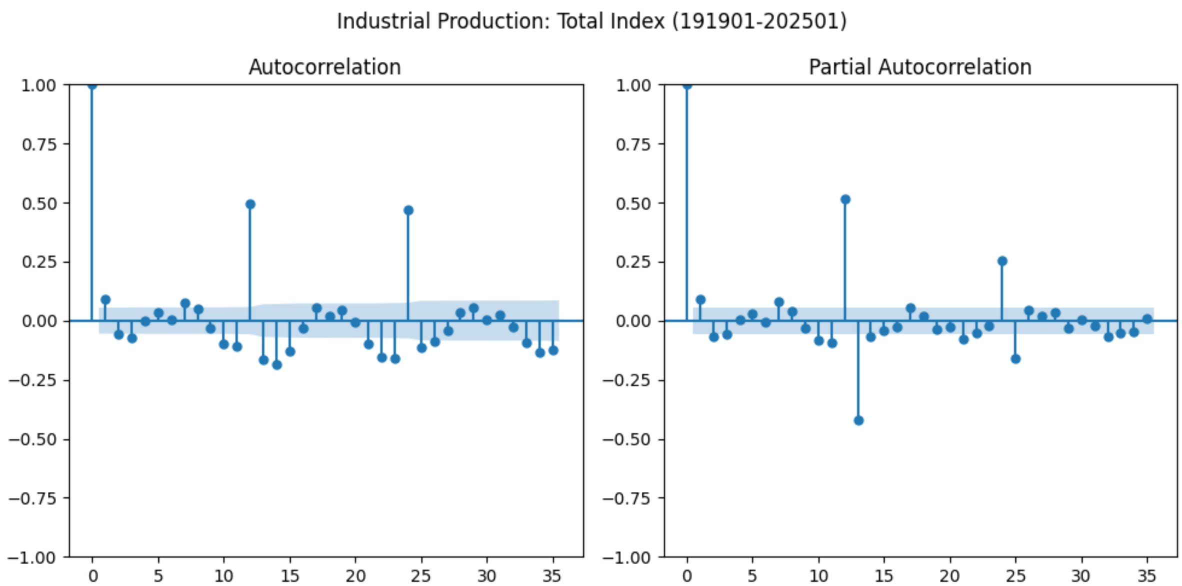


10.3 Autocorrelation

Autocorrelation measures the relationship between time series observations at different points in time. The autocorrelation function (ACF) at lag h is given by:
\$$\rho_h = \frac{\gamma_h}{\gamma_0} where$ \gamma_h $is the autocovariance at lag$ h , and \gamma_0 \$ is the variance.

The **partial autocorrelation function (PACF)** isolates the direct relationship between observations separated by lag h, removing the influence of intermediate lags (i.e., $Y_{t-1}, Y_{t-2}, ..., Y_{t-h+1}$).

```
values = df.diff().dropna().values.squeeze()
fig, axes = plt.subplots(1, 2, clear=True, figsize=(10,5))
plot_acf(values, lags=35, ax=axes[0])
plot_pacf(values, lags=35, ax=axes[1], method='ywm')
plt.suptitle(alf.header(result.observed.name) + date_title)
plt.tight_layout()
```



10.4 Time Series Models

10.4.1 Autoregressive (AR) models

Autoregressive (AR) models describe a time series in terms of its past values. An AR(1) process follows:
\$$Y_t = \delta + \phi Y_{t-1} + \epsilon_t where$ \delta $is the intercept,$ \phi $is the autoregressive coefficient, and$ \epsilon_t \$ is white noise.

For a higher-order AR(p) model:
\$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ... + \phi_p Y_{t-p} + \epsilon_t$\$

The PACF for an AR process is nonzero only for the first p lags, while the ACF gradually decays ($\rho(h) = \phi^{|h|}$)

10.4.2 Moving average (MA) models**

A moving average model expresses Y_t as a function of past shocks:
$Y_t = \mu + \theta_1 \epsilon_{t-1} + \epsilon_t AnMA(q)modelextendsthistoinclude$ q $lagsof$ \epsilon $: Y_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + ... + \theta_q \epsilon_{t-q}$

The ACF of an MA process is zero for lags greater than q, while the PACF has a more complex structure.

10.4.3 ARMA models

An **ARMA(p, q)** model combines AR and MA components. For example, a simple ARMA(1,1) evolves according to: $Y_t = \mu + \phi Y_{t-1} + \theta \epsilon_{t-1} + \epsilon_t$

- The mean of this process is $\mu = \delta/(1-\phi)$
- The variance is $\gamma_0 = \sigma^2(1 + 2\phi\theta + \theta^2)/(1-\phi^2)$

An ARMA(1,1) process is covariance-stationary if $|\phi| <> 1$. The MA coefficient plays no role in determining whether the process is covariance-stationary, because any MA is covariance-stationary as the MA component only affects a single lag of the shock. The AR component, however, affects all lagged shocks and so if ϕ_1 is too large, then the time series is not covariance-stationary.

10.4.4 Lag lengths

Determining the appropriate lag lengths for the AR and MA components (i.e., p and q, respectively) is a key challenge when building an ARMA model. The first step in model building is to inspect the sample autocorrelation and sample PACFs.

The **Box-Pierce** test statistic is the sum of the squared autocorrelations scaled by the sample size T: $Q_{BP} = T \sum_{i=1}^{h} (\frac{T+1}{T-1}) \hat{\rho}_i^2$

When the null is true, Q_{BP} is asymptotically distributed as a χ^2_h variable.

The **Ljung-Box** statistic is a modified version of the Box-Pierce statistic that works better in smaller samples, and is defined as: $Q_{LB} = T \sum_{i=1}^{h} (\frac{T+2}{T-i}) \hat{\rho}_i^2$

10.4.5 Seasonal component

Seasonal components can be added to the short-term components of an ARMA(p,q) model, by using lags only at the seasonal frequency. A seasonal ARMA combines these two components into a single specification:

$$ARMA(p, q) \times (p_s, q_s)_f$$

where p and q are the orders of the short-run lag polynomials, p_s and q_s represent seasonal lag orders, and f denotes the seasonal horizon (e.g., every 3 or 12 months with monthly observations).

10.4.6 Unit Roots

Random walks are most important source of **non-stationarity** in economic time series. A simple random walk process evolves according to:

$$Y_t = Y_{t-1} + \epsilon_t$$

Unit roots generalize random walks by adding short-run stationary dynamics to the long-run random walk.

Spurious relationships can occur when non-stationary series are regressed against each other; this produces produces a coefficient estimate that is large and seemingly statistically different from zero when using conventional statistical distributions to obtain the critical values.

Differencing removes unit roots and prevents such misleading results. If Y_t has a unit root (i.e. has **integration order** equal to 1), then the difference: $\Delta Y_t = Y_t - Y_{t-1}$ does not.

10.4.7 SARIMAX model

The **Seasonal AutoRegressive Integrated Moving Average with eXogenous Regressors (SARIMAX)** model is denoted:
\$$(p, d, q) \times (P, D, Q)_s$\$
where:

- (p, d, q) are the orders of AR, differencing, and MA components.
- (P, D, Q, s) are the seasonal counterparts and periodicity.

```
split_date = df.index[-12]   # train/test split date
# df_train = df.loc[:split_date].dropna()
```

```
# Fit a SARIMA(1,1,3) with seasonal order (0, 0, 0, 12)
pdq = (1, 1, 1)   #(12, 1, 0)
seasonal_pdqs = (0, 0, 0, 12)
arima = SARIMAX(df, order=pdq, seasonal_order=seasonal_pdqs, trend='c').fit()
fig = arima.plot_diagnostics(figsize=(10, 6), lags=36)
plt.tight_layout()
arima.summary()
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =             4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f= -2.09476D+00    |proj g|=  3.46166D-01

At iterate    5    f= -2.09484D+00    |proj g|=  2.67321D-02

At iterate   10    f= -2.09486D+00    |proj g|=  1.29274D-01

           * * *
```

(continues on next page)

(continued from previous page)

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     14     18      1     0     0   8.413D-04  -2.095D+00
  F =  -2.0948681083990448

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
```

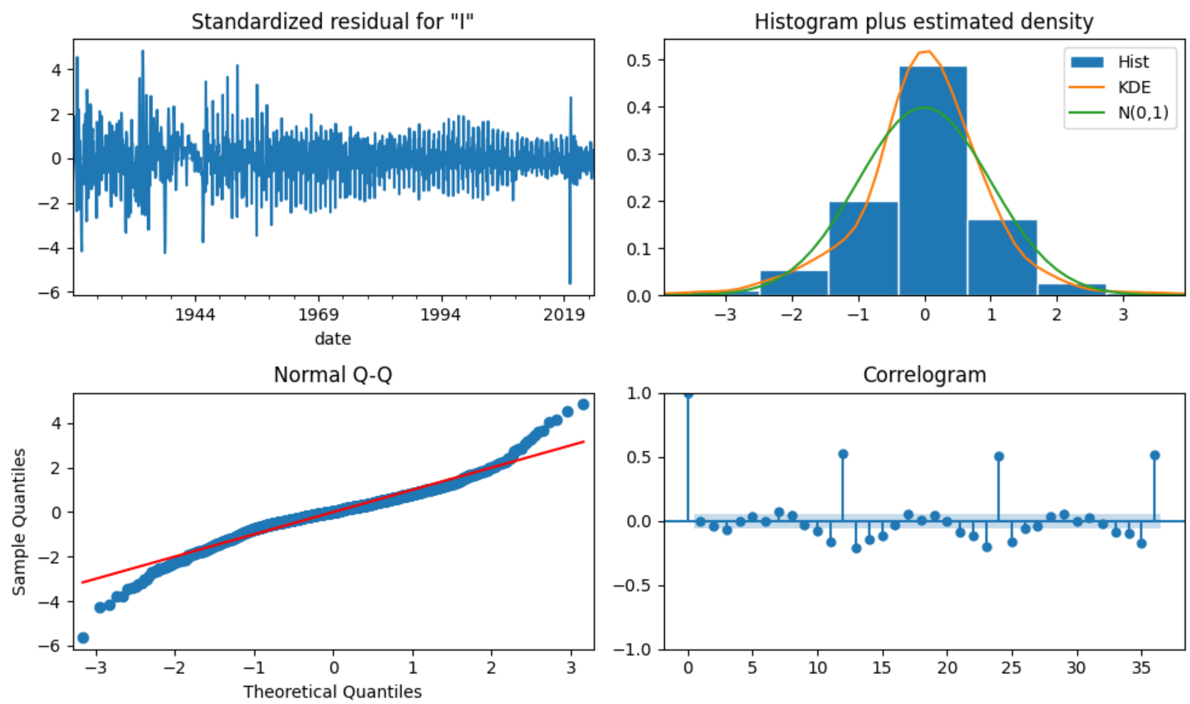
```
 This problem is unconstrained.
```

Dep. Variable:	IPB50001N	**No. Observations:**	1273
Model:	SARIMAX(1, 1, 1)	**Log Likelihood**	2666.767
Date:	Mon, 03 Mar 2025	**AIC**	-5325.534
Time:	10:54:09	**BIC**	-5304.941
Sample:	01-31-1919 - 01-31-2025	**HQIC**	-5317.799
Covariance Type:	opg		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
intercept	0.0027	0.001	2.396	0.017	0.000	0.005
ar.L1	-0.1167	0.223	-0.524	0.600	-0.553	0.320
ma.L1	0.2195	0.222	0.987	0.323	-0.216	0.655
sigma2	0.0009	2.21e-05	39.946	0.000	0.001	0.001

Ljung-Box (L1) (Q):	0.02	**Jarque-Bera (JB):**	563.24
Prob(Q):	0.87	**Prob(JB):**	0.00
Heteroskedasticity (H):	0.31	**Skew:**	-0.20
Prob(H) (two-sided):	0.00	**Kurtosis:**	6.23

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).



10.5 Forecasting

```
series_id, start = 'INDPRO', 0
df_all = alf(series_id, log=1, diff=1, start=start).dropna()
df_all.index = pd.to_datetime(df_all.index, format='%Y%m%d')
df_all.index.freq = freq
df_train = df_all[df_all.index <= split_date]
df_test = df_all[df_all.index > split_date]
```

10.5.1 AR lag order selection

The most natural measure of fit is the sample variance of the estimated residuals, also known as the Mean Squared Error (MSE) of the model. Unfortunately, choosing a model to minimize MSE also selects a specification that is far too large. The solution to this overfitting problem is to add a penalty to the MSE that increases each time a new parameter is added, employing criteria like **Akaike Information Criterion (AIC)** and **Bayesian Information Criterion (BIC)**.

- AIC is calculated by $T \ln \hat{\sigma}^2 + 2k$, where T is the sample size and k is the number of parameters.
- BIC alters the penalty and is computed by $T \ln \hat{\sigma}^2 2 + k \ln T$

Unlike the AIC, the BIC has a cost per parameter that slowly increases with T. Hence BIC always selects a model that is no larger than the model selected by the AIC (assuming $\ln T > 2$), and is a consistent model selection criterion (i.e., the true model is selected as $T \rightarrow \infty$).

```
lags = ar_select_order(df_train, maxlag=36, ic='bic', old_names=False).ar_lags
print('(BIC) lags= ', len(lags), ':', lags)
```

```
(BIC) lags=  1 : [1]
```

```
# Train final model on train split
model = AutoReg(df_train, lags=lags, old_names=False).fit()
print(model.summary())
```

```
                            AutoReg Model Results
==============================================================================
Dep. Variable:                 INDPRO   No. Observations:                 1261
Model:                     AutoReg(1)   Log Likelihood                3366.312
Method:               Conditional MLE   S.D. of innovations              0.017
Date:                Mon, 03 Mar 2025   AIC                          -6726.625
Time:                        10:54:10   BIC                          -6711.208
Sample:                    03-31-1919   HQIC                         -6720.831
                         - 02-29-2024
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0013      0.000      2.705      0.007       0.000       0.002
INDPRO.L1      0.4859      0.025     19.794      0.000       0.438       0.534
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            2.0581           +0.0000j            2.0581            0.0000
-----------------------------------------------------------------------------
```

10.5.2 One-step forecast

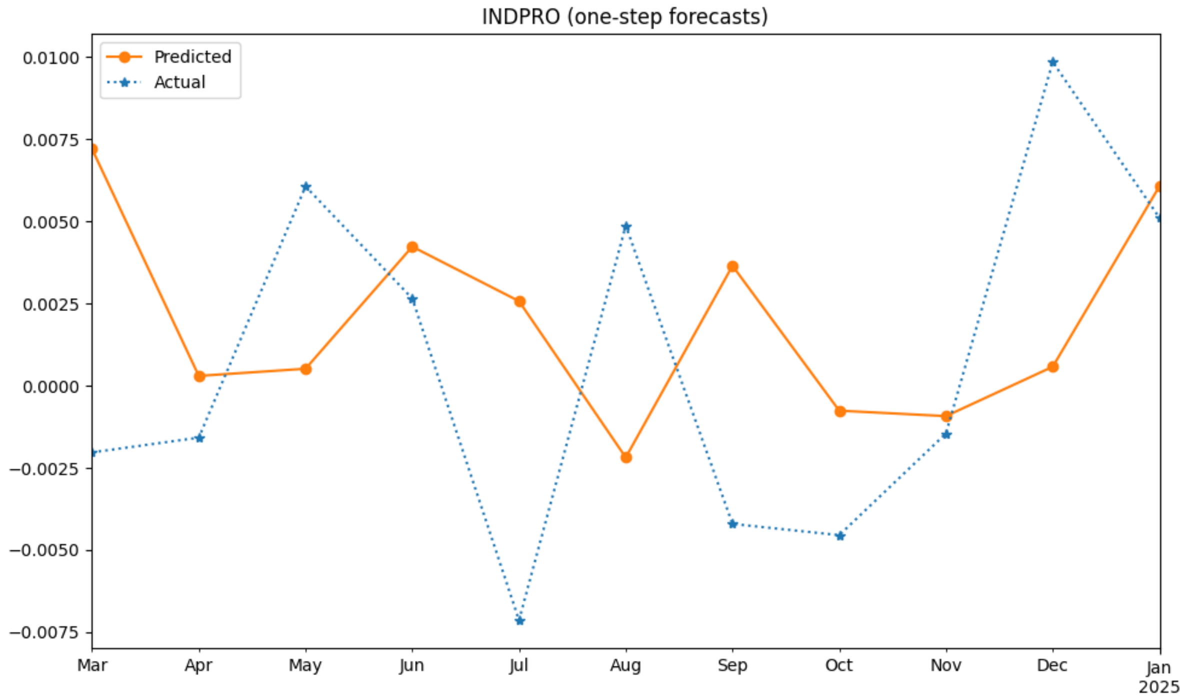
One-step forecast predicts Y_{T+1} using all available data up to time T, including the entire prior history of Y ($Y_T, Y_{T-1}, ...$), as well as all values of any other variables that occurred at time T or earlier.

```
# Observations to predict are from the oos test split
test = AutoReg(df_all, lags=lags, old_names=False)
```

```
# Use model params from train split, start predictions from last train row
df_pred = test.predict(model.params, start=df_test.index[0])
mse = mean_squared_error(df_test, df_pred)
#var = np.mean(np.square(df_test - df_train.mean()))
print(f"ST Forecast({len(df_pred)}): rmse={np.sqrt(mse)}")
```

```
ST Forecast(11): rmse=0.006247508053669628
```

```
fig, ax = plt.subplots(clear=True, num=1, figsize=(10, 6))
df_pred.plot(ax=ax, c='C1', ls='-', marker='o', label='Predicted')
df_test.plot(ax=ax, c='C0', ls=':', marker='*', label='Actual')
ax.legend()
ax.set_title(series_id + " (one-step forecasts)")
ax.set_xlabel('')
plt.tight_layout()
```

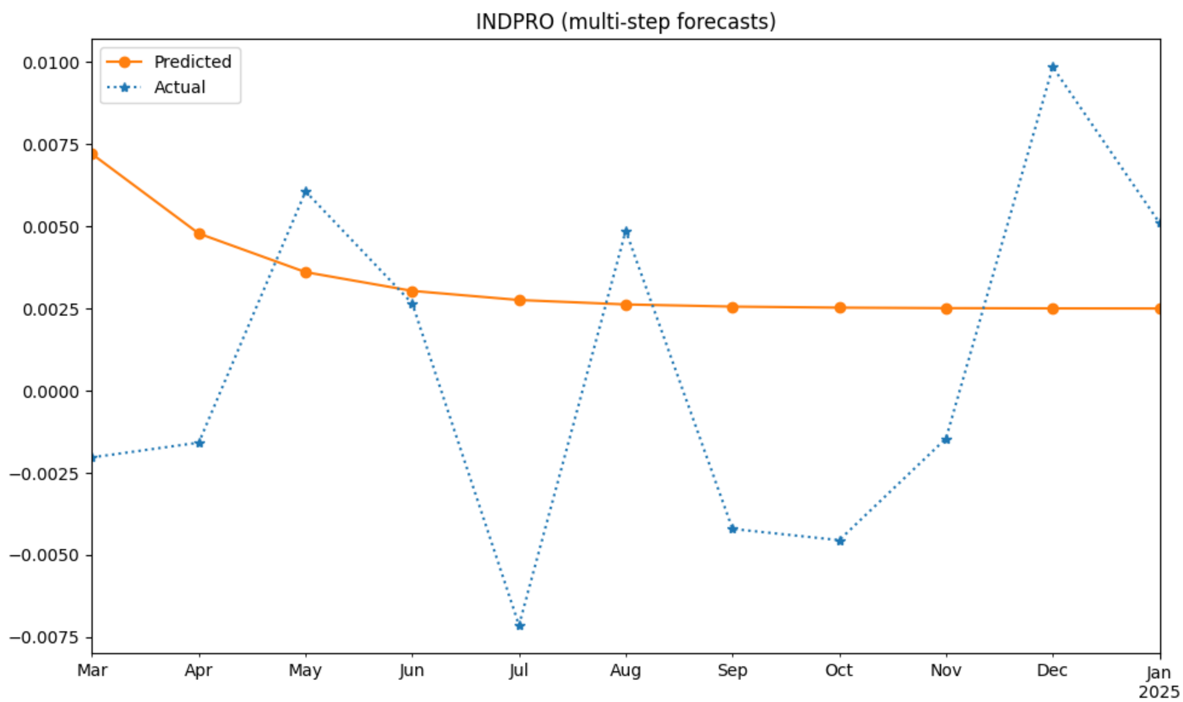


10.5.3 Multi-step forecast

Multi-step forecasts recursively predict values at future horizons, starting with $E_T[Y_{T+1}].Theforecastathorizonhdependsonforecastsfromearliersteps$(E_T[Y_{T+ 1}], $\cdots$, E_T [Y_{T+ h - 1}]). When these quantities appear in the forecast for period $T + h$, they are replaced by the forecasts computed for horizons $1, 2, ..., h - 1$.

```
# set dynamic=True for multi-step ahead predictions
df_pred = test.predict(model.params, dynamic=True,
                       start=df_test.index[0], end=df_test.index[-1])
mse = mean_squared_error(df_test, df_pred)
#var = np.mean(np.square(df_test - df_train.mean()))
print(f"Long-term Forecasts:  rmse={np.sqrt(mse):.6f}")
fig, ax = plt.subplots(clear=True, num=2, figsize=(10, 6))
df_pred.plot(ax=ax, c='C1', ls='-', marker='o', label='Predicted')
df_test.plot(ax=ax, c='C0', ls=':', marker='*', label='Actual')
ax.legend()
ax.set_title(series_id + " (multi-step forecasts)")
ax.set_xlabel('')
plt.tight_layout()
```

```
Long-term Forecasts:  rmse=0.006085
```



10.5.4 Granger causality

Granger causality tests whether past values of one time series help predict another.

```
# Granger Causality: INDPRO vs CPI
variables = ['INDPRO', 'CPIAUCSL']
start = 19620101
for series_id, exog_id in zip(variables, list(reversed(variables))):
    df = pd.concat([alf(s, start=start, log=1)
                    for s in [series_id, exog_id]], axis=1)
    df.index = pd.DatetimeIndex(df.index.astype(str))
    df.index.freq = freq
    data = df.diff().dropna()

    print(f"Null Hypothesis: {exog_id} granger-causes {series_id}")
    res = grangercausalitytests(data, maxlag=3)
    print()

    dmf = (f'{series_id} ~ {series_id}.shift(1) '
           f' + {exog_id}.shift(1) '
           f' + {exog_id}.shift(2) '
           f' + {exog_id}.shift(3) ')
    model = smf.ols(formula=dmf, data=data).fit()
    robust = model.get_robustcov_results(cov_type='HAC', use_t=None, maxlags=0)
    print(robust.summary())
```

```
Null Hypothesis: CPIAUCSL granger-causes INDPRO

Granger Causality
number of lags (no zero) 1
```

(continues on next page)

(continued from previous page)

```
ssr based F test:         F=0.4754  , p=0.4907  , df_denom=752, df_num=1
ssr based chi2 test:   chi2=0.4773  , p=0.4896  , df=1
likelihood ratio test: chi2=0.4772  , p=0.4897  , df=1
parameter F test:         F=0.4754  , p=0.4907  , df_denom=752, df_num=1

Granger Causality
number of lags (no zero) 2
ssr based F test:         F=7.1786  , p=0.0008  , df_denom=749, df_num=2
ssr based chi2 test:   chi2=14.4530 , p=0.0007  , df=2
likelihood ratio test: chi2=14.3162 , p=0.0008  , df=2
parameter F test:         F=7.1786  , p=0.0008  , df_denom=749, df_num=2

Granger Causality
number of lags (no zero) 3
ssr based F test:         F=5.4544  , p=0.0010  , df_denom=746, df_num=3
ssr based chi2 test:   chi2=16.5167 , p=0.0009  , df=3
likelihood ratio test: chi2=16.3381 , p=0.0010  , df=3
parameter F test:         F=5.4544  , p=0.0010  , df_denom=746, df_num=3

                            OLS Regression Results
==============================================================================
Dep. Variable:                 INDPRO   R-squared:                       0.090
Model:                            OLS   Adj. R-squared:                  0.085
Method:                 Least Squares   F-statistic:                     2.729
Date:                Mon, 03 Mar 2025   Prob (F-statistic):             0.0283
Time:                        10:54:12   Log-Likelihood:                 2472.9
No. Observations:                 753   AIC:                            -4936.
Df Residuals:                     748   BIC:                            -4913.
Df Model:                           4
Covariance Type:                  HAC
=====================================================================================
                        coef    std err          t      P>|t|      [0.025      0.
↪975]
-----------------------------------------------------------------------------------
↪--
Intercept             0.0019      0.001      1.993      0.047    2.94e-05       0.
↪004
INDPRO.shift(1)       0.2553      0.141      1.817      0.070      -0.020       0.
↪531
CPIAUCSL.shift(1)     0.4106      0.208      1.973      0.049       0.002       0.
↪819
CPIAUCSL.shift(2)    -0.4210      0.193     -2.178      0.030      -0.800      -0.
↪042
CPIAUCSL.shift(3)    -0.1652      0.134     -1.228      0.220      -0.429       0.
↪099
==============================================================================
Omnibus:                      721.664   Durbin-Watson:                   1.981
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           120296.612
Skew:                          -3.763   Prob(JB):                         0.00
Kurtosis:                      64.462   Cond. No.                         565.
==============================================================================

Notes:
[1] Standard Errors are heteroscedasticity and autocorrelation robust (HAC) using␣
↪0 lags and without small sample correction
Null Hypothesis: INDPRO granger-causes CPIAUCSL
```

(continues on next page)

(continued from previous page)

```
Granger Causality
number of lags (no zero) 1
ssr based F test:         F=0.0450  , p=0.8321  , df_denom=752, df_num=1
ssr based chi2 test:   chi2=0.0452  , p=0.8317  , df=1
likelihood ratio test: chi2=0.0452  , p=0.8317  , df=1
parameter F test:         F=0.0450  , p=0.8321  , df_denom=752, df_num=1

Granger Causality
number of lags (no zero) 2
ssr based F test:         F=0.0150  , p=0.9851  , df_denom=749, df_num=2
ssr based chi2 test:   chi2=0.0301  , p=0.9850  , df=2
likelihood ratio test: chi2=0.0301  , p=0.9850  , df=2
parameter F test:         F=0.0150  , p=0.9851  , df_denom=749, df_num=2

Granger Causality
number of lags (no zero) 3
ssr based F test:         F=0.0926  , p=0.9641  , df_denom=746, df_num=3
ssr based chi2 test:   chi2=0.2804  , p=0.9637  , df=3
likelihood ratio test: chi2=0.2803  , p=0.9637  , df=3
parameter F test:         F=0.0926  , p=0.9641  , df_denom=746, df_num=3

                            OLS Regression Results
==============================================================================
Dep. Variable:               CPIAUCSL   R-squared:                       0.386
Model:                            OLS   Adj. R-squared:                  0.383
Method:                 Least Squares   F-statistic:                     51.93
Date:                Mon, 03 Mar 2025   Prob (F-statistic):           1.28e-38
Time:                        10:54:12   Log-Likelihood:                 3452.5
No. Observations:                 753   AIC:                            -6895.
Df Residuals:                     748   BIC:                            -6872.
Df Model:                           4
Covariance Type:                  HAC
=====================================================================================
                       coef    std err          t      P>|t|      [0.025      0.
↪975]
-----------------------------------------------------------------------------------
↪--
Intercept            0.0012      0.000      6.847      0.000       0.001       0.
↪002
CPIAUCSL.shift(1)    0.6210      0.046     13.623      0.000       0.532       0.
↪711
INDPRO.shift(1)     -0.0021      0.014     -0.147      0.883      -0.030       0.
↪025
INDPRO.shift(2)      0.0005      0.017      0.032      0.975      -0.032       0.
↪033
INDPRO.shift(3)      0.0061      0.009      0.676      0.499      -0.012       0.
↪024
==============================================================================
Omnibus:                       88.983   Durbin-Watson:                   2.119
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              750.324
Skew:                           0.030   Prob(JB):                    1.17e-163
Kurtosis:                       7.890   Cond. No.                         319.
==============================================================================

Notes:
```

(continues on next page)

(continued from previous page)

```
[1] Standard Errors are heteroscedasticity and autocorrelation robust (HAC) using
↪0 lags and without small sample correction
```

10.5.5 Impulse response function

Impulse response functions (IRF) analyze how shocks propagate through a system, often in **Vector Autoregression (VAR)** models.

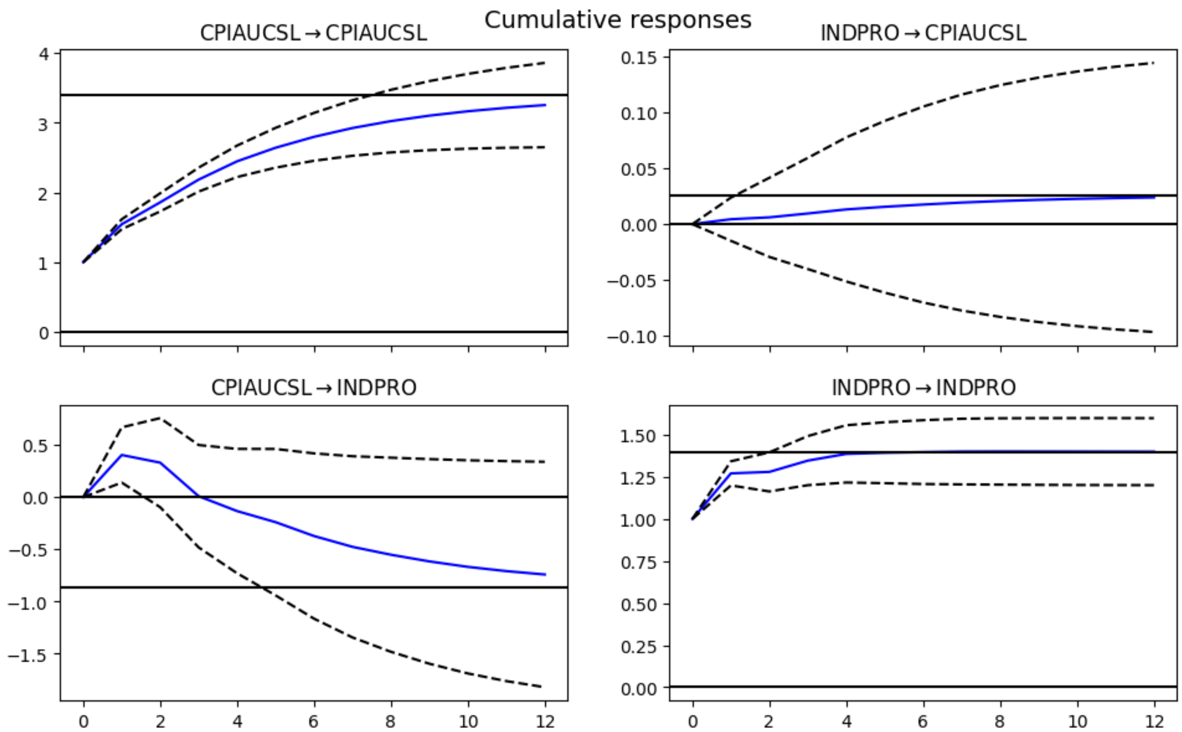
```python
# Vector Autoregression: Impulse Response Function
model = VAR(data)
results = model.fit(maxlags=3)
print(results.summary())
irf = results.irf(12)
#irf.plot(orth=False)
irf.plot_cum_effects(orth=False, figsize=(10, 6))
```

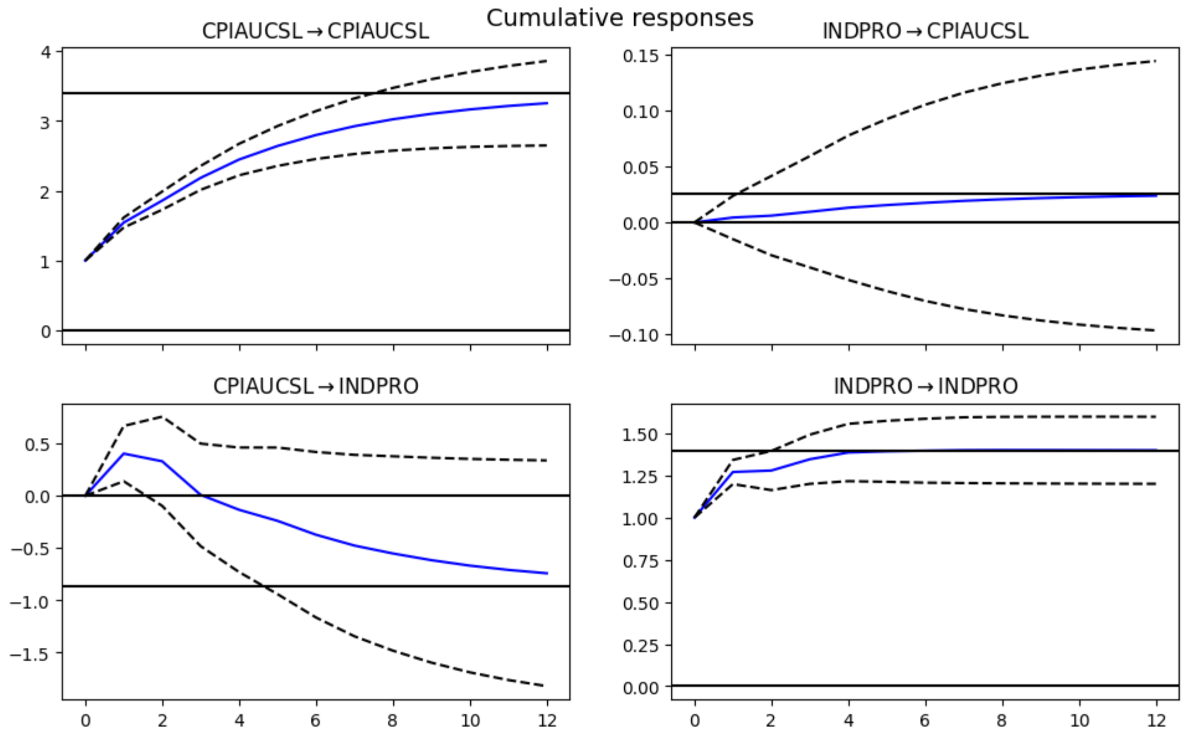
```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Mon, 03, Mar, 2025
Time:                     10:54:12
--------------------------------------------------------------------
No. of Equations:         2.00000    BIC:                   -21.3411
Nobs:                     753.000    HQIC:                  -21.3939
Log likelihood:           5944.36    FPE:                4.94710e-10
AIC:                     -21.4270    Det(Omega_mle):     4.85639e-10
--------------------------------------------------------------------
Results for equation CPIAUCSL
==============================================================================
                 coefficient       std. error           t-stat            prob
------------------------------------------------------------------------------
const               0.000909         0.000139            6.528           0.000
L1.CPIAUCSL         0.544275         0.036383           14.960           0.000
L1.INDPRO           0.004382         0.009861            0.444           0.657
L2.CPIAUCSL         0.016181         0.041512            0.390           0.697
L2.INDPRO          -0.001898         0.010116           -0.188           0.851
L3.CPIAUCSL         0.147062         0.036710            4.006           0.000
L3.INDPRO           0.002953         0.009745            0.303           0.762
==============================================================================

Results for equation INDPRO
==============================================================================
                 coefficient       std. error           t-stat            prob
------------------------------------------------------------------------------
const               0.001899         0.000517            3.673           0.000
L1.CPIAUCSL         0.404130         0.135092            2.992           0.003
L1.INDPRO           0.270682         0.036614            7.393           0.000
L2.CPIAUCSL        -0.402895         0.154135           -2.614           0.009
L2.INDPRO          -0.066262         0.037563           -1.764           0.078
L3.CPIAUCSL        -0.182529         0.136306           -1.339           0.181
L3.INDPRO           0.083606         0.036182            2.311           0.021
==============================================================================
```

(continues on next page)

(continued from previous page)

```
Correlation matrix of residuals
            CPIAUCSL    INDPRO
CPIAUCSL    1.000000  0.101927
INDPRO      0.101927  1.000000
```





References:

FRM Part I Exam Book Quantitative Analysis Ch 10-11

CHAPTER

ELEVEN

APPROXIMATE FACTOR MODELS

It is better to be vaguely right than precisely wrong - Carveth Read

Approximate factor models provide a simplified yet effective way to represent time series data by identifying common factors that explain variation across observed variables. A critical step in time series analysis is ensuring stationarity, typically assessed using tests like the Augmented Dickey-Fuller (ADF) test. Transformations such as differencing and logarithmic scaling are often applied to remove non-stationary components. Principal Component Analysis (PCA) is commonly used for factor extraction, and the optimal number of components can be selected using criteria like the Bayesian Information Criterion (BIC). To handle missing data, imputation methods such as the Expectation-Maximization (EM) algorithm are used.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA
from sklearn.preprocessing import StandardScaler
from finds.readers import Alfred, fred_md, fred_qd
from finds.recipes import (approximate_factors, mrsq, select_baing, integration_order,
                           remove_outliers, is_outlier)
from secret import credentials
VERBOSE = 0
#%matplotlib qt
```

```
alf = Alfred(api_key=credentials['fred']['api_key'], verbose=VERBOSE)
```

```
## Retrieve recession periods from FRED
vspans = alf.date_spans('USREC')
DataFrame(vspans, columns=['Start', 'End'])
```

```
        Start        End
0  1854-12-31 1854-12-31
1  1857-06-30 1858-12-31
2  1860-10-31 1861-06-30
3  1865-04-30 1867-12-31
4  1869-06-30 1870-12-31
5  1873-10-31 1879-03-31
6  1882-03-31 1885-05-31
7  1887-03-31 1888-04-30
8  1890-07-31 1891-05-31
9  1893-01-31 1894-06-30
```

(continues on next page)

(continued from previous page)

```
10 1895-12-31 1897-06-30
11 1899-06-30 1900-12-31
12 1902-09-30 1904-08-31
13 1907-05-31 1908-06-30
14 1910-01-31 1912-01-31
15 1913-01-31 1914-12-31
16 1918-08-31 1919-03-31
17 1920-01-31 1921-07-31
18 1923-05-31 1924-07-31
19 1926-10-31 1927-11-30
20 1929-08-31 1933-03-31
21 1937-05-31 1938-06-30
22 1945-02-28 1945-10-31
23 1948-11-30 1949-10-31
24 1953-07-31 1954-05-31
25 1957-08-31 1958-04-30
26 1960-04-30 1961-02-28
27 1969-12-31 1970-11-30
28 1973-11-30 1975-03-31
29 1980-01-31 1980-07-31
30 1981-07-31 1982-11-30
31 1990-07-31 1991-03-31
32 2001-03-31 2001-11-30
33 2007-12-31 2009-06-30
34 2020-02-29 2020-04-30
```

11.1 Integration order

11.1.1 Augmented Dickey-Fuller test

The **Augmented Dickey-Fuller (ADF) test** is one of the most widely used methods for detecting unit roots in a time series. It operates by performing an ordinary least squares (OLS) regression, where the first difference of the series is regressed on its lagged level, along with deterministic components and lagged differences. The general form of the ADF regression is:

$$\Delta Y_t = \gamma Y_{t-1} + (\delta_0 + \delta_1 t) + \lambda_1 \Delta Y_{t-1} + ... + \lambda_p \Delta Y_{t-p}$$

If $\gamma = 0$, then Y_t follows a **random walk** and is non-stationary, implying the presence of a unit root. The alternative hypothesis, $\gamma < 0$, suggests that Y_t is **covariance-stationary**. This is a one-sided test, as positive values of γ would indicate an autoregressive coefficient greater than one, leading to instability.

When the test fails to reject the null hypothesis of a unit root, differencing is required to transform the series into a stationary form. Best practice involves repeatedly applying the ADF test to the differenced data until stationarity is achieved.

```
qd_df, qd_codes = fred_qd() # 202004
md_df, md_codes = fred_md() # 201505
qd_date = max(qd_df.index)
md_date = max(md_df.index)
```

```
FRED-QD vintage: quarterly/current.csv
FRED-MD vintage: monthly/current.csv
```

11.1.2 Transformations

The FRED-MD and FRED-QD datasets include transformation codes that indicate appropriate preprocessing steps, such as applying logarithmic transformations or differencing, to ensure stationarity. The number of differences required depends on the presence of unit roots.

```
print(f"Number of series by suggested tcode transformations ({md_date}):")
tcodes = DataFrame.from_dict({i: alf.tcode[i] for i in range(1, 8)},
                             orient='index')\
                  .join(qd_codes['transform'].value_counts().rename('fred-qd'))\
                  .join(md_codes['transform'].value_counts().rename('fred-md'))\
                  .fillna(0)\
                  .astype(int)\
                  .rename_axis(index='tcode')
tcodes
```

```
Number of series by suggested tcode transformations (20250131):
```

tcode	diff	log	pct_change	fred-qd	fred-md
1	0	0	0	22	11
2	1	0	0	32	19
3	2	0	0	0	0
4	0	1	0	0	10
5	1	1	0	140	52
6	2	1	0	49	33
7	1	0	1	1	1

```
# For each series, compare fitted integration order with tcode
out = {}
stationary_out = {}
for label, df, transforms in [['md', md_df, md_codes['transform']],
                              ['qd', qd_df, qd_codes['transform']]]:
    stationary = dict()
    for series_id, tcode in transforms.items():
        # apply transformation if series tcode is valid
        if tcode in range(1, 8):
            # take logs of series if specified by tcode
            s = np.log(df[series_id]) if tcode in [4, 5, 6] else df[series_id]

            # estimate integration order
            order = integration_order(s.dropna(), pvalue=0.05)

            # expected order specified by tcode
            expected_order = 2 if tcode == 7 else ((tcode - 1) % 3)

            # accumulate results for this series
            stationary[series_id] = {'tcode': tcode,
                                     'I(p)': order,
                                     'different': order - expected_order,
                                     'title': alf.header(series_id)}
#             print(series_id, tcode, expected_order, order)

    # collect results for display
    stationary = DataFrame.from_dict(stationary, orient='index')
```

(continues on next page)

(continued from previous page)

```
    stationary = stationary.sort_values(stationary.columns.to_list())
    c = stationary.groupby(['tcode','I(p)'])['title'].count().reset_index()
    out[label] = c.pivot(index='tcode', columns='I(p)', values='title')\
                 .fillna(0).astype(int)
    out[label].columns=[f"I({p})" for p in out[label].columns]
    stationary_out[label] = stationary[stationary['different'] > 0]
```

```
print('Series by tcode, transformations and estimated order of integration:')
results = pd.concat([tcodes.drop(columns='fred-md'),
                     out['qd'],
                     tcodes['fred-md'],
                     out['md']], axis=1).fillna(0).astype(int)
print('Integration order by transformation')
results
```

```
Series by tcode, transformations and estimated order of integration:
Integration order by transformation
```

```
       diff  log  pct_change  fred-qd  I(0)  I(1)  I(2)  fred-md  I(0)  I(1)  \
tcode
1         0    0           0       22    17     5     0       11    11     0
2         1    0           0       32    11    19     2       19     4    15
3         2    0           0        0     0     0     0        0     0     0
4         0    1           0        0     0     0     0       10     7     3
5         1    1           0      140    30   107     3       52    14    38
6         2    1           0       49     0    29    20       33     0    30
7         1    0           1        1     0     1     0        1     0     1

       I(2)
tcode
1         0
2         0
3         0
4         0
5         0
6         3
7         0
```

```
print('FRED-MD Series with unit root after transformation')
stationary_out['md']
```

```
FRED-MD Series with unit root after transformation
```

```
          tcode  I(p)  different  \
PERMITMW      4     1          1
HOUSTMW       4     1          1
HOUSTNE       4     1          1

                                                      title
PERMITMW  New Privately-Owned Housing Units Authorized i...
HOUSTMW   New Privately-Owned Housing Units Started: Tot...
HOUSTNE   New Privately-Owned Housing Units Started: Tot...
```

```
print('FRED-QD Series with unit root after transformation')
stationary_out['qd']
```

```
FRED-QD Series with unit root after transformation
```

```
                tcode  I(p)  different  \
GS1TB3M             1     1          1
NWPI                1     1          1
TLBSNNBBDI          1     1          1
TLBSNNCBBDI         1     1          1
HWI                 1     1          1
GFDEBTN             2     2          1
S&P div yield       2     2          1
CES2000000008       5     2          1
TLBSHNO             5     2          1
OPHMFG              5     2          1

                                                          title
GS1TB3M                                         *** GS1TB3M ***
NWPI                                               *** NWPI ***
TLBSNNBBDI                                   *** TLBSNNBBDI ***
TLBSNNCBBDI                                 *** TLBSNNCBBDI ***
HWI                              Help Wanted Index for United States
GFDEBTN                              Federal Debt: Total Public Debt
S&P div yield        S&P's Composite Common Stock: Dividend Yield
CES2000000008  Average Hourly Earnings of Production and Nons...
TLBSHNO        Households and Nonprofit Organizations; Total ...
OPHMFG         Manufacturing Sector: Labor Productivity (Outp...
```

11.2 Factor selection

Bai and Ng (2002) provide a systematic approach for extracting and determining the optimal number of factors in an approximate factor model of time series, and imputing missing or outlier observations in the dataset.

11.2.1 Principal component analysis (PCA)

Principal component analysis finds a low-dimensional representation of a data set that contains as much as possible of the variation of its p features. Each principal component is a linear combination of the original features, capturing the most significant patterns in the data.

The first principal component is the normalized linear combination of features:

$$Z_1 = \phi_{11}X_1 + \phi_{21}X_2 + ... + \phi_{p1}X_p$$

which maximizes variance. Subsequent principal components, such as Z_2 and Z_3, are determined iteratively, ensuring that each is uncorrelated with the previously computed components while explaining the next highest level of variance.

11.2.2 BIC criterion

Determining the appropriate number of factors is a crucial step in approximate factor modeling. The **Bayesian Information Criterion (BIC)** selects the optimal number of components by balancing model fit and complexity. Lower BIC values indicate a better trade-off between variance explanation and model parsimony.

11.2.3 Data imputation

When dealing with missing data in time series, the **Expectation-Maximization (EM) algorithm** can be employed to estimate missing values from factors iteratively. After each imputation iteration, factors are recovered as projections from PCA, ensuring that the underlying structure of the data is captured.

```
# Verify BaiNg implemention on original FRED-MD and FRED-QD
qd_df, qd_codes = fred_qd('assets/FRED-QD_2020m04.csv', url='')
md_df, md_codes = fred_md('assets/FRED-MD_2015m5.csv', url='')
for freq, df, transforms in [['monthly', md_df, md_codes['transform']],
                             ['quarterly', qd_df, qd_codes['transform']]]:
    # Apply tcode transformations
    transformed = []
    for col in df.columns:
        transformed.append(alf.transform(df[col],
                                         tcode=transforms[col],
                                         freq=freq[0]))
    data = pd.concat(transformed, axis=1).iloc[2:]
    cols = list(data.columns)
    sample = data.index[((np.count_nonzero(np.isnan(data), axis=1)==0)
                         | (data.index <= 20141231))
                        & (data.index >= 19600301)]

    # set missing and outliers to NaN
    x = data.loc[sample]
    x = remove_outliers(x)     # default fence 'iq10' is 10 times IQ

    # compute factors EM and auto select number of components, r
    Z = approximate_factors(x, p=2, verbose=VERBOSE)
    r = select_baing(Z, p=2)

    # show marginal R2's of series to each component
    mR2 = mrsq(Z, r).to_numpy()
    print(f"FRED-{freq[0].upper()}D {freq} series:")
    print(DataFrame({'selected': r,
                     'variance explained': np.sum(np.mean(mR2[:, :r], axis=0)),
                     'start': min(sample),
                     'end': max(sample),
                     'obs': Z.shape[0],
                     'series': Z.shape[1]},
                    index=[f'factors']))

    for k in range(r):
        args = np.argsort(-mR2[:, k])
        print(f"Factor:{1+k} Variance Explained={np.mean(mR2[:,k]):.4f}")
        print(DataFrame.from_dict({mR2[arg, k].round(4):
                                   {'series': cols[arg],
                                    'description': alf.header(cols[arg])}
                                   for arg in args[:10]},
                                  orient='index'))
```

```
FRED-QD vintage: assets/FRED-QD_2020m04.csv
FRED-MD vintage: assets/FRED-MD_2015m5.csv
FRED-MD monthly series:
         selected  variance explained     start       end  obs  series
factors         8            0.485832  19600331  20141231  658     134
Factor:1 Variance Explained=0.1613
           series                              description
0.7424     USGOOD             All Employees, Goods-Producing
0.7235     PAYEMS               All Employees, Total Nonfarm
0.7002     MANEMP               All Employees, Manufacturing
0.6565       NAPM                               *** NAPM ***
0.6540  IPMANSICS  Industrial Production: Manufacturing (SIC)
0.6513    DMANEMP               All Employees, Durable Goods
0.6314     INDPRO          Industrial Production: Total Index
0.6037    NAPMNOI                            *** NAPMNOI ***
0.6026     NAPMPI                             *** NAPMPI ***
0.5601     CUMFNS   Capacity Utilization: Manufacturing (SIC)
Factor:2 Variance Explained=0.0703
          series                                        description
0.6259   T10YFFM  10-Year Treasury Constant Maturity Minus Feder...
0.6164    AAAFFM  Moody's Seasoned Aaa Corporate Bond Minus Fede...
0.5856    BAAFFM  Moody's Seasoned Baa Corporate Bond Minus Fede...
0.5832    T5YFFM  5-Year Treasury Constant Maturity Minus Federa...
0.4785  TB6SMFFM     6-Month Treasury Bill Minus Federal Funds Rate
0.4779  TB3SMFFM     3-Month Treasury Bill Minus Federal Funds Rate
0.4322    T1YFFM  1-Year Treasury Constant Maturity Minus Federa...
0.2367  COMPAPFF            3-Month Commercial Paper Minus FEDFUNDS
0.2258    BUSINV                         Total Business Inventories
0.1896   NAPMPRI                                    *** NAPMPRI ***
Factor:3 Variance Explained=0.0652
                 series                                        description
0.7192      CUSR0000SAC  Consumer Price Index for All Urban Consumers: ...
0.7066  DNDGRG3M086SBEA  Personal consumption expenditures: Nondurable ...
0.6700         CPIAUCSL  Consumer Price Index for All Urban Consumers: ...
0.6392    CUSR0000SA0L5  Consumer Price Index for All Urban Consumers: ...
0.6115    CUUR0000SA0L2  Consumer Price Index for All Urban Consumers: ...
0.5923            PCEPI  Personal Consumption Expenditures: Chain-type ...
0.5907         CPITRNSL  Consumer Price Index for All Urban Consumers: ...
0.5482         CPIULFSL  Consumer Price Index for All Urban Consumers: ...
0.4863           PPIFCG  Producer Price Index by Commodity for Finished...
0.4779           PPIITM  Producer Price Index by Commodity Intermediate...
Factor:4 Variance Explained=0.0547
        series                                        description
0.4294     GS1  Market Yield on U.S. Treasury Securities at 1-...
0.4228     GS5  Market Yield on U.S. Treasury Securities at 5-...
0.4123     AAA          Moody's Seasoned Aaa Corporate Bond Yield
0.3995   TB6MS  6-Month Treasury Bill Secondary Market Rate, D...
0.3893    GS10  Market Yield on U.S. Treasury Securities at 10...
0.3621     BAA          Moody's Seasoned Baa Corporate Bond Yield
0.3179   TB3MS  3-Month Treasury Bill Secondary Market Rate, D...
0.3139    CP3M        3-Month AA Financial Commercial Paper Rates
0.2681   HOUST  New Privately-Owned Housing Units Started: Tot...
0.2562  HOUSTW  New Privately-Owned Housing Units Started: Tot...
Factor:5 Variance Explained=0.0425
          series                                        description
0.2481   PERMITW  New Privately-Owned Housing Units Authorized i...
0.2374    PERMIT  New Privately-Owned Housing Units Authorized i...
```

(continues on next page)

(continued from previous page)

```
0.2316    HOUSTW  New Privately-Owned Housing Units Started: Tot...
0.2225       GS5  Market Yield on U.S. Treasury Securities at 5-...
0.2221       GS1  Market Yield on U.S. Treasury Securities at 1-...
0.2146     HOUST  New Privately-Owned Housing Units Started: Tot...
0.2028      GS10  Market Yield on U.S. Treasury Securities at 10...
0.1951  PERMITMW  New Privately-Owned Housing Units Authorized i...
0.1949    T1YFFM  1-Year Treasury Constant Maturity Minus Federa...
0.1932     TB6MS  6-Month Treasury Bill Secondary Market Rate, D...
Factor:6 Variance Explained=0.0365
          series                                      description
0.2186   IPCONGD             Industrial Production: Consumer Goods
0.1793   ISRATIO        Total Business: Inventories to Sales Ratio
0.1736    NAPMEI                                    *** NAPMEI ***
0.1628  IPDCONGD  Industrial Production: Durable Consumer Goods:...
0.1577   IPFINAL             Industrial Production: Final Products
0.1545  TB6SMFFM     6-Month Treasury Bill Minus Federal Funds Rate
0.1425      NAPM                                      *** NAPM ***
0.1416    NAPMII                                    *** NAPMII ***
0.1414    ACOGNO         Manufacturers' New Orders: Consumer Goods
0.1373   IPFPNSS  Industrial Production: Final Products and Noni...
Factor:7 Variance Explained=0.0292
               series                                      description
0.5165        S&P 500          S&P's Common Stock Price Index: Composite
0.5159    S&P: indust        S&P's Common Stock Price Index: Industrials
0.4002  S&P div yield       S&P's Composite Common Stock: Dividend Yield
0.2764   S&P PE ratio  S&P's Composite Common Stock: Price-Earnings R...
0.2564        UMCSENT         University of Michigan: Consumer Sentiment
0.1030        IPCONGD              Industrial Production: Consumer Goods
0.1019         EXCAUS  Canadian Dollars to U.S. Dollar Spot Exchange ...
0.0728        IPFINAL              Industrial Production: Final Products
0.0644       IPDCONGD  Industrial Production: Durable Consumer Goods:...
0.0565        IPFPNSS  Industrial Production: Final Products and Noni...
Factor:8 Variance Explained=0.0262
               series                                      description
0.5375       TWEXMMTH  Nominal Major Currencies U.S. Dollar Index (Go...
0.2309         EXSZUS     Swiss Francs to U.S. Dollar Spot Exchange Rate
0.2111         EXUSUK  U.S. Dollars to U.K. Pound Sterling Spot Excha...
0.1497         EXJPUS     Japanese Yen to U.S. Dollar Spot Exchange Rate
0.1332         SRVPRD                All Employees, Service-Providing
0.1199  CES0600000008  Average Hourly Earnings of Production and Nons...
0.1156         ACOGNO         Manufacturers' New Orders: Consumer Goods
0.1128  CES3000000008  Average Hourly Earnings of Production and Nons...
0.1104         USGOVT                      All Employees, Government
0.1010        USTRADE                    All Employees, Retail Trade
FRED-QD quarterly series:
         selected  variance explained     start       end  obs  series
factors         7              0.4981  19600331  20191231  240     248
Factor:1 Variance Explained=0.2014
           series                                      description
0.8382     USPRIV                     All Employees, Total Private
0.8184     USGOOD                   All Employees, Goods-Producing
0.8165      OUTMS  Manufacturing Sector: Real Sectoral Output for...
0.8124  IPMANSICS         Industrial Production: Manufacturing (SIC)
0.8120     PAYEMS                     All Employees, Total Nonfarm
0.8057     INDPRO               Industrial Production: Total Index
0.7758     MANEMP                     All Employees, Manufacturing
```

(continues on next page)

(continued from previous page)

```
0.7717     HOANBS  Nonfarm Business Sector: Hours Worked for All ...
0.7667    DMANEMP                       All Employees, Durable Goods
0.7651     UNRATE                                  Unemployment Rate
Factor:2 Variance Explained=0.0824
               series                                        description
0.4995         AAAFFM  Moody's Seasoned Aaa Corporate Bond Minus Fede...
0.4761         T5YFFM  5-Year Treasury Constant Maturity Minus Federa...
0.4622         PERMIT  New Privately-Owned Housing Units Authorized i...
0.4404         BUSINV                         Total Business Inventories
0.4231          HOUST  New Privately-Owned Housing Units Started: Tot...
0.4055        PERMITS  New Privately-Owned Housing Units Authorized i...
0.3925  S&P div yield       S&P's Composite Common Stock: Dividend Yield
0.3849            TCU                 Capacity Utilization: Total Index
0.3674      CPF3MTB3M  3-Month Commercial Paper Minus 3-Month Treasur...
0.3633       GS10TB3M                                   *** GS10TB3M ***
Factor:3 Variance Explained=0.0727
                 series                                        description
0.7569    CUSR0000SA0L2  Consumer Price Index for All Urban Consumers: ...
0.7405      CUSR0000SAC  Consumer Price Index for All Urban Consumers: ...
0.7368  DGDSRG3Q086SBEA  Personal consumption expenditures: Goods (chai...
0.7212          PCECTPI  Personal Consumption Expenditures: Chain-type ...
0.7065         CPITRNSL  Consumer Price Index for All Urban Consumers: ...
0.6963  DNDGRG3Q086SBEA  Personal consumption expenditures: Nondurable ...
0.6798    CUSR0000SA0L5  Consumer Price Index for All Urban Consumers: ...
0.6712         CPIAUCSL  Consumer Price Index for All Urban Consumers: ...
0.6472          WPSID61  Producer Price Index by Commodity: Intermediat...
0.6352         CPIULFSL  Consumer Price Index for All Urban Consumers: ...
Factor:4 Variance Explained=0.0467
               series                                        description
0.4044          IMFSL    Institutional Money Market Funds (DISCONTINUED)
0.3457  CES9093000001                   All Employees, Local Government
0.3259  CES9092000001                   All Employees, State Government
0.2484         EXUSEU            U.S. Dollars to Euro Spot Exchange Rate
0.2482         USGOVT                         All Employees, Government
0.2364        GFDEBTN                   Federal Debt: Total Public Debt
0.2251        REVOLSL    Revolving Consumer Credit Owned and Securitized
0.2202         USSERV                     All Employees, Other Services
0.2128        COMPRMS  Manufacturing Sector: Real Hourly Compensation...
0.2126           NWPI                                       *** NWPI ***
Factor:5 Variance Explained=0.0375
             series                                        description
0.3664       OPHMFG  Manufacturing Sector: Labor Productivity (Outp...
0.3093          HWI              Help Wanted Index for United States
0.2989         NWPI                                       *** NWPI ***
0.2961       AWHMAN  Average Weekly Hours of Production and Nonsupe...
0.2703       OPHPBS  Business Sector: Labor Productivity (Output pe...
0.2358       OPHNFB  Nonfarm Business Sector: Labor Productivity (O...
0.2307     UNRATELT                                   *** UNRATELT ***
0.2189      UNLPNBS  Nonfarm Business Sector: Unit Nonlabor Payment...
0.2131       ULCMFG  Manufacturing Sector: Unit Labor Costs for All...
0.1989  TLBSNNCBBDI                                *** TLBSNNCBBDI ***
Factor:6 Variance Explained=0.0303
                 series                                        description
0.2669           CONSPI    Nonrevolving consumer credit to Personal Income
0.2388            ULCBS  Business Sector: Unit Labor Costs for All Workers
0.2379           ULCNFB  Nonfarm Business Sector: Unit Labor Costs for ...
```

(continues on next page)

(continued from previous page)

```
0.2173          CONSUMER              Consumer Loans, All Commercial Banks
0.2016            EXUSEU           U.S. Dollars to Euro Spot Exchange Rate
0.1979            AHETPI  Average Hourly Earnings of Production and Nons...
0.1865          NONREVSL  Nonrevolving Consumer Credit Owned and Securit...
0.1613           TOTALSL        Total Consumer Credit Owned and Securitized
0.1453  B020RE1Q156NBEA  Shares of gross domestic product: Exports of g...
0.1361           UMCSENT         University of Michigan: Consumer Sentiment
Factor:7 Variance Explained=0.0270
             series                                         description
0.2663   USEPUINDXM  Economic Policy Uncertainty Index for United S...
0.1954     TNWBSHNO  Households and Nonprofit Organizations; Net Wo...
0.1889      TABSHNO  Households and Nonprofit Organizations; Total ...
0.1872      S&P 500          S&P's Common Stock Price Index: Composite
0.1856  S&P: indust        S&P's Common Stock Price Index: Industrials
0.1808    TFAABSHNO  Households and Nonprofit Organizations; Total ...
0.1756    NASDAQCOM                            NASDAQ Composite Index
0.1623     GS10TB3M                                    *** GS10TB3M ***
0.1394       OPHPBS  Business Sector: Labor Productivity (Output pe...
```

```python
# Compute approximate factors from current FRED-MD
data, t = fred_md()     # fetch dataframe of current fred-md and transform codes
data.index = pd.to_datetime(data.index, format='%Y%m%d')
transforms = t['transform']
data = pd.concat([alf.transform(data[col], tcode=transforms[col], freq='m')
                  for col in data.columns],  axis=1)

# remove outliers and impute using Bai-Ng approach
r = 8   # fix number of factors = 8
X = remove_outliers(data, method='farout')
X = approximate_factors(X, kmax=r, p=0, verbose=VERBOSE)
```

```
FRED-MD vintage: monthly/current.csv
```

Recover factors as the projections from PCA

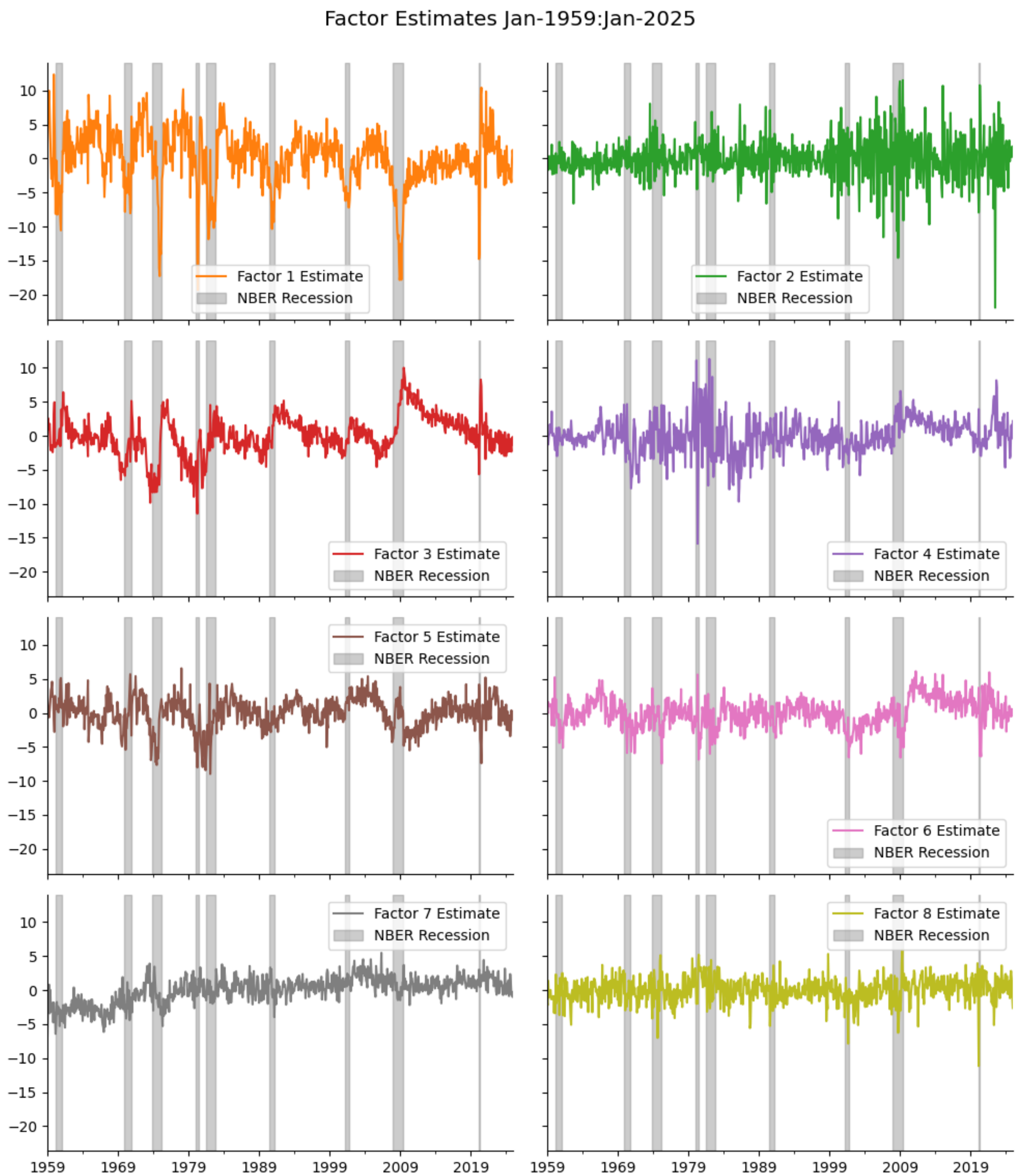
```python
# Extract factors from PCA projections on imputed data
y = StandardScaler().fit_transform(X)
pca = PCA(n_components=r).fit(y)
factors = DataFrame(pca.transform(y), index=data.index, columns=range(1, 1+r))
```

```python
# Plot extracted factors
fig, axes = plt.subplots(nrows=4, ncols=2, figsize=(10, 12), sharex=True, sharey=True)
axes = axes.flatten()
for col, ax in zip(factors.columns, axes):
    flip = -np.sign(max(factors[col]) + min(factors[col])) # try match sign
    (flip*factors[col]).plot(ax=ax, color=f"C{col}")
    for a,b in vspans:
        if b >= min(factors.index):
            ax.axvspan(max(a, min(factors.index)),
                       min(b, max(factors.index)),
                       alpha=0.4,
                       color='grey')
    ax.spines["top"].set_visible(False)
    ax.spines["right"].set_visible(False)
```

(continues on next page)

(continued from previous page)

```
    ax.legend([f"Factor {col} Estimate", 'NBER Recession'])
plt.tight_layout(rect=[0, 0.03, 1, 0.95])
plt.suptitle(f"Factor Estimates {factors.index[0]:%b-%Y}:"
             f"{factors.index[-1]:%b-%Y}", fontsize='x-large')
plt.show()
```



References:

Bai, Jushan and Ng, Serena, 2002, Determining the Number of Factors in Approximate Factor Models, Econometrica

70:1, 191-2211

McCracken, Michael W., and Serena Ng, 2016. FRED-MD: A monthly database for macroeconomic research, Journal of Business & Economic Statistics, 34(4), 574-589.

Michael W. McCracken, Serena Ng, 2020, FRED-QD: A Quarterly Database for Macroeconomic Research.

CHAPTER

TWELVE

STATE SPACE MODELS

If you want to understand today, you have to search yesterday - Pearl Buck

State space models provide a powerful framework for modeling time series data, particularly when the observed data are generated by an underlying system of **hidden states**.

A fundamental example of a state space model is the **Hidden Markov Model (HMM)**, which represents a system where an unobserved sequence of states follows a **Markov process**, and each state emits observations according to some probability distribution.

The **Gaussian Mixture Model (GMM)** is a probabilistic method that represents data as a combination of multiple Gaussian distributions, allowing for the estimation of the likelihood of an unknown state.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import seaborn as sns
from hmmlearn import hmm
from sklearn.preprocessing import StandardScaler
from sklearn.mixture import GaussianMixture
from finds.readers import fred_qd, fred_md, Alfred
from finds.recipes import approximate_factors, remove_outliers
from secret import paths, credentials
VERBOSE = 0
# %matplotlib qt
```

```
# Load and pre-process time series from FRED
alf = Alfred(api_key=credentials['fred']['api_key'], verbose=VERBOSE)
vspans = alf.date_spans('USREC')  # to indicate recession periods in the plots
```

```
# FRED-MD
freq = 'M'
df, t = fred_md() if freq == 'M' else fred_qd()
transforms = t['transform']
data = pd.concat([alf.transform(df[col], tcode=transforms[col], freq=freq)
                  for col in df.columns],
                 axis=1)\
        .iloc[1:]   # apply transforms and drop first row
cols = list(data.columns)

# remove outliers and impute using Bai-Ng approach
r = 8   # fix number of factors = 8
```

(continues on next page)

(continued from previous page)

```
data = remove_outliers(data)
data = approximate_factors(data, kmax=r, p=0, verbose=VERBOSE)
```

```
FRED-MD vintage: monthly/current.csv
```

```
# standardize inputs
X = StandardScaler().fit_transform(data.values)
```

12.1 Hidden Markov Model

A **Hidden Markov Model (HMM)** is a type of multivariate time series model where the system is governed by a set of **unobserved (hidden) states** that follow a Markov process. The model is characterized by three key components:

1. **Transition probabilities**: The likelihood of moving from one state to another (or remaining in the same state), represented by a **transition matrix** A.
2. **Emission probabilities**: The probability of observing a certain value given a hidden state, denoted as θ.
3. **Initial state distribution**: A probability vector π that defines the starting state of the system.

Thus, an HMM is fully specified by the parameter set $\lambda = (A, \theta, \pi)$. The observed sequence X is generated by an unobserved state sequence Z, making inference about Z a central problem in HMM applications.

To work with HMMs in Python, we use the `hmmlearn` package, which provides efficient algorithms for solving the three fundamental HMM tasks:

- **Evaluation**: Computing the likelihood of an observation sequence given the model, $P(X|\lambda)$. This is solved using the **Forward-Backward algorithm**, implemented via the `.score()` method.
- **Decoding**: Determining the most probable sequence of hidden states Z given the observations X. This is accomplished using the **Viterbi algorithm**, accessed through the `.predict()` method.
- **Training**: Estimating the model parameters $\lambda = (A, \theta, \pi)$ by maximizing the likelihood of the observed data. This is done using the **Baum-Welch algorithm**, a specific case of the **Expectation-Maximization (EM) algorithm**, via the `.fit()` method.

The `GaussianHMM` model in `hmmlearn` assumes a **Gaussian distribution** for emissions. The `covariance_type` parameter controls the structure of the covariance matrix, with the following options (ordered by increasing complexity):

- `"spherical"`: A single variance value is used for all features within each state.
- `"diag"`: Each state has a diagonal covariance matrix, allowing feature-specific variances.
- `"tied"`: A single full covariance matrix is shared across all states.
- `"full"`: Each state has its own full covariance matrix, providing the most flexibility.

To select the optimal model, an **information criterion** such as the **Bayesian Information Criterion (BIC)** can be used to balance model complexity and goodness of fit.

```
def hmm_summary(markov, X, lengths, matrix=False):
    """Helper to return summary statistics from fitting Hidden Markov Model

    Args:
        markov: Fitted GaussianHMM
        X: Input data of shape (nsamples, nfeatures)
```

(continues on next page)

(continued from previous page)

```
        lengths: Lengths of the individual sequences in X, sum is nsamples
        matrix: Whether to return the transition and stationary matrices

    Returns:
        Dictionary of results in {'aic', 'bic', 'parameters', 'NLL'}
    """
    logL = markov.score(X, lengths)
    T = np.sum(lengths)       # n_samples
    n = markov.n_features     # number of features ~ dim of covariance matrix
    m = markov.n_components   # number of states
    k = dict(diag=m*n,        # parms in mean and cov matrix
             full=m*n*(n-1)/2,
             tied=n*(n-1)/2,
             spherical=m)[markov.covariance_type] + markov.n_features
    p = m**2 + (k * m) - 1    # number of indepedent parameters of the model

    results = {'aic': -2 * logL + (2 * p),
               'bic': -2 * logL + (p * np.log(T)),
               'parameters': p,
               'NLL' : -logL}
    if matrix:   # whether to return the transition and stationary matrix
        matrix = DataFrame(markov.transmat_)\
            .rename_axis(columns='Transition Matrix:')
        matrix['Stationary'] = markov.get_stationary_distribution()
        results.update({'matrix': matrix})    # return matrix as DataFrame
    return results
```

```
# Compare covariance types in Gaussian HMM models
out = []
for covariance_type in ["full", "diag", "tied", "spherical"]:
    for n_components in range(1, 8):
        if VERBOSE:
            print('=============', covariance_type, n_components, "==========")
        markov = hmm.GaussianHMM(n_components=n_components,
                                 covariance_type=covariance_type,
                                 verbose=VERBOSE,
                                 tol=1e-6,
                                 random_state=0,
                                 n_iter=500)\
                    .fit(X, [X.shape[0]])
        result = hmm_summary(markov, data, [X.shape[0]])
        #print(n_components, Series(results, name=covariance_type).to_frame().T)
        result.update({'cov_type': covariance_type,
                       'n_components': n_components})
        out.append(Series(result))
results = pd.concat(out, axis=1).T.convert_dtypes()
```

```
Model is not converging.  Current: -48374.028067592284 is not greater than -48374.
↪02806594439. Delta is -1.6478952602483332e-06
Model is not converging.  Current: -125173.83118863673 is not greater than -125173.
↪83118797389. Delta is -6.628397386521101e-07
Model is not converging.  Current: -123662.93419332648 is not greater than -123662.
↪93417683932. Delta is -1.648715988267213e-05
```

```
# Show best bic's
best_bic = []
for covariance_type in ["full", "diag", "tied", "spherical"]:
    result = results[results['cov_type'] == covariance_type]
    argmin = np.argmin(result['bic'])
    best_bic.append(result.iloc[[argmin]])
best_bic = pd.concat(best_bic, axis=0)
best_type = best_bic.iloc[np.argmin(best_bic['bic'])]['cov_type']
print(f"HMM best bic type: {best_type}")
best_bic.round(0)
```

```
HMM best bic type: spherical
```

```
            aic          bic  parameters          NLL    cov_type  n_components
0   212991056.0  213028448.0        8001  106487527.0        full             1
12   16508012.0   16532906.0        5327    8248679.0        diag             6
14  212991056.0  213028448.0        8001  106487527.0        tied             1
23   14544672.0   14546518.0         395    7271941.0   spherical             3
```

```
# fit model with best_bic
n_components = best_bic[best_bic['cov_type'] == best_type]['n_components']
n_components = int(n_components.iloc[0])
markov = hmm.GaussianHMM(n_components=n_components, covariance_type=best_type,
                         verbose=False, tol=1e-6, random_state=0, n_iter=100)\
            .fit(X, [X.shape[0]])
pred_markov = Series(markov.predict(X),
                     name='state',
                     index=pd.to_datetime(data.index, format="%Y%m%d"))
matrix = hmm_summary(markov, X, [X.shape[0]], matrix=True)['matrix']
```

```
# Compute average change in INDPRO by state
df = alf('INDPRO', freq=freq, log=1, diff=1)
df.index = pd.DatetimeIndex(df.index.astype(str), freq='infer')
indpro = pd.concat([df, pred_markov], join='inner', axis=1).groupby('state').mean(
↪'INDPRO')

# and HMM transition and stationary probabilities
print("HMM transition and stationary probabilities, and average INDPRO value by state
↪")
matrix.join(indpro).round(4)
```

```
HMM transition and stationary probabilities, and average INDPRO value by state
```

```
        0       1       2  Stationary   INDPRO
0  0.6485  0.1775  0.1740      0.1720  -0.0020
1  0.0677  0.8595  0.0728      0.4482   0.0053
2  0.0793  0.0854  0.8352      0.3798  -0.0002
```

Plot predicted states

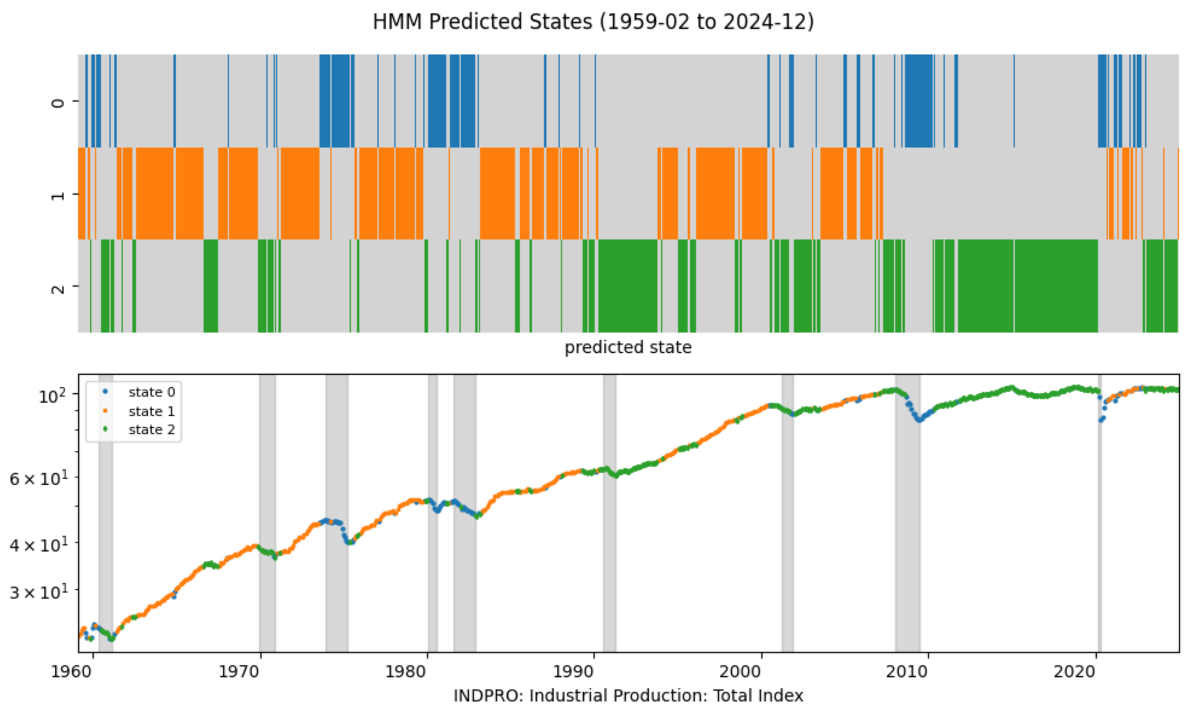
```
def plot_states(modelname, labels, num=1, series_id='INDPRO', freq='M'):
    """helper to plot predicted states 'IPMANSICS'"""
```

(continues on next page)

(continued from previous page)

```
    # n_components markers
    n_components = len(np.unique(labels))
    markers = ["o", "s", "d", "X", "P", "8", "H", "*", "x", "+"][:n_components]

    fig, (bx, ax) = plt.subplots(nrows=2, ncols=1, figsize=(10, 6))

    # plot series, with states colored
    df = alf(series_id, freq=freq)
    df.index = pd.DatetimeIndex(df.index.astype(str), freq='infer')
    df = df[(df.index >= min(labels.index)) & (df.index <= max(labels.index))]
    for i, marker in zip(range(n_components), markers):
        df.loc[labels == i].plot(ax=ax, style=marker, markersize=2, color=f"C{i}",
↪rot=0)
    ax.set_xlabel(f"{series_id}: {alf.header(series_id)}")
    ax.set_xlim(left=min(df.index), right=max(df.index))
    for a,b in vspans:   # shade economic recession periods
        if (b > min(df.index)) & (a < max(df.index)):
            ax.axvspan(max(a, min(df.index)), min(b, max(df.index)),
                       alpha=0.3, color='grey')
    ax.legend([f"state {i}" for i in range(n_components)], fontsize=8)
    ax.set_yscale('log')

    s = np.zeros((n_components, len(labels)))
    for i, j in enumerate(labels.values.flatten()):
        s[j][i] = j + 1
    sns.heatmap(s, vmin=0, vmax=n_components, ax=bx, cbar=False, xticklabels=False,
                cmap=["lightgrey"] + [f"C{i}" for i in range(n_components)])
    bx.set_xlabel('predicted state')
    date_str = f" ({str(df.index[0])[:7]} to {str(df.index[-1])[:7]})"
    fig.suptitle(f"{modelname.upper()} Predicted States" + date_str)

    plt.tight_layout()
```

```
plot_states('HMM', pred_markov, num=1, freq=freq)
```



12.1.1 Markov Chains

Markov Chains are a fundamental class of state space models where the future state of a system depends only on the current state and not on past states. Key concepts in Markov Chains include:

- Geometric Runs: The probability distribution of the time until a particular state recurs, which often follows a geometric distribution.
- Irreducibility: A property of a Markov Chain where it is possible to reach any state from any other state, ensuring long-run stability in the system.

The `Graphviz` python package can be used to visualize the states and transition probabilities of a Hidden Markov Model (HMM), providing a representation of how states evolve over time.

```
# Visualize HMM transitions and states
from sklearn.preprocessing import minmax_scale
colors = minmax_scale(indpro).flatten()

import graphviz
def fillcolor(r, g, b):
    def scale(x):
        return int((1 - x) * 256 * .5 + 64)
    return f"#{scale(r):02x}{scale(g):02x}{scale(b):02x}"
```

```
dot = graphviz.Digraph(engine='circo', graph_attr={'splines': 'true'})
dot.graph_attr['size'] = '10,10'
dot.graph_attr['ratio'] = 'expand'
for i in range(len(matrix)):  # Add nodes
    dot.node(name=str(i), label=f"{matrix.iloc[i, i]:.03f}",
             style='filled', fillcolor=fillcolor(0, colors[i], colors[i]))
for i in range(len(matrix)):  # Add edges
```

(continues on next page)

(continued from previous page)

```
    for j in range(len(matrix)):
        if i != j:
            dot.edge(tail_name=str(i), head_name=str(j),
                     label=f"{matrix.iloc[i, j]:.03f}", color='red')
```

```
matrix
```

```
Transition Matrix:        0         1         2  Stationary
0                  0.648460  0.177506  0.174035    0.171962
1                  0.067656  0.859514  0.072830    0.448240
2                  0.079320  0.085432  0.835248    0.379799
```

```
dot
# dot.format = 'png'
# dot.view(filename='digraph')  # Visualize the graph
```

```
<graphviz.graphs.Digraph at 0x7f3131652c10>
```

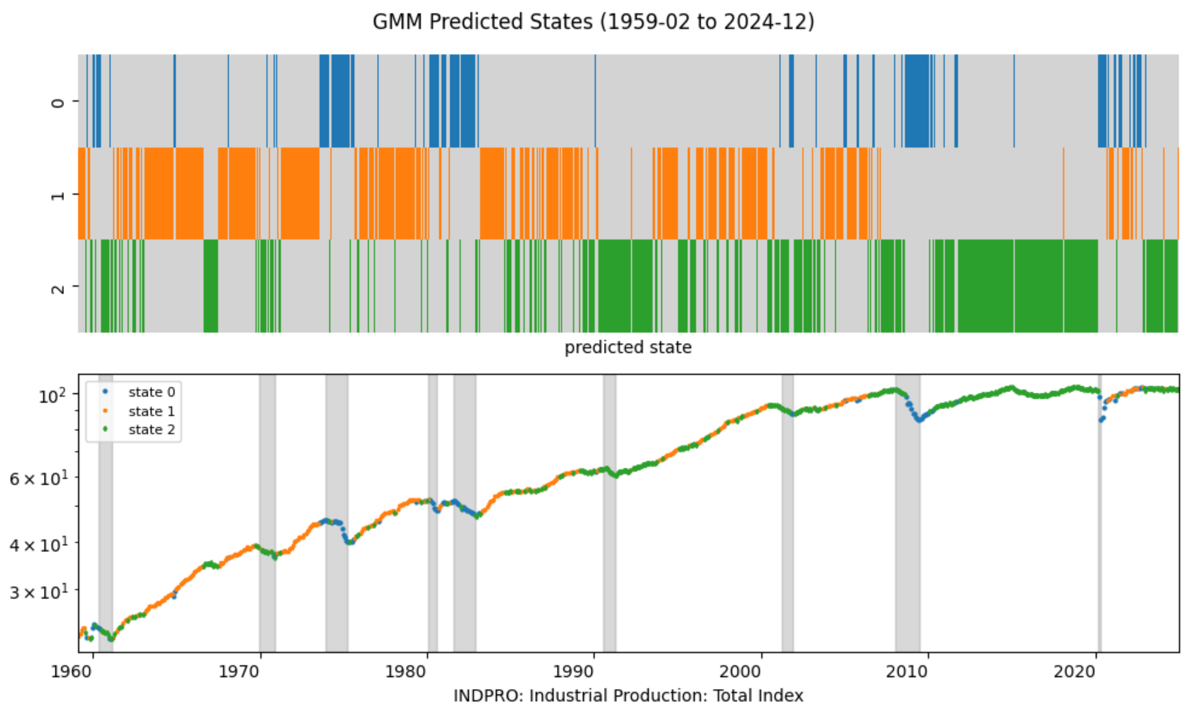
12.2 Gaussian Mixture Model

A **Gaussian Mixture Model (GMM)** is a probabilistic model that assumes that the data is generated from a mixture of multiple Gaussian distributions. Each component in the mixture represents a different subpopulation within the data. The following are tje parameters that need to be estimated:

1. Mixing Coefficients (Weights) π_k –These represent the proportion of each Gaussian component in the overall mixture and must sum to 1.
2. Mean Vectors μ_k –The center of each Gaussian component in the feature space, indicating where each cluster is located.
3. Covariance Matrices Σ_k –These define the shape and spread of each Gaussian distribution, determining how data points are dispersed around the mean.

These parameters are typically estimated using the **Expectation-Maximization (EM) algorithm**, by iteratively optimizing the likelihood of the observed data.

```
gmm = GaussianMixture(n_components=n_components,
                      random_state=0,
                      covariance_type=best_type).fit(X)
pred_gmm = Series(gmm.predict(X),
                  name='state',
                  index=pd.to_datetime(data.index, format="%Y%m%d"))
plot_states('GMM', pred_gmm, num=2, freq=freq)
```



12.2.1 Persistance

Persistence in an HMM refers to the likelihood that the system remains in the same hidden state over time rather than transitioning to a different state. It is determined by the self-transition probabilities on the diagonal of the transition matrix.

In GMMs, persistence refers to the likelihood of a data point belonging to the same Gaussian component across different observations. Since GMMs do not model temporal dependencies like HMMs, persistence is inferred from the posterior probability of a point belonging a particular cluster.

```
# Compare persistance of HMM and GMM
dist = DataFrame({
    'Hidden Markov': ([np.mean(pred_markov[:-1].values == pred_markov[1:].values)]
                     + matrix.iloc[:,-1].tolist()),
    'Gaussian Mixture': ([np.mean(pred_gmm[:-1].values == pred_gmm[1:].values)]
                        + (pred_gmm.value_counts().sort_index()/len(pred_gmm)).
↪tolist())},
                index=(['Average persistance of states']
                       + [f'Stationary prob of state {s}' for s in range(n_
↪components)]))
print("Compare HMM with GMM:")
dist
```

```
Compare HMM with GMM:
```

	Hidden Markov	Gaussian Mixture
Average persistance of states	0.820253	0.732911
Stationary prob of state 0	0.171962	0.152971
Stationary prob of state 1	0.448240	0.424779
Stationary prob of state 2	0.379799	0.422250

CHAPTER

THIRTEEN

TERM STRUCTURE OF INTEREST RATES

The best thing about the future is that it comes one day at a time - Abraham Lincoln

The term structure of interest rates describes how interest rates and bond yields vary across different maturities, typically illustrated using a yield curve. We examine basic concepts such as spot rates, forward rates, par rates, and yield-to-maturity, along with techniques for modeling the term structure, including yield curve construction, splines, and bootstrapping. To gain intuitive insights into the dynamics of interest rate movements, we also apply low-rank approximation methods such as Principal Component Analysis (PCA) and Singular Value Decomposition (SVD).

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import re
from typing import List, Dict
from datetime import datetime
import numpy as np
import numpy.linalg as la
from scipy.interpolate import CubicSpline
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
from sklearn.preprocessing import StandardScaler
from sklearn.pipeline import Pipeline
from sklearn.decomposition import PCA
from finds.readers import Alfred
from secret import credentials, paths
VERBOSE = 0
# %matplotlib qt
```

```
alf = Alfred(api_key=credentials['fred']['api_key'], convert_date=0, verbose=VERBOSE)
```

13.1 Interest Rates

The frequency at which interest is compounded determines how an interest rate is measured. For example, an annual compounding rate assumes interest is compounded once per year, while a semi-annual compounding rate assumes two compounding periods per year.

The **Annualized Percentage Yield (APY)** or **effective annual yield** accounts for the effects of compounding. The conversion formula is:

$$APY = \left(1 + \frac{R_m}{m}\right)^m - 1$$

where m represents the number of compounding periods per year.

To convert an interest rate R_m with compounding frequency m to an equivalent **continuously compounded rate** r, we use the formula:

$$e^{rt} = \left(1 + \frac{R_m}{m}\right)^{mt}$$

The **spot rate** for a given maturity is the zero-coupon rate applicable to that period. It is directly related to the discount factor:

$$\text{Discount factor} = \left(1 + \frac{R_m}{m}\right)^{-t}$$

The **forward rate** is the implied future interest rate derived from observed spot rates. Longer-term spot rates can be determined by compounding forward rates.

A bond's **yield to maturity (YTM)** is the single discount rate that equates the bond's present value of cash flows to its market price.

The **par rate** is the coupon rate at which a bond is issued at par value. A bond priced at par has a yield to maturity equal to its coupon rate.

The **term structure of interest rates** represents the relationship between yields and maturities. When plotted, it forms a **yield curve**. An upward-sloping curve indicates that forward rates are higher than spot and par rates, while a downward-sloping curve suggests the opposite.

Swap rates represent fixed interest rates exchanged for floating rates in an interest rate swap and are considered par rates.

```
# retrieve Constant Maturity Treasuries, excluding inflation-indexed and discontinued
cat = alf.get_category(115)  # Fed H.15 Selected Interest Rates
print('Retrieved category:', cat['id'], cat['name'])
```

```
Retrieved category: 115 Treasury Constant Maturity
```

```
treas = DataFrame.from_dict(
    {s['id']: [s['observation_start'], s['frequency'], s['title'].split(',')[0][44:]]
     for s in cat['series'] if 'Inflation' not in s['title'] and
     'DISCONT' not in s['title'] and s['frequency'] in ['Daily', 'Monthly']},
    columns = ['start', 'freq', 'title'],
    orient='index').sort_values(['freq', 'start'])
print("Constant Maturity Treasuries in FRED")
pd.set_option('display.max_colwidth', None)
treas
```

```
Constant Maturity Treasuries in FRED
```

```
              start     freq                      title
DGS1     1962-01-02    Daily    1-Year Constant Maturity
DGS10    1962-01-02    Daily   10-Year Constant Maturity
DGS20    1962-01-02    Daily   20-Year Constant Maturity
DGS3     1962-01-02    Daily    3-Year Constant Maturity
DGS5     1962-01-02    Daily    5-Year Constant Maturity
DGS7     1969-07-01    Daily    7-Year Constant Maturity
DGS2     1976-06-01    Daily    2-Year Constant Maturity
DGS30    1977-02-15    Daily   30-Year Constant Maturity
DGS3MO   1981-09-01    Daily   3-Month Constant Maturity
DGS6MO   1981-09-01    Daily   6-Month Constant Maturity
```

(continues on next page)

(continued from previous page)

```
DGS1MO  2001-07-31    Daily  1-Month Constant Maturity
GS1     1953-04-01  Monthly   1-Year Constant Maturity
GS10    1953-04-01  Monthly  10-Year Constant Maturity
GS20    1953-04-01  Monthly  20-Year Constant Maturity
GS3     1953-04-01  Monthly   3-Year Constant Maturity
GS5     1953-04-01  Monthly   5-Year Constant Maturity
GS7     1969-07-01  Monthly   7-Year Constant Maturity
GS2     1976-06-01  Monthly   2-Year Constant Maturity
GS30    1977-02-01  Monthly  30-Year Constant Maturity
GS3M    1981-09-01  Monthly  3-Month Constant Maturity
GS6M    1981-09-01  Monthly  6-Month Constant Maturity
GS1M    2001-07-01  Monthly  1-Month Constant Maturity
```

```python
# infer maturity from label
daily = pd.concat([alf(s, freq='D')
                   for s in treas.index if treas.loc[s, 'freq']=='Daily'],
                  axis=1, join='outer')
daily.columns = [int(re.sub('\D', '', col)) * (1 if col[-1].isalpha() else 12)   # MO␣
 ↪or M
                 for col in daily.columns]  # infer maturity in months from label
daily = daily.rename_axis(columns='maturity').sort_index(axis=1)
daily
```

```
maturity     1     3     6    12    24    36    60    84   120   240   360
date
1962-01-02   NaN   NaN   NaN  3.22   NaN  3.70  3.88   NaN  4.06  4.07   NaN
1962-01-03   NaN   NaN   NaN  3.24   NaN  3.70  3.87   NaN  4.03  4.07   NaN
1962-01-04   NaN   NaN   NaN  3.24   NaN  3.69  3.86   NaN  3.99  4.06   NaN
1962-01-05   NaN   NaN   NaN  3.26   NaN  3.71  3.89   NaN  4.02  4.07   NaN
1962-01-08   NaN   NaN   NaN  3.31   NaN  3.71  3.91   NaN  4.03  4.08   NaN
...          ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
2025-02-21  4.36  4.32  4.30  4.15  4.19  4.19  4.26  4.35  4.42  4.69  4.67
2025-02-24  4.36  4.31  4.30  4.15  4.13  4.17  4.23  4.32  4.40  4.69  4.66
2025-02-25  4.34  4.30  4.28  4.12  4.07  4.08  4.12  4.21  4.30  4.59  4.55
2025-02-26  4.35  4.31  4.28  4.12  4.05  4.04  4.06  4.16  4.25  4.55  4.51
2025-02-27  4.38  4.32  4.28  4.13  4.07  4.05  4.09  4.19  4.29  4.59  4.56

[15774 rows x 11 columns]
```

```python
monthly = pd.concat([alf(s, freq='M')
                     for s in treas.index if treas.loc[s, 'freq']=='Monthly'],
                    axis=1, join='outer')
monthly.columns = [int(re.sub('\D', '', col)) * (1 if col[-1].isalpha() else 12)   #␣
 ↪MO or M
                   for col in monthly.columns]  # infer maturity in months from label
monthly = monthly.rename_axis(columns='maturity').sort_index(axis=1)
monthly
```

```
maturity     1     3     6    12    24    36    60    84   120   240   360
date
1953-04-30   NaN   NaN   NaN  2.36   NaN  2.51  2.62   NaN  2.83  3.08   NaN
1953-05-31   NaN   NaN   NaN  2.48   NaN  2.72  2.87   NaN  3.05  3.18   NaN
1953-06-30   NaN   NaN   NaN  2.45   NaN  2.74  2.94   NaN  3.11  3.21   NaN
```

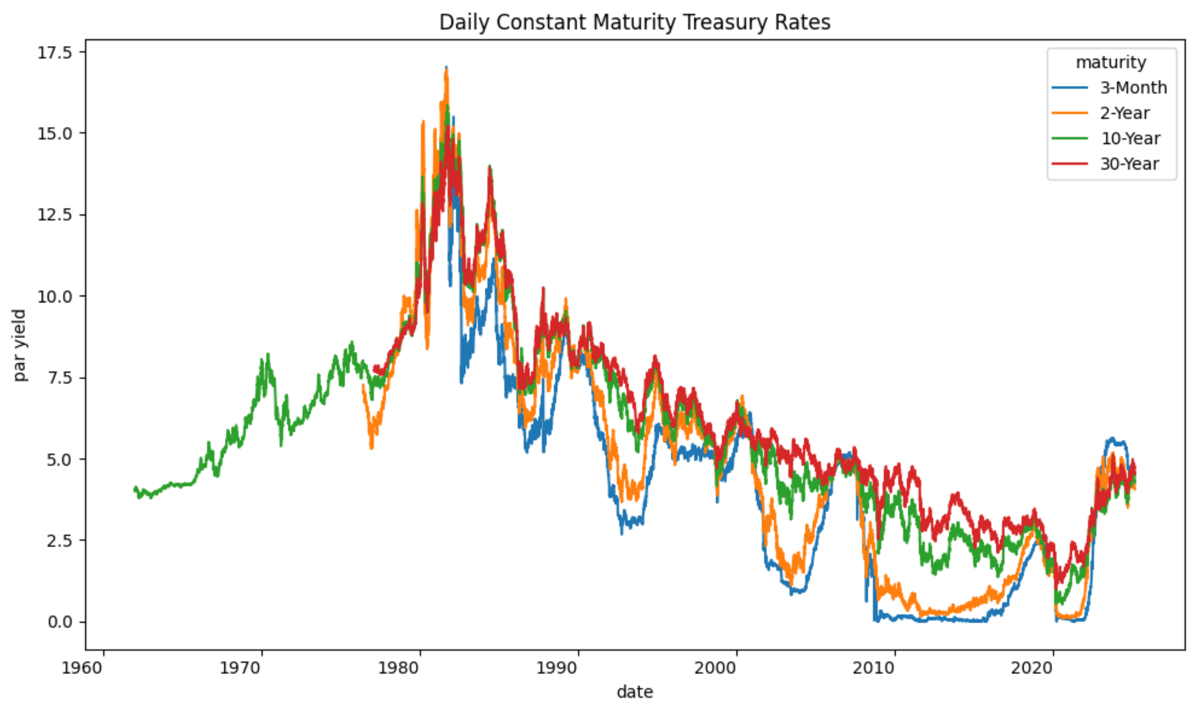
(continues on next page)

(continued from previous page)

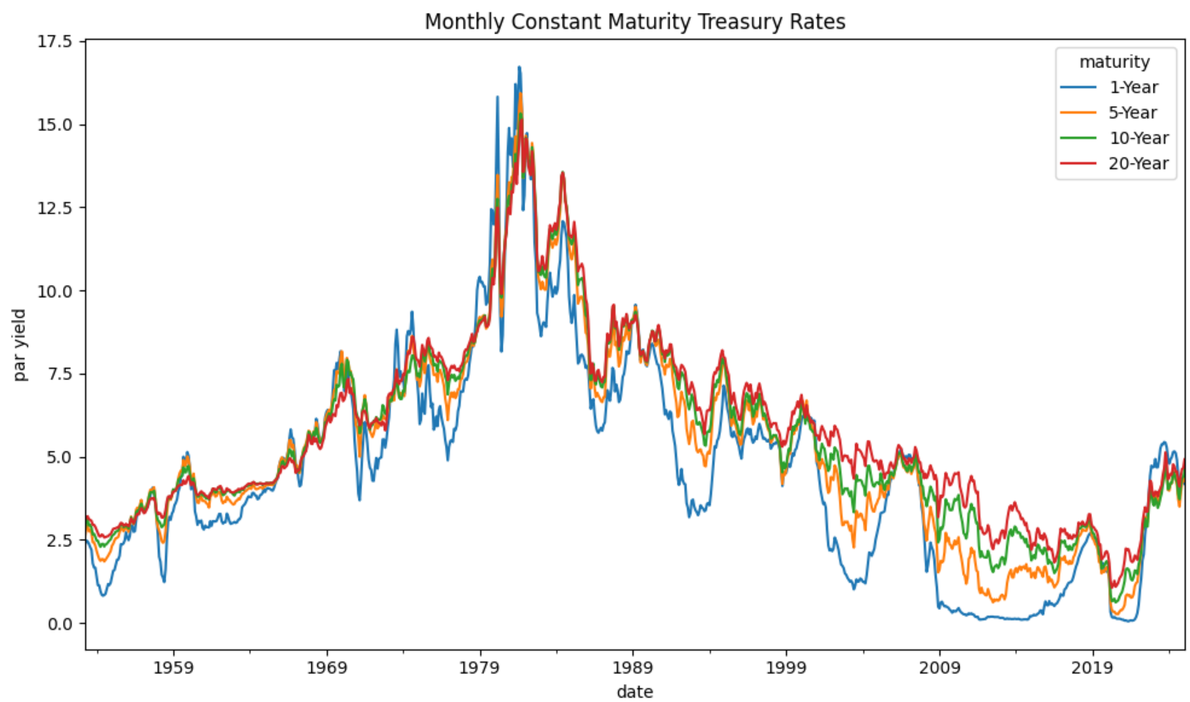
```
1953-07-31   NaN   NaN   NaN  2.38   NaN  2.62  2.75   NaN  2.93  3.12   NaN
1953-08-31   NaN   NaN   NaN  2.28   NaN  2.58  2.80   NaN  2.95  3.10   NaN
...          ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
2024-09-30  5.06  4.92  4.55  4.03  3.62  3.51  3.50  3.60  3.72  4.10  4.04
2024-10-31  4.92  4.72  4.44  4.20  3.97  3.90  3.91  3.99  4.10  4.44  4.38
2024-11-30  4.71  4.62  4.43  4.33  4.26  4.21  4.23  4.29  4.36  4.63  4.54
2024-12-31  4.50  4.39  4.32  4.23  4.23  4.22  4.25  4.32  4.39  4.66  4.58
2025-01-31  4.42  4.34  4.26  4.18  4.27  4.33  4.43  4.53  4.63  4.92  4.85

[862 rows x 11 columns]
```

```python
# mapper to display maturity months as labels
mapper = lambda month: f"{month}-Month" if month < 12 else f"{int(month/12)}-Year"
```

```python
cols = [3, 24, 120, 360]
fig, ax = plt.subplots(figsize=(10, 6))
daily[cols].rename(columns=mapper).plot(ax=ax, rot=0)
plt.title('Daily Constant Maturity Treasury Rates')
plt.ylabel('par yield')
plt.tight_layout()
```




```python
cols = [12, 60, 120, 240]
fig, ax = plt.subplots(figsize=(10, 6))
monthly[cols].rename(columns=mapper).plot(ax=ax, rot=0)
plt.title('Monthly Constant Maturity Treasury Rates')
plt.ylabel('par yield')
plt.tight_layout()
```



13.2 Yield Curve

The U.S. Treasury's official **yield curve** is a par yield curve constructed using a monotone convex method. This curve is based on indicative bid-side price quotations collected by the Federal Reserve Bank of New York at approximately 3:30 PM each trading day.

Historically, the Treasury used a quasi-cubic Hermite spline (HS) method to construct the yield curve. This approach interpolated yields directly from observed market data under the assumption that the resulting curve was a par yield curve. However, since December 6, 2021, the monotone convex method has been the standard.

Constant Maturity Treasury (CMT) yields are derived from the Treasury's par yield curve and represent bond-equivalent yields for semiannual interest-paying securities. These yields are expressed on a simple annualized basis rather than an effective annual yield (APY) basis, which incorporates compounding effects. To convert a CMT yield to APY, use:

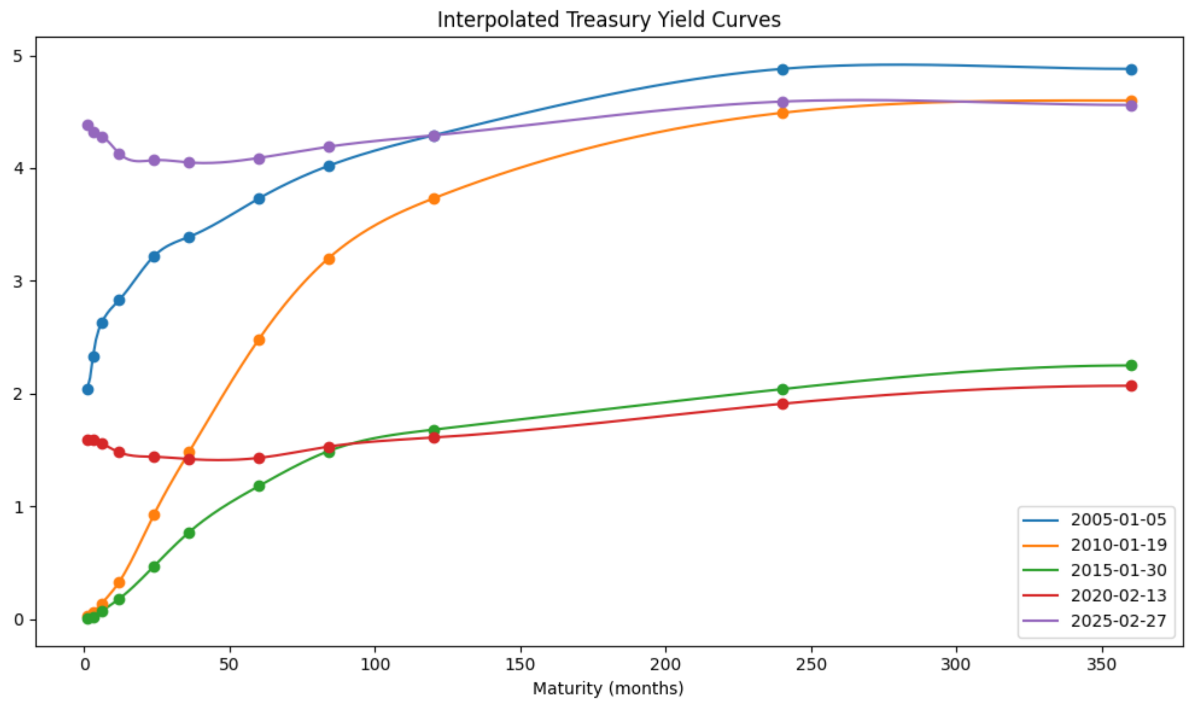
$$APY = \left(1 + \frac{y}{2}\right)^2 - 1$$

13.2.1 Splines

Spline interpolation is used to estimate yields at maturities not directly observed in the market. A **piecewise cubic polynomial** ensures smooth transitions between maturities and maintains twice continuous differentiability.

```
yield_curve = dict()
curve_dates = sorted(daily.dropna().index[-1:0:-(5*252)])
for date in curve_dates:
    yield_curve[date] = CubicSpline(
        x=daily.columns.to_list(), y=daily.loc[date].values, bc_type='clamped')
```

```
# Plot historical yield curves
fig, ax = plt.subplots(figsize=(10, 6))
X = list(range(1, 361))
for col, (date, curve) in enumerate(yield_curve.items()):
    ax.plot(X, curve(X), label=date.strftime('%Y-%m-%d'), color=f"C{col}")
plt.legend()
for col, (date, curve) in enumerate(yield_curve.items()):
    daily.loc[date].plot(ax=ax, marker='o', ls='', color=f"C{col}", label=None)
plt.title('Interpolated Treasury Yield Curves')
plt.xlabel('Maturity (months)')
plt.tight_layout()
```



13.2.2 Bootstrap Method

Bootstrapping is a process used to derive spot rates by progressively solving for zero-coupon rates using available bond data. This iterative approach fits spot rates to increasingly longer maturities.

```
# To bootstrap 6-month spot rates from 6-month par yields
m = 2   # compounding periods per year
```

```
# list of 6-monthly maturities
maturities = list(range(int(12/m), daily.columns[-1]+1, int(12/m)))
```

```
# Helper to bootstrap spot rates from par yield curve
def bootstrap_spot(coupon: float, spots: List[float], m: int, price: float=1) ->
 ↪float:
    """Compute spot rate to maturity of par bond from yield and sequence of spots
```

(continues on next page)

(continued from previous page)

```
    Args:
      coupon : Annual coupon rate
      spots : Simple annual spot rates each period (excluding last period before␣
↪maturity
      m : Number of compounding periods per year
      price: Present value of bond

    Returns:
      Simple spot interest rate till maturity
    """
    if not spots:           # trivial one-period bond
        return coupon / price
    n = len(spots) + 1      # number of coupons through maturity

    # discount factors from given spot rates
    discount = [(1 + spot/m)**(-(1+t)) for t, spot in enumerate(spots)]

    # nominal amount of last payment
    last_payment = 1 + coupon/m

    # infer present value of last coupon and principal
    last_pv = price - np.sum(discount) * coupon/m

    # compute discount factor and annualize the effective rate
    spot = ((last_payment/last_pv)**(1/n) - 1) * m
    return spot
```

```
# select most recent yield curve
curve_date = curve_dates[-1]
yields = [yield_curve[curve_date](t) / 100 for t in maturities]
spots = []
for coupon in yields:
    spots.append(bootstrap_spot(coupon=coupon, spots=spots, m=m))
DataFrame({curve_date.strftime('%Y-%m%d'): spots}, index=maturities).head()
```

```
     2025-0227
6     0.042800
12    0.041285
18    0.040591
24    0.040684
30    0.040639
```

Helper to compute bond prices

```
def bond_price(coupon: float, n: int, m: int, yields: float | List[float],
               par: float = 1) -> float:
    """Compute present value of bond given spot rates or yield-to-maturity

    Args:
      coupon : Annual coupon rate
      n : Number of remaining coupons
      m : Number of compounding periods per year
      yields : Simple annual yield-to-maturity or spot rates each period
      par : face or par value of bond
```

(continues on next page)

(continued from previous page)

```
    Returns:
      Present value of bond
    """
    if not pd.api.types.is_list_like(yields):
        yields = [yields] * n        # same yield-to-maturity is spot rate every␣
↪period
    assert len(yields) == n, "Number of yields must equal number of couponds"
    pv = [(1 + yields[t-1]/m)**(-t) * (coupon/m + (par if t == n else 0))
          for t in range(1, n+1)]    # discount every period's payment, plus last face
    return np.sum(pv)
```

```
# Sanity-check par bond price given spots
for t in range(len(yields)):
    price = bond_price(coupon=yields[t], n=t+1, m=2, yields=spots[:(t+1)])
    assert np.allclose(price, 1.0)    # discounted payments at spot rates must equal␣
↪price
```

```
# Compute forward rates from spot rates
def forwards_from_spots(spots: List[float], m: int, skip: int=0) -> List[float]:
    """Compute forward rates given spot rates

    Args:
      spots : Sequence of simple annual spot rates
      m : Number of compounding periods per year
      skip: Number of initial periods skipped

    Returns:
      List of forward rates, excluding first period of spot rates input
    """
    result = []
    assert len(spots) >= 2, "Require at least two spot rates as input"
    for t in range(1, len(spots)):
        n = skip + t
        numerator = (1 + spots[n]/m)**n          # discounted value of period n
        denominator = (1 + spots[n-1]/m)**(n-1)    # discounter value of period n-1
        result.append(((numerator / denominator) - 1) * m)
    return result
```

```
forwards = [spots[0]] + forwards_from_spots(spots=spots, m=m)
```

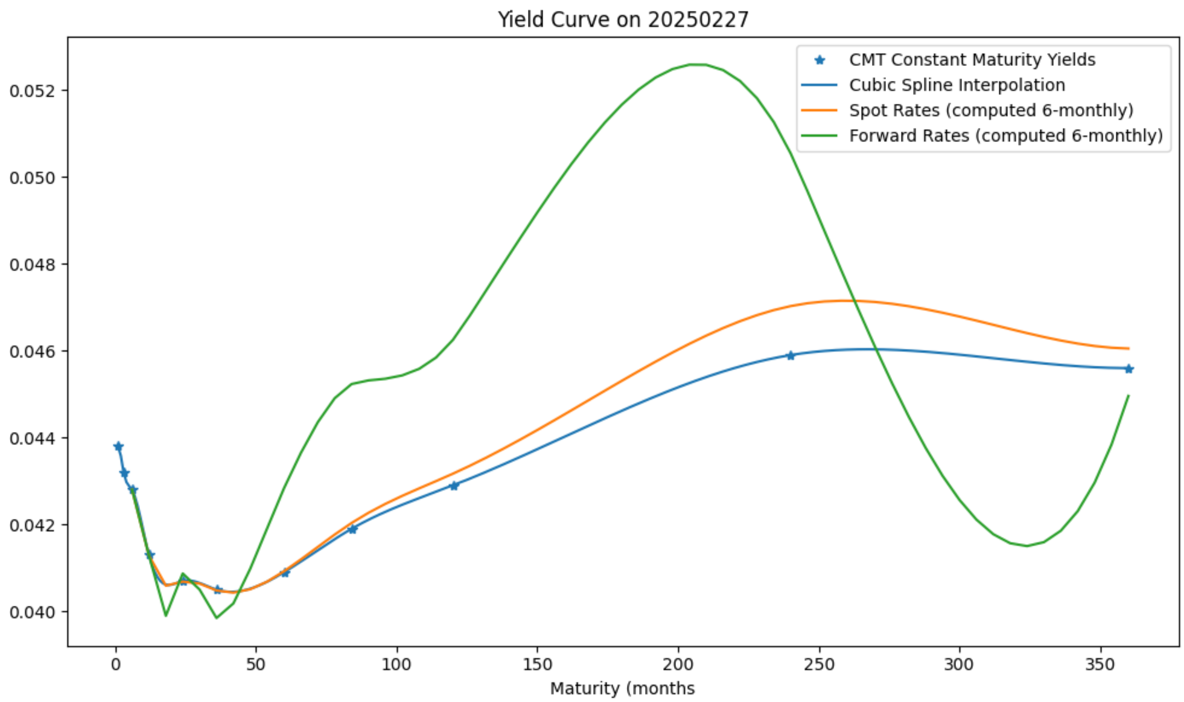
Plot current yield curve

```
fig, ax = plt.subplots(figsize=(10, 6))
(daily.loc[curve_date] / 100).plot(marker='*', ls='', color="C0")
X = range(1, maturities[-1]+1)
ax.plot(X, [yield_curve[curve_date](t) / 100 for t in X], marker='', ls='-', color="C0
↪")
ax.plot(maturities, spots, marker='', ls='-', color="C1")
ax.plot(maturities, forwards, marker='', ls='-', color="C2")
plt.legend(['CMT Constant Maturity Yields', 'Cubic Spline Interpolation',
            'Spot Rates (computed 6-monthly)', 'Forward Rates (computed 6-monthly)'])
plt.title(f"Yield Curve on {curve_date.strftime('%Y%m%d')}")
plt.xlabel('Maturity (months')
```

(continues on next page)

(continued from previous page)

```
plt.tight_layout()
```



There are more sophisticated methods for fitting yield curves to treasuries prices, such as this model by Liu and Wu (2021)

```
# Download reconstructed yield curve data by Liu and Wu (2021)
# file_id from https://sites.google.com/view/jingcynthiawu/yield-data
file_id = '1-wmStGZHLx55dSYi3gQK2vb3F8dMw_Nb'   # monthly
#file_id = '11HsxLl_u2tBNt3FyN5iXGsIKLwxvVz7t'  # daily
src = "https://drive.google.com/uc?export=download&id={}".format(file_id) # to load␣
↪from gdrive
df = pd.ExcelFile(src).parse()
dates = np.where(df.iloc[:, 0].astype(str).str[0].str.isdigit())[0] # locate first␣
↪date cell
```

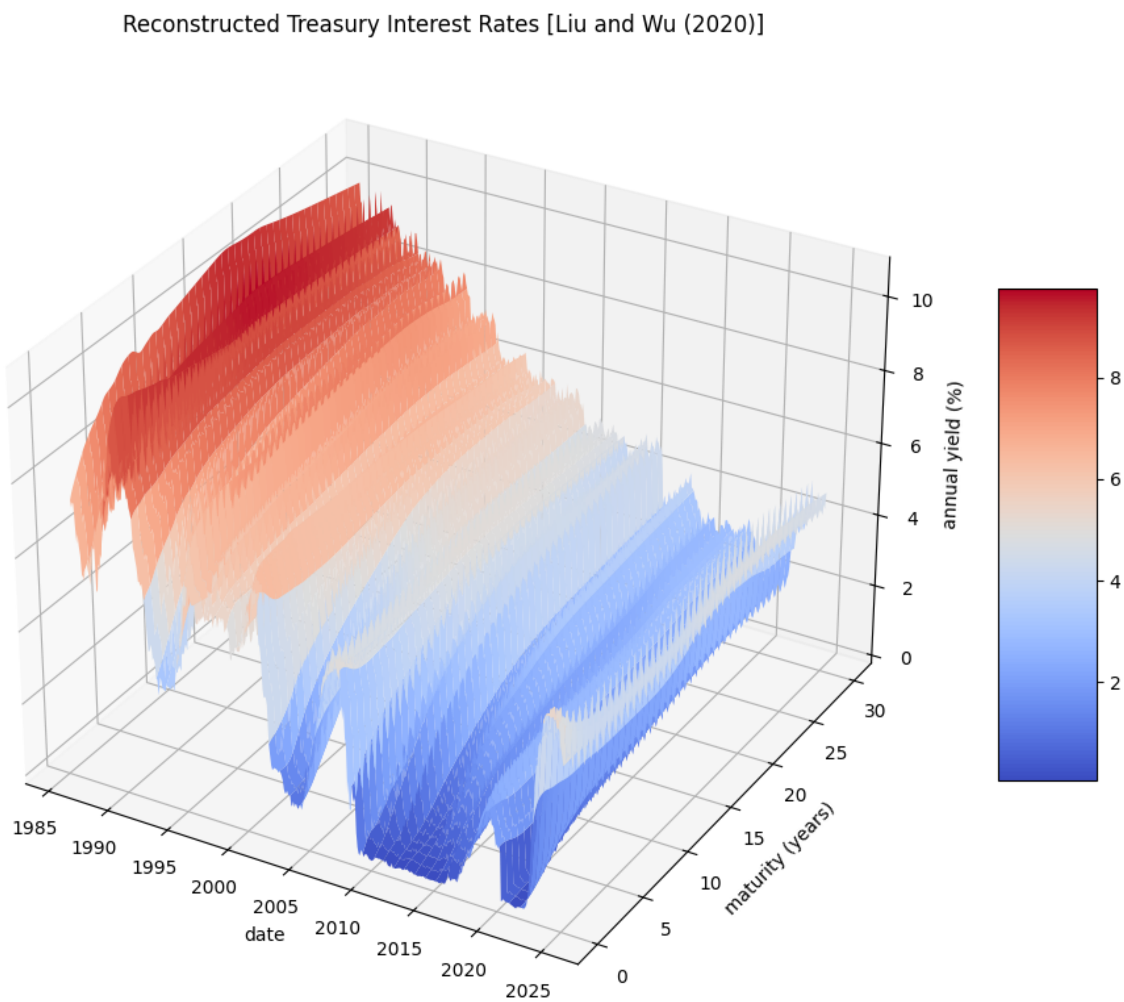
```
liuwu = DataFrame(np.exp(df.iloc[dates,1:361].astype(float).values/100) - 1,
                  index=(pd.to_datetime(df.iloc[dates, 0], format="%Y%m")
                         + pd.offsets.MonthEnd(1)),
                  columns=np.arange(1, 361))
```

```
# Plot historical reconstructed rates as 3D surface
from mpl_toolkits.mplot3d import Axes3D
r = liuwu.dropna()
X, Y = np.meshgrid(r.index.map(lambda x: float(str(x)[:4]) + float(str(x)[5:7])/12),
                   r.columns.astype(float)/12)
#X, Y = np.meshgrid((r.index//10000) + ((((r.index//100)%100)-1)/12),
#                    r.columns.astype(float)/12)
Z = r.T.to_numpy()*100
fig = plt.figure(num=1, clear=True, figsize=(10, 8))
ax = plt.axes(projection='3d')
```

(continues on next page)

(continued from previous page)

```
f = ax.plot_surface(X, Y, Z, cmap='coolwarm', linewidth=0, antialiased=True)
ax.set_title('Reconstructed Treasury Interest Rates [Liu and Wu (2020)]')
ax.set_xlabel('date')
ax.set_ylabel('maturity (years)')
ax.set_zlabel('annual yield (%)')
fig.colorbar(f, shrink=0.5, aspect=5)
plt.tight_layout()
```



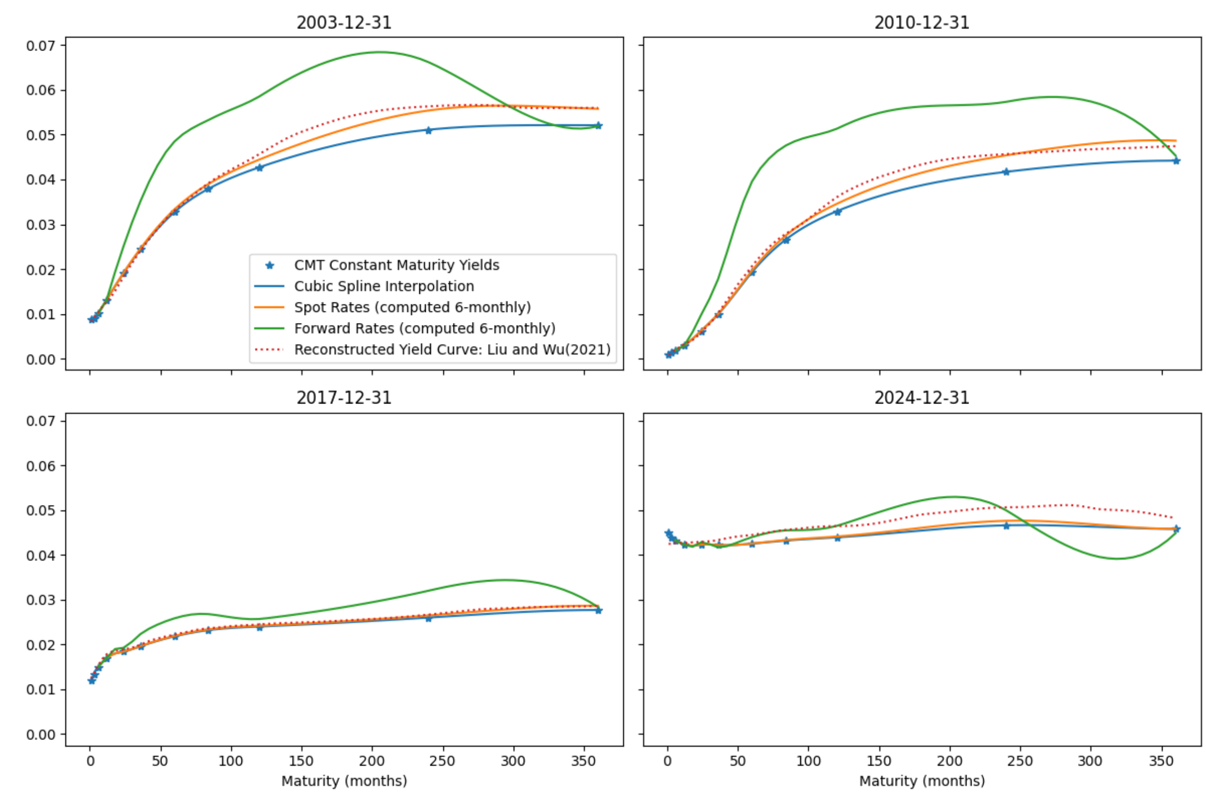

```
# Plot historical Yield Curves
curve_dates = sorted(liuwu.index[-1:0:-(7*12)])[-4:]
for date in curve_dates:
    yield_curve[date] = CubicSpline(
        x=monthly.columns.to_list(), y=monthly.loc[date].values, bc_type='clamped')
```

```
fig, axes = plt.subplots(2,2, figsize=(12, 8), sharey=True, sharex=True)
axes = axes.flatten()
for num, (curve_date, ax) in enumerate(zip(curve_dates, axes)):
    # fit yields
    yields = [yield_curve[curve_date](t) / 100 for t in maturities]
```

(continues on next page)

(continued from previous page)

```
    # compute spots
    spots = []
    for coupon in yields:
        spots.append(bootstrap_spot(coupon=coupon, spots=spots, m=m))

    # compute forwards
    forwards = [spots[0]] + forwards_from_spots(spots=spots, m=m)

    # plot
    (monthly.loc[curve_date] / 100).plot(marker='*', ls='', color="C0", ax=ax) # CMT␣
↪yield
    X = range(1, maturities[-1]+1)
    ax.plot(X, [yield_curve[curve_date](t) / 100 for t in X], marker='', ls='-',␣
↪color="C0")
    ax.plot(maturities, spots, marker='', ls='-', color="C1")
    ax.plot(maturities, forwards, marker='', ls='-', color="C2")
    liuwu.loc[curve_date].plot(ax=ax, marker='', ls=':', color="C3")   # Liu and Wu
    ax.set_title(f"{curve_date.strftime('%Y-%m-%d')}")
    ax.set_xlabel('Maturity (months)')
    if not num:
        ax.legend(['CMT Constant Maturity Yields', 'Cubic Spline Interpolation',
                   'Spot Rates (computed 6-monthly)', 'Forward Rates (computed 6-
↪monthly)',
                   'Reconstructed Yield Curve: Liu and Wu(2021)'])
plt.tight_layout()
```



13.3 Principal Component Analysis (PCA)

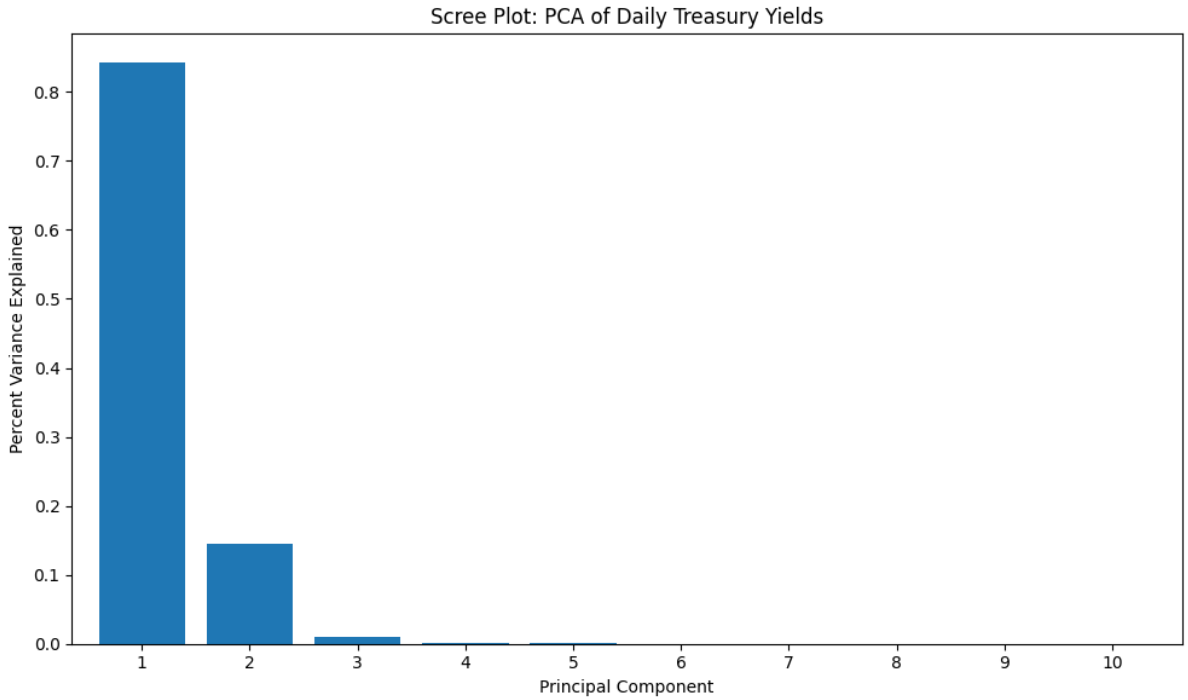
Principal Component Analysis (PCA), also referred to as **eigendecomposition**, involves rotating the data space so that each axis captures the maximum variance. We apply PCA to identify the key factors influencing daily interest rate in Constant Maturity Treasury (CMT) rates.

The number of factors in PCA equals the number of interest rate maturities analyzed. The first principal component accounts for 84% of daily yield variance, while the first two components explain almost 99%. The first three components together account for approximately 99.9%, meaning most of the uncertainty in yield changes can be attributed to these factors.

```
X = daily.dropna()
Y = StandardScaler().fit_transform(X)
pca = PCA().fit(Y)
DataFrame({'Cumulative Variance Explained': pca.explained_variance_ratio_.cumsum()},
           index=[f"PC {c+1}" for c in range(pca.n_components_)])
```

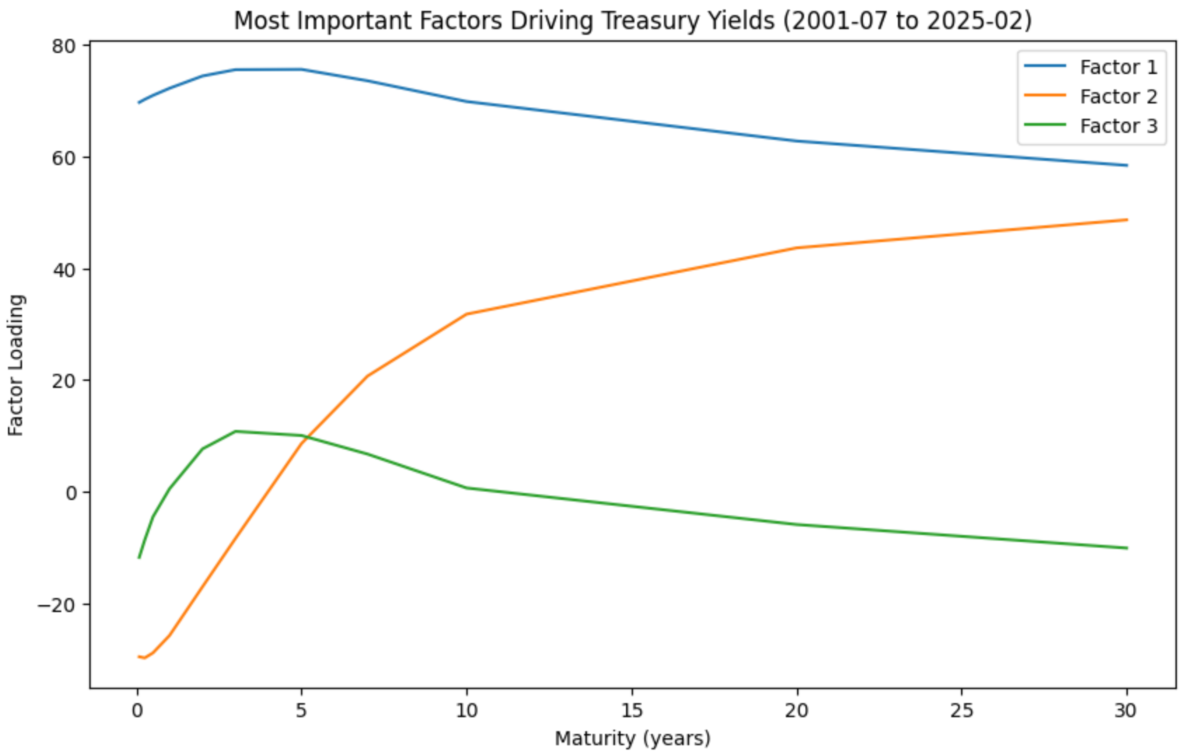
	Cumulative Variance Explained
PC 1	0.841980
PC 2	0.986916
PC 3	0.997595
PC 4	0.998925
PC 5	0.999401
PC 6	0.999727
PC 7	0.999866
PC 8	0.999929
PC 9	0.999965
PC 10	0.999986
PC 11	1.000000

```
# Scree plot
scree = Series(pca.explained_variance_ratio_,
               index=np.arange(1, Y.shape[1] + 1))
fig, ax = plt.subplots(figsize=(10, 6))
scree[:10].plot(kind='bar', rot=0, width=.8, ax=ax)
ax.set_title('Scree Plot: PCA of Daily Treasury Yields')
ax.set_ylabel("Percent Variance Explained")
ax.set_xlabel("Principal Component")
plt.tight_layout()
```




```
# Factor Loadings
Z = pca.components_[:3, :].T * pca.singular_values_[:3].reshape(1, 3)
loadings = DataFrame(Z, columns=[f"Factor {n+1}" for n in range(3)], index=X.columns/
↪12)
fig, ax = plt.subplots(figsize=(10, 6))
loadings.plot(ax=ax)
ax.set_title(f"Most Important Factors Driving Treasury Yields"
             f" ({str(X.index[0])[:7]} to {str(X.index[-1])[:7]})")
ax.set_xlabel('Maturity (years)')
ax.set_ylabel('Factor Loading')
```

```
Text(0, 0.5, 'Factor Loading')
```



Key Factors Affecting Treasury Yields

1. **Level Factor** –Represents a parallel shift in yields across all maturities.
2. **Slope Factor** –Indicates changes in the steepness of the yield curve, where short-term and long-term rates move in opposite directions.
3. **Twist Factor** –Describes changes in the curvature of the yield curve, where intermediate rates move differently from short- and long-term rates.

13.4 Singular Value Decomposition (SVD)

Singular Value Decomposition (SVD) is a factorization method that decomposes a matrix into three components:

$$A = USV^T$$

where:

- U and V are orthogonal matrices representing rotations.
- S is a diagonal matrix containing singular values (scaling factors).

SVD generalizes eigenvalue decomposition:

$$A^T A = V \Lambda V^T$$

where Λ contains the eigenvalues.

A goal of both PCA and SVD is to approximate the original data matrix with a lower-dimensional presentation, with the most important eigen- and singular vectors associated with the largest eigenvalues and singular values respectively.

- **Eigenvalues** (λ) in PCA correspond to squared singular values in SVD.

- **Principal components** (columns of V) are the **eigenvectors** or **right singular vectors**.
- **Loadings** are obtained by multiplying each principal component by its corresponding singular value.
- **Projections (scores)** are computed by projecting data onto the principal components.

```
# A is num_samples (N) by num_features (K) data matrix, standardized by column
A = daily.dropna().values
A = (A - A.mean(axis=0)) / A.std(axis=0)  # subtract mean, divide by std
N, K = A.shape
A.shape
```

```
(5896, 11)
```

```
# svd x is related to pca of x'x
u, s, vT = np.linalg.svd(A, full_matrices=False)
v = vT.T   # transposed right singular vector was returned
```

The **eigenvalues** (λ) of PCA

- can be retrieved as (N-1) times pca.explained_variance_
- are equal to the squares of the singular values (s from SVD)

```
# s**2 = lambda = N * explained_variance
assert np.allclose((N-1) * pca.explained_variance_, s**2), 'eigenvalues values'
assert np.allclose(pca.singular_values_, s), "singular values"
```

The **components** (columns of V) of an eigendecomposition

- are also called the **eigenvectors**, or right **singular vectors** from the SVD
- can be retrieved from the rows of pca.components_, or
- are the rows of V^T (identically, the columns of V from SVD)

Relatedly:

- **loadings** are computed by multiplying each component by its corresponding singular value $v \cdot s$

```
# components and right singular vectors are identical up to sign flip
for pc in range(K):
    assert (np.allclose(vT[pc, :], pca.components_[pc, :]) or
            np.allclose(vT[pc, :], -pca.components_[pc, :]))
```

```
# square of loadings is same as square of data matrix, i.e. the covariance matrix
loadings = np.diag(pca.singular_values_) @ pca.components_
assert np.allclose(A.T @ A, loadings.T @ loadings), 'square matrix'
```

The **projections** of PCA

- are also known as the **scores** or **co-ordinates**
- are computed by projecting the data matrix on the components $A \cdot V$
- or by scaling each left singular vector by its corresponding singular value $u \cdot s$
- can be retrieved by calling pca.transform() on the data matrix

```python
# assert: x @ v == transform(x) (aka projection on components)
y = pca.transform(A)
for pc in range(K):
    assert np.allclose((A@v)[:,pc], -y[:,pc]) or np.allclose((A@v)[:,pc], y[:,pc])
    assert np.allclose(u[:,pc]*s[pc], -y[:,pc]) or np.allclose(u[:,pc]*s[pc], y[:,pc])
```

13.5 Low-Rank Approximations

PCA and SVD allow approximating the original data matrix with a lower-dimensional representation. A rank-k approximation is a technique find a matrix A_k with lower rank k that is as close as possible to the original matrix A in terms of some measure, typically the Frobenius norm. This approximation reduces the complexity of the data while retaining its most important features.

13.5.1 Low-rank approximation by PCA

$$A \approx A_k' A_k = V_{[:k]} D_{[:k]} V_{[:k]}'$$

```python
ATA = A.T.dot(A)
eigval, eigvec = (N-1)*pca.explained_variance_, pca.components_.T
assert np.allclose(eigvec.dot(np.diag(eigval)).dot(eigvec.T), ATA), "pca error"
```

```python
print('rank-K PCA approximation:')
DataFrame.from_dict({k: (la.norm(
    eigvec[:, :k].dot(np.diag(eigval[:k])).dot(eigvec[:, :k].T) - ATA)/la.norm(ATA))
    for k in range(1, 5)}, orient='index', columns=['Frobenius Norm'])\
    .rename_axis(index='K')
```

```
rank-K PCA approximation:
```

```
   Frobenius Norm
K
1        0.170097
2        0.012614
3        0.001707
4        0.000700
```

13.5.2 Low-rank approximation by SVD

$A \approx A_k = U_{[:k]} S_{[:k]} V_{[:k]}'$

```python
assert np.allclose(u.dot(np.diag(s)).dot(v.T), A), "svd error"
```

```python
print('rank-K SVD approximation:')
DataFrame.from_dict({k: la.norm(
    u[:, :k].dot(np.diag(s[:k])).dot(v[:, :k].T) - A) / la.norm(A)
    for k in range(1, 5)}, orient='index', columns=['Frobenius Norm'])\
    .rename_axis(index='K')
```

```
rank-K SVD approximation:
```

```
   Frobenius Norm
K
1        0.397517
2        0.114383
3        0.049039
4        0.032781
```

References:

FRM Part 1 Exam Book Valuation and Risk Models, Chapter 12-13

Yan Liu and Jing Cynthia Wu “Reconstructing the Yield Curve”, Journal of Financial Economics, 2021, 142 (3), 1395-1425.

https://home.treasury.gov/policy-issues/financing-the-government/interest-rate-statistics/treasury-yield-curve-methodology

CHAPTER

FOURTEEN

INTEREST RATE RISK

The essence of investment management is the management of risks, not the management of returns - Benjamin Graham

Changes in interest rates directly affect the value of bonds and fixed-income portfolios. Various measures of interest rate sensitivity, such as duration, convexity, DV01, and key rate shifts help quantify how bond prices respond to fluctations in the yield curve. Additionally, we explore the statistical approach to identifying key risk factors that explain interest rate movements.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
from typing import List
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
from sklearn.preprocessing import StandardScaler
from sklearn.pipeline import Pipeline
from sklearn.decomposition import PCA
import statsmodels.formula.api as smf
from finds.readers import Alfred
from finds.recipes import bond_price
from secret import credentials
# %matplotlib qt
VERBOSE = 0
```

14.1 Interest rate sensitivity

14.1.1 Duration

Duration, or **Macaulay Duration**, measures how a bond's price P changes in response to an instantaneous change in its yield y.

Consider a bond with price P and yield y. To simplify the duration formula, we first express the yield using continuous compounding. The bond's price is given by:

$$P = \sum_{t=1}^{n} c_t e^{-yt}$$

Differentiating this equation, we obtain:

$$\Delta P = -\sum_{t=1}^{n} c_t t e^{-yt} \Delta y$$

Yield-based duration is defined as the proportional change in bond price for a small change in yield:

$$D = \frac{\Delta P}{P \Delta y} = \sum_{t=1}^{n} t \frac{c_t e^{-yt}}{P}$$

This formulation provides an alternative interpretation of duration: it represents the weighted average time at which cash flows are received, with each time weighted by the proportion of the bond's total value received at that time. This explains why the term "duration" is used to describe sensitivity to yield changes—duration effectively measures how long an investor must wait to receive the bond's cash flows.

Modified duration accounts for different compounding conventions. If yields are measured with semi-annual compounding instead of continuous compounding, Macaulay duration must be adjusted by dividing by $(1 + y_2/2)$ (or $1 + y_m/m$ for m-thly compounding).

Effective duration, on the other hand, measures the percentage change in the price of a bond with embedded options due to a small shift in all interest rates.

14.1.2 DV01

DV01 (Dollar Value of a 01) quantifies the impact of a one-basis-point change in interest rates on a bond or portfolio's value. It is given by:

$$DV01 = -\frac{\Delta P}{\Delta r}$$

where Δr represents a small parallel shift in the interest rate term structure (expressed in basis points).

14.1.3 Convexity

Duration and convexity appear as the first two terms in a Taylor expansion of a bond's price with respect to interest rates. While duration provides a linear estimate of price sensitivity, convexity measures the curvature of the bond price-yield relationship, refining estimates for larger rate movements.

Convexity accounts for the fact that bond price changes are not perfectly linear with respect to interest rate movements. **Yield-based convexity**, when yields are expressed with continuous compounding, is given by:

$$C = \frac{1}{P} \frac{1}{(1 + r/m)^2} \sum_{t=1}^{T} (t/2m + (t/m)^2) \frac{C_t}{(1 + r/m)^t}$$

This formula represents a weighted average of the squared time to maturity. When yields are expressed with semi-annual compounding, these expressions must be divided by $(1 + y_2/2)^2$ (or $(1 + y_m/m)^2$ for m-thly compounding), and the result is known as **modified convexity**.

Effective convexity measures how duration itself changes in response to interest rate shifts and is calculated as:

$$C = \frac{1}{P} [\frac{P^+ + P^- - 2P}{(\Delta r)^2}]$$

```
# Helpers to calculate duration and convexity
def macaulay_duration(coupon: float, n: int, m: int, price: float,
                      yields: float | List[float], par: float = 1, **kwargs) -> float:
    """Compute macaulay duration of a bond given spot rates or yield-to-maturity

    Args:
      coupon : Annual coupon rate
```

(continues on next page)

(continued from previous page)

```
      n : Number of remaining coupons
      m : Number of compounding periods per year
      price : current market price of bond
      yields : Simple annual yield-to-maturity or spot rates each period
      par : face or par value of bond

    Returns:
      Macaulay duration
    """
    if not pd.api.types.is_list_like(yields):
        yields = [yields] * n         # same spot rate every period
    assert len(yields) == n, "Number of spot rates must equal number of couponds"
    pv = [(1 + yields[t-1]/m)**(-t) * (t/m) * (coupon/m + par*(t == n))
          for t in range(1, n+1)]     # discount every period's time-weighted payment
    return np.sum(pv) / price
```

```
def modified_duration(coupon: float, n: int, m: int, price: float,
                      yields: float | List[float], par: float = 1, **kwargs) -> float:
    """Compute modified duration of a bond given spot rates or yield-to-maturity

    Args:
      coupon : Annual coupon rate
      n : Number of remaining coupons
      m : Number of compounding periods per year
      price : current market price of bond
      yields : Simple annual yield-to-maturity or spot rates each period
      par : face or par value of bond

    Returns:
      Modified duration
    """
    assert not pd.api.types.is_list_like(yields), "Not Implemented"
    ytm = yields
    return (macaulay_duration(coupon=coupon, n=n, m=m, price=price, yields=yields,␣
↪par=par)
            / (1 + ytm/2))
```

```
def modified_convexity(coupon: float, n: int, m: int, price: float,
                       yields: float | List[float], par: float = 1, **kwargs) ->␣
↪float:
    """Compute mocified convexity of a bond given spot rates or yield-to-maturity

    Args:
      coupon : Annual coupon rate
      n : Number of remaining coupons
      m : Number of compounding periods per year
      price : current market price of bond
      yields : Simple annual yield-to-maturity or spot rates each period
      par : face or par value of bond

    Returns:
      Modified convexity
    """
    assert not pd.api.types.is_list_like(yields), "Not Implemented"
    ytm = yields
```

(continues on next page)

(continued from previous page)

```
    if not pd.api.types.is_list_like(yields):
        yields = [yields] * n          # same spot rate every period
    assert len(yields) == n, "Number of spot rates must equal number of coupons"
    pv = [(1 + yields[t-1]/m)**(-t) * ((t/m)**2 + t/(2*m)) * (coupon/m + par*(t == n))
          for t in range(1, n+1)]     # discount every period's time-weighted payment
    return np.sum(pv) / (price * (1 + ytm/m)**2)
```

14.1.4 Barbells and bullets

Positive convexity benefits bondholders when there is a parallel shift in interest rates. Consider two fixed-income strategies:

- A barbell strategy, which consists of holding short- and long-maturity bonds
- A bullet strategy, which focuses on medium-term bonds

Both strategies may have the same yield (4%) and duration (8.1758), but the barbell strategy typically outperforms when interest rates shift in parallel. This creates an arbitrage opportunity:

1. Invest a given USD amount in the barbell strategy
2. Short the same USD amount in the bullet strategy

If term structure shifts were always parallel, this approach would be consistently profitable.

```
# Compute prices, duration and convexity for 3 bonds (FRM Valuation and Risk Models␣
↪12.7)
bond5Y = dict(coupon=2, m=2, n=2*5, par=100, yields=0.04)
bond10Y = dict(coupon=4, m=2, n=2*10, par=100, yields=0.04)
bond20Y = dict(coupon=6, m=2, n=2*20, par=100, yields=0.04)
for bond in [bond5Y, bond10Y, bond20Y]:
    bond |= dict(price=bond_price(**bond))
    bond |= dict(duration=modified_duration(**bond))
    bond |= dict(convexity=modified_convexity(**bond))
bonds = DataFrame.from_dict(
    {f"{b['n']/b['m']:.0f}-year, {b['coupon']}% coupon": [
        b['price'], b['duration'], b['convexity']]
     for b in [bond5Y, bond10Y, bond20Y]},
    orient='index',
    columns=['Value', 'Effective Duration', 'Effective Convexity'])
```

```
print("Table 12.4 Effective Durations and Convexities of Three Bonds")
bonds.round(4)
```

```
Table 12.4 Effective Durations and Convexities of Three Bonds
```

	Value	Effective Duration	Effective Convexity
5-year, 2% coupon	91.0174	4.6764	24.8208
10-year, 4% coupon	100.0000	8.1757	78.8979
20-year, 6% coupon	127.3555	12.6233	212.4587

```
# Compute 5Y and 20Y weights of barbell portfolio, with same duration as bullet 10Y
barbell = ((bond10Y['duration'] - bond20Y['duration']) /
```

(continues on next page)

(continued from previous page)

```
            (bond5Y['duration'] - bond20Y['duration']))
print(f"Barbell: weight in 5Y = {barbell:.4f}, weight in 20Y = {1-barbell:.4f}")
```

```
Barbell: weight in 5Y = 0.5597, weight in 20Y = 0.4403
```

```
# Compare durations and convexities
DataFrame.from_dict(dict(Bullet=np.array([0,1,0]).dot(bonds),
                         Barbell=np.array([barbell, 0, 1-barbell]).dot(bonds)),
                    orient='index', columns=bonds.columns)\
                    [['Effective Duration', 'Effective Convexity']]
```

	Effective Duration	Effective Convexity
Bullet	8.175717	78.897925
Barbell	8.175717	107.444902

14.1.5 Key rate shifts

Consider three key spot rates: the two-year, five-year, and ten-year rates. Each influences rates in its surrounding maturity range, and together, their combined movements contribute to an overall one-basis-point shift in the yield curve.

These shifts, known as **key rate shifts**, allow for a more detailed decomposition of DV01. The impact of these shifts is captured by **partial 01s** or **key rate 01s (KR01s)**, defined as follows:

- **$KR01_1$**: The reduction in portfolio value from a one-basis-point increase in the two-year spot rate
- **$KR01_2$**: The reduction in portfolio value from a one-basis-point increase in the five-year spot rate
- **$KR01_3$**: The reduction in portfolio value from a one-basis-point increase in the ten-year spot rate

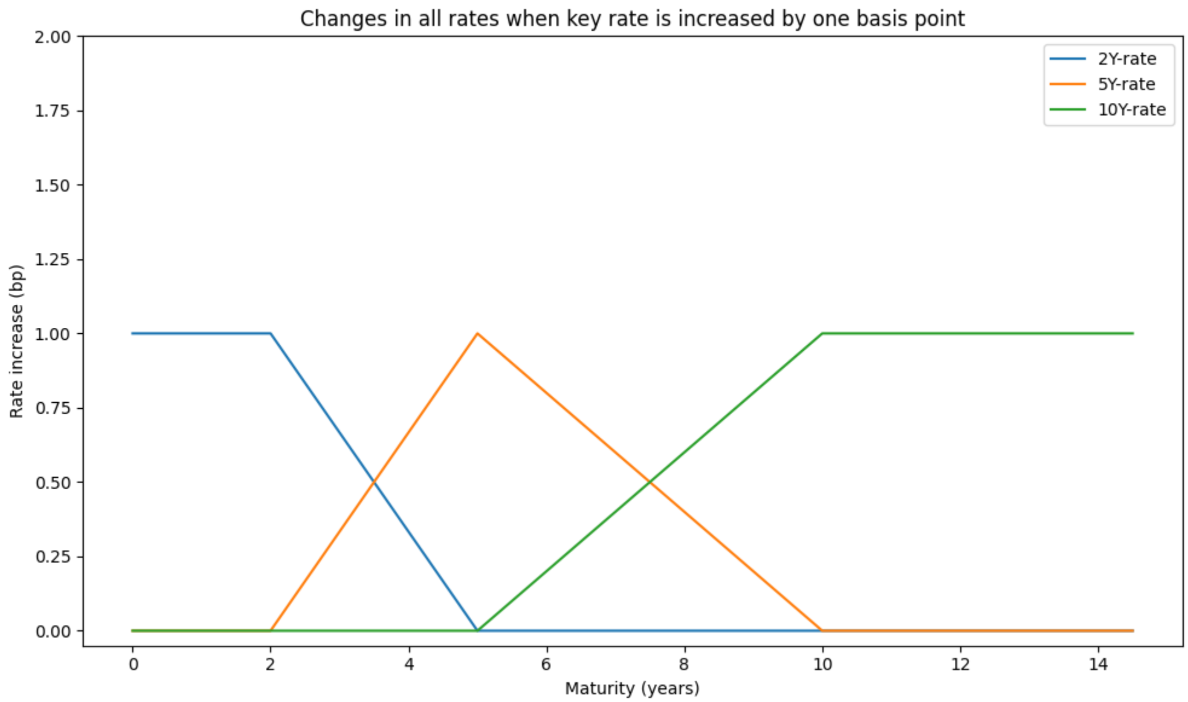
Since these shifts sum to the overall DV01, we have:

$$DV01 = KR01_1 + KR01_2 + KR01_3$$

By analyzing these key rate sensitivities, investors can better estimate portfolio values under various term structure movements and hedge against specific interest rate risks.

```
KR = [2, 5, 10]
DV01 = dict()
for maturity in np.arange(0, 15, 0.5):
    if maturity <= KR[0]:
        changes = [1, 0, 0]
    elif maturity >= KR[2]:
        changes = [0, 0, 1]
    elif maturity < KR[1]:
        diff = (maturity - KR[0])/(KR[1] - KR[0])
        changes = [1 - diff, diff, 0]
    else:
        diff = (KR[2] - maturity)/(KR[2] - KR[1])
        changes = [0, diff, 1-diff]
    DV01[maturity] = changes
```

```
fig, ax = plt.subplots(figsize=(10, 6))
DataFrame.from_dict(DV01, orient='index', columns=[f"{y}Y-rate" for y in KR])\
         .plot(ax=ax)
ax.legend()
ax.set_title('Changes in all rates when key rate is increased by one basis point')
ax.set_ylabel('Rate increase (bp)')
ax.set_xlabel('Maturity (years)')
ax.set_ylim(top=2)
plt.tight_layout()
```



14.2 Risk factors

We examine the risk factors which drive the daily returns of the Merrill Lynch Total Bond Indexes.

```
alf = Alfred(api_key=credentials['fred']['api_key'], verbose=VERBOSE)
```

```
# get Merrill Lynch bond indexes
freq = 'D'    # periodicity 'M' or 'D'
cat = alf.get_category(32413)
print(cat['id'], cat['name'])
```

```
32413 BofA Merrill Lynch Total Bond Return Index Values
```

```
# get bond index returns
bonds = []   # to accumulate bond returns
for s in cat['series']:
```

(continues on next page)

(continued from previous page)

```
    bonds.append(alf(s['id'], start=19961231, freq=freq) )
bonds_df = pd.concat(bonds, axis=1).sort_index()
```

```
# show blocks of data availability
counts = bonds_df.notna().sum(axis=1).rename('count')
counts = pd.concat([counts, (counts != counts.shift()).cumsum().rename('notna')],␣
↪axis=1)
counts = counts.reset_index().groupby(['notna', 'count'])['date'].agg(['first','last
↪'])
counts
```

```
                first      last
notna count
1     15     19961231  19971230
2     16     19971231  19981230
3     33     19981231  20031230
4     48     20031231  20181108
5     46     20181109  20181109
6     48     20181112  20250228
```

Choose start date with good data availability

```
start_date = 19981231
rets = bonds_df.loc[bonds_df.index >= start_date,
                    bonds_df.loc[start_date].notna().values].iloc[:-1]
rets = pd.concat([alf.transform(rets[col], log=1, diff=1)
                  for col in rets.columns], axis=1).dropna()
date_str = f" ({rets.index[0]}-{rets.index[-1]})"
rets
```

```
          BAMLCC0A0CMTRIV  BAMLCC0A1AAATRIV  BAMLCC0A2AATRIV  BAMLCC0A3ATRIV  \
date
19990104        -0.001053         -0.001421        -0.000660       -0.001166
19990105        -0.003846         -0.004194        -0.003500       -0.003857
19990106         0.002388          0.002099         0.002102        0.002495
19990107        -0.002694         -0.002813        -0.002413       -0.002573
19990108        -0.002583         -0.002941        -0.002419       -0.002658
...                   ...               ...              ...             ...
20250221         0.004058          0.005418         0.004519        0.004090
20250224         0.001879          0.002273         0.001855        0.001842
20250225         0.005450          0.007181         0.006098        0.005468
20250226         0.002036          0.002643         0.002200        0.002056
20250227        -0.002092         -0.003094        -0.002047       -0.002056

          BAMLCC0A4BBBTRIV  BAMLCC1A013YTRIV  BAMLCC2A035YTRIV  \
date
19990104         -0.000995          0.000076          0.000473
19990105         -0.003952         -0.000844         -0.002452
19990106          0.002419          0.000466          0.000901
19990107         -0.002996          0.000101         -0.000853
19990108         -0.002504         -0.000920         -0.002006
...                    ...               ...               ...
20250221          0.003918          0.001278          0.002482
20250224          0.001917          0.000683          0.001165
```

(continues on next page)

(continued from previous page)

```
20250225         0.005274         0.001025         0.002846
20250226         0.001964         0.000422         0.001190
20250227        -0.002106         0.000216        -0.000200

          BAMLCC3A057YTRIV  BAMLCC4A0710YTRIV  BAMLCC7A01015YTRIV  ...  \
date                                                               ...
19990104          0.000534          -0.000519           -0.001209  ...
19990105         -0.003343          -0.004276           -0.004990  ...
19990106          0.001695           0.001967            0.002537  ...
19990107         -0.001561          -0.002662           -0.003371  ...
19990108         -0.003309          -0.003773           -0.004368  ...
...                    ...                ...                 ...  ...
20250221          0.003648           0.004107            0.005264  ...
20250224          0.001481           0.001797            0.002408  ...
20250225          0.004326           0.005516            0.007383  ...
20250226          0.001797           0.002089            0.002762  ...
20250227         -0.000742          -0.001944           -0.002688  ...

          BAMLEMPTPRVICRPITRIV  BAMLEMRACRPIASIATRIV  BAMLEMRECRPIEMEATRIV  \
date
19990104              0.001599             -0.000700             -0.005616
19990105              0.002692              0.000400              0.008113
19990106              0.007144              0.003395              0.009135
19990107             -0.001484             -0.003095              0.007386
19990108             -0.000594              0.001998              0.000392
...                        ...                   ...                   ...
20250221              0.001815              0.002360              0.001548
20250224              0.001124              0.001355              0.001095
20250225              0.002267              0.003170              0.002267
20250226              0.001417              0.001305              0.001557
20250227             -0.000023              0.000111              0.000705

          BAMLEMRLCRPILATRIV  BAMLEMUBCRPIUSTRIV  BAMLHE00EHYITRIV  \
date
19990104            0.002297            0.001099          0.000195
19990105            0.002392            0.002394          0.012287
19990106            0.006251            0.006060          0.000577
19990107            0.000198           -0.000694          0.003932
19990108           -0.002079           -0.000892          0.000000
...                      ...                 ...               ...
20250221            0.001954            0.002056          0.000502
20250224            0.000737            0.001161          0.000422
20250225            0.002544            0.002939          0.000026
20250226            0.002419            0.001755          0.000659
20250227           -0.000991           -0.000062          0.000527

          BAMLHYH0A0HYM2TRIV  BAMLHYH0A1BBTRIV  BAMLHYH0A2BTRIV  \
date
19990104            0.001490          0.000250         0.001999
19990105            0.000674         -0.000834         0.000781
19990106            0.001487          0.001583         0.001041
19990107           -0.000112         -0.001250         0.000520
19990108            0.001737         -0.000918         0.002509
...                      ...               ...              ...
20250221           -0.000726         -0.000524        -0.001033
20250224            0.001040          0.001016         0.001033
```

(continues on next page)

(continued from previous page)

```
20250225           0.001028          0.001456         0.000758
20250226           0.001984          0.001797         0.001809
20250227          -0.000006         -0.000065         0.000063

          BAMLHYH0A3CMTRIV
date
19990104          0.003234
19990105          0.005428
19990106          0.003480
19990107          0.001005
19990108          0.007643
...                    ...
20250221         -0.000715
20250224          0.001213
20250225         -0.000062
20250226          0.003319
20250227          0.000046

[6830 rows x 33 columns]
```

Select these bond return indexes

```
pd.set_option('display.max_colwidth', None)
print("Bond Index Total Returns")
Series(alf.header(rets.columns), index=rets.columns, name='title')\
    .to_frame().rename_axis('series')
```

```
Bond Index Total Returns
```

```
                                                                                       ␣
↪                                   title
series
BAMLCC0A0CMTRIV                                                          ICE BofA US␣
↪Corporate Index Total Return Index Value
BAMLCC0A1AAATRIV                                                     ICE BofA AAA US␣
↪Corporate Index Total Return Index Value
BAMLCC0A2AATRIV                                                       ICE BofA AA US␣
↪Corporate Index Total Return Index Value
BAMLCC0A3ATRIV                                                  ICE BofA Single-A US␣
↪Corporate Index Total Return Index Value
BAMLCC0A4BBBTRIV                                                     ICE BofA BBB US␣
↪Corporate Index Total Return Index Value
BAMLCC1A013YTRIV                                                ICE BofA 1-3 Year US␣
↪Corporate Index Total Return Index Value
BAMLCC2A035YTRIV                                                ICE BofA 3-5 Year US␣
↪Corporate Index Total Return Index Value
BAMLCC3A057YTRIV                                                ICE BofA 5-7 Year US␣
↪Corporate Index Total Return Index Value
BAMLCC4A0710YTRIV                                              ICE BofA 7-10 Year US␣
↪Corporate Index Total Return Index Value
BAMLCC7A01015YTRIV                                            ICE BofA 10-15 Year US␣
↪Corporate Index Total Return Index Value
BAMLCC8A015PYTRIV                                               ICE BofA 15+ Year US␣
↪Corporate Index Total Return Index Value
```

(continues on next page)

(continued from previous page)

```
BAMLEM1BRRAAA2ACRPITRIV                      ICE BofA AAA-A Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEM2BRRBBBCRPITRIV                          ICE BofA BBB Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEM3BRRBBCRPITRIV                            ICE BofA BB Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEM4BRRBLCRPITRIV                    ICE BofA B & Lower Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEM5BCOCRPITRIV                      ICE BofA Crossover Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEMCBPITRIV                                     ICE BofA Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEMEBCRPIETRIV                             ICE BofA Euro Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEMFSFCRPITRIV        ICE BofA Private Sector Financial Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEMHBHYCRPITRIV                     ICE BofA High Yield Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEMIBHGCRPITRIV                     ICE BofA High Grade Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEMNSNFCRPITRIV                  ICE BofA Non-Financial Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEMPBPUBSICRPITRIV       ICE BofA Public Sector Issuers Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEMPTPRVICRPITRIV       ICE BofA Private Sector Issuers Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEMRACRPIASIATRIV                          ICE BofA Asia Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEMRECRPIEMEATRIV                          ICE BofA EMEA Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEMRLCRPILATRIV                   ICE BofA Latin America Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLEMUBCRPIUSTRIV                              ICE BofA US Emerging Markets␣
↪Corporate Plus Index Total Return Index Value
BAMLHE00EHYITRIV                                                 ICE BofA Euro␣
↪High Yield Index Total Return Index Value
BAMLHYH0A0HYM2TRIV                                                 ICE BofA US␣
↪High Yield Index Total Return Index Value
BAMLHYH0A1BBTRIV                                                ICE BofA BB US␣
↪High Yield Index Total Return Index Value
BAMLHYH0A2BTRIV                                           ICE BofA Single-B US␣
↪High Yield Index Total Return Index Value
BAMLHYH0A3CMTRIV                                       ICE BofA CCC & Lower US␣
↪High Yield Index Total Return Index Value
```

14.2.1 Statistical risk factors

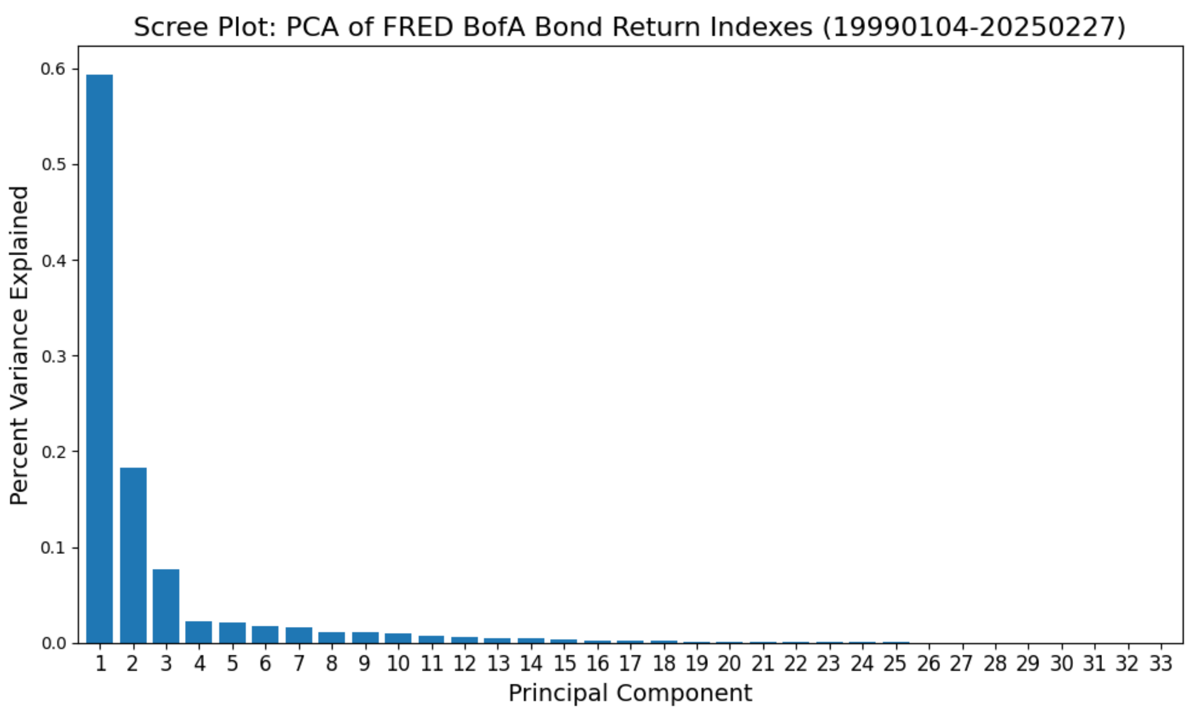
Extract principal components of bond index returns

```
pipe = Pipeline([('scaler', StandardScaler()), ('pca', PCA())])
pipe.fit(rets)
print(pipe.named_steps['pca'].explained_variance_ratio_)  # sanity check
scree = Series(pipe.named_steps['pca'].explained_variance_ratio_,
               index=np.arange(1, rets.shape[1]+1))
DataFrame(scree.cumsum().rename('Cumulative Variance Ratio Explained')).iloc[:10]
```

```
[5.93480704e-01 1.83414224e-01 7.60999397e-02 2.29460554e-02
 2.08374926e-02 1.73252063e-02 1.58102554e-02 1.11222261e-02
 1.07270153e-02 9.46337690e-03 7.12268856e-03 5.78660305e-03
 5.08617829e-03 4.56329291e-03 3.87130739e-03 2.53236096e-03
 2.40159698e-03 2.17255450e-03 1.28094767e-03 9.31393646e-04
 7.70330791e-04 6.01123712e-04 3.93600105e-04 3.60165273e-04
 3.03684121e-04 2.70804862e-04 9.27216331e-05 8.30786301e-05
 5.32780919e-05 3.92254907e-05 3.55714421e-05 1.53907425e-05
 5.60549950e-06]
```

```
    Cumulative Variance Ratio Explained
1                              0.593481
2                              0.776895
3                              0.852995
4                              0.875941
5                              0.896778
6                              0.914104
7                              0.929914
8                              0.941036
9                              0.951763
10                             0.961226
```

```
# Scree plot
fig, ax = plt.subplots(num=1, clear=True, figsize=(10, 6))
scree.plot(kind='bar', rot=0, width=.8, ax=ax)
ax.set_title('Scree Plot: PCA of FRED BofA Bond Return Indexes' + date_str,␣
↪fontsize=16)
ax.xaxis.set_tick_params(labelsize=12)
ax.set_ylabel("Percent Variance Explained", fontsize=14)
ax.set_xlabel("Principal Component", fontsize=14)
plt.tight_layout()
```



14.2.2 Explainability of statistical risk factors

```
# Extract factor returns from bond indexes with PCA projection
K = 4
factors = DataFrame(pipe.transform(rets)[:, :K],
                    columns=[f"PC{c+1}" for c in range(K)],
                    index=pd.DatetimeIndex(rets.index.astype(str), freq='infer'))
```

Construct interest rate spreads to compare with the statistical factors:

- **Level**: The average of the two-year and ten-year Treasury rates
- **Slope**: The difference between the ten-year and two-year Treasury rates
- **Twist**: The difference between (ten-year minus five-year) and (five-year minus two-year) rates
- **Credit Spread**: The difference between BAA corporate bond yields and ten-year Treasury rates

```
# Construct interest rate spread changes
spreads = pd.concat([alf(s, freq=freq) for s in ['BAA10Y', 'DGS10', 'DGS5', 'DGS2']],
                     axis=1)\
            .sort_index()
spreads.index = pd.DatetimeIndex(spreads.index.astype(str), freq='infer')
spreads['level'] = 0.5 * (spreads['DGS2'] + spreads['DGS10'])
spreads['credit'] = spreads['BAA10Y']
spreads['slope'] = spreads['DGS10'] - spreads['DGS2']
spreads['twist'] = ((spreads['DGS10'] - spreads['DGS5']) -
                    (spreads['DGS5'] - spreads['DGS2']))
spreads = spreads.drop(columns=['BAA10Y', 'DGS10', 'DGS5', 'DGS2'])\
                 .ffill()\
                 .diff()\
                 .dropna()
spreads
```

```
            level  credit         slope         twist
date
1986-01-03  0.010   -0.04  1.776357e-15  1.776357e-15
1986-01-06  0.015   -0.01  1.000000e-02 -1.000000e-02
1986-01-07 -0.100    0.06 -6.000000e-02 -8.881784e-16
1986-01-08  0.155   -0.14  7.000000e-02  3.000000e-02
1986-01-09  0.160   -0.05 -4.000000e-02 -8.000000e-02
...           ...     ...           ...           ...
2025-02-24 -0.040   -0.01  4.000000e-02 -2.000000e-02
2025-02-25 -0.080    0.02 -4.000000e-02  6.000000e-02
2025-02-26 -0.035    0.01 -3.000000e-02  5.000000e-02
2025-02-27  0.030    0.01  2.000000e-02  0.000000e+00
2025-02-28 -0.065    0.05  3.000000e-02 -1.000000e-02

[9795 rows x 4 columns]
```

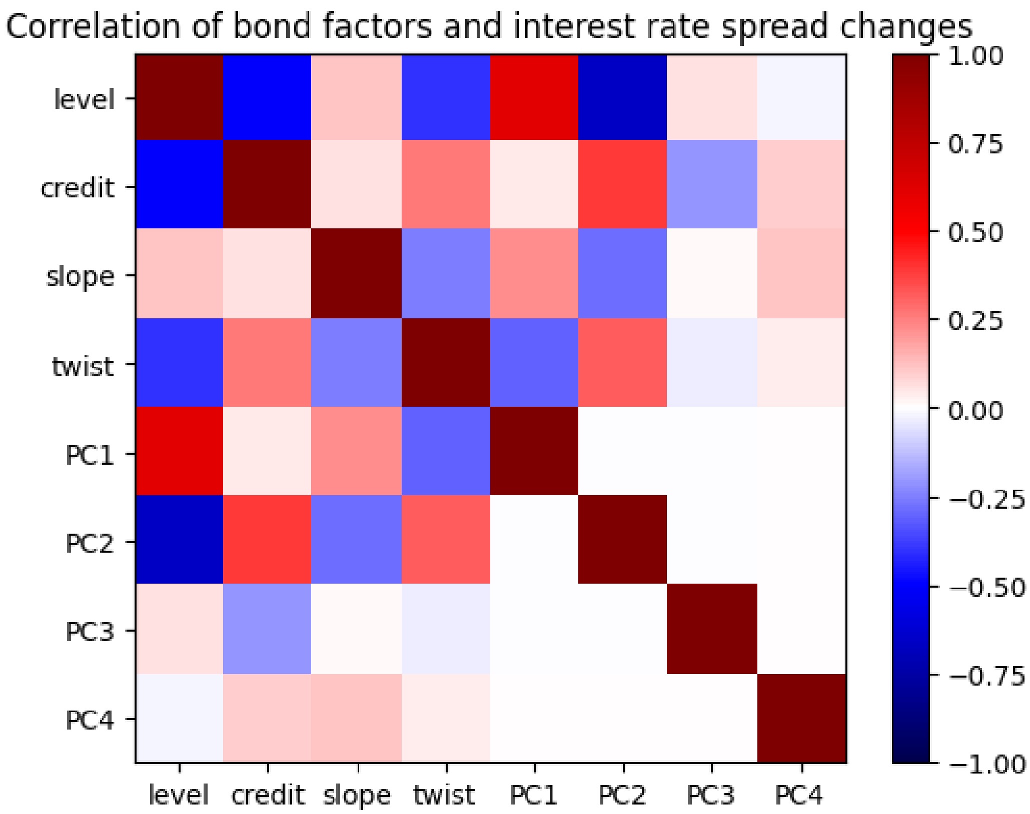
```
# Show correlations between bond factor returns and spread changes
data = pd.concat([spreads, factors], axis=1, join='inner')
corr = data.corr()
#plt.imshow(corr**2, vmin=0, vmax=1, cmap='Purples')
plt.imshow(corr, vmin=-1, vmax=1, cmap='seismic')
plt.xticks(range(len(corr)), corr.index)
plt.yticks(range(len(corr)), corr.index)
```

(continues on next page)

(continued from previous page)

```
plt.colorbar()
plt.title('Correlation of bond factors and interest rate spread changes')
```

```
Text(0.5, 1.0, 'Correlation of bond factors and interest rate spread changes')
```




```
# Show regression fits
for pc in range(K):
    print(smf.ols(f"PC{pc+1} ~ credit + level + slope + twist", data=data)\
          .fit(cov_type='HAC', cov_kwds={'maxlags': 63})\
          .summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    PC1   R-squared:                       0.547
Model:                            OLS   Adj. R-squared:                  0.546
Method:                 Least Squares   F-statistic:                     449.8
Date:                Mon, 03 Mar 2025   Prob (F-statistic):               0.00
Time:                        17:15:31   Log-Likelihood:                -16545.
No. Observations:                6533   AIC:                         3.310e+04
Df Residuals:                    6528   BIC:                         3.313e+04
Df Model:                           4
Covariance Type:                  HAC
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
```

(continues on next page)

(continued from previous page)

```
Intercept      0.0260      0.089      0.293      0.770      -0.148       0.200
credit        67.2252     10.644      6.316      0.000      46.363      88.087
level         67.5805      3.099     21.810      0.000      61.507      73.654
slope          9.3162      2.473      3.767      0.000       4.469      14.164
twist        -13.9266      2.182     -6.384      0.000     -18.202      -9.651
==============================================================================
Omnibus:                     4287.230   Durbin-Watson:                   1.169
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           232868.764
Skew:                           2.485   Prob(JB):                         0.00
Kurtosis:                      31.823   Cond. No.                         40.8
==============================================================================

Notes:
[1] Standard Errors are heteroscedasticity and autocorrelation robust (HAC) using
↪63 lags and without small sample correction
                            OLS Regression Results
==============================================================================
Dep. Variable:                    PC2   R-squared:                       0.486
Model:                            OLS   Adj. R-squared:                  0.486
Method:                 Least Squares   F-statistic:                     276.0
Date:                Mon, 03 Mar 2025   Prob (F-statistic):          1.62e-219
Time:                        17:15:31   Log-Likelihood:                -13117.
No. Observations:                6533   AIC:                         2.624e+04
Df Residuals:                    6528   BIC:                         2.628e+04
Df Model:                           4
Covariance Type:                  HAC
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.0026      0.048      0.055      0.956      -0.092       0.097
credit         8.5003      3.413      2.491      0.013       1.811      15.190
level        -27.6696      1.648    -16.794      0.000     -30.899     -24.440
slope        -14.4220      1.193    -12.090      0.000     -16.760     -12.084
twist          0.0005      1.291      0.000      1.000      -2.530       2.531
==============================================================================
Omnibus:                     4199.401   Durbin-Watson:                   1.263
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           212780.844
Skew:                           2.428   Prob(JB):                         0.00
Kurtosis:                      30.534   Cond. No.                         40.8
==============================================================================

Notes:
[1] Standard Errors are heteroscedasticity and autocorrelation robust (HAC) using
↪63 lags and without small sample correction
                            OLS Regression Results
==============================================================================
Dep. Variable:                    PC3   R-squared:                       0.048
Model:                            OLS   Adj. R-squared:                  0.048
Method:                 Least Squares   F-statistic:                     9.825
Date:                Mon, 03 Mar 2025   Prob (F-statistic):           6.37e-08
Time:                        17:15:32   Log-Likelihood:                -12252.
No. Observations:                6533   AIC:                         2.451e+04
Df Residuals:                    6528   BIC:                         2.455e+04
Df Model:                           4
Covariance Type:                  HAC
==============================================================================
```

(continues on next page)

(continued from previous page)

```
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept     -0.0047      0.025     -0.189      0.850      -0.053       0.044
credit       -12.9602      2.326     -5.571      0.000     -17.519      -8.401
level         -2.1745      1.302     -1.671      0.095      -4.726       0.377
slope          1.7327      1.218      1.422      0.155      -0.655       4.120
twist          0.4779      1.097      0.436      0.663      -1.671       2.627
==============================================================================
Omnibus:                     3598.010   Durbin-Watson:                   1.563
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           548862.079
Skew:                           1.601   Prob(JB):                         0.00
Kurtosis:                      47.789   Cond. No.                         40.8
==============================================================================

Notes:
[1] Standard Errors are heteroscedasticity and autocorrelation robust (HAC) using␣
↪63 lags and without small sample correction
                            OLS Regression Results
==============================================================================
Dep. Variable:                    PC4   R-squared:                       0.024
Model:                            OLS   Adj. R-squared:                  0.024
Method:                 Least Squares   F-statistic:                     13.23
Date:                Mon, 03 Mar 2025   Prob (F-statistic):           9.77e-11
Time:                        17:15:32   Log-Likelihood:                -8402.4
No. Observations:                6533   AIC:                         1.681e+04
Df Residuals:                    6528   BIC:                         1.685e+04
Df Model:                           4
Covariance Type:                  HAC
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept     -0.0016      0.013     -0.125      0.901      -0.027       0.024
credit         2.5655      1.691      1.517      0.129      -0.748       5.879
level          0.5677      0.753      0.753      0.451      -0.909       2.044
slope          2.7974      0.428      6.532      0.000       1.958       3.637
twist          1.6629      0.452      3.679      0.000       0.777       2.549
==============================================================================
Omnibus:                     2769.335   Durbin-Watson:                   1.940
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           176343.662
Skew:                           1.201   Prob(JB):                         0.00
Kurtosis:                      28.339   Cond. No.                         40.8
==============================================================================

Notes:
[1] Standard Errors are heteroscedasticity and autocorrelation robust (HAC) using␣
↪63 lags and without small sample correction
```

References:

FRM Part I Exam Book Valuation and Risk Models Ch12-13

CHAPTER

FIFTEEN

OPTIONS PRICING

Derivatives are financial weapons of mass destruction - Warren Buffett

We explore the basics of options, common strategies, and the foundational pricing models used in the financial markets. The price of an option is influenced by various factors, including the underlying asset's price, the time remaining until expiration, the volatility of the asset, interest rates, and the strike price. The valuation of options can be approached through techniques like binomial tree pricing and Monte Carlo simulations, or models like the Black-Scholes-Merton model.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
from pandas import DataFrame, Series
import pandas as pd
import numpy as np
from scipy import stats
import matplotlib.pyplot as plt
from tqdm import tqdm
import time
from finds.utils import row_formatted
from finds.readers import Alfred
from secret import credentials
VERBOSE = 0
#%matplotlib qt
```

15.1 Options

An **European** call or put option grants the buyer the right to buy or sell an asset at a specified price on a given expiration date. An **American** call or put option, on the other hand, allows the buyer to exercise the option at any time before or on the expiration date. The specified date is known as the **expiration** (or **maturity**) date, while the price at which the asset can be traded is referred to as the **strike** (or **exercise**) price.

Options can be categorized based on their **moneyness**, meaning the relationship between the option's strike price and the current price of the underlying asset. An option is considered **in-the-money** if it would result in a positive payoff if exercised immediately, **out-of-the-money** if it would lead to a negative payoff, and **at-the-money** if the strike price equals the current market price.

The value of an option depends on several factors, including:

- The price of the underlying asset (S)
- The strike price (K)
- The risk-free rate (r)
- The volatility of the asset's price (σ)

- The time to maturity (T)
- Any dividends to be paid during the life of the option (q)

Put-call parity defines the relationship between the prices of European call and put options with identical strike prices and expiration dates: $S - C = PV(K) - P$.

15.1.1 Option strategies

Various trading strategies involve entering positions in multiple options simultaneously, and sometimes incorporating the underlying asset.

- A **protective put** strategy involves buying a put option while holding the underlying asset. This combined position offers a payoff similar to that of a call option, along with the amount of cash equivalent to the present value of the strike price.
- A **covered call** strategy entails owning an asset and selling a call option on it. Typically, the call option is out-of-the-money, and the strategy generates income from the option premium while sacrificing potential upside gains beyond the strike price.
- A **bull spread** is employed by an investor expecting an increase in the asset's price. The strategy involves buying a European call with a lower strike price (K1) and selling a European call with a higher strike price (K2).
- A **bear spread** involves buying a European put option with a higher strike price (K2) and selling a European put option with a lower strike price (K1).
- A **box spread** is a portfolio consisting of a bull spread (using call options) and a bear spread (using put options). Both spreads use the same strike prices and expiration dates.
- A **butterfly spread** involves a combination of three options, created using either calls or puts. A common version involves one long call with a lower strike price (K1), one long call with a higher strike price (K3), and two short calls at a strike price (K2), where $K2 = (K1 + K3)/2$.
- A **calendar spread** strategy consists of buying a long call option with an expiration date at time T* and selling a short call option with an earlier expiration date, both at the same strike price (K).
- A **straddle** is created by holding both a long call and a long put with the same strike price and expiration date.
- A **strangle** is similar to a straddle but reduces the cost by selecting a call option with a higher strike price than the put option.

```python
# define call and put payoffs at maturity
def call_payoff(K):
    return lambda s: s - K if s > K else 0

def put_payoff(K):
    return lambda s: K - s if s < K else 0
```

```python
# helpers to plot final payoff of option strategy
def plot_payoff(payoff, S=np.linspace(70, 130, 60), ax=None, label=''):
    """helper to plot final payoff over range of stock price S"""
    ax = ax or plt.gca()
    df = DataFrame([payoff(s) for s in S], index=S)
    zeros = (df == 0).all(axis=0)
    df = df.drop(columns=df.columns[zeros])
    df = df + np.sign(df.sum(axis=0))*0.4     # jigger so can display separate
    y = df.sum(axis=1).rename(label)
    y.plot(ax=ax, lw=3)
```

(continues on next page)

(continued from previous page)

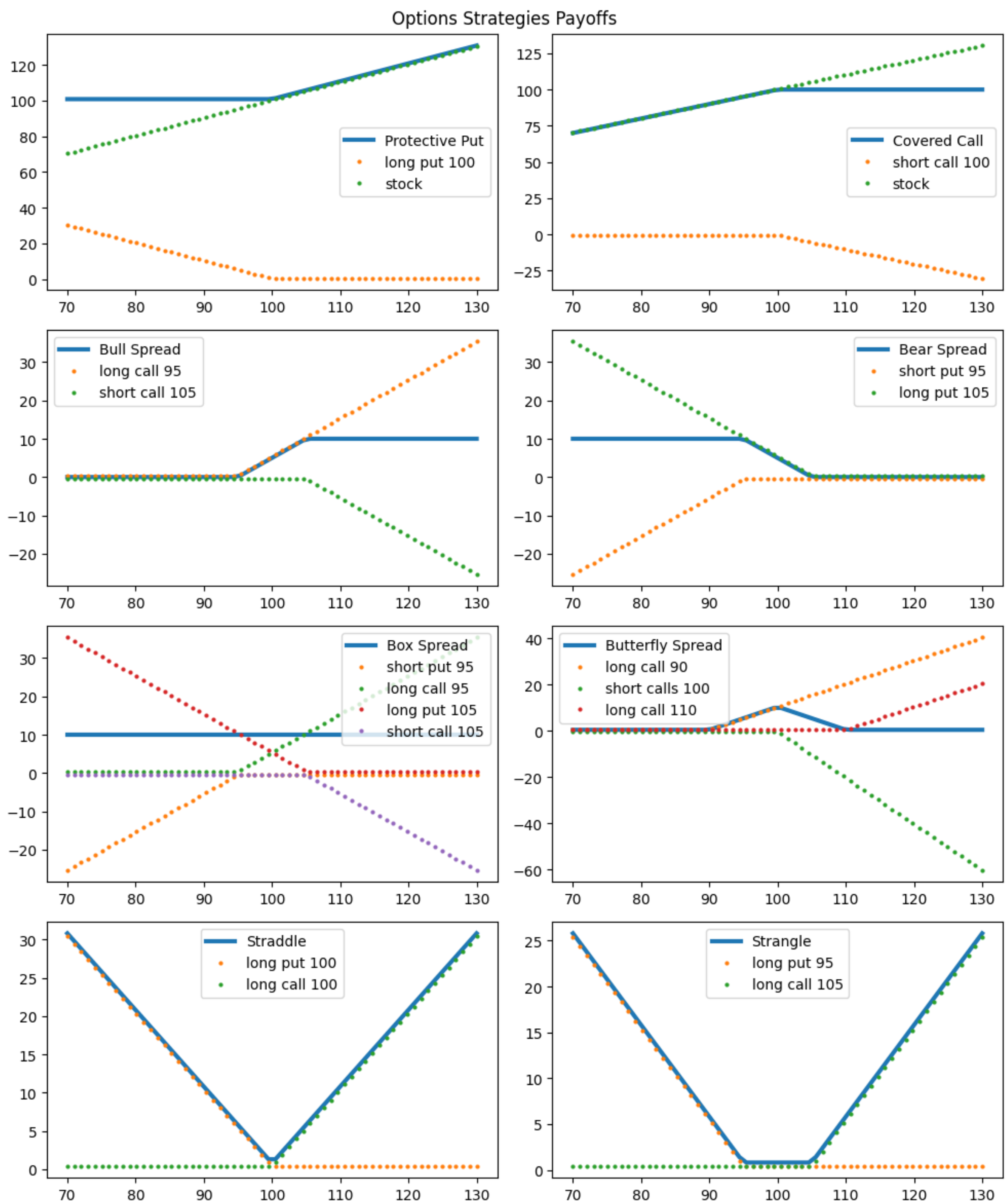
```
    df.plot(marker='.', ls='', ms=4, ax=ax)
    ax.legend([label] + list(df.columns))

def _ls(n):
    """helper to label as long or short position"""
    return 'long' if n >=0 else 'short'
```

```
# Define and plot the options strategies
fig, ax = plt.subplots(nrows=4, ncols=2, figsize=(10,12))
ax = ax.flatten()

def options_strategy(K, calls=0, puts=0, stocks=0):
    return lambda s: {f"{_ls(calls)} call {K}": calls*call_payoff(K)(s),
                      f"{_ls(puts)} put {K}": puts*put_payoff(K)(s),
                      "stock": stocks*s}

plot_payoff(options_strategy(K=100, puts=1, stocks=1), ax=ax[0], label='Protective Put
↪')

plot_payoff(options_strategy(K=100, calls=-1, stocks=1), ax=ax[1], label='Covered Call
↪')

def bull_spread(K1, K2):
    assert K2 > K1, "K2 must be greater than K1"
    return lambda s: {f"long call {K1}": call_payoff(K1)(s),
                      f"short call {K2}": -call_payoff(K2)(s)}

plot_payoff(bull_spread(K1=95, K2=105), ax=ax[2], label='Bull Spread')

def bear_spread(K1, K2):
    assert K2 > K1, "K2 must be greater than K1"
    return lambda s: {f"short put {K1}": -put_payoff(K1)(s),
                      f"long put {K2}": put_payoff(K2)(s)}

plot_payoff(bear_spread(K1=95, K2=105), ax=ax[3], label='Bear Spread')

def box_spread(K1, K2):
    assert K2 > K1, "K2 must be greater than K1"
    return lambda s: {f"short put {K1}": -put_payoff(K1)(s),
                      f"long call {K1}": call_payoff(K1)(s),
                      f"long put {K2}": put_payoff(K2)(s),
                      f"short call {K2}": -call_payoff(K2)(s)}

plot_payoff(box_spread(K1=95, K2=105), ax=ax[4], label='Box Spread')

def butterfly_spread(K1, K3):
    assert K3 > K1, "K3 must be greater than K1"
    K2 = (K1 + K3) / 2
    return lambda s: {f"long call {K1}": call_payoff(K1)(s),
                      f"short calls {K2:g}": -2*call_payoff(K2)(s),
                      f"long call {K3}": call_payoff(K3)(s)}

plot_payoff(butterfly_spread(K1=90, K3=110), ax=ax[5], label='Butterfly Spread')

def straddle(K):
    return lambda s: {f"long put {K}": put_payoff(K)(s),
```

(continues on next page)

(continued from previous page)

```
                      f"long call {K}": call_payoff(K)(s)}
plot_payoff(straddle(K=100), ax=ax[6], label='Straddle')

def strangle(K1, K2):
    assert K2 > K1, "K2 must be greater than K1"
    return lambda s: {f"long put {K1}": put_payoff(K1)(s),
                      f"long call {K2}": call_payoff(K2)(s)}
plot_payoff(strangle(K1=95, K2=105), ax=ax[7], label='Strangle')

plt.suptitle('Options Strategies Payoffs')
plt.tight_layout()
```


Options Strategies Payoffs
Protective Put
long put 100
stock
Covered Call
short call 100
stock
Bull Spread
long call 95
short call 105
Bear Spread
short put 95
long put 105
Box Spread
short put 95
long call 95
long put 105
short call 105
Butterfly Spread
long call 90
short calls 100
long call 110
Straddle
long put 100
long call 100
Strangle
long put 95
long call 105

15.1.2 Exotic options

In addition to standard European and American options (which are termed plain vanilla options), there are **exotic options** (or simply **exotics**) which have more complex structures and non-standard features.

- A **Bermudan** option is one where the exercise of the option is restricted to certain predetermined dates.
- A **forward start** option is an option that starts at a future date and is usually at-the-money at the time it begins.
- A **gap** option is a European option where the price triggering a payoff is different from the price used to calculate the payoff.
- A **cliquet** option is a series of forward start options, each with its own rules for determining strike prices.
- **Binary** options (also called **digital** options) pay a fixed amount or asset if the option' s price exceeds (or falls below) the strike price, otherwise, they pay nothing.
- **Asian** options provide payoffs based on the arithmetic average of the asset' s price during the option' s life.
- A **lookback** option' s payoff depends on the maximum or minimum price reached by the asset during its lifetime. A floating lookback option gives a payoff based on the difference between the final asset price and the minimum (or maximum) price reached, while a fixed lookback option bases the payoff on the difference between the maximum (or minimum) price and the strike price.
- **Barrier** options have payoffs that depend on whether the price of the asset reaches a specific level, with various types like down-and-out, down-and-in, up-and-out, and up-and-in.
- A **compound** option is an option on another option, which results in two strike prices and two expiration dates.
- An **asset-exchange** option allows the holder to exchange one asset for another.
- A **volatility swap** is a forward contract based on the realized volatility of an asset during a specified period. Traders exchange the realized volatility at the end of the period for a pre-specified volatility rate.
- The exercise of a **Bermudan** option is restricted to certain dates.
- A **forward start** option is an option that will begin at a future time. It is usually stated that the option will be at-the-money at the time it starts.
- A **gap** option is a European call or put option where the price triggering a payoff is different from the price used in calculating the payoff.
- A **cliquet** option is a series of forward start options with certain rules for determining the strike prices.
- **Binary** call (put) options may a fixed amount of cash or an asset when its price is above (below) the strike price, or nothing otherwise. Cash-or-nothing optinos are sometimes referred to as **digital** options.
- **Asian** options provide a payoff dependent on an arithmetic aver-age of the underlying asset price during the life of the option.
- The payoff from a **lookback** option depends on the maximum or minimum asset price reached during the life of the option. A floating lookback call (put) gives a payoff equal difference between the final asset price and minimum (maximum) price. The payoff of a fixed lookback call (put) is based on the difference betwen the maximum (minimum) price and the strike price.
- **Barrier** options come into existence or ceases to exist depending on whether the asset price reaches a particular barrier. There are down-and-out, down-and-in, up-and-out and up-and-in variants.
- A **compound** option is an option on another option. Thus, there are two strike prices and two maturity dates.
- In an **asset-exchange** option, the holder has the right to exchange one asset for another.
- A **volatility swap** is a forward contract on the realized volatility of an asset during a certain period. A trader agrees to exchange a pre-specified volatility for the realized volatility at the end of the period.

15.2 Binomial option pricing

The **binomial option pricing model**, proposed by Cox, Ross, and Rubinstein (1979), is widely used to value American-style options and other derivatives.

15.2.1 No-arbitrage

The no-arbitrage principle assumes there are no opportunities for riskless profit in the market. The **law of one price** stipulates that if two portfolios produce the same cash flows at the same times, they should have the same price. Binomial trees use this no-arbitrage principle to model option pricing.

For example, assuming a non-dividend-paying stock with price S, the stock can either increase to Su or decrease to Sd within time T. The portfolio is structured with:

- A short position in the derivative, and
- A position in the stock which we set equal to $\Delta = \dfrac{f_u - f_d}{Su - Sd}$

The value of the portfolio at time T is

- $Su\Delta - f_u$ if the stock price increases, and
- $Sd\Delta - f_d$ if the stock price decreases.

The value of the portfolio today is $S\Delta - f$, where f is the value of the derivative today. Suppose r is the risk-free rate for maturity T. For no arbitrage, we must have

$S\Delta - f = \dfrac{f_u d - f_d u}{u - d} e^{-rT}$

Substituting for Δ, gives:

$$f = e^{-rT}[pf_u + (1-p)f_d]$$

where $p = \dfrac{e^{rT} - d}{u - d}$

15.2.2 Risk neutral pricing

Suppose we choose to interpret the variable p as the probability of an upward movement (with 1 - p being the probability of a downward movement), then the expected stock price grows at the risk-free rate. It also means that p is the probability of an upward movement in a risk-neutral world.

The **risk-neutral** valuation approach applies a probability p for upward movements and values the option by its expected payoff, discounted at the risk-free rate. Risk-neutral pricing assumes that all assets earn the risk-free rate of return in the market. Put another way, a risk-neutral world is one where all tradable assets have an expected return equal to the risk-free interest rate. The probabilities of different outcomes in a risk-neutral world are therefore based on this assumption, and a risk-neutral investor has no preference between assets with different risks. This methodology simplifies the valuation process by treating all market participants as indifferent to risk.

It should be emphasized that the risk-neutral valuation is nothing more than an artificial way of valuing derivatives. We are not assuming that the world is actually risk-neutral. We are instead arguing that the price of a derivative is the same in the real world as it would be in the risk-neutral world

15.2.3 Binomial tree

In practice, the binomial model is extended to multi-step trees, where each step reflects changes in the asset' s price over time. Parameters like Δt (time between steps), and u, d (upward and downward movements) are chosen based on the asset' s volatility:

- the length of a tree step as Δt
- the parameters u and d should be chosen to reflect the volatility of the stock price. If we denote the volatility per year by σ, then appropriate values for the parameters are
  - $u = e^{\sigma\sqrt{\Delta t}}$
  - $u = e^{-\sigma\sqrt{\Delta t}}$

  where Δt is measured in years.
- hence \$ $f = e^{-r\Delta t}[pf_u + (1-p)f_d] where$ p = \dfrac{e^{r \Delta t} - d}{u - d}\$
- the delta, or position taken in the stock to hedge a short posion in the derivative, is $\Delta = \dfrac{f_u - f_d}{Su - Sd}$

Dividends: For assets paying dividends, the probability p is adjusted to account for the dividend yield q:

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

Currency Options: A currency can be considered as an asset providing a yield at the foreign risk-free rate. Therefore, the analysis we presented for a stock paying a continuous dividend yield applies, with q equal to the foreign risk-free rate.

Futures: Because it costs nothing to enter into a futures contract, the return on a futures contract in a risk-neutral world must be zero. This means we can treat a futures contract like a stock, paying a continuous dividend yield equal to the risk free rate.

```
def binomial_tree(S, sigma, r, T, steps, payoff=None, q=0, american=False,
                  verbose=True):
    delta_t = T / steps
    u = np.exp(sigma * np.sqrt(delta_t))
    d = np.exp(-sigma * np.sqrt(delta_t))
    p = (np.exp((r - q) * delta_t) - d) / (u - d)
    result = dict(value=None, u=u, d=d, p=p, delta_t=delta_t)
    if payoff is not None:
        label = "STEP {:<5d}".format     # to label output of each step

        # initialize price vectors at last step
        prices = DataFrame(0.0, index=np.arange(steps+1),
                           columns=['stock', 'option', 'delta'])
        for downs in range(steps+1):
            s = d**downs * u**(steps-downs) * S       # price after number of downs
            prices.loc[downs, 'stock'] = s
            prices.loc[downs, 'option'] = payoff(s)  # option value at expiry
        print(row_formatted(prices.T.rename_axis(columns=label(steps)),
                            default="{:.4f}"))

        # roll back one time step at a time
        for step in range(steps - 1, -1, -1):

            # update all scenarios in this time step
            for downs in range(step+1):

                # stock price after this number of downs
```

(continues on next page)

(continued from previous page)

```
                s = d**downs * u**(step - downs) * S
                prices.loc[downs, 'stock'] = s

                # value of option is max of roll back or exercise (if American)
                exercise = payoff(s) if american else 0   # if exercise American
                f_u = prices.loc[downs, 'option']         # option value after up
                f_d = prices.loc[downs + 1, 'option']     # option value after down
                f = np.exp(-r * delta_t) * (p * f_u + (1 - p) * f_d)
                prices.loc[downs, 'option'] = max(f, exercise)
                prices.loc[downs, 'delta'] = (f_u - f_d) / (S*u - S*d)

            # Display this time step
            prices = prices.iloc[:-1]
            print()
            print(row_formatted(prices.T.rename_axis(columns=label(step)),
                                default="{:.4f}"))
        result['value'] = prices.loc[0, 'option']
    return DataFrame(result, index=[steps]).rename_axis(columns='STEPS').round(4)
```

European call option in 2 steps

```
# Figure 14.3 of FRM Part I Exam Book "Valuation and Risk Models"
binomial_tree(S=29, sigma=0.25, r=0.03, T=1, steps=2,
              payoff=call_payoff(K=30))
```

```
STEP 2           0        1        2
stock      41.2995  29.0000  20.3635
option     11.2995   0.0000   0.0000
delta       0.0000   0.0000   0.0000

STEP 1           0        1
stock      34.6076  24.3010
option      5.5483   0.0000
delta       1.0963   0.0000

STEP 0           0
stock      29.0000
option      2.7243
delta       0.5383
```

```
STEPS   value       u      d       p  delta_t
2      2.7243  1.1934  0.838  0.4984      0.5
```

European put option in 2 steps

```
# Figure 14.4 of FRM Part I Exam Book "Valuation and Risk Models"
binomial_tree(S=29, sigma=0.25, r=0.03, T=1, steps=2,
              payoff=put_payoff(K=30))
```

```
STEP 2           0        1        2
stock      41.2995  29.0000  20.3635
option      0.0000   1.0000   9.6365
delta       0.0000   0.0000   0.0000
```

(continues on next page)

(continued from previous page)

```
STEP 1           0        1
stock      34.6076  24.3010
option      0.4941   5.2523
delta      -0.0970  -0.8380

STEP 0           0
stock      29.0000
option      2.8377
delta      -0.4617
```

```
STEPS   value       u      d       p  delta_t
2      2.8377  1.1934  0.838  0.4984      0.5
```

American put option in 4 steps

```
# Figure 14.5 of FRM Part I Exam Book "Valuation and Risk Models"
binomial_tree(S=29, sigma=0.25, r=0.03, T=1, steps=2,
              payoff=put_payoff(K=30), american=True)
```

```
STEP 2           0        1        2
stock      41.2995  29.0000  20.3635
option      0.0000   1.0000   9.6365
delta       0.0000   0.0000   0.0000

STEP 1           0        1
stock      34.6076  24.3010
option      0.4941   5.6990
delta      -0.0970  -0.8380

STEP 0           0
stock      29.0000
option      3.0584
delta      -0.5050
```

```
STEPS   value       u      d       p  delta_t
2      3.0584  1.1934  0.838  0.4984      0.5
```

American put option in 4 steps

```
# Figure 14.6 of FRM Part I Exam Book "Valuation and Risk Models"
binomial_tree(S=29, sigma=0.25, r=0.03, T=1, steps=4,
              payoff=put_payoff(K=30), american=True)
```

```
STEP 4           0        1        2        3        4
stock      47.8129  37.2367  29.0000  22.5852  17.5894
option      0.0000   0.0000   1.0000   7.4148  12.4106
delta       0.0000   0.0000   0.0000   0.0000   0.0000

STEP 3           0        1        2        3
stock      42.1948  32.8613  25.5924  19.9314
option      0.0000   0.4974   4.4076  10.0686
delta       0.0000  -0.1376  -0.8825  -0.6873
```

(continues on next page)

(continued from previous page)

```
STEP 2            0         1         2
stock       37.2367   29.0000   22.5852
option       0.2474    2.4387    7.4148
delta       -0.0684   -0.5379   -0.7788

STEP 1            0         1
stock       32.8613   25.5924
option       1.3356    4.8958
delta       -0.3015   -0.6846

STEP 0            0
stock       29.0000
option       3.0966
delta       -0.4898
```

```
STEPS    value        u        d        p  delta_t
4       3.0966   1.1331   0.8825   0.4988     0.25
```

European call on index with dividend yield

```
# Figure 14.7 of FRM Part I Exam Book "Valuation and Risk Models"
binomial_tree(S=2500, sigma=0.15, r=0.03, q=0.02, T=0.5, steps=3,
              payoff=call_payoff(K=2500))
```

```
STEP 3              0           1           2           3
stock       3004.1734   2657.8778   2351.5002   2080.4391
option       504.1734    157.8778      0.0000      0.0000
delta          0.0000      0.0000      0.0000      0.0000

STEP 2              0           1           2
stock       2825.7257   2500.0000   2211.8212
option       328.7911     78.2792      0.0000
delta          1.1303      0.5153      0.0000

STEP 1              0           1
stock       2657.8778   2351.5002
option       202.0979     38.8125
delta          0.8177      0.2555

STEP 0              0
stock       2500.0000
option       119.5793
delta          0.5330
```

```
STEPS      value        u        d        p  delta_t
3       119.5793   1.0632   0.9406   0.4983   0.1667
```

American call option on foreign currency

```
# Figure 14.8 of FRM Part I Exam Book "Valuation and Risk Models"
binomial_tree(S=0.78, sigma=0.12, r=0.02, q=0.06, T=1, steps=4,
              payoff=call_payoff(K=0.8))
```

```
STEP 4          0       1       2       3       4
stock      0.9916  0.8794  0.7800  0.6918  0.6136
option     0.1916  0.0794  0.0000  0.0000  0.0000
delta      0.0000  0.0000  0.0000  0.0000  0.0000

STEP 3          0       1       2       3
stock      0.9338  0.8282  0.7346  0.6515
option     0.1239  0.0318  0.0000  0.0000
delta      1.1972  0.8483  0.0000  0.0000

STEP 2          0       1       2
stock      0.8794  0.7800  0.6918
option     0.0685  0.0127  0.0000
delta      0.9837  0.3394  0.0000

STEP 1          0       1
stock      0.8282  0.7346
option     0.0350  0.0051
delta      0.5955  0.1358

STEP 0          0
stock      0.7800
option     0.0170
delta      0.3191
```

```
STEPS  value       u       d       p  delta_t
4      0.017  1.0618  0.9418  0.4021     0.25
```

American put option on futures contract

```
# Figure 14.9 of FRM Part I Exam Book "Valuation and Risk Models"
binomial_tree(S=38, sigma=0.2, r=0.04, q=0.04, T=9/12, steps=3,
              payoff=put_payoff(K=40), american=True)
```

```
STEP 3           0        1        2        3
stock      51.2946  41.9965  34.3838  28.1511
option      0.0000   0.0000   5.6162  11.8489
delta       0.0000   0.0000   0.0000   0.0000

STEP 2           0        1        2
stock      46.4133  38.0000  31.1118
option      0.0000   2.9190   8.8882
delta       0.0000  -0.7377  -0.8187

STEP 1           0        1
stock      41.9965  34.3838
option      1.5172   5.9925
delta      -0.3834  -0.7841

STEP 0           0
stock      38.0000
option      3.8282
delta      -0.5879
```

```
STEPS   value       u       d      p  delta_t
3      3.8282  1.1052  0.9048  0.475     0.25
```

15.3 Black-Scholes-Merton model

The **Black-Scholes-Merton model**, introduced in two papers in 1973, revolutionized options pricing and remains one of the most widely used models for pricing European options. Based on assumptions like lognormal stock price distributions, continuous trading, and no-arbitrage conditions, the model provides a formula for calculating the price of call and put options. In one of the papers, Black and Scholes used the capital asset pricing model (CAPM) to derive the relationship between the return from a stock and the return from an option on the stock. In the other, Merton used no-arbitrage arguments similar to those of the binomial tree approach. Both derived that the price evolution of derivatives satisfies the same partial differential equation:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 S}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

To value an option, we simply apply boundary conditions. For a European call option with time to maturity T and strike price K, the boundary condition is that the value of the option is max(S - K, 0) at time T. For a European put, this boundary condition is max(K - S, 0). Other derivatives give rise to other boundary conditions.

The solutions for the prices of a European call and put are the Black-Scholes-Merton formulas:

$$C = Se^{-qT}\Phi(d_1) - Ke^{-rT}\Phi(d_2)$$

$$p = Ke^{-rT}\Phi(-d_2) - Se^{-qT}\Phi(-d_1)$$

where:

- $d_1 = \dfrac{\ln(S_0/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$
- $d_2 = \dfrac{\ln(S_0/K) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$
- S is the current stock price,
- K is the strike price,
- T is the time to maturity in years,
- r is the risk-free rate per year (continuously compounded),
- q is the dividend yield (or foreign risk-free rate for currency options)
- σ is an estimated volatility per year over the next T years,
- Φ is the cumulative normal distribution function

Suppose discreate dividends are expected bo be paid with ex-dates during the life of the option. The option can be valued by replacing stock price S with S - PV(D), the present value of those dividends.

Since American call options on a non-dividend paying stock should never be exercised early, the pricing formula for American call options on non-dividend paying stocks as well as for European call options are the same. But it may be optimal to exercise early when there are discrete dividends, but only immediately before an ex-dividend date. American put options on stocks and all American options on stock indices, currencies, and futures should not be valued as European options. Binomial trees can be used in these cases.

Black-Scholes-Merton option pricing formulas

```
def _d1(S, K, sigma, r, T, q):
    """helper to compute d1 term in Black-Scholes-Merton formula"""
    return (np.log(S/K) + (r - q + sigma**2/2) * T) / (sigma * np.sqrt(T))
```

```
def call(S, K, sigma, r, T, q=0.):
    """Black-Scholes-Merton call option value"""
    d1 = _d1(S=S, K=K, sigma=sigma, r=r, T=T, q=q)
    d2 = d1 - sigma * np.sqrt(T)
    return S*np.exp(-q*T)*stats.norm.cdf(d1) - K*np.exp(-r*T)*stats.norm.cdf(d2)
```

```
def put(S, K, sigma, r, T, q=0.):
    """Black-Scholes-Merton put option value"""
    d1 = _d1(S=S, K=K, sigma=sigma, r=r, T=T, q=q)
    d2 = d1 - sigma * np.sqrt(T)
    return K*np.exp(-r*T)*stats.norm.cdf(-d2) - S*np.exp(-q*T)*stats.norm.cdf(-d1)
```

```
print(call(S=56, K=60, r=0.05, sigma=0.3, T=18/12))  # 8.3069
print(put(S=56, K=60, r=0.05, sigma=0.3, T=18/12))   # 7.9715
```

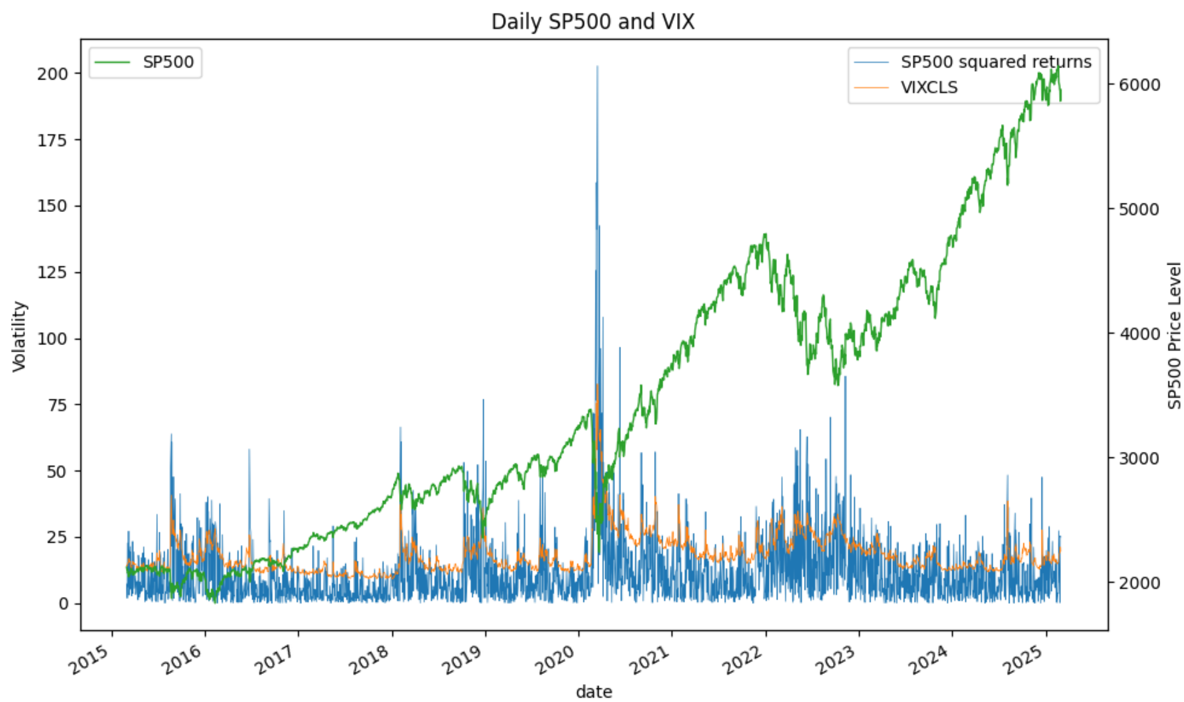
```
8.306909593824336
7.971518773537515
```

15.3.1 Implied volatility

Implied volatility is the volatility figure derived from an option’s market price and used to infer expectations about future price movements.

The Chicago Board Options Exchange has developed indices that track volatilities. The most popular of these is the SPX VIX index, which tracks the volatilities of 30-day options on the S&P 500. Traders monitor implied volatilities carefully and often use them to communicate prices.

```
alf = Alfred(api_key=credentials['fred']['api_key'], verbose=VERBOSE, convert_date=0)
vix = alf('VIXCLS')
sp500_sq = alf('SP500', log=1, diff=1).rename('SP500 squared returns')
sp500 = alf('SP500')
df = pd.concat([100*np.sqrt(252*sp500_sq**2), vix], axis=1, join='inner').dropna()

fig, ax = plt.subplots(figsize=(10, 6))
df.plot(ax=ax, lw=.5)
ax.set_ylabel('Volatility')
bx = ax.twinx()
sp500.plot(ax=bx, lw=1, color="C2")
bx.set_ylabel('SP500 Price Level')
plt.title('Daily SP500 and VIX')
plt.legend()
plt.tight_layout()
```



15.3.2 Volatility smile

If the assumptions underlying the Black-Scholes-Merton model held exactly, all options on an asset would have the same implied volatility at all times. In practice, implied volatility varies with the strike price and time to maturity. The **volatility smile** refers to this observed pattern, where options with extreme strike prices (both high and low) tend to have higher implied volatilities. Because of put-call parity, the implied volatility of a European call option is the same as that of a European put option when they have the same strike price and time to maturity.

In equity options, the volatility smile generally slopes downward. This means that out-of-the-money puts and in-the-money calls tend to have higher implied volatilities, while out-of-the-money calls and in-the-money puts have lower implied volatilities. This phenomenon is often referred to as a volatility skew. There is a negative correlation between equity prices and volatility, which means that as stock prices fall, volatility increases, and as stock prices rise, volatility decreases.

For foreign currency options, the volatility smile takes a U-shape. Both out-of-the-money and in-the-money options tend to have higher implied volatilities compared to at-the-money options. The volatility of exchange rates is not constant and can be subject to sudden jumps, often driven by central bank actions. These nonconstant volatilities and jumps make extreme price movements more likely.

Traders also often use a volatility term structure, where the implied volatility of an option depends on its time to maturity. When volatility smiles and term structures are combined, they form a **volatility surface**. This surface shows implied volatility as a function of both the strike price and the time to maturity. When quoting option prices, traders interpolate between known implied volatilities to estimate the implied volatility for the option in question. This estimated volatility is then plugged into the Black-Scholes-Merton model to calculate the option price. This approach helps address the fact that the market does not always price options in line with the assumptions of the Black-Scholes-Merton model.

15.3.3 The "Greeks"

The Greek letters, or **Greeks** as they are often called, are metrics used to measure the sensitivity of option prices to different factors such as changes in the price of the underlying asset, volatility, time decay, and interest rates.

- **Delta** measures the sensitivity to changes in the price of the underlying asset:

$\Delta_c = e^{-qt}\Phi(d_1)$

$\Delta_p = e^{-qt}[\Phi(d_1) - 1]$

- **Gamma** measures the sensitivity of a portfolio' s delta to changes in the price of the underlying asset:

$\Gamma = \dfrac{e^{-qt}}{S\sigma\sqrt{t}}\phi(d_1)$

- **Theta** measures the sensity to time to expiration

$\Theta_c = -\dfrac{S\sigma e^{-qt}}{2\sqrt{t}}\phi(d_1) - rKe^{-rt}\Phi(d_2) + qSe^{-qt}\Phi(d_1)$

$\Theta_p = -\dfrac{S\sigma e^{-qt}}{2\sqrt{t}}\phi(d_1) + rKe^{-rt}\Phi(-d_2) - qSe^{-qt}\Phi(-d_1)$

- **Vega** measures the sensitivity to the implied volatility

$V = Se^{-qt}\sqrt{t}\phi(d_1)$

- **Rho** measures the sensitivity to changes in the level of interest rate

$\rho_c = Kte^{-rt}\Phi(d_2)$

$\rho_p = Kte^{-rt}\Phi(-d_2)$

Any of the Greek letters for a portfolio of derivatives dependent on the same asset can be calculated as the weighted sum of the Greek letters for each portfolio component.

The Black-Scholes-Merton analysis can be used to show that $\Theta + (r-q)S\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma = (r-q)C$

```
# Helper to plot a greek over range of stock price S"""
def plot_greek(K, sigma, r, T, greek, S=np.linspace(50, 200, 100),
               ax=None, label='', q=0., c="C0"):
    """helper to plot a greek over range of stock price S"""
    y = [greek(S=s, K=K, sigma=sigma, r=r, T=T, q=q) for s in S]
    ax = ax or plt.gca()
    ax.plot(S, y, color=c)
    #ax.set_xlabel('Stock Price')
    ax.legend([label])
```

Define and plot options sensitivities

```
# Plot options sensitivities
fig, ax = plt.subplots(nrows=4, ncols=2, figsize=(10,12))
ax = ax.flatten()

# call option parameters
opt = dict(K=105, r=0.04, sigma=0.25, T=1)

def delta_call(S, K, sigma, r, T, q=0.):
    """Black-Scholes-Merton call option delta"""
    d1 = _d1(S=S, K=K, sigma=sigma, r=r, T=T, q=q)
```

(continues on next page)

(continued from previous page)

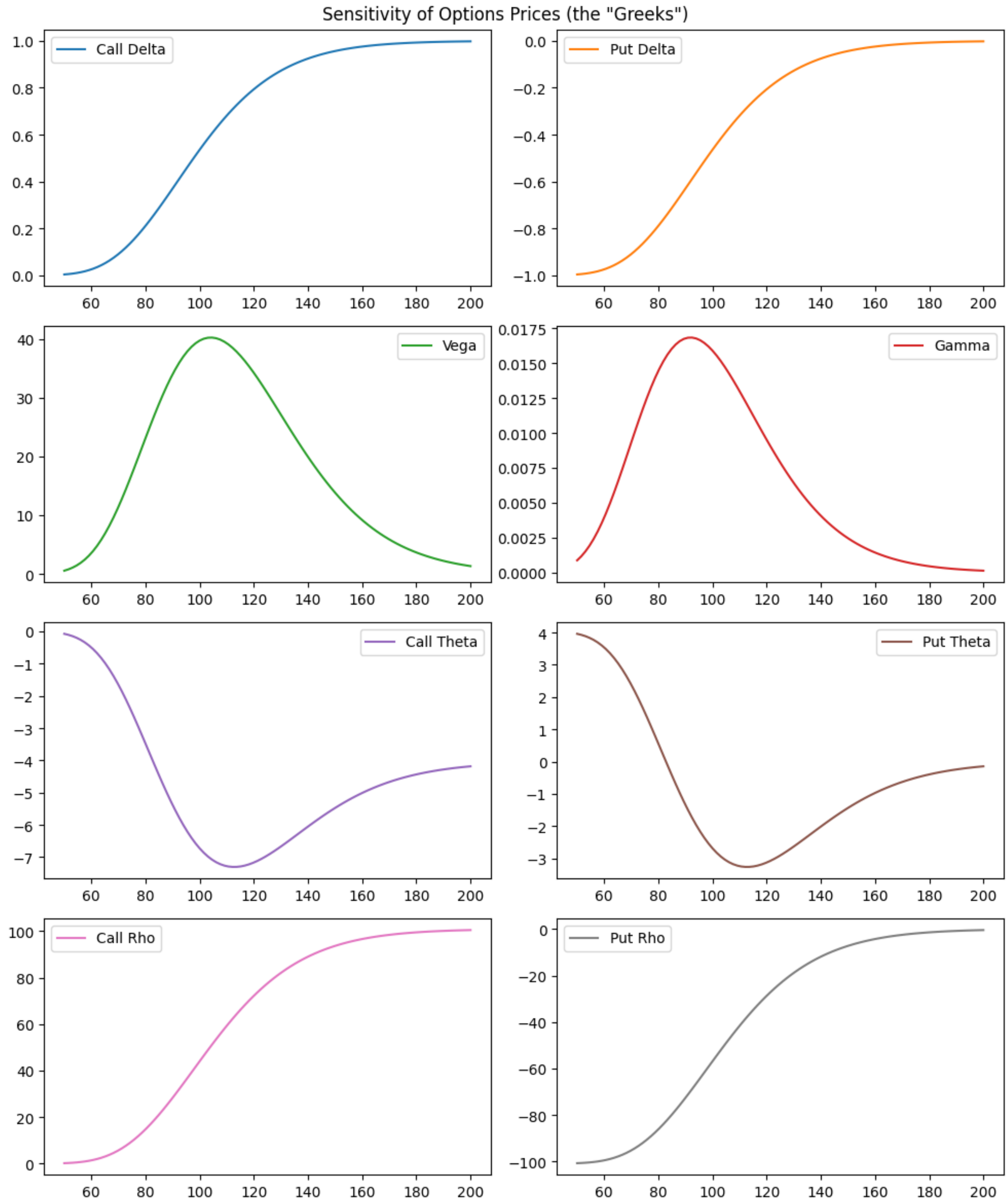
```
    return np.exp(-q*T) * stats.norm.cdf(d1)
def delta_put(S, K, sigma, r, T, q=0.):
    """Black-Scholes-Merton put option delta"""
    d1 = _d1(S=S, K=K, sigma=sigma, r=r, T=T, q=q)
    return -np.exp(-q*T) * stats.norm.cdf(-d1)
print(delta_call(S=100, **opt))  # 0.5358

plot_greek(greek=delta_call, label='Call Delta', ax=ax[0], c="C0", **opt)
plot_greek(greek=delta_put, label='Put Delta', ax=ax[1], c="C1", **opt)

def vega(S, K, sigma, r, T, q=0.):
    """Black-Scholes-Merton call or put option delta"""
    d1 = _d1(S=S, K=K, sigma=sigma, r=r, T=T, q=q)
    return S * np.exp(-q*T) * np.sqrt(T) * stats.norm.pdf(d1)
print(vega(S=100, **opt))  # 39.73

plot_greek(greek=vega, label='Vega', ax=ax[2], c="C2", **opt)

def gamma(S, K, sigma, r, T, q=0.):
    """Black-Scholes-Merton call or put option gamma"""
    d1 = _d1(S=S, K=K, sigma=sigma, r=r, T=T, q=q)
    return np.exp(-q*T) * stats.norm.pdf(d1) / (S * sigma * np.sqrt(T))
print(gamma(S=100, **opt))  # 0.0159

plot_greek(greek=gamma, label='Gamma', ax=ax[3], c="C3", **opt)

def theta_call(S, K, sigma, r, T, q=0.):
    """Black-Scholes-Merton call option theta"""
    d1 = _d1(S=S, K=K, sigma=sigma, r=r, T=T, q=q)
    d2 = d1 - sigma * np.sqrt(T)
    return ((-S * np.exp(-q*T) * stats.norm.pdf(d1) * sigma / (2 * np.sqrt(T)))
            - (r * K * np.exp(-r*T) * stats.norm.cdf(d2)) +
            + (q * S * np.exp(-q*T) * stats.norm.cdf(d1)))
def theta_put(S, K, sigma, r, T, q=0.):
    """Black-Scholes-Merton put option theta"""
    d1 = _d1(S=S, K=K, sigma=sigma, r=r, T=T, q=q)
    d2 = d1 - sigma * np.sqrt(T)
    return ((-S * np.exp(-q*T) * stats.norm.pdf(d1) * sigma / (2 * np.sqrt(T)))
            + (r * K * np.exp(-r*T) * stats.norm.cdf(-d2)) +
            - (q * S * np.exp(-q*T) * stats.norm.cdf(-d1)))
print(theta_call(S=100, **opt))  # 6.73

plot_greek(greek=theta_call, label='Call Theta', ax=ax[4], c="C4", **opt)

plot_greek(greek=theta_put, label='Put Theta', ax=ax[5], c="C5", **opt)

def rho_call(S, K, sigma, r, T, q=0.):
    """Black-Scholes-Merton call option rho"""
    d1 = _d1(S=S, K=K, sigma=sigma, r=r, T=T, q=q)
    d2 = d1 - sigma * np.sqrt(T)
    return K * T * np.exp(-r * T) * stats.norm.cdf(d2)
def rho_put(S, K, sigma, r, T, q=0.):
    """Black-Scholes-Merton put option rho"""
    d1 = _d1(S=S, K=K, sigma=sigma, r=r, T=T, q=q)
    d2 = d1 - sigma * np.sqrt(T)
    return -K * T * np.exp(-r * T) * stats.norm.cdf(-d2)
```

(continues on next page)

(continued from previous page)

```
print(rho_call(S=100, **opt))  # 44

plot_greek(greek=rho_call, label='Call Rho', ax=ax[6], c="C6", **opt)

plot_greek(greek=rho_put, label='Put Rho', ax=ax[7], c="C7", **opt)

plt.suptitle('Sensitivity of Options Prices (the "Greeks")')
plt.tight_layout()
```

```
0.5357925584332519
39.73355715246575
0.0158934228609863
-6.727614629584682
44.02299963816157
```




```
# Check that: theta + r S delta + 1/2 sigma^2 S^2 gamma = r call
S = 100
print(theta_call(S=S, **opt) + opt['r'] * S * delta_call(100, **opt) +
      0.5 * opt['sigma']**2 * S**2 * gamma(S=S, **opt),
      opt['r'] * call(S=S, **opt))
```

```
0.3822502482065442 0.38225024820654485
```

15.4 Monte Carlo simulation

Simulation is an effective way to estimate expectations that are difficult or impossible to compute analytically. Consider a random variable X that can be simulated (for example, from a normal distribution), and a function g that can be evaluated at realizations of X. To estimate the expected value of $g(X)$, we take multiple simulated draws. Since these draws are independent and identically distributed (iid), the expectation can be approximated as:

$$\hat{E}[g(X)] = \frac{1}{b}\sum_{i=1}^{b} g(X_i)$$

where b is the number of simulated samples. By the Law of Large Numbers (LLN), the approximation improves as b increases:

$$\lim_{b\to\infty} \hat{E}[g(X)] = E[g(X)]$$

Additionally, the Central Limit Theorem (CLT) implies that the distribution of the simulated estimate approaches a normal distribution as the number of simulations grows. The variance of the simulated estimate can be approximated by:

$$V[\hat{E}[g(X)]] = \frac{\sigma_b^2}{b}$$

where σ_b^2 is the sample variance. This variance can be estimated as:

$$\sigma_g^2 = \frac{1}{b}\sum_{i=1}^{b}(g(X_i) - E[g(X_i)])^2$$

which is the standard variance estimator for iid samples. The standard error of the estimate, $\frac{\sigma_g}{\sqrt{b}}$, indicates the accuracy of the approximation. This allows us to adjust b to achieve any desired level of precision.

15.4.1 Antithetic Variates

Antithetic variates are a technique to improve the accuracy of simulation by generating a second set of random variables that are negatively correlated with the original set of random variables. These pairs of variables are created using a single uniform random number. Each uniform variable U_i generates its counterpart $1 - U_i$, both of which are then mapped through the inverse cumulative distribution function (CDF) to generate correlated random variables.

Using antithetic variates reduces the simulation error, but only if the function $g(X)$ is monotonic, which ensures that the negative correlation between the paired variables results in reduced error. The benefit of this approach comes from the negative correlation, which decreases the standard error and thus increases the accuracy of the simulation.

15.4.2 Control Variates

Control variates are another technique to enhance simulation accuracy. A control variate is a derived random variable $h(X_i)$, which is correlated with the primary variable $g(X_i)$ but has a known mean, typically zero. To be effective, a good control variate must satisfy two criteria:

1. It should be computationally inexpensive to construct from the data x_i, offering a more cost-effective alternative to increasing the number of simulations.

2. It should exhibit a high correlation with the function $g(X)$, making it useful for reducing the error in the primary estimate.

The optimal parameter β, which minimizes the approximation error, is typically found using regression:

$$g(x_i) = \alpha + \beta h(x_i)$$

By adjusting β, the control variate technique helps improve the accuracy of the estimated value of $g(X)$.

15.4.3 Simulating option prices

To simulate the price of a financial option, we assume that the logarithm of the stock price follows a normal distribution. The log of the stock price at time T, denoted s_T, is given by:

$$s_T = s_0 + T\left(r_f - \frac{\sigma^2}{2}\right) + \sqrt{T}x_i$$

where x_i is a random variable sampled from a normal distribution $N(0, \sigma^2)$, r_f is the risk-free rate, and σ^2 is the variance of the stock return. The final stock price is then $S_T = \exp(s_T)$, and the present value of the option's payoff is:

$$C = \exp(-r_f T)\max(S_T - K, 0)$$

where K is the strike price of the option. The expected price of a call option can be estimated by averaging the simulated payoffs:

$$E[C] = \overline{C} = \frac{1}{b}\sum_{i=1}^{b} C_i$$

where C_i represents the simulated payoff for each instance. This approach is particularly useful for complex pricing scenarios where analytical solutions may not exist.

```python
# helpers for random number generator and monte carlo simulation
class RNG:
    """Helper to generate random normal variables, with optional antithetic variates""
↪"
    def __init__(self, seed=None, antithetic=False, ppf=stats.norm.ppf):
        self.ppf = ppf
        self.antithetic = antithetic
        self._prev = None   # to track if antithetic pair available to return next
        self.seed = seed
        self.rng = np.random.default_rng(seed)

    def __call__(self, shape=1, **kwargs):
        _shape = (shape, ) if isinstance(shape, int) else shape
        n = np.prod(_shape)
        if self.antithetic:  # generate half as many rv's, by returning 1-rv
            new = int((n + 1) / 2) if self._prev is None else int(n / 2)
            new = self.rng.random((new,))
            rem = 1 - new[:n - len(new) - int(self._prev is not None)]
            last = new[-1] if len(rem) < len(new) else None   # if last pair unused
            out = [] if self._prev is None else [1 - self._prev]
            #out.extend(new)
            #out.extend(rem)
            for x, y in zip(new[:len(rem)], rem):
                out.extend([x, y])
```

(continues on next page)

(continued from previous page)

```
            if last is not None:
                out.extend([last])
            out = np.array(out)
            self._prev = last
        else:
            out = self.rng.random(_shape)
        if shape == 1:
            return self.ppf(out[0], **kwargs)
        else:
            return self.ppf(out.reshape(_shape), **kwargs)

def monte_carlo(rng, S, K, sigma, r, T, control=None):
    """Helper to price European call option by Monte Carlo Simulation
    Args:
      control : True price of European put option, as control variate
    """
    if rng.antithetic:
        label = 'Both' if control else 'Antithetic'
    else:
        label = 'Control' if control else 'Standard'
    result = {}
    for b in [50*4**i for i in range(9)]:
        tic = time.time()
        x = rng(b, scale=sigma)
        s = S * np.exp(T * (r - sigma**2/2) + np.sqrt(T) * x)
        c = np.exp(-r * T) * np.maximum(s - K, 0)
        if control is not None:   # to apply control variate method
            error = np.exp(-r * T) * np.maximum(K - s, 0) - control   # error of put␣
↪price
            ols = stats.linregress(x=error, y=c)   # compute best hedge to minimize␣
↪error
            c = c - ols.slope * error
        result[b] = dict(Price=np.mean(c).round(2),
                         StdErr=(np.std(c) / np.sqrt(b)).round(2),
                         elapsed = np.round(time.time() - tic, 4))
    return DataFrame.from_dict(result, orient='index').rename_axis(columns=label)

RNG(antithetic=True)((2, 3))    # generate normal r.v.'s with antithetic variates
```

```
array([[-0.72886259,  0.72886259,  0.25403329],
       [-0.25403329, -0.81873243,  0.81873243]])
```

Standard simulation

```
S = 2500
K = 2500
sigma = 0.164
r = 0.02
T = 2

seed = 42
rng = RNG(seed=seed)
monte_carlo(rng, S=S, K=K, sigma=sigma, r=r, T=T)
```

```
Standard   Price  StdErr  elapsed
50        297.91   54.77   0.0005
200       247.65   25.97   0.0003
800       283.95   15.23   0.0003
3200      279.62    7.59   0.0004
12800     278.25    3.73   0.0009
51200     279.45    1.85   0.0027
204800    278.93    0.93   0.0112
819200    278.31    0.46   0.0491
3276800   278.59    0.23   0.2490
```

Antithetic Variates

```
rng = RNG(seed=seed, antithetic=True)
monte_carlo(rng, S=S, K=K, sigma=sigma, r=r, T=T)
```

```
Antithetic   Price  StdErr  elapsed
50          280.86   54.41   0.0004
200         242.12   25.04   0.0006
800         276.97   14.15   0.0013
3200        284.38    7.63   0.0013
12800       277.30    3.72   0.0014
51200       279.28    1.87   0.0052
204800      278.15    0.93   0.0225
819200      278.46    0.46   0.1279
3276800     278.48    0.23   0.4977
```

Control Variates

```
control = put(S, K, sigma, r, T)   # Black-Scholes-Merton put price
rng = RNG(seed=seed)
monte_carlo(rng, S=S, K=K, sigma=sigma, r=r, T=T, control=control)
```

```
Control   Price  StdErr  elapsed
50       263.59   47.04   0.0010
200      251.85   22.76   0.0003
800      287.49   13.52   0.0002
3200     279.92    6.76   0.0003
12800    278.43    3.32   0.0010
51200    279.01    1.64   0.0022
204800   279.02    0.83   0.0109
819200   278.22    0.41   0.0441
3276800  278.64    0.21   0.3454
```

Both Antithetic and Control Variates

```
control = put(S, K, sigma, r, T)    # Black-Scholes-Merton put price
rng = RNG(seed=seed, antithetic=True)
monte_carlo(rng, S=S, K=K, sigma=sigma, r=r, T=T, control=control)
```

```
Both      Price  StdErr  elapsed
50       286.14   45.71   0.0006
200      227.29   22.03   0.0008
800      277.91   12.32   0.0003
```

(continues on next page)

(continued from previous page)

```
3200     286.51    6.79   0.0008
12800    276.56    3.31   0.0018
51200    279.47    1.66   0.0058
204800   277.88    0.83   0.0244
819200   278.41    0.41   0.1019
3276800  278.41    0.21   0.5569
```

References:

F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities," Journal of Political Economy, 81 (May/June 1973): 637–59

R. C. Merton, "Theory of Rational Option Pricing," Bell Journal of Economics and Management Science 4 (Spring 1973): 141–183.

Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option Pricing: A Simplified Approach. Journal of Financial Economics, 7(3), 229-263.

Terence Lim, Andrew W. Lo, Robert C. Merton and Myron S. Scholes (2006), "The Derivatives Sourcebook", Foundations and Trends in Finance: Vol. 1: No. 5–6, pp 365-572. http://dx.doi.org/10.1561/0500000005

FRM Part I Exam Book Financial Markets and Products Ch. 14-15

FRM Part I Exam Book Quantitative Analysis Ch. 13

FRM Part I Exam Book Valuation and Risk Models Ch. 14-16

FRM Part II Exam Book Market Risk Measurement and Management Ch. 15

CHAPTER

SIXTEEN

VALUE AT RISK

The only constant in life is change - Heraclitus

Value at Risk (VaR) is a widely used risk measure in financial risk management that quantifies the potential loss in a portfolio over a given time period with a specified confidence level. However, VaR has limitations, including its inability to capture the severity of losses beyond its threshold. To address this, alternative measures such as Expected Shortfall (ES) have been introduced. We explore different methodologies for calculating VaR, including parametric, historical, and Monte Carlo simulation approaches, as well as advanced techniques such as stressed VaR and bootstrapping. Additionally, we examine conditional volatility models, including EWMA and GARCH, to account for changing market conditions. Finally, we discuss backtesting methods, such as the Kupiec Likelihood Ratio test and conditional coverage tests, to validate the accuracy of VaR models in real-world scenarios.

```python
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
from typing import Dict
import numpy as np
from scipy import stats
import pandas as pd
from pandas import DataFrame, Series
import statsmodels.api as sm
import matplotlib.pyplot as plt
from finds.readers import Alfred
from finds.utils import row_formatted
from secret import credentials
#pd.set_option('display.max_rows', None)
VERBOSE = 0
#%matplotlib qt
```

16.1 Risk measures

Value at Risk (VaR) is a fundamental risk measure that estimates the maximum potential loss in a portfolio over a specific period at a given confidence level. A VaR at an (\alpha%) confidence level represents the loss threshold that has an (\alpha%) probability of being exceeded.

16.1.1 Coherence

A major limitation of VaR is that it does not provide insight into the magnitude of losses beyond its threshold. Artzner et al. proposed four essential properties that a risk measure should satisfy:

1. **Monotonicity:** If one portfolio consistently performs worse than another under all conditions, it should have a higher risk measure.
2. **Translation Invariance:** Adding a risk-free cash amount K to a portfolio should reduce its risk measure by K.
3. **Homogeneity:** Scaling a portfolio by a factor of X should scale its risk measure by the same factor.
4. **Subadditivity:** The risk measure of a merged portfolio should not exceed the sum of the individual risk measures, ensuring diversification benefits.

A risk measure that satisfies all four properties is considered **coherent**. **Expected Shortfall (ES)** is a coherent risk measure, whereas VaR lacks subadditivity. ES is calculated as the probability-weighted average of losses beyond the VaR threshold. Other coherent risk measures can be derived by applying a risk aversion function to weight quantiles, with ES being a special case where tail quantiles receive equal weighting.

Retrieve crypto currency index returns from FRED

```python
# retrieve crypto currency index from FRED, as log returns
alf = Alfred(api_key=credentials['fred']['api_key'], verbose=VERBOSE, convert_date=0)
cat = alf.get_category(33913)
cryptos = [alf(s['id'], log=1, diff=1)
           for s in cat['series'] if 'DISCONT' not in s['title']]
cryptos = pd.concat(cryptos, axis=1).sort_index()
date_str = f" ({str(cryptos.index[0])[:10]} to {str(cryptos.index[-1])[:10]})"
titles = Series({s['id']: s['title']
                 for s in cat['series'] if 'DISCONT' not in s['title']},
                name=cat['name'])
```

```python
# crypto index names
titles.to_frame().rename_axis(index=cat['id'])
```

```
               Cryptocurrencies
33913
CBBCHUSD  Coinbase Bitcoin Cash
CBBTCUSD       Coinbase Bitcoin
CBETHUSD      Coinbase Ethereum
CBLTCUSD      Coinbase Litecoin
```

```python
# recent crypto log returns
cryptos
```

```
            CBBCHUSD  CBBTCUSD  CBETHUSD  CBLTCUSD
date
2014-12-01       NaN       NaN       NaN       NaN
2014-12-02       NaN  0.021391       NaN       NaN
2014-12-03       NaN  0.000000       NaN       NaN
2014-12-04       NaN -0.002384       NaN       NaN
2014-12-06       NaN  0.002384       NaN       NaN
...              ...       ...       ...       ...
2025-02-26  0.010099  0.006085 -0.011953  0.018231
2025-02-27  0.062384 -0.003877 -0.030322  0.009735
2025-02-28 -0.014295  0.020210 -0.008916 -0.029413
2025-03-01  0.071640  0.091550  0.127686  0.028240
2025-03-02 -0.003326 -0.010081 -0.019098  0.015676

[3710 rows x 4 columns]
```

16.1.2 Parametric method

Assuming returns follow a normal distribution, VaR and ES at a confidence level α are given by:

$$VaR = -\mu + \sigma z_{1-\alpha}$$

$$ES = -\mu + \sigma \frac{\exp(-z_{1-\alpha}^2/2)}{(1-\alpha)\sqrt{2\pi}}$$

where z_α is the standard normal quantile corresponding to α.

For example, at a 95% confidence level ($\alpha = 0.95$), $z_\alpha = -1.645$, which represents the lower 5% quantile. In practice, μ and σ are estimated from historical data, but since short-term mean returns are difficult to measure accurately, μ is often assumed to be zero.

Under the assumption that geometric returns are normally distributed, then arithmetic returns follow a **lognormal distribution**. The skewness of the lognormal $(\exp(\sigma^2)+2)\sqrt{(\exp(\sigma^2)-1)}$ is always positive, hence the lognormal has a long right-tail. The lognormal has kurtosis $\exp(\sigma^2)^4 + 2\exp(\sigma^2)^3 + 3\exp(\sigma^2)^2 - 3$ which exceeds 3 and increases with volatility, hence exhibits fatter tails than the normal distribution.

```python
# Helper to compute parametric VaR and ES
def parametric_risk(sigma: float | Series, alpha: float) -> Dict:
    """Calculate parametric gaussian VaR and ES"""
    var = -sigma * stats.norm.ppf(1 - alpha)
    es = sigma * stats.norm.pdf(stats.norm.ppf(1 - alpha)) / (1 - alpha)
    return dict(value_at_risk=var, expected_shortfall=es)
```

```python
alpha = 0.95
volatility = {label: np.std(cryptos[label]) for label in cryptos}
parametric = DataFrame({label: Series(parametric_risk(std, alpha=alpha))
                        for label, std in volatility.items()})
print(f"Parametric Risk Measures (alpha={alpha})")
pd.concat([DataFrame.from_dict({'volatility': volatility}, orient='index'),
           parametric], axis=0).round(4)
```

```
Parametric Risk Measures (alpha=0.95)
```

```
                      CBBCHUSD   CBBTCUSD   CBETHUSD   CBLTCUSD
volatility              0.0576     0.0396     0.0505     0.0543
value_at_risk           0.0948     0.0651     0.0831     0.0894
expected_shortfall      0.1189     0.0817     0.1042     0.1121
```

16.1.3 Delta-Normal method

The delta-normal method provides an approximation for non-linear portfolios by assuming that underlying asset returns are normally distributed. The risk of the portfolio is modeled using **delta**, which measures the sensitivity of portfolio value to changes in underlying asset prices. A more refined approximation includes **gamma**, the second derivative of portfolio value with respect to asset prices, known as the **delta-gamma method**.

16.1.4 Monte Carlo simulation method

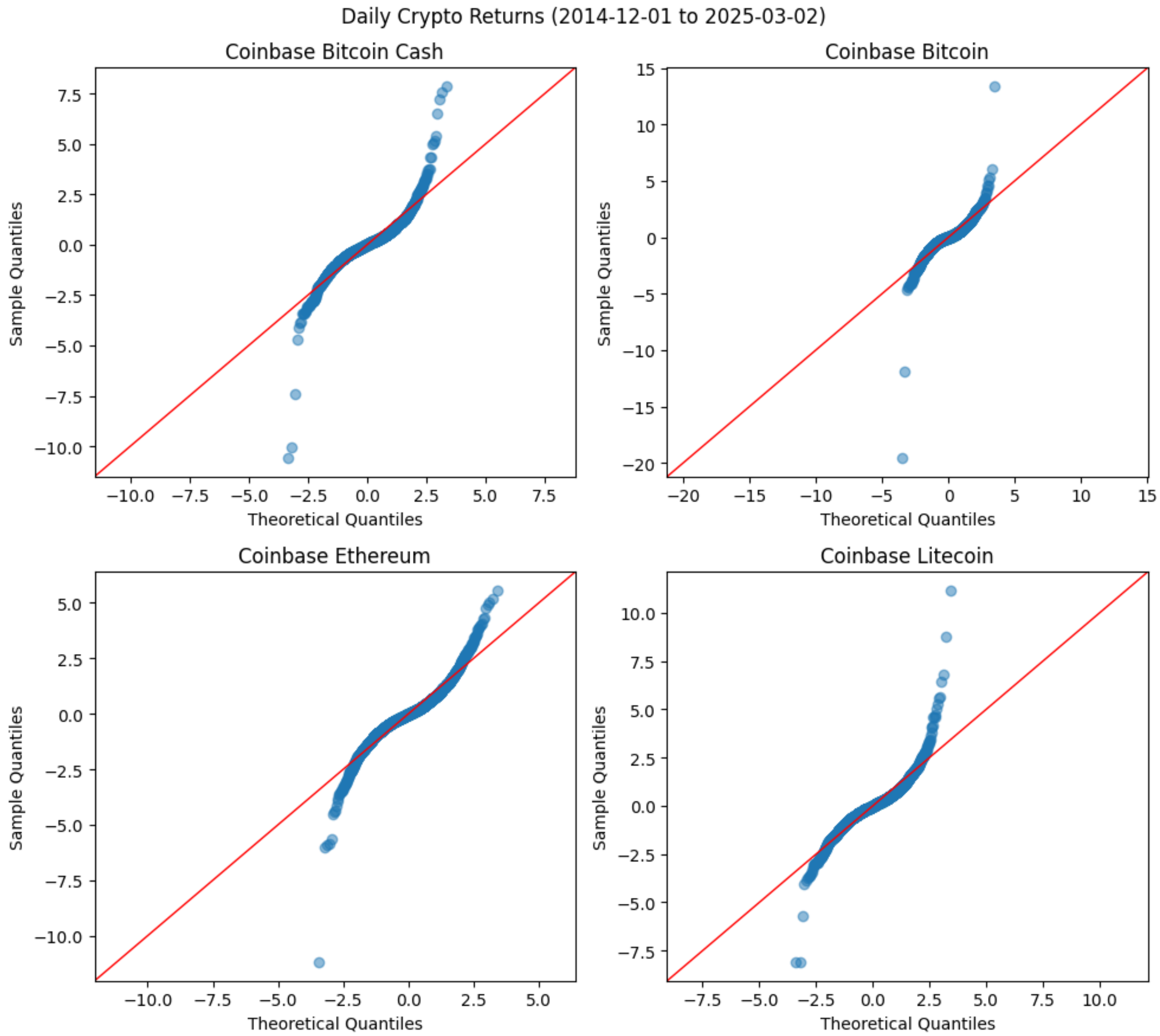
Monte Carlo simulation applies to both linear and non-linear portfolios. This approach generates random scenarios based on an assumed distribution for underlying risk factors. If 1,000 scenarios are simulated, the 95% VaR is estimated as the 50th worst loss (i.e., the 5th percentile), while Expected Shortfall is calculated as the average of the 49 worst losses.

However, standard Monte Carlo models assume normality and independence, which may not always reflect real-world financial data, necessitating the use of **non-parametric approaches**.

A **QQ plot** compares the empirical distribution of returns to a theoretical normal distribution. If returns exhibit heavier tails than the reference distribution, the QQ plot will have steeper slopes at the extremes.

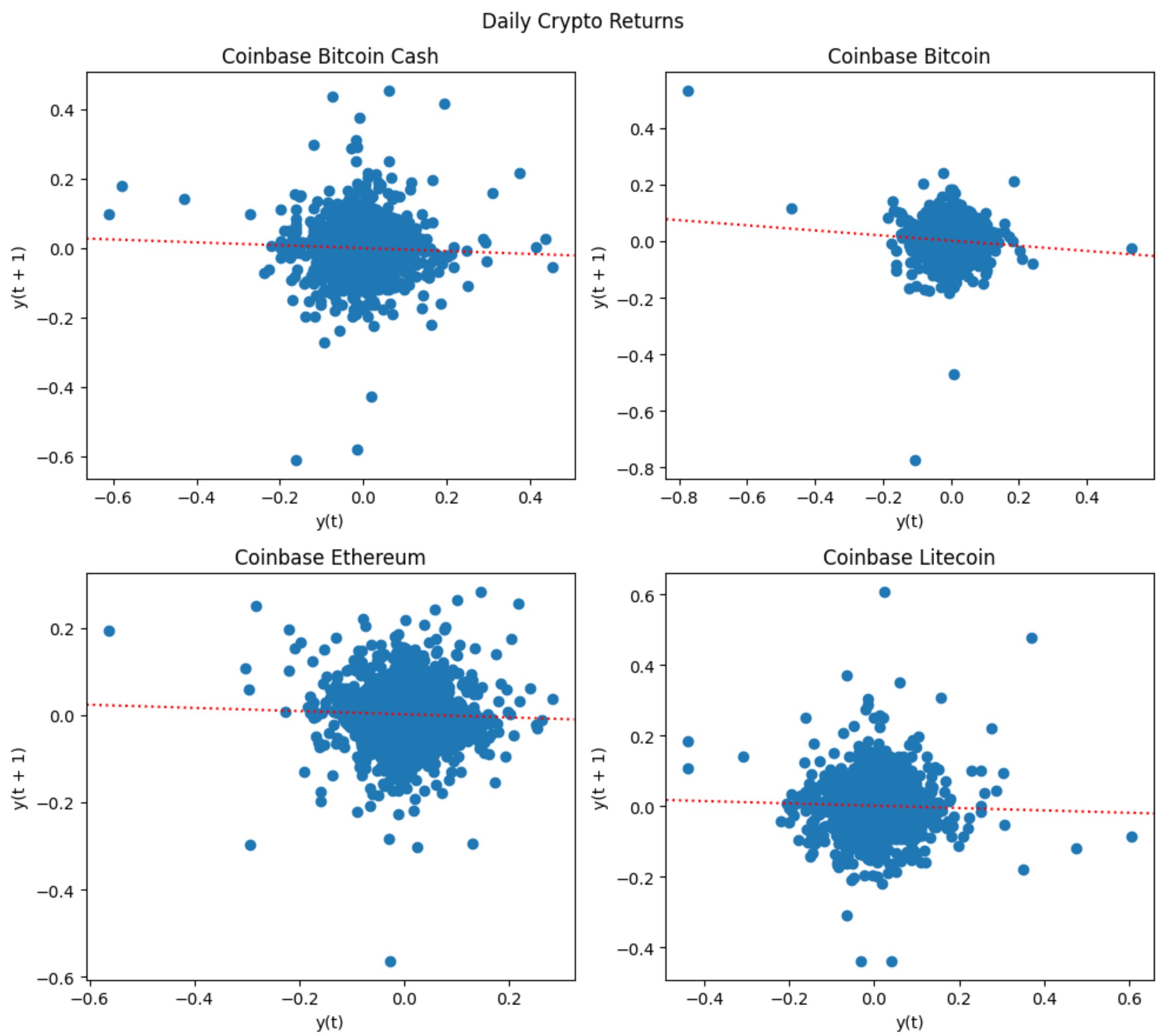
```python
# QQ Plot for Gaussian assumption
from statsmodels.graphics.gofplots import ProbPlot
fig, axes = plt.subplots(2, 2, figsize=(10, 9))
for label, ax in zip(cryptos, axes.flatten()):
    pp = ProbPlot(cryptos[label].dropna(), fit=True)
    pp.qqplot(ax=ax, color='C0', alpha=.5)
    sm.qqline(ax=ax, fmt='r--', line='45', lw=1)
    ax.set_title(f"{titles[label]}")
plt.suptitle(f"Daily Crypto Returns" + date_str)
plt.tight_layout()
```

```
/home/terence/env3.11/lib/python3.11/site-packages/statsmodels/graphics/gofplots.
↪py:1043: UserWarning: color is redundantly defined by the 'color' keyword␣
↪argument and the fmt string "b" (-> color=(0.0, 0.0, 1.0, 1)). The keyword␣
↪argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```



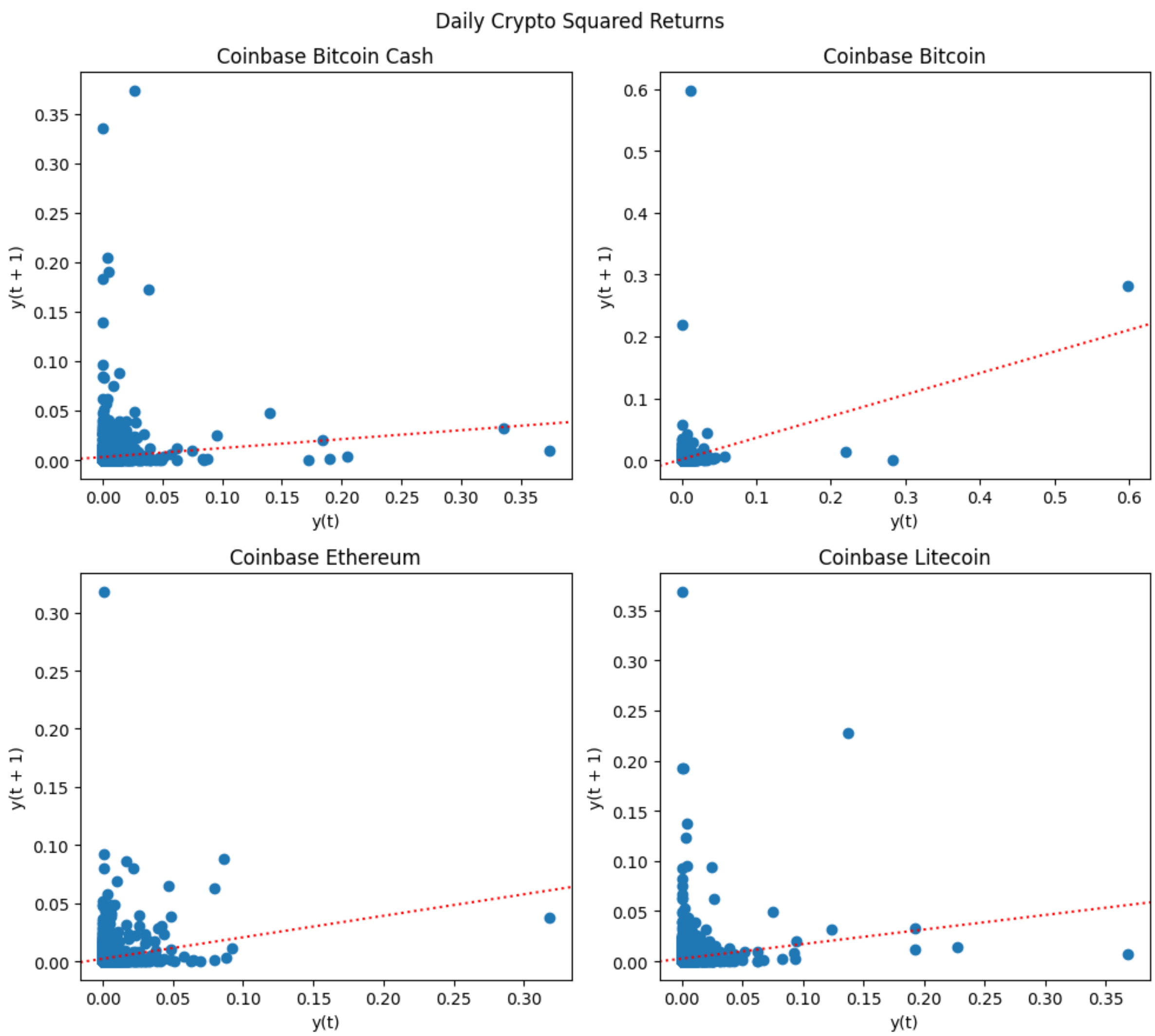
Lag plot of daily returns

```python
# Autocorrelation of returns
import statsmodels.api as sm
fig, axes = plt.subplots(2, 2, figsize=(10, 9))
for label, ax in zip(cryptos, axes.flatten()):
    X = cryptos[label].dropna()
    pd.plotting.lag_plot(X, lag=1, ax=ax)
    r = stats.linregress(X.values[1:], X.values[:-1])
    ax.axline((0, r.intercept), slope=r.slope, ls=':', color="red")
    ax.set_title(f"{titles[label]}")
plt.suptitle(f"Daily Crypto Returns")
plt.tight_layout()
```



Lag plot of squared daily returns

```
# Autocorrelation of squared returns
fig, axes = plt.subplots(2, 2, figsize=(10, 9))
for label, ax in zip(cryptos, axes.flatten()):
    X = cryptos[label].dropna()**2
    pd.plotting.lag_plot(X, lag=1, ax=ax)
    r = stats.linregress(X.values[1:], X.values[:-1])
    ax.axline((0, r.intercept), slope=r.slope, ls=':', color="red")
    ax.set_title(f"{titles[label]}")
plt.suptitle(f"Daily Crypto Squared Returns")
plt.tight_layout()
```



16.1.5 Historical Simulation method

Historical simulation (HS) is the simplest non-parametric approach that estimates VaR by ordering historical losses and selecting the appropriate quantile. Expected Shortfall is calculated as the average of losses beyond the VaR threshold. This method accommodates non-normal features such as skewness and fat tails, making it robust in capturing market risks. However, its accuracy depends on whether the historical dataset adequately represents future market conditions.

```
# Helper to compute VaR, ES and sample moments from historical simulation
def historical_risk(X: Series, alpha: float):
    """Calculate historical VaR, ES, and sample moments"""
    X = X.dropna()
    N = len(X)
    var = -np.percentile(X, 100 * (1 - alpha))
    es = -np.mean(X[X < var])
    vol = np.std(X, ddof=0)
    skew = stats.skew(X)
    kurt = stats.kurtosis(X)
    jb = stats.jarque_bera(X)[0]
```

(continues on next page)

(continued from previous page)

```
    jbp = stats.jarque_bera(X)[1]
    return dict(N=N, value_at_risk=var, expected_shortfall=es, volatility=vol,
                skewness=skew, excess_kurtosis=kurt-3, jb_statistic=jb, jb_pvalue=jbp)
```

```
hist = DataFrame({label: historical_risk(cryptos[label], alpha=alpha)
                  for label in cryptos})
print(f"Historical Risk Measures (alpha={alpha})")
row_formatted(hist.round(4), {'N': "{:.0f}"})
```

```
Historical Risk Measures (alpha=0.95)
```

	CBBCHUSD	CBBTCUSD	CBETHUSD	CBLTCUSD
N	2623	3709	3207	3117
value_at_risk	0.0831	0.0583	0.074	0.0822
expected_shortfall	0.0078	0.0032	0.0055	0.0061
volatility	0.0576	0.0396	0.0505	0.0543
skewness	-0.2394	-1.822	-0.4136	0.7306
excess_kurtosis	13.159	52.3855	6.0638	10.3101
jb_statistic	28562.5232	476116.6693	11069.1899	23285.8936
jb_pvalue	0.0	0.0	0.0	0.0

16.1.6 Stressed VaR method

During periods of market stress, volatility and correlations tend to rise, often leading to *correlation breakdowns* where asset prices move more synchronously. It is sometimes stated that "in stressed markets all correlations go to one." Standard VaR models may not capture these dynamics accurately. **Stressed VaR** is calculated using historical periods of extreme market distress, such as the 2008 Financial Crisis or the 2021–2022 Crypto Winter, as opposed to simply the most recent number of years. Stress testing is designed to identify vulnerabilities, particularly those involving periods of high volatility.

```
beg, end = '2021-11-01', '2022-11-21'  # dubbed "crypto winter"
stress = DataFrame({label: historical_risk(cryptos.loc[beg:end, label], alpha=alpha)
                    for label in cryptos})
print(f"Stressed Risk Measures ({beg} to {end})")
row_formatted(stress.round(4).rename_axis(index='(alpha=0.05)'), {'N': "{:.0f}"})
```

```
Stressed Risk Measures (2021-11-01 to 2022-11-21)
```

(alpha=0.05)	CBBCHUSD	CBBTCUSD	CBETHUSD	CBLTCUSD
N	386	386	386	386
value_at_risk	0.0818	0.0624	0.0773	0.0911
expected_shortfall	0.0078	0.0058	0.0074	0.0053
volatility	0.0464	0.0348	0.0461	0.0478
skewness	-0.3194	-0.5904	-0.3785	-0.417
excess_kurtosis	-0.5048	0.7985	-0.6065	-1.0258
jb_statistic	106.6976	254.4822	101.3559	73.8708
jb_pvalue	0.0	0.0	0.0	0.0

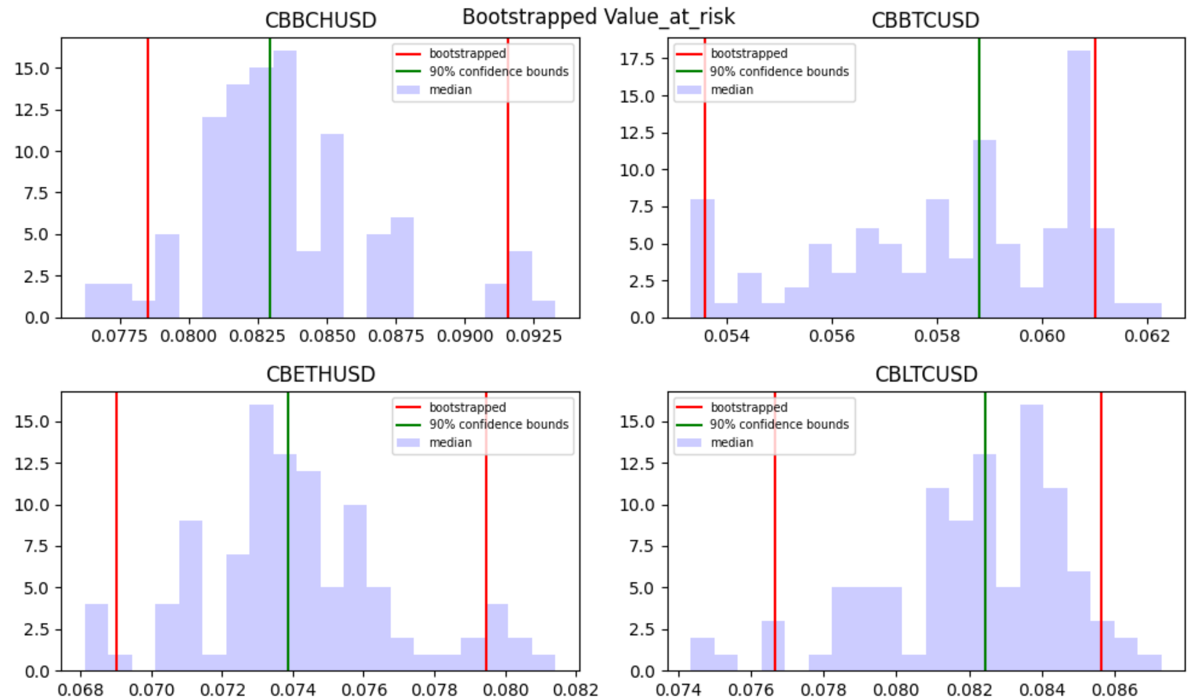
16.1.7 Bootstrap method

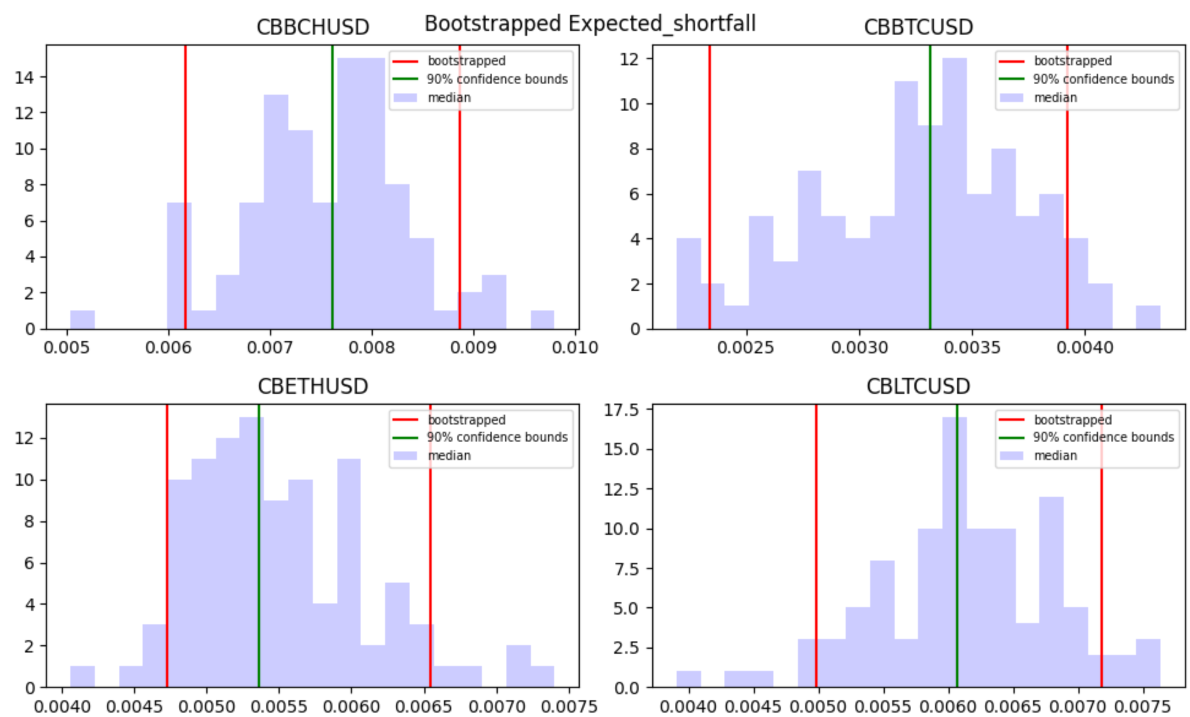
Bootstrap resampling improves historical simulation by repeatedly drawing random samples from the dataset with replacement. Each resampled dataset provides a new VaR or ES estimate, and the distribution of these estimates is used to compute confidence intervals. However, basic bootstrap methods assume that observations are independent over time, which may not hold for financial returns. Modifications such as the **block bootstrap** preserve time dependencies by sampling blocks of consecutive observations.

```
def bootstrap_risk(X: Series, alpha: float, n: int) -> dict:
    """Calculate bootstrap VaR, ES, confidence and plot VaR histogram"""
    X = X.dropna()
    N = len(X)
    bootstraps = []
    for _ in range(n):
        Z = Series(np.random.choice(X, N), index=X.index)
        bootstraps.append(historical_risk(Z, alpha=alpha))
    bootstraps = DataFrame.from_records(bootstraps)
    return bootstraps
```

```
def confidence_intervals(X: Series, confidence: float) -> dict:
    """Extracts confidence intervals and median from a series"""
    lower = (1 - confidence) / 2
    upper = lower + confidence
    return np.quantile(X, [lower, 0.5, upper], method='inverted_cdf')
```

```
# Run and plot bootstrapped VaR and ES
n = 100
confidence = 0.9
intervals = dict()
for measure in ['value_at_risk', 'expected_shortfall']:
    intervals[measure] = dict()
    fig, axes = plt.subplots(2, 2, figsize=(10, 6))
    for label, ax in zip(cryptos, axes.flatten()):
        bootstraps = bootstrap_risk(cryptos[label].dropna(), alpha=alpha, n=n)
        interval = confidence_intervals(bootstraps[measure], confidence=confidence)
        intervals[measure][label] = interval.tolist()
        ax.hist(bootstraps[measure], color='blue', alpha=0.2, bins=int(n/5))
        ax.axvline(x=interval[0], color='red')
        ax.axvline(x=interval[1], color='green')
        ax.legend(['bootstrapped', f"{confidence*100:.0f}% confidence bounds",
                   'median'], fontsize='x-small')
        ax.axvline(x=interval[2], color='red')
        ax.set_title(label)
    plt.tight_layout()
    plt.suptitle('Bootstrapped ' + measure.capitalize())
```






```
# display confidence intervals of VaR
DataFrame(intervals['value_at_risk'], index=['lower', 'median', 'upper'])\
    .rename_axis(index='Value at Risk')
```

```
                 CBBCHUSD  CBBTCUSD  CBETHUSD  CBLTCUSD
Value at Risk
```

(continues on next page)

(continued from previous page)

```
lower           0.078517  0.053589  0.069026  0.076687
median          0.082939  0.058818  0.073875  0.082435
upper           0.091576  0.061019  0.079481  0.085629
```

```
# display confidence intervals of VaR
DataFrame(intervals['expected_shortfall'], index=['lower', 'median', 'upper'])\
    .rename_axis(index='Expected Shortfall')
```

```
                    CBBCHUSD  CBBTCUSD  CBETHUSD  CBLTCUSD
Expected Shortfall
lower               0.006168  0.002339  0.004729  0.004981
median              0.007616  0.003314  0.005362  0.006072
upper               0.008870  0.003924  0.006546  0.007182
```

16.2 Conditional volatility models

A return distribution's characteristics may change over time. A mixture model of normal distributions with varying volatilities produces more peakedness and fatter tails than a simple normal distribution. In a model where returns re conditionally normal, the distribution is normal each day, while the standard deviation of the return varies over time. This leads to an unconditional distribution with fat tails.

16.2.1 EWMA model

Volatility can be estimated using an equal-weighted moving average of squared returns. However, this method suffers from sudden jumps when large returns enter or exit the dataset.

The **Exponentially Weighted Moving Average (EWMA)** model addresses this by applying exponentially decreasing weights to past returns:

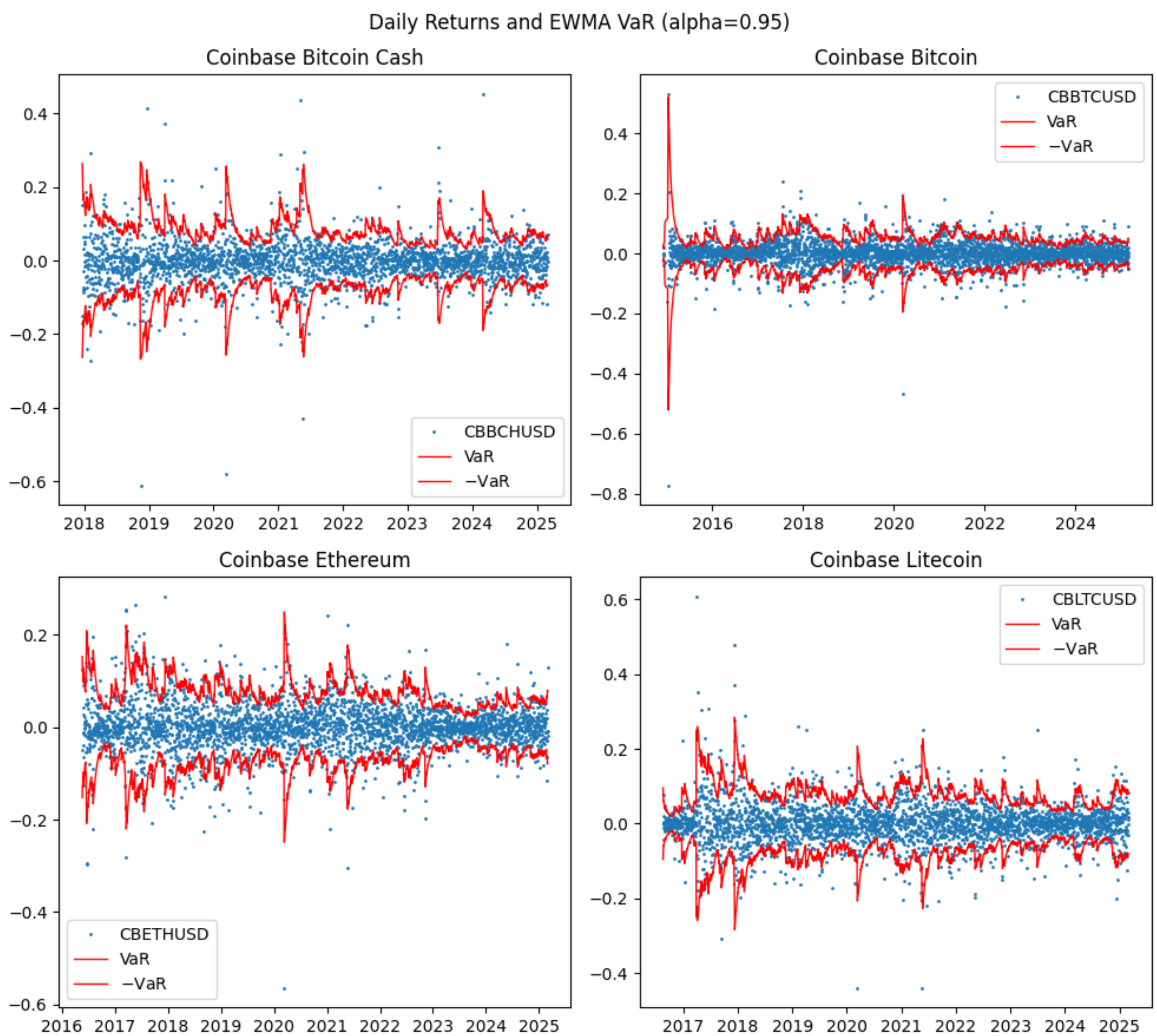
$$\sigma_t^2 = (1-\lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2$$

where λ is a positive value less than 1 which determines the decay rate of past observations. This formula provides a very simple way of implementing EWMA. The new estimate of the variance rate on day t is a weighted average of the estimate of the variance rate made for the previous day $t-1$, and the most recent observation of the squared return on day $t-1$. In the 1990s, JP Morgan's **RiskMetrics** suggests $\lambda = 0.94$ for daily market volatility estimation.

```
# Estimate EWMA (lambda=0.94) rolling model predictions for all cryptos
lambda_ = 0.94
ewma = {label: np.sqrt((cryptos[label]**2).dropna().ewm(alpha=1-lambda_).mean())
        for label in cryptos}
```

```
# Helper to plot predicted VaR vs actual returns
def plot_var(X: Series, VaR: Series, ax: plt.Axes):
    """Helper to plot returns and VaR predictions"""
    ax.plot(X, ls='', marker='.', markersize=2)
    ax.plot(-VaR.shift(-1), lw=1, ls='-', c='r')
    ax.plot(VaR.shift(-1), lw=1, ls='-', c='r')
    ax.legend([X.name, 'VaR', '$-$VaR'])
```

```
# Plot daily returns and EWMA predicted VaR of all cryptos
fig, axes = plt.subplots(2, 2, figsize=(10, 9))
for label, ax in zip(cryptos, axes.flatten()):
    Z = pd.concat([cryptos[label].dropna(),
                   parametric_risk(ewma[label], alpha=alpha)['value_at_risk']\
                   .rename('VaR')],
                  join='inner', axis=1).dropna()
    plot_var(Z[label], VaR=Z['VaR'], ax=ax)
    ax.set_title(titles[label])
plt.suptitle(f"Daily Returns and EWMA VaR (alpha={alpha})")
plt.tight_layout()
```




```
# Properties of EWMA normalized returns for all cryptos
ewma_hist = dict()
for label in cryptos:
    X = (cryptos[label] / ewma[label].shift(-1))  # normalize by predict vol
    ewma_hist[label] = Series(historical_risk(X, alpha=0.95)).rename(label)
print("Normalized by EWMA predicted volatility (alpha=0.95)")
DataFrame(ewma_hist).round(4)
```

```
Normalized by EWMA predicted volatility (alpha=0.95)
```

```
                        CBBCHUSD   CBBTCUSD   CBETHUSD   CBLTCUSD
N                      2622.0000  3708.0000  3206.0000  3116.0000
value_at_risk             1.5056     1.4695     1.5831     1.5697
expected_shortfall        0.1241     0.0717     0.0828     0.0896
volatility                0.9269     0.9252     0.9432     0.9425
skewness                  0.1035    -0.0464    -0.0177     0.0048
excess_kurtosis          -1.5298    -1.5609    -2.1157    -1.8466
jb_statistic            240.8139   321.2816   104.6335   172.7197
jb_pvalue                 0.0000     0.0000     0.0000     0.0000
```

16.2.2 GARCH model

The **Generalized Autoregressive Conditional Heteroskedasticity (GARCH)** model, developed by Robert Engel and Tim Bollerslev, can be intuitively regarded as an extension of EWMA. In **GARCH (1,1)**, we also give some weight to a long run average variance $\hat{\sigma}$.

$$\sigma_t^2 = \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \hat{\sigma}$$

where $\alpha + \beta + \gamma = 1$. This introduces **mean reversion**, where the $\hat{\sigma}$ term provides a "pull" toward the long-run average, ensuring that volatility stabilizes over time. And just as with ARMA specifications of time series models, more complex **GARCH(p,q)** models can incorporate additional lags of q squared returns and p variance estimates for improved accuracy.

rugarch package in R

```python
# Estimate GARCH(1, 1) by calling R's rugarch library
import rpy2.robjects as ro
from rpy2.robjects.packages import importr
from finds.utils import PyR
def rugarch(X: Series, savefig: str = '', verbose=VERBOSE) -> Series:
    """GARCH(1,1) wrapper over rugarch"""
    rugarch_ro = importr('rugarch')  # to use library rugarch
    c_ = ro.r['c']
    list_ = ro.r['list']
    spec = ro.r['ugarchspec'](mean_model=list_(armaOrder=c_(0,0), include_mean=False))
    model = ro.r['ugarchfit'](spec, data=PyR(X.values).ro)
    if verbose:
        ro.r['show'](model)
    if savefig:
        for which in [4, 5, 10, 11]:
            ro.r['plot'](model, which=which)
            PyR.savefig(f"{savefig}{which}.png", display=None)
    return Series(PyR(ro.r['sigma'](model)).values.flatten(),
                  index=X.index, name=X.name)
```

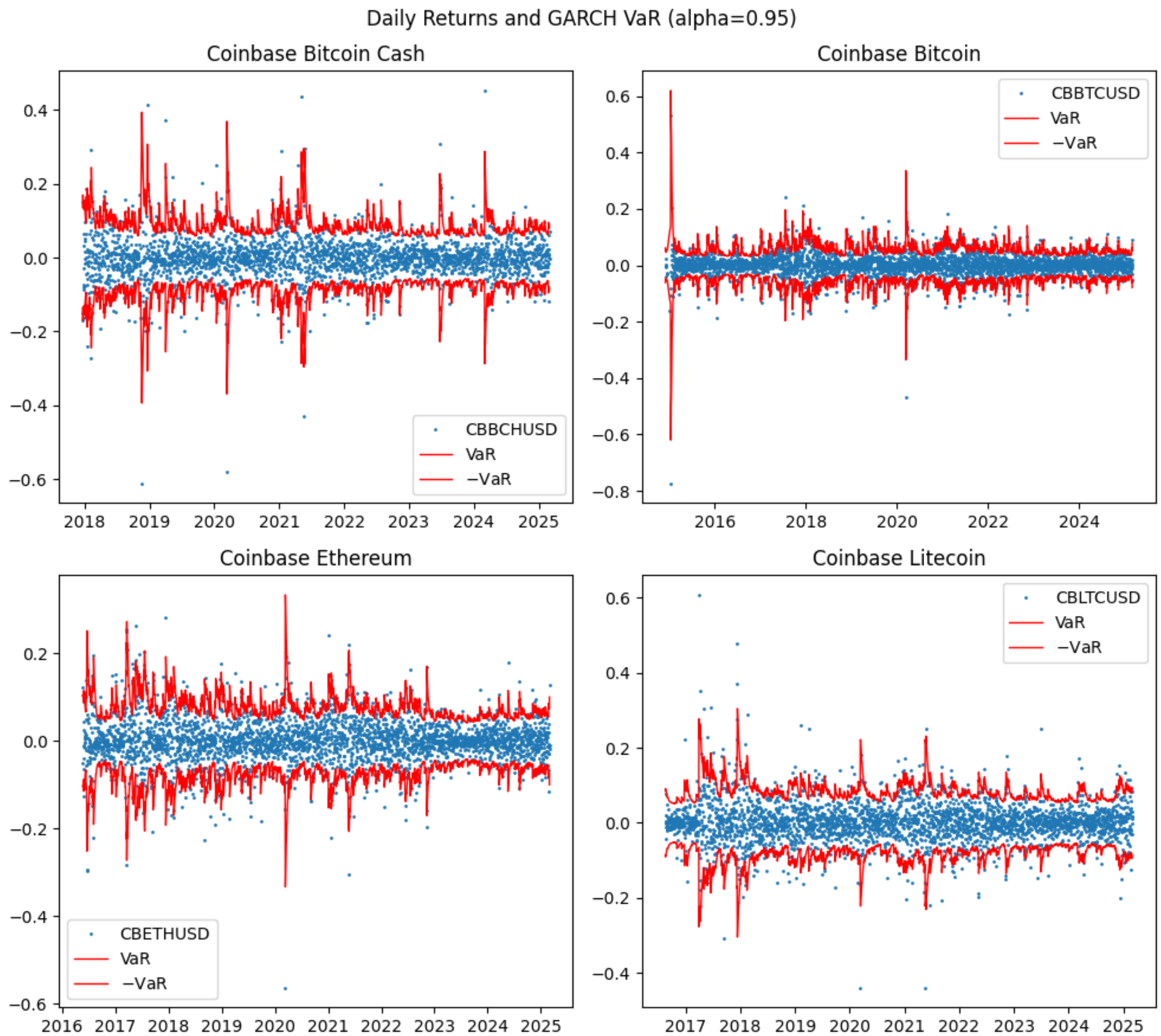
```python
# Estimate GARCH(1,1) full period model for all cryptos
garch = {label: rugarch(cryptos[label].dropna()) for label in cryptos}
```

```python
# Plot daily returns and GARCH predicted VaR
alpha = 0.95   # VaR parameter
```

(continues on next page)

(continued from previous page)

```
fig, axes = plt.subplots(2, 2, figsize=(10, 9))
for label, ax in zip(cryptos, axes.flatten()):
    Z = pd.concat([cryptos[label],
                   parametric_risk(garch[label], alpha=alpha)['value_at_risk']\
                   .rename('VaR')],
                  join='inner', axis=1).dropna()
    plot_var(Z[label], VaR=Z['VaR'], ax=ax)
    ax.set_title(titles[label])
plt.suptitle(f"Daily Returns and GARCH VaR (alpha={alpha})")
plt.tight_layout()
```




```
# Properties of GARCH normalized returns
garch_hist = dict()
for label in cryptos:
    X = (cryptos[label] / garch[label].shift(-1))  # normalize by predict vol
    garch_hist[label] = Series(historical_risk(X, alpha=0.95)).rename(label)
print("Normalized by GARCH predicted volatility (alpha=0.95)")
DataFrame(garch_hist).round(4)
```

```
Normalized by GARCH predicted volatility (alpha=0.95)
```

```
                     CBBCHUSD   CBBTCUSD   CBETHUSD   CBLTCUSD
N                   2622.0000  3708.0000  3206.0000  3116.0000
value_at_risk          1.3382     1.2965     1.4123     1.3956
expected_shortfall     0.1062     0.0504     0.0789     0.0843
volatility             0.7901     0.7740     0.8411     0.8454
skewness               0.0185    -0.0364    -0.0033     0.0350
excess_kurtosis       -2.5903    -2.9028    -2.7553    -1.9190
jb_statistic          18.4912     2.2795     8.0058   152.3519
jb_pvalue              0.0001     0.3199     0.0183     0.0000
```

16.3 Backtesting VaR

The simplest method for validating a VaR model is failure rate analysis, which counts the proportion of times actual losses exceed the VaR estimate. If the model is accurate, these exceedances should follow a binomial distribution with probability $p = 1 - \alpha$, where α is the VaR confidence level. The VaR model can be rejected in two regions, both when the number of observed violations is too few or too many.

16.3.1 Kupiec Likelihood Ratio test

Kupiec (1995) proposed a likelihood ratio test for backtesting VaR based on the number of observed violations:
\$$LR = -2[(N - S)\ln(1 - p) + S\ln(p)] + 2[(N - S)\ln(1 - S/N) + S\ln(S/N)]$\$

where S is the number of VaR exceedances in N observations.

```
def kupiec_LR(alpha: float, s: int, n: int):
    """Compute Kupiec likelihood ratio given s violations in n trials

    Returns:
        Dict of likelihood statistic and pvalue
    """
    p = 1 - alpha        # prob of violation
    num = np.log(1 - p)*(n - s) + np.log(p)*s
    den = np.log(1 - (s/n))*(n - s) + np.log(s/n)*s
    lr = -2 * (num - den)
    return {'lr': lr, 'violations': s, 'N': n,
            # '5%_critical': stats.chi2.ppf(0.95, df=1),
            'pvalue': 1 - stats.chi2.cdf(lr, df=1)}
```

```
def kupiec(X: Series, VaR: Series, alpha: float) -> Dict:
    """Kupiec Likelihood Ratio test of VaR

    Returns:
        Dict of likelihood statistic and pvalue
    """
    Z = pd.concat([X, VaR], axis=1).dropna()
    n = len(Z)
    s = np.sum(Z.iloc[:, 0] < -Z.iloc[:, 1])  # number of violations < -VaR
    return kupiec_LR(alpha=alpha, s=s, n=n)
```

```
# Kupiec likelihood ratio test for EWMA
row_formatted(DataFrame(
    {label: kupiec(cryptos[label],
                   VaR=parametric_risk(ewma[label], alpha=alpha)['value_at_risk'],
                   alpha=alpha) for label in cryptos})\
               .rename_axis(index=f"EWMA({lambda_})", columns="Kupiec LR Test:").
↪round(4),
               {'N': '{:.0f}', 'violations': '{:.0f}'})
```

```
Kupiec LR Test: CBBCHUSD  CBBTCUSD  CBETHUSD  CBLTCUSD
EWMA(0.94)
lr                1.9136    3.8477    1.3918    3.0693
violations           116       160       146       135
N                   2623      3709      3207      3117
pvalue            0.1666    0.0498    0.2381    0.0798
```

```
# Kupiec likelihood ratio test for GARCH(1,1)
row_formatted(DataFrame(
    {label: kupiec(cryptos[label],
                   VaR=parametric_risk(garch[label], alpha=alpha)['value_at_risk'],
                   alpha=alpha) for label in cryptos})\
               .rename_axis(index=f"GARCH(1,1)", columns="Kupiec LR Test:").round(4),
               {'N': '{:.0f}', 'violations': '{:.0f}'})
```

```
Kupiec LR Test: CBBCHUSD  CBBTCUSD  CBETHUSD  CBLTCUSD
GARCH(1,1)
lr                6.8446    5.5607    3.4345   12.3497
violations           103       155       138       115
N                   2623      3709      3207      3117
pvalue            0.0089    0.0184    0.0638    0.0004
```

16.3.2 Conditional coverage tests

Value-at-Risk (VaR) violations should occur randomly over time; if violations cluster, it may suggest that the model is misspecified. Conditional coverage tests, which extend Kupiec's test, evaluate not only the frequency but also the independence of exceptions over time.

```
# TODO: conditional likelihood test
```

References:

Jorion, Phillippe. Value at Risk.

P. Artzner, F. Delbaen, J.-M. Eber, and D. Heath, "Coherent Measures of Risk", Mathematical Finance 9 (1999): 203–228.

Kupiec, P. (1995). Techniques for Verifying the Accuracy of Risk Measurement Models. Journal of Derivatives, 3, 73-84.

FRM Part I Exam Book Ch. 1-3

FRM Part II Exam Book Market Risk Measurement and Management Ch. 1-2

CHAPTER

SEVENTEEN

COVARIANCE MATRIX

Alone, we can do so little; together, we can do so much - Helen Keller

The covariance matrix quantifies the relationships between asset returns, enabling investors to assess diversification benefits and manage portfolio risk. We begin by exploring risk budgeting, which breaks down a portfolio's overall risk contribution by asset, and specifically risk parity portfolios, which aim to distribute risk evenly across assets. We then cover techniques for covariance matrix estimation, including principal component analysis (PCA), Exponentially Weighted Moving Average (EWMA), and shrinkage methods.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
from sklearn.decomposition import PCA
from pandas import DataFrame
import cvxpy as cp
from tqdm import tqdm
import matplotlib.pyplot as plt
import matplotlib.colors as mcolors
from sklearn.covariance import LedoitWolf, OAS, EmpiricalCovariance
from sklearn import cluster
from finds.utils import ColorMap
from finds.readers import FFReader
import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)

# %matplotlib qt
VERBOSE = 0
```

17.1 Portfolio risk

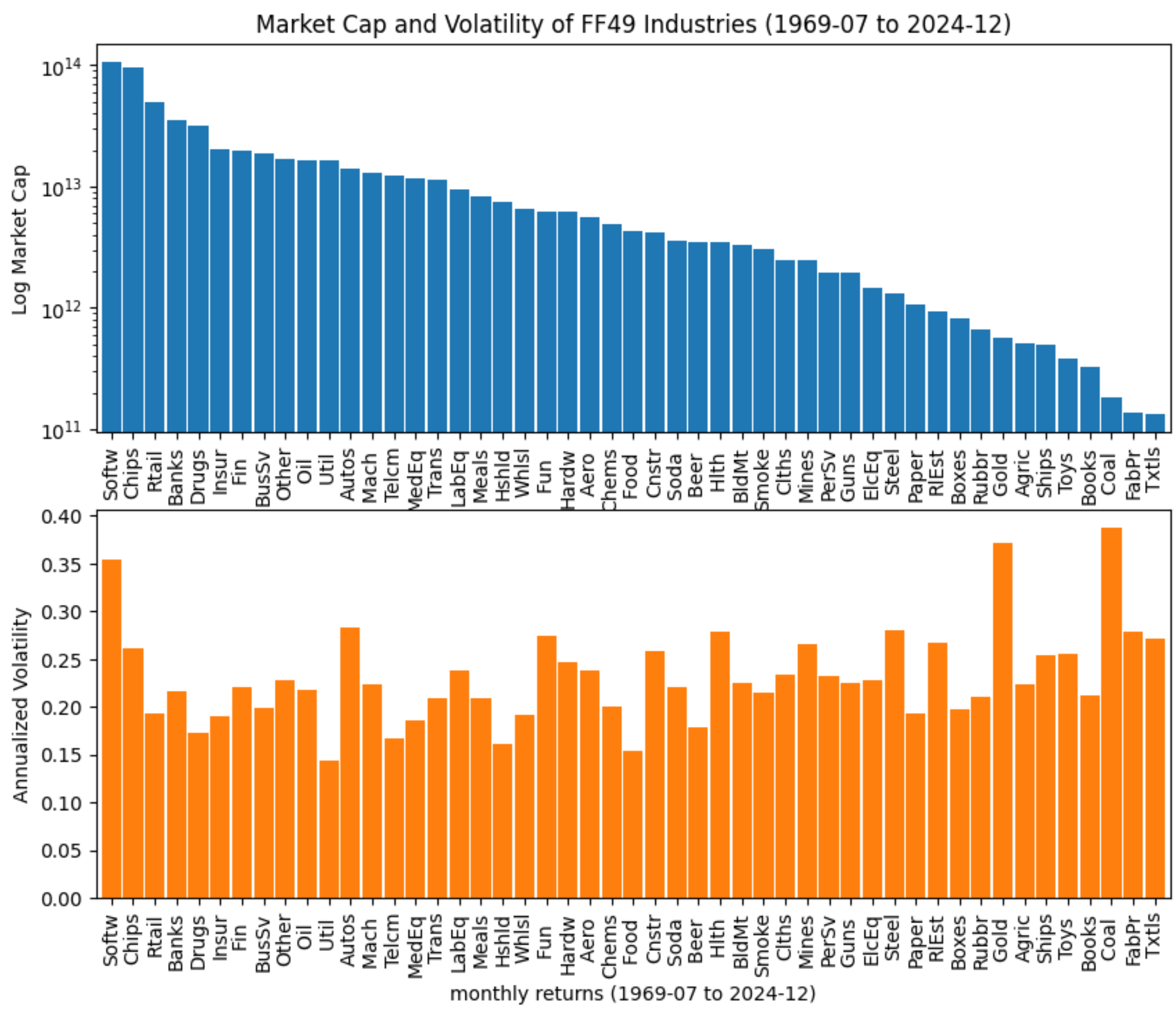
The covariance matrix is essential for portfolio risk analysis, helping investors evaluate diversification benefits and effectively manage overall risk. In this analysis, we use monthly returns from the 49 industry indexes compiled by Ken French.

```
# Retrieve industry returns from Ken French Data Library website
symbol = '49_Industry_Portfolios'
ff = FFReader(symbol)
keep = ff[0].index[(ff[0] > -99).all(axis=1)]
rets = (ff[0] / 100).reindex(keep)
caps = (ff[4] * ff[5] * 10e6).reindex(keep)    # number of firms x avg market cap
weights = caps.iloc[-1] / caps.iloc[-1].sum()
```

```python
X = (rets - rets.mean(axis=0))      # demean by industry
sigma = 12 * X.T @ X / len(X)       # annualized covariance matrix
Y = caps.iloc[-1]\
        .rename('cap')\
        .to_frame()\
        .join(DataFrame({'vol': np.sqrt(np.diag(sigma))}, index=X.columns))\
        .sort_values('cap', ascending=False)
```

```python
# Plot annualized volatility and market caps of industries
fig, ax = plt.subplots(nrows=2, figsize=(10, 8))
Y['cap'].plot.bar(ax=ax[0], color="C0", width=0.9)
ax[0].set_yscale('log')
ax[0].set_title(f"Market Cap and Volatility of FF49 Industries ({X.index[0]} to {X.
↪index[-1]})")
ax[0].set_ylabel(f"Log Market Cap")
Y['vol'].plot.bar(ax=ax[1], color="C1", width=0.9)
ax[1].set_ylabel(f"Annualized Volatility")
ax[1].set_xlabel(f"monthly returns ({X.index[0]} to {X.index[-1]})")
```

```
Text(0.5, 0, 'monthly returns (1969-07 to 2024-12)')
```



17.1.1 Risk budgetting

Risk budgeting breaks down a portfolio's overall risk contribution by asset.

- **Portfolio Risk**: The volatility of a market portfolio is given by:
  $\sigma_P = W^T\Sigma W$ where W represents the vector of asset weights in the portfolio, and Σ is the covariance matrix of asset returns.
- **Covariance**: The covariance of an individual security i with the portfolio P is expressed as:
  $\sigma_{iP} = \rho_{iP} \cdot \sigma_i \cdot \sigma_P$ where ρ_{iP} is the correlation between the security and the portfolio.
- **Beta** (β): The systematic risk of a security relative to the portfolio, defined as:
  $\beta_i = \frac{\sigma_{iP}}{\sigma_P^2}$
  This represents the regression slope of the security's returns against the portfolio's returns.
- **Marginal Contribution to Risk (MCR)**: Measures the sensitivity of portfolio volatility to small changes in an asset's weight:
  $\frac{\partial \sigma_P}{\partial w_i} = \frac{\sigma_{iP}}{\sigma_P}$
- **Percent Contribution to Risk (PCR)**: Indicates the fraction of total portfolio risk attributable to a security:
  $\beta_i \cdot w_i$
  The sum of all asset contributions equals 1.
- **Contribution to Portfolio Risk**: The total portfolio risk can also be expressed as:
  $\beta_i \cdot w_i \cdot \sigma_P = w_i \cdot \sigma_i \cdot \rho_{iP}$ This highlights how an asset's weight, volatility, and correlation with the portfolio drive its risk contribution.

```python
# Helper to compute portfolio risk budget
def risk_budget(w, sigma, labels):
    """Compute portfolio risk analytics"""
    sigma_ = np.array(sigma) * 100 * 100   # express as percent returns
    w_ =  np.array(w)

    # Portfolio volatility (percent)
    vol = np.sqrt(w_.T @ sigma_ @ w_)

    # Covariance of each security wrt market portfolio
    cov = sigma_ @ w_

    # Beta of each security wrt market portfolio
    beta = cov / (w_.T @ sigma_ @ w_)

    # Marginal Contribution to Risk of each security
    marginal = beta * vol

    # Percent Contribution to Risk
    percent = beta * w_ * 100

    # Contribution to Risk
    contrib = marginal * w_

    return DataFrame({'weight': list(w),
                      'Beta': list(beta),
                      'MCR(%)': list(marginal),
                      'PCR(%)': list(percent),
                      'CR': list(contrib)},
                     index=labels)
```

```
vol = np.sqrt(weights.T @ sigma @ weights)  # market portfolio risk
print(f"Market Portfolio Risk Budget (vol={vol*100:.2f}%)")
risk_budget(weights, sigma, weights.index).sort_values('CR', ascending=False)
```

```
Market Portfolio Risk Budget (vol=19.34%)
```

	weight	Beta	MCR(%)	PCR(%)	CR
Softw	0.182276	1.555931	30.097357	28.360924	5.486032
Chips	0.161610	1.184836	22.919038	19.148127	3.703943
Rtail	0.085470	0.828755	16.031140	7.083394	1.370186
Banks	0.059413	0.843213	16.310804	5.009761	0.969070
Fin	0.033877	0.971510	18.792521	3.291171	0.636632
Drugs	0.053875	0.581038	11.239382	3.130335	0.605520
BusSv	0.031733	0.953210	18.438533	3.024847	0.585115
Other	0.028863	0.906179	17.528786	2.615469	0.505927
Insur	0.034341	0.726098	14.045377	2.493480	0.482330
Autos	0.023701	0.997867	19.302368	2.365052	0.457487
Mach	0.022131	0.974029	18.841261	2.155620	0.416975
Trans	0.019526	0.884115	17.101985	1.726316	0.333932
LabEq	0.015992	1.048768	20.286986	1.677156	0.324423
Oil	0.028253	0.592003	11.451492	1.672590	0.323540
MedEq	0.019788	0.718659	13.901468	1.422059	0.275078
Telcm	0.020845	0.597110	11.550276	1.244649	0.240760
Meals	0.014186	0.862779	16.689278	1.223961	0.236759
Fun	0.010471	1.140790	22.067019	1.194478	0.231055
Util	0.027976	0.375849	7.270286	1.051464	0.203391
Hardw	0.010442	0.988086	19.113163	1.031731	0.199574
Whlsl	0.011183	0.869373	16.816831	0.972226	0.188064
Aero	0.009449	0.927079	17.933076	0.875994	0.169449
Hshld	0.012793	0.598646	11.579978	0.765832	0.148140
Cnstr	0.007072	1.050526	20.320993	0.742934	0.143710
Chems	0.008270	0.808718	15.643534	0.668792	0.129369
Hlth	0.005851	0.968313	18.730690	0.566569	0.109595
BldMt	0.005606	0.956894	18.509810	0.536398	0.103759
Clths	0.004255	0.938755	18.158938	0.399468	0.077272
Food	0.007388	0.497495	9.623355	0.367564	0.071100
Soda	0.006012	0.610836	11.815793	0.367218	0.071033
Mines	0.004196	0.870379	16.836284	0.365172	0.070638
Beer	0.005945	0.567434	10.976238	0.337364	0.065259
PerSv	0.003352	0.919656	17.789492	0.308300	0.059636
Smoke	0.005225	0.477864	9.243631	0.249665	0.048294
ElcEq	0.002505	0.982095	18.997290	0.245969	0.047579
Steel	0.002262	1.052579	20.360697	0.238090	0.046055
Guns	0.003317	0.646421	12.504129	0.214389	0.041471
RlEst	0.001590	1.038549	20.089312	0.165107	0.031938
Paper	0.001822	0.736880	14.253940	0.134245	0.025968
Boxes	0.001378	0.731479	14.149461	0.100793	0.019497
Rubbr	0.001124	0.872047	16.868545	0.098007	0.018958
Ships	0.000854	0.858852	16.613317	0.073364	0.014191
Toys	0.000644	0.991057	19.170634	0.063808	0.012343
Agric	0.000859	0.704508	13.627738	0.060501	0.011703
Books	0.000555	0.874586	16.917676	0.048503	0.009382
Gold	0.000960	0.429793	8.313752	0.041262	0.007982
Coal	0.000311	0.879600	17.014650	0.027317	0.005284
FabPr	0.000234	0.912134	17.643973	0.021316	0.004123
Txtls	0.000225	0.946333	18.305513	0.021250	0.004111

17.1.2 Risk parity portfolios

A **risk parity portfolio (RPP)** aims to equalize the risk contribution of each asset class before leveraging to reach a target volatility. The allocation can be framed as a convex optimization problem, as proposed by Spinu (2013): \$min$_w \frac{1}{2} w^T \Sigma W - \sum_i b_i \log w_i subjecttonon-negativeassetweights$ w_i \geq 0 , $where$ b_i = 1/N \$ represents the desired risk budget for an equally weighted risk-parity portfolio.

We use the `cvxpy` convex optimization package. The risk parity portfolio has the greatest weight in the Utilities industry and lowest weight in Software. By contrast, Software has the most weight in the market.

```
# Set up variables and constraints
N = len(sigma)
b = np.ones(N)/N              # risk parity
W = cp.Variable(N)            # portfolio weights to solve for
constraints = [W >= 0]        # non-negative weights constraint
```

```
# Solve objective
obj = 0.5 * cp.quad_form(W, sigma) - cp.sum(cp.multiply(b, cp.log(W)))
prob = cp.Problem(cp.Minimize(obj), constraints)
prob.solve()
```

```
2.6135631335344156
```

```
# normalize solution weights
rpp = (W/cp.sum(W)).value
```

```
vol = np.sqrt(rpp.T @ sigma @ rpp)
print(f"Risk Parity Portfolio (vol={vol*100:.2f}%)")
risk_budget(rpp, sigma, weights.index).sort_values('weight', ascending=False)
```

```
Risk Parity Portfolio (vol=16.47%)
```

	weight	Beta	MCR(%)	PCR(%)	CR
Util	0.038404	0.531408	8.750135	2.040813	0.336039
Food	0.030511	0.668941	11.014738	2.040994	0.336069
Smoke	0.029554	0.690540	11.370397	2.040830	0.336042
Drugs	0.028845	0.707521	11.650005	2.040838	0.336043
Telcm	0.028641	0.712525	11.732403	2.040757	0.336030
Gold	0.028104	0.726190	11.957409	2.040872	0.336049
Beer	0.027832	0.733279	12.074135	2.040854	0.336046
Hshld	0.026951	0.757247	12.468779	2.040819	0.336040
Soda	0.024589	0.829944	13.665817	2.040752	0.336029
Oil	0.024289	0.840218	13.834978	2.040816	0.336040
Guns	0.023380	0.872899	14.373107	2.040875	0.336049
MedEq	0.023346	0.874162	14.393900	2.040858	0.336046
Agric	0.022982	0.888036	14.622356	2.040846	0.336045
Insur	0.021825	0.934992	15.395521	2.040612	0.336006
Rtail	0.021405	0.953405	15.698710	2.040790	0.336035
Boxes	0.021403	0.953503	15.700322	2.040788	0.336035
Paper	0.021003	0.971719	16.000274	2.040923	0.336057
Hardw	0.019636	1.039324	17.113457	2.040800	0.336037
Meals	0.019328	1.055907	17.386507	2.040808	0.336038

(continues on next page)

(continued from previous page)

```
Banks  0.019318  1.056439  17.395264  2.040815  0.336039
Whlsl  0.019181  1.063981  17.519442  2.040796  0.336036
Chems  0.019173  1.064438  17.526974  2.040834  0.336042
Books  0.018802  1.085413  17.872354  2.040809  0.336038
Rubbr  0.018759  1.087921  17.913644  2.040859  0.336047
Other  0.018749  1.088522  17.923535  2.040857  0.336046
Trans  0.018642  1.094754  18.026153  2.040836  0.336043
BusSv  0.018231  1.119418  18.432270  2.040768  0.336032
PerSv  0.018229  1.119511  18.433801  2.040785  0.336035
Fin    0.017894  1.140526  18.779843  2.040823  0.336041
Clths  0.017822  1.145101  18.855166  2.040831  0.336042
Ships  0.017659  1.155699  19.029669  2.040855  0.336046
Aero   0.017440  1.170157  19.267736  2.040774  0.336033
Mines  0.017359  1.175643  19.358068  2.040800  0.336037
FabPr  0.017234  1.184134  19.497876  2.040778  0.336033
ElcEq  0.017124  1.191811  19.624284  2.040799  0.336037
LabEq  0.017067  1.195775  19.689563  2.040793  0.336036
Mach   0.016969  1.202645  19.802683  2.040811  0.336039
Autos  0.016837  1.212055  19.957625  2.040782  0.336034
Toys   0.016776  1.216480  20.030489  2.040790  0.336035
Chips  0.016721  1.220482  20.096388  2.040783  0.336034
BldMt  0.016630  1.227147  20.206127  2.040795  0.336036
Hlth   0.016604  1.229089  20.238114  2.040814  0.336039
Coal   0.016511  1.236048  20.352700  2.040804  0.336038
Txtls  0.016357  1.247656  20.543830  2.040811  0.336039
RlEst  0.015599  1.308309  21.542534  2.040793  0.336036
Cnstr  0.015534  1.313753  21.632185  2.040810  0.336039
Fun    0.015339  1.330599  21.909558  2.040955  0.336062
Steel  0.015297  1.334155  21.968126  2.040802  0.336037
Softw  0.014115  1.445795  23.806377  2.040797  0.336037
```

17.2 Covariance matrix estimation

The simplest way to estimate the covariance matrix is to calculate the average product of return deviations for each pair of assets. Additionally, we examine other techniques designed to improve estimation accuracy, particularly in high-dimensional settings.

17.2.1 Principal components

By retaining only the largest principal components, the **Principal Component Analysis (PCA)** method provides a lower-dimensional but more stable approximation of the covariance matrix that captures most of the data's variability.

Each principal component (PC) can be interpreted as a weighted portfolio of industry assets. The projection of industry returns onto a given component effectively computes the returns of the corresponding portfolio.

```
# Fit PCA
pca = PCA().fit(X)

# Retrieve components, and sign flip if necessary
loadings = (np.diag(pca.singular_values_) @ pca.components_).T  # compute loadings by
↪column
components = DataFrame.from_records(pca.components_.T, index=X.columns)
```

(continues on next page)

(continued from previous page)

```
components *= np.sign(components.sum(axis=0))

# Compute projections, can be interpreted as portfolio returns
proj = pca.transform(X)
proj *= np.sign(np.sum(pca.components_, axis=1))  # flip signs if necessary

# Equal weighted market average return
avg = X.mean(axis=1)
```

From the summaries of the weights, the first principal component (PC1) resembles a broadly diversified portfolio, capturing more than half of the total variance. Higher-order principal components represent long-short spread portfolios, which explain incremental variance.

```
K = 20
print(f"Top {K} principal components")
DataFrame({
    'frac weights +ve': np.mean(components.iloc[:, :K].values >= 0, axis=0),
    'sum weights': np.sum(components.iloc[:, :K].values, axis=0),
    'sum sqr weights': np.sum((components.iloc[:, :K].values)**2, axis=0),
    'sum abs weights': np.sum(np.abs(components.iloc[:, :K].values), axis=0),
    'corr avg ret':  [np.corrcoef(avg, proj[:, i])[0, 1] for i in range(K)],
    'cumulative expl ratio': np.cumsum(pca.explained_variance_ratio_[:K])},
          index=[f"PC{i+1}" for i in range(K)]).round(4)
```

```
Top 20 principal components
```

```
      frac weights +ve  sum weights  sum sqr weights  sum abs weights  \
PC1             1.0000       6.8350              1.0           6.8350
PC2             0.2449       0.0942              1.0           4.1476
PC3             0.5306       0.3638              1.0           3.5694
PC4             0.6735       0.9018              1.0           5.1037
PC5             0.4694       0.5504              1.0           5.2080
PC6             0.5510       0.7125              1.0           5.5172
PC7             0.5714       0.0807              1.0           5.5623
PC8             0.5510       0.0678              1.0           4.9024
PC9             0.5102       0.1727              1.0           5.1930
PC10            0.4898       0.2090              1.0           4.8891
PC11            0.4490       0.2954              1.0           5.0131
PC12            0.5306       0.0517              1.0           5.2657
PC13            0.6531       0.1003              1.0           4.8573
PC14            0.5102       0.1026              1.0           4.8301
PC15            0.4898       0.1416              1.0           4.8642
PC16            0.5102       0.0216              1.0           5.1350
PC17            0.4898       0.0855              1.0           5.3457
PC18            0.5714       0.1669              1.0           5.1840
PC19            0.5102       0.1752              1.0           5.1187
PC20            0.4694       0.1086              1.0           5.1526

      corr avg ret  cumulative expl ratio
PC1         0.9988                 0.5590
PC2         0.0046                 0.6218
PC3         0.0142                 0.6620
PC4         0.0341                 0.6994
PC5         0.0185                 0.7290
```

(continues on next page)

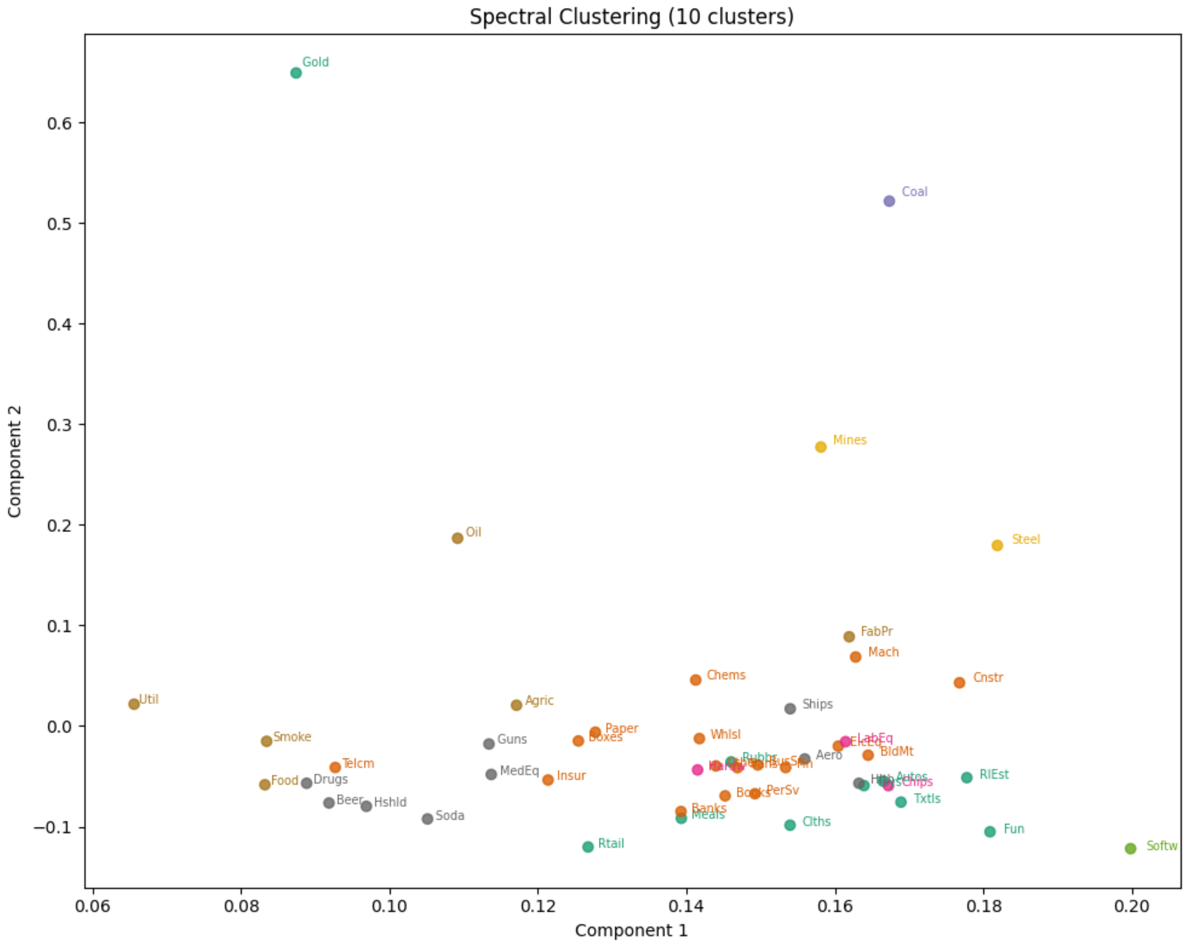
(continued from previous page)

```
PC6         0.0207                  0.7510
PC7         0.0021                  0.7689
PC8         0.0016                  0.7831
PC9         0.0040                  0.7971
PC10        0.0045                  0.8091
PC11        0.0063                  0.8210
PC12        0.0011                  0.8321
PC13        0.0020                  0.8423
PC14        0.0020                  0.8521
PC15        0.0027                  0.8615
PC16        0.0004                  0.8703
PC17        0.0015                  0.8788
PC18        0.0028                  0.8864
PC19        0.0029                  0.8936
PC20        0.0018                  0.9007
```

Spectral Clustering

Spectral clustering is a technique that groups data points based on the eigenvalues and eigenvectors of a similarity matrix. From the results, Component 1 primarily appears to capture market-wide beta risk sensitivity. Component 2 appears to reflect exposure to commodity-related factors (e.g. gold, coal and mining)

```
# Spectral clustering
spectral = cluster.SpectralClustering(
    n_clusters=10,
    eigen_solver="arpack",
    #affinity="nearest_neighbors",
    random_state=42,
)
spectral.fit(X.T)   # number of features equals the number observations dates
cmap = ColorMap(spectral.n_clusters, colormap='Dark2')
fig, ax = plt.subplots(figsize=(10, 8))
ax.scatter(components.iloc[:, 0], components.iloc[:, 1],
           c=cmap[spectral.labels_], alpha=.8)
for t, c, xy in zip(components.index, spectral.labels_,
                    components.iloc[:, :2].values):
    ax.annotate(text=t, xy=xy, xytext=xy * 1.01, color=cmap[c], fontsize='x-small')
ax.set_xlabel('Component 1')
ax.set_ylabel('Component 2')
ax.set_title(f"Spectral Clustering ({spectral.n_clusters} clusters)")
plt.tight_layout()
plt.show()
```




```
DataFrame({'cluster': spectral.labels_}, index=components.index).sort_values('cluster
↪')
```

```
        cluster
Fun           0
Toys          0
Txtls         0
Clths         0
Rubbr         0
Autos         0
RlEst         0
Rtail         0
Meals         0
Gold          1
BldMt         2
ElcEq         2
Books         2
Telcm         2
Cnstr         2
Chems         2
Boxes         2
Paper         2
BusSv         2
```

(continues on next page)

(continued from previous page)

```
Other        2
Fin          2
Insur        2
Banks        2
Trans        2
PerSv        2
Mach         2
Whlsl        2
Coal         3
Chips        4
LabEq        4
Hardw        4
Softw        5
Steel        6
Mines        6
Smoke        7
Food         7
FabPr        7
Oil          7
Agric        7
Util         7
Aero         8
Ships        8
Guns         8
Hlth         9
Hshld        9
Drugs        9
Soda         9
Beer         9
MedEq        9
```

17.2.2 EWMA

The **Exponentially-Weighted Moving Average (EWMA) method** assigns greater importance to recent data when estimating the covariance matrix, which is particularly useful for tracking market risk changes. JP Morgan RiskMetrics (1996) proposed a decay parameter $\lambda = 0.97$, implying a monthly decay rate of 0.03. The **half-life** of the weighting function represents the time required for a weight to decrease by 50%.

```
# Compute half-life of risk metrics' lambda for monthly data
def halflife(decay, half=0.5):
    """Returns halflife (t) from its definition: 0.5 = (1-decay)^t"""
    return -np.log(1/half)/np.log(1 - decay)
```

```
risk_metrics_lambda = 0.97   # for monthly data
print('Half-life: ', halflife(decay=1-risk_metrics_lambda).round(1), 'months')
```

```
Half-life:  22.8 months
```

```
def ewma(X, decay):
    """Helper to compute EWMA covariance matrix estimate"""
    weights = (1 - decay)**np.arange(len(X))[::-1]
    return (weights.reshape((1, -1)) * X.T) @ X / weights.sum()
```

17.2.3 Shrinkage methods

Covariance matrix estimates can be regularized using shrinkage. Ledoit and Wolf (1993) proposed shrinking the sample covariance matrix towards an identity matrix, and derived a closed-form formula to compute the asymptotically optimal shrinkage parameter β by minimizing a MSE criterion:

$$(1-\beta)\Sigma + \beta\frac{tr(\Sigma)}{N}I_n$$

Chen et al. (2010) proposed the **Oracle Approximating Shrinkage (OAS) Estimator** whose convergence is significantly better under the assumption that the data are Gaussian.

17.2.4 Volatility of the GMV portfolio

The out-of-sample (OOS) realized volatility of the **Global Minimum Variance Portfolio (GMV)** serves as a benchmark for evaluating covariance estimation accuracy. The GMV portfolio is designed to minimize total portfolio volatility. Since it relies on accurate covariance estimates, errors can lead to suboptimal diversification and higher realized volatility.

Beginning in January 2000, we update the covariance matrix estimate monthly using a rolling 30-year data window, and track the month-ahead return of the GMV portfolio. By the end of the test period, we expect the most accurate covariance matrix estimator to achieve the lowest return volatility.

```
# Helper method to compute Minimum Variance Portfolio and realized volatility
def gmv(cov, ret):
    """Compute minimum variance portfolio and return"""
    w = np.linalg.inv(cov) @ np.ones((cov.shape[1], 1))
    return float((np.array(ret) @ w/sum(w)).flatten()[0])
```

```
# Rolling monthly evaluation
decay = 0.03      # risk metrics monthly decay rate for EWMA
n_components = 10   # for PCA
split = '2000-01' # start rolling OOS prediction tests from this date
```

```
r = {}     # collect realized returns
for date in tqdm(rets.index[rets.index >= split]):
    X_train = rets.iloc[rets.index < date, :][-30*12:]   # 30 years of training data
    X_test = rets.iloc[rets.index == date, :]      # predict one month ahead returns
    r[date] = {}

    # Empirical covariance
    cov = EmpiricalCovariance().fit(X_train).covariance_
    r[date]['Covariance'] = gmv(cov, X_test)
    r[date]['Diagonal'] = gmv(np.diag(np.diag(cov)), X_test)
    r[date]['EWMA'] = gmv(ewma(X_train, decay=decay), X_test)
    r[date][f"PCA-{n_components}"] = gmv(PCA(n_components).fit(X_train).get_
↪covariance(), X_test)
    r[date]['Identity'] = gmv(np.identity(len(cov)), X_test)
    r[date]['LW'] = gmv(LedoitWolf().fit(X_train).covariance_, X_test)
    r[date]['OAS'] = gmv(OAS().fit(X_train).covariance_, X_test)
```

```
100%|██████████| 300/300 [01:01<00:00,  4.86it/s]
```

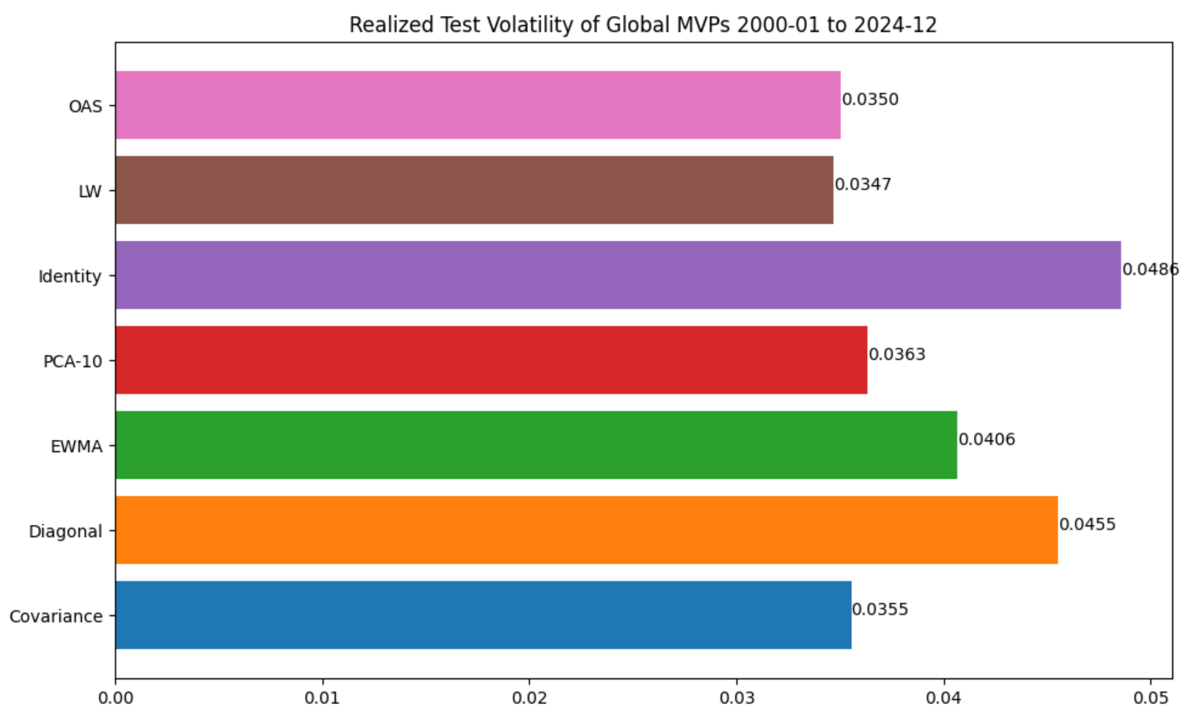
```
ts = DataFrame.from_dict(r, orient='index')
vol = ts.std().rename('realized volatility').to_frame()
print('Realized volatility of minimum variance portfolio')
vol.T
```

```
Realized volatility of minimum variance portfolio
```

```
                     Covariance  Diagonal      EWMA    PCA-10  Identity  \
realized volatility    0.035531  0.045541  0.040645  0.036339  0.048569

                           LW       OAS
realized volatility  0.034681  0.035041
```

Plot evaluation period realized volatility of minimum variance portfolios

```
fig, ax = plt.subplots(figsize=(10, 6))
values = vol.values.flatten()
ax.barh(vol.index, width=values, color=list(mcolors.TABLEAU_COLORS.values()))
for row, val in enumerate(values):
    ax.annotate(f"{val:.4f}", xy=(val, row))
ax.set_title(f"Realized Test Volatility of Global MVPs {split} to {rets.index[-1]}")
plt.tight_layout()
```



CHAPTER

EIGHTEEN

MARKET MICROSTRUCTURE

Beware of little expenses. A small leak will sink a great ship - Benjamin Franklin

Market microstructure focuses on the mechanics of how securities are traded, analyzing factors such as price formation, liquidity, and trading costs. The NYSE Trade and Quote (TAQ) dataset is a widely used source of tick data, containing detailed records of executed trades and best bid and offer quotes. We analyze how key liquidity measures vary across market capitalization and during the trading day. We also examine intraday volatility patterns through the variance ratio and high-frequency estimators.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
import time
import os
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
from tqdm import tqdm
import multiprocessing
from finds.database import SQL, RedisDB
from finds.structured import CRSP, BusDay
from finds.readers import opentaq, itertaq, bin_trades, bin_quotes, TAQ, \
    clean_trade, clean_nbbo, align_trades, plot_taq
from finds.utils import plot_time, Store, row_formatted
from finds.recipes import weighted_average, hl_vol, ohlc_vol
from secret import credentials, paths
import warnings
VERBOSE = 0
if not VERBOSE:  # Suppress FutureWarning messages
    warnings.simplefilter(action='ignore', category=FutureWarning)

%matplotlib inline
```

```
sql = SQL(**credentials['sql'], verbose=VERBOSE)
user = SQL(**credentials['user'], verbose=VERBOSE)
bday = BusDay(sql)
rdb = RedisDB(**credentials['redis'])
crsp = CRSP(sql, bday, rdb=rdb, verbose=VERBOSE)
```

```
taqdir = paths['taq']
storedir = paths['scratch'] / 'ticks'
open_t = pd.to_datetime('1900-01-01T9:30')     # exclude <=
close_t = pd.to_datetime('1900-01-01T16:00')   # exclude >
```

(continues on next page)

(continued from previous page)

```
EPSILON = 1e-15
dates = [20191007, 20191008, 20180305, 20180306]
```

18.1 Tick data

18.1.1 NYSE TAQ

The **NYSE Trade and Quote (TAQ) dataset** contains tick-by-tick intraday trading data for U.S. equities, providing information on executed trades and best bid/offer quotes from various exchanges. There are three primary types of TAQ daily files:

1. Trades (EQY_US_ALL_TRADE_YYYYMMDD.gz) –containing executed trades data, including price, volume, microsecond timestamps, sale conditions and trade correction indicators
2. National Best Bid and Offer NBBO (EQY_US_ALL_NBBO_YYYYMMDD.gz) - containing best bid/offer price, size, microsecond timestamps, quote conditions and market center identifiers
3. Master (EQY_US_ALL_REF_MASTER_YYYYMMDD.gz) –containing reference information about securities, including symbol, CUSIP, security description, shares outstanding and primary exchange

The `taq` module in the FinDS package provides tools for processing NYSE TAQ data, with functionalities including:

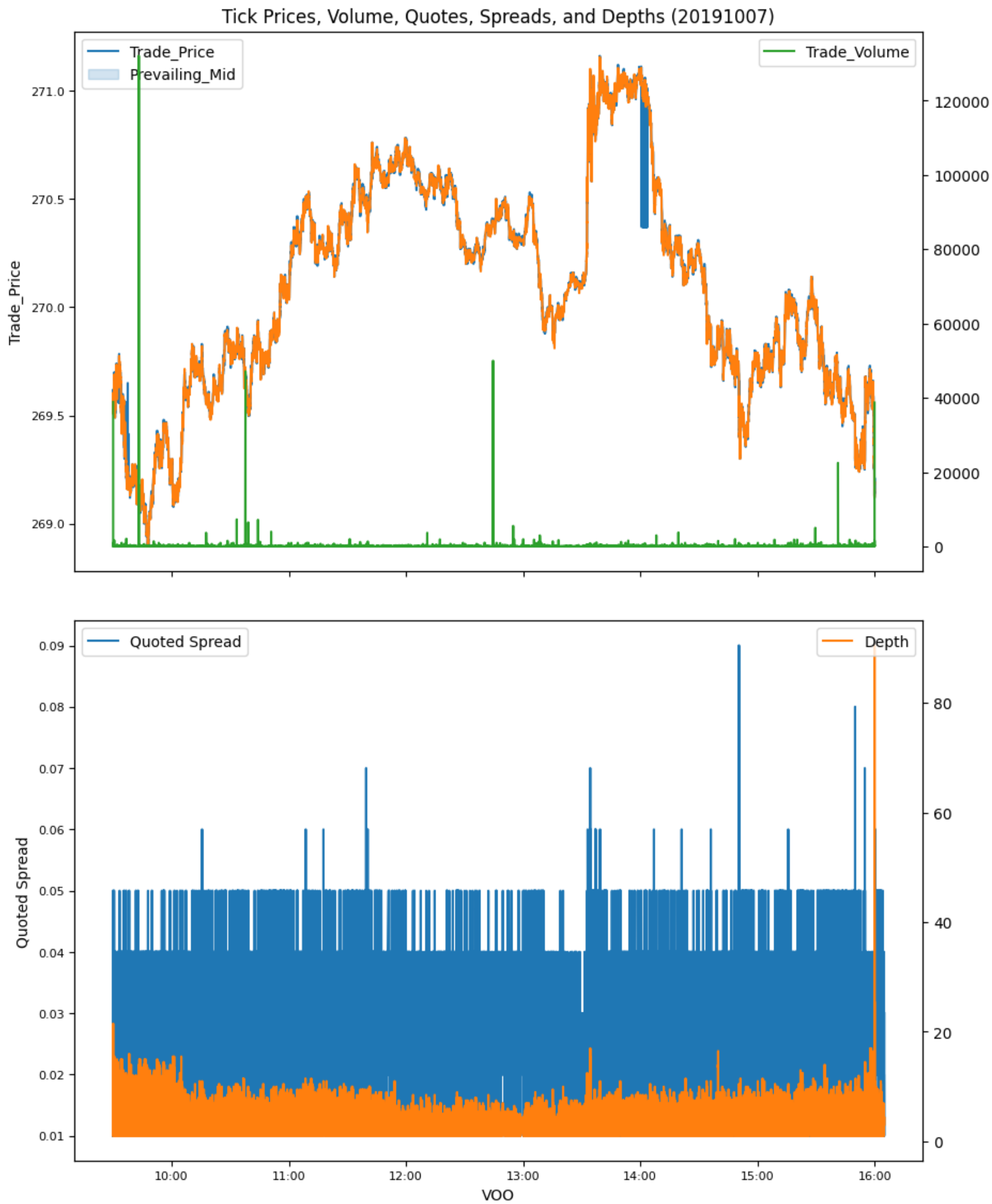
- File reading & indexing: `open_taq`, `itertaq`, `taq_from_csv`
- Data cleaning & filtering: `clean_trades`, `clean_nbbo`
- Tick-level analysis: `TAQ` class
- Resampling & binning: `bin_trades`, `bin_quotes`, `align_trades`
- Visualization: `plot_taq`

Bad trades and quotes records, such as invalid prices, duplicate records, and specific sale conditions, can be filtered out to ensure data integrity.

Retrieve and visualize tick-level trades and quotes for a selected stock (e.g., `VOO`, an ETF tracking the S&P 500).

```
# Plot VOO tick-by-tick
master, trades, quotes = opentaq(dates[0], taqdir)
symbol = "VOO"
t = trades[symbol]
q = quotes[symbol]
ct = clean_trade(t, close_t=close_t + np.timedelta64('5','m'))
cq = clean_nbbo(q)
align_trades(ct, cq, inplace=True)

plot_taq(ct[['Trade_Price', 'Prevailing_Mid']].groupby(level=0).last(),
            ct['Trade_Volume'].groupby(level=0).last(),
            (cq['Best_Offer_Price'] - cq['Best_Bid_Price'])\
            .rename('Quoted Spread').groupby(level=0).last(),
            ((cq['Best_Bid_Size'] + cq['Best_Offer_Size']) / 2)\
            .rename('Depth').groupby(level=0).last(),
            open_t=open_t,
            close_t=close_t + np.timedelta64('5','m'),
            num=1,
            title=f"Tick Prices, Volume, Quotes, Spreads, and Depths ({dates[0]})"
)
```

```
(<Axes: title={'center': 'Tick Prices, Volume, Quotes, Spreads, and Depths␣
↪(20191007)'}, xlabel='VOO', ylabel='Trade_Price'>,
 <Axes: xlabel='VOO', ylabel='Quoted Spread'>)
```



The data is preprocessed to extract a universe of U.S.-domiciled common stocks, and indexed by ticker symbol for efficient access.

```
for d, date in enumerate(dates):
    store = Store(storedir / str(date), verbose=VERBOSE)
    master, trades, quotes = opentaq(date, taqdir)

    # screen on CRSP universe
    univ = crsp.get_universe(date)\
               .join(crsp.get_section(dataset='names',
                                      fields=['ncusip', 'permco', 'exchcd'],
                                      date_field='date',
                                      date=date,
                                      start=0), how='inner')\
               .sort_values(['permco', 'ncusip'])

    # drop duplicate share classes (same permco): keep largest cap
    dups = master['CUSIP'].str\
                          .slice(0, 8)\
                          .isin(univ.loc[univ.duplicated(['permco'],keep=False),
                                         'ncusip'])
    #shareclass.extend(master[dups].to_dict(orient='index').values())
    univ = univ.sort_values(['permco', 'cap'], na_position='first')\
               .drop_duplicates(['permco'], keep='last')\
               .reset_index()\
               .set_index('ncusip', drop=False)

    # Iterate by symbol over Daily Taq trades, nbbo and master files
    for ct, cq, mast in itertaq(trades,
                                quotes,
                                master,
                                cusips=univ['ncusip'],
                                open_t=open_t,
                                close_t=None,
                                verbose=VERBOSE):
        header = {'date':date}
        header.update(univ.loc[mast['CUSIP'][:8],
                               ['permno','decile','exchcd','siccd']])
        header.update(mast[['Symbol', 'Round_Lot']])
        store[header['Symbol']] = dict(header=header, ct=ct, cq=cq, mast=mast)
    quotes.close()
    trades.close()
```

18.2 Liquidty measures

- **Depth**: The average bid and ask size at the best quotes, indicating the available liquidity at the current market price.
- **Quoted Spread**: The difference between the best ask and best bid prices, representing the cost of immediacy for traders.
- **Effective Spread**: A trade-based measure of execution cost, calculated as twice the absolute difference between the trade price and the midquote.
- **Price Impact**: The change in the midquote price after a trade, reflecting how much a trade moves the market.
- **Realized Spread**: The difference between the trade price and the midquote price a few minutes later, measuring the profitability of liquidity providers.
- **Lee-Ready Tick Test**: A method for classifying trades as buyer- or seller-initiated, using trade price movements relative to the prevailing midquote.

- **Volume Weighted Average Price (VWAP)** –A common trade execution benchmark, though it can be influenced by the trade itself –achieving zero slippage to VWAP while accounting for 100% of market volume does not necessarily indicate good execution.

18.2.1 Intraday liquidity

Intraday liquidity is analyzed computing liquidity measures across various time intervals: 1-second, 2-second, 5-second, 15-second, 30-second, 1-minute, 2-minute, and 5-minute bins.

```
intervals = ([(v, 's') for v in [1, 2, 5, 15, 30]] + [(v, 'm') for v in [1, 2, 5]])
max_num = 100000
bin_keys = ['effective', 'realized', 'impact',
            'quoted', 'volume', 'offersize', 'bidsize',
            'ret', 'retq', 'counts']
```

```
# helper call run liquidity calculations by date, parallelizable
def intraday(date):
    """Compute intraday liquidity for a date"""
    store = Store(storedir / str(date), verbose=VERBOSE)
    symbols = sorted(store)
    daily_all = []
    bins_all = {k: [] for k in bin_keys}
    for num, symbol in enumerate(symbols):
        if num >= max_num:    # set small max_num for debugging
            break
        header, ct, cq, mast = store[symbol].values()

        # Compute and collect daily and bin statistics at all intervals
        daily = header.copy()   # to collect current stock's daily stats

        # Compute effective spreads by large and small trade sizes
        med_volume = mast['Round_Lot'] * (cq['Best_Bid_Size'].median()
                                          + cq['Best_Offer_Size'].median()) / 2.
        data = ct.loc[(ct.index > open_t) & (ct.index < close_t),
                      ['Trade_Price', 'Prevailing_Mid', 'Trade_Volume']]
        eff_spr = data['Trade_Price'].div(data['Prevailing_Mid']).sub(1).abs()
        eff_large = eff_spr[data['Trade_Volume'].ge(med_volume).to_numpy()]
        daily['large_trades'] = len(eff_large)
        daily['large_volume'] = data.loc[data['Trade_Volume'].ge(med_volume),
                                         'Trade_Volume'].mean()
        daily['large_spread'] = eff_large.mean()
        eff_small = eff_spr[data['Trade_Volume'].lt(med_volume)]
        daily['small_trades'] = len(eff_small)
        daily['small_volume'] = data.loc[data['Trade_Volume'].lt(med_volume),
                                         'Trade_Volume'].mean()
        daily['small_spread'] = eff_small.mean()

        v, u = intervals[-1]
        for (v, u) in intervals:
            bt = bin_trades(ct, v, u, open_t=open_t, close_t=close_t)
            bq = bin_quotes(cq, v, u, open_t=open_t, close_t=close_t)
            daily[f"tvar{v}{u}"] = bt['ret'].var(ddof=0) * len(bt)
            daily[f"tvarHL{v}{u}"] = ((hl_vol(bt['maxtrade'], bt['mintrade'])**2)
                                      * len(bt))
            daily[f"tvarOHLC{v}{u}"] = ((ohlc_vol(bt['first'],
```

(continues on next page)

(continued from previous page)

```
                                            bt['maxtrade'],
                                            bt['mintrade'],
                                            bt['last'])**2)
                                  * len(bt))
        daily[f"qvar{v}{u}"] = bq['retq'].var(ddof=0) * len(bq)
        daily[f"qvarHL{v}{u}"] = ((hl_vol(bq['maxmid'], bq['minmid'])**2)
                                * len(bq))
        daily[f"qvarOHLC{v}{u}"] = ((ohlc_vol(bq['firstmid'],
                                            bq['maxmid'],
                                            bq['minmid'],
                                            bq['mid'])**2)
                                  * len(bq))
        daily[f"tunch{v}{u}"] = np.mean(np.abs(bt['ret']) < EPSILON)
        daily[f"qunch{v}{u}"] = np.mean(np.abs(bq['retq']) < EPSILON)
        daily[f"tzero{v}{u}"] = np.mean(bt['counts'] == 0)


    # Collect final (i.e. 5 minute bins) bt and bq intraday series
    df = bq.join(bt, how='left')
    for s in ['effective', 'realized', 'impact', 'quoted']:
        bins_all[s].append({**header,
                            **(df[s]/df['mid']).to_dict()})
    for s in ['volume', 'offersize', 'bidsize', 'ret', 'retq', 'counts']:
        bins_all[s].append({**header,
                            **df[s].to_dict()})

    # Collect daily means
    daily.update(df[['bidsize', 'offersize', 'quoted', 'mid']].mean())
    daily.update(df[['volume', 'counts']].sum())
    daily.update(weighted_average(df[['effective', 'impact', 'realized',
                                      'vwap', 'volume']],
                                  weights='volume'))
    daily_all.append(daily)

return DataFrame(daily_all), {k: DataFrame(bins_all[k]) for k in bin_keys}
```

To optimize performance, **multiprocessing** package is used to parallelize computations, allowing efficient distribution of input data across multiple processes using its **Pool** API.

```
# Run each day as a parallel thread, must be called as context manager or with close␣
↪command
with multiprocessing.Pool(processes=min(os.cpu_count(), len(dates))) as p:
    data = p.map(intraday, dates)
```

```
# Combine data
daily_df = pd.concat([data[j][0] for j in range(len(data))], ignore_index=True)
bins_df = {k: pd.concat([data[j][1][k] for j in range(len(data))], ignore_index=True)
           for k in bin_keys}
```

```
# Store in scratch folder
store = Store(paths['scratch'])
```

```
store['tick.daily'] = daily_df
store['tick.bins'] = bins_df
```

```
# Fetch extracted data
daily_df = store['tick.daily']
bins_df = store['tick.bins']
```

18.3 By market capitalization

The daily averages of liquidity measures are analyzed across different market capitalization categories.

```
# group by market cap (NYSE deciles 1-3, 4-6, 7-9, 10) and exchange listed
daily_df['Size'] = pd.cut(daily_df['decile'],
                          [0, 3.5, 6.5, 9.5, 11],
                          labels=['large', 'medium', 'small', 'tiny'])
groupby = daily_df.groupby(['Size'], observed=False)
```

```
# collect results for each metric
results = {}    # to collect results as dict of {column: Series}
formats = {}    # and associated row formatter string
results.update(groupby['mid']\
               .count()\
               .rename('Number of Stock/Days').to_frame())
formats.update({'Number of Stock/Days': '{:.0f}'})
```

```
result = groupby[['mid', 'vwap']].mean()    # .quantile(), and range
result.columns = ['Midquote Price', "VWAP"]
formats.update({k: '{:.2f}' for k in result.columns})
results.update(result)
```

```
result = groupby[['counts', 'volume']].mean()
result.columns = ['Number of trades', "Volume (shares)"]
formats.update({k: '{:.0f}' for k in result.columns})
results.update(result)
```

```
# volatility from 5m intervals
result = np.sqrt(groupby[['tvar5m', 'qvar5m', 'tvarHL5m', 'qvarHL5m',
                          'tvarOHLC5m', 'qvarOHLC5m']].mean())
result.columns = ['Volatility(trade price)', "Volatility(midquote)",
                  'Volatility(HL trade price)', "Volatility(HL midquote)",
                  'Volatility(OHLC trade price)', "Volatility(OHLC midquote)"]
formats.update({k: '{:.4f}' for k in result.columns})
results.update(result)
```

```
result = groupby[['offersize', 'bidsize']].mean()
result.columns = [s.capitalize() + ' (lots)' for s in result.columns]
formats.update({k: '{:.1f}' for k in result.columns})
results.update(result)
```

```
spr = ['quoted', 'effective', 'impact', 'realized']
result = groupby[spr].mean()
result.columns = [s.capitalize() + ' $ spread' for s in spr]
```

(continues on next page)

(continued from previous page)

```
formats.update({k: '{:.4f}' for k in result.columns})
results.update(result)
```

```
rel = [s.capitalize() + ' (% price)' for s in spr]
daily_df[rel] = daily_df[spr].div(daily_df['mid'], axis=0)  # scale spreads
result = 100*groupby[rel].mean()
formats.update({k: '{:.4f}' for k in result.columns})
results.update(result)
```

```
# summarize large and small trade effective spreads
spr = ['large_spread', 'small_spread']
result = 100*groupby[spr].mean()
result.columns = ['Large trade (% spread) ', 'Small trade (% spread) ']
formats.update({k: '{:.4f}' for k in result.columns})
results.update(result)
```

```
spr = ['large_trades', 'small_trades']
result = groupby[spr].mean()
result.columns = ['Large trade (# trades) ', 'Small trade (# trades) ']
formats.update({k: '{:.0f}' for k in result.columns})
results.update(result)
```

```
spr = ['large_volume', 'small_volume']
result = groupby[spr].mean()
result.columns = ['Large trade (avg volume) ', 'Small trade (avg volume) ']
formats.update({k: '{:.0f}' for k in result.columns})
results.update(result)
```

```
# display table of results
print("Average Liquidity by Market Cap")
row_formatted(DataFrame(results).T, formats)
```

```
Average Liquidity by Market Cap
```

```
Size                              large  medium   small    tiny
Number of Stock/Days               2063    2572    4297    4618
Midquote Price                   128.98   64.91   27.76    7.92
VWAP                             128.97   64.92   27.75    7.90
Number of trades                  25178    8407    3658     837
Volume (shares)                 3031056  995483  533736  254773
Volatility(trade price)          0.0145  0.0211  0.0359  0.0807
Volatility(midquote)             0.0152  0.0223  0.0355  0.0918
Volatility(HL trade price)       0.0176  0.0217  0.0335  0.0690
Volatility(HL midquote)          0.0144  0.0198  0.0288  0.0594
Volatility(OHLC trade price)     0.0184  0.0218  0.0324  0.0612
Volatility(OHLC midquote)        0.0138  0.0183  0.0255  0.0452
Offersize (lots)                    8.8     9.8    11.9    13.6
Bidsize (lots)                      8.9    17.0    14.1    16.5
Quoted $ spread                  0.0630  0.0672  0.0781  0.0841
Effective $ spread               0.0379  0.0442  0.0406  0.0457
Impact $ spread                  0.0273  0.0244  0.0248  0.0186
```

(continues on next page)

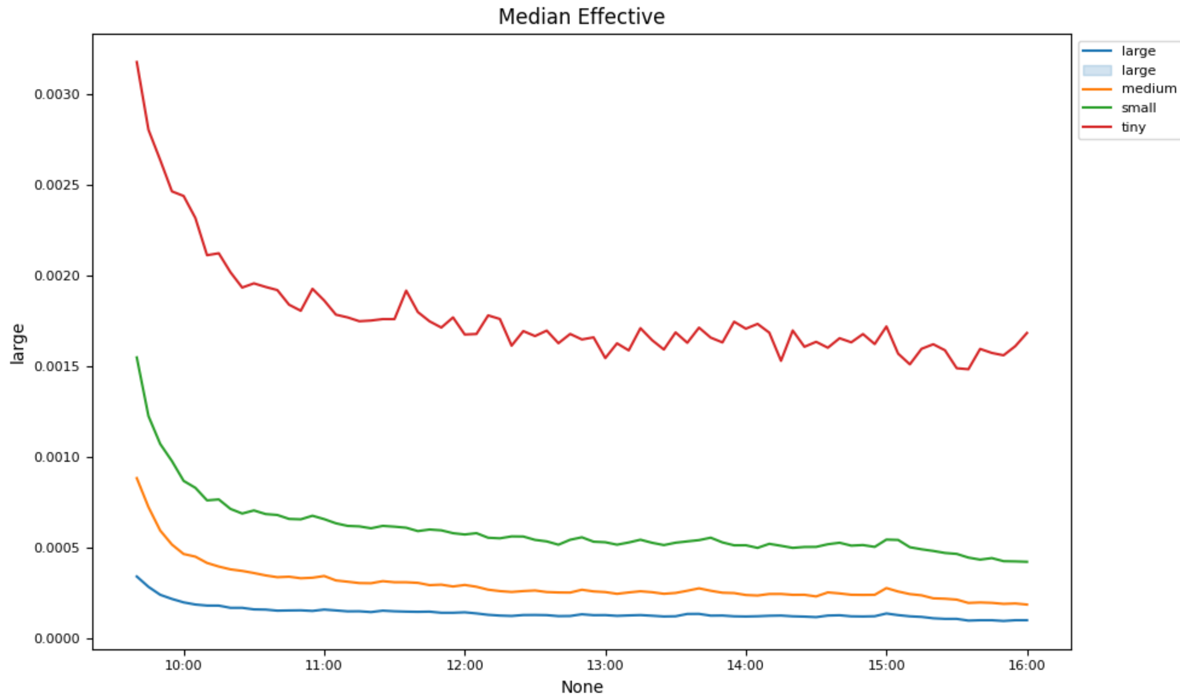
(continued from previous page)

```
Realized $ spread              0.0106  0.0199  0.0158  0.0270
Quoted (% price)               0.0350  0.0834  0.2500  1.1582
Effective (% price)            0.0200  0.0421  0.1287  0.6638
Impact (% price)               0.0165  0.0345  0.0904  0.2669
Realized (% price)             0.0036  0.0077  0.0384  0.3973
Large trade (% spread)         0.0191  0.0420  0.1310  0.6398
Small trade (% spread)         0.0190  0.0400  0.1304  0.6755
Large trade (# trades)           3133    1140     423     108
Small trade (# trades)          22045    7267    3236     729
Large trade (avg volume)         1734    1608    2635    2324
Small trade (avg volume)           61      57      71      85
```

18.3.1 By time of day

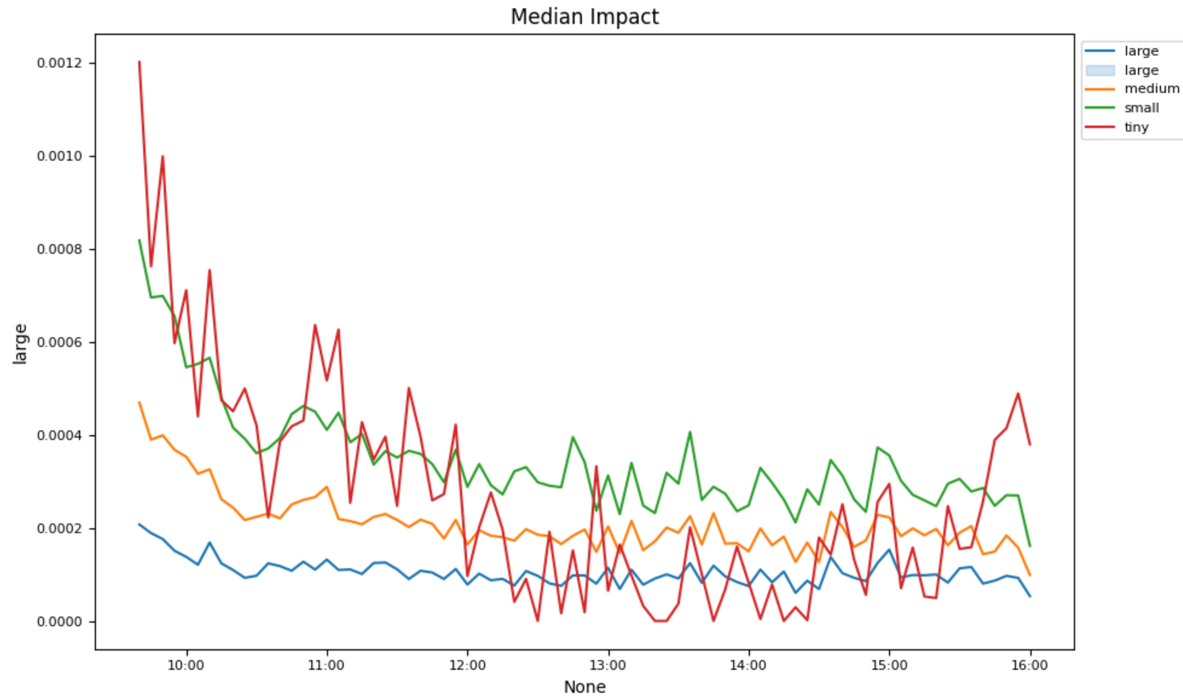
Market liquidity changes are examined over the trading day.

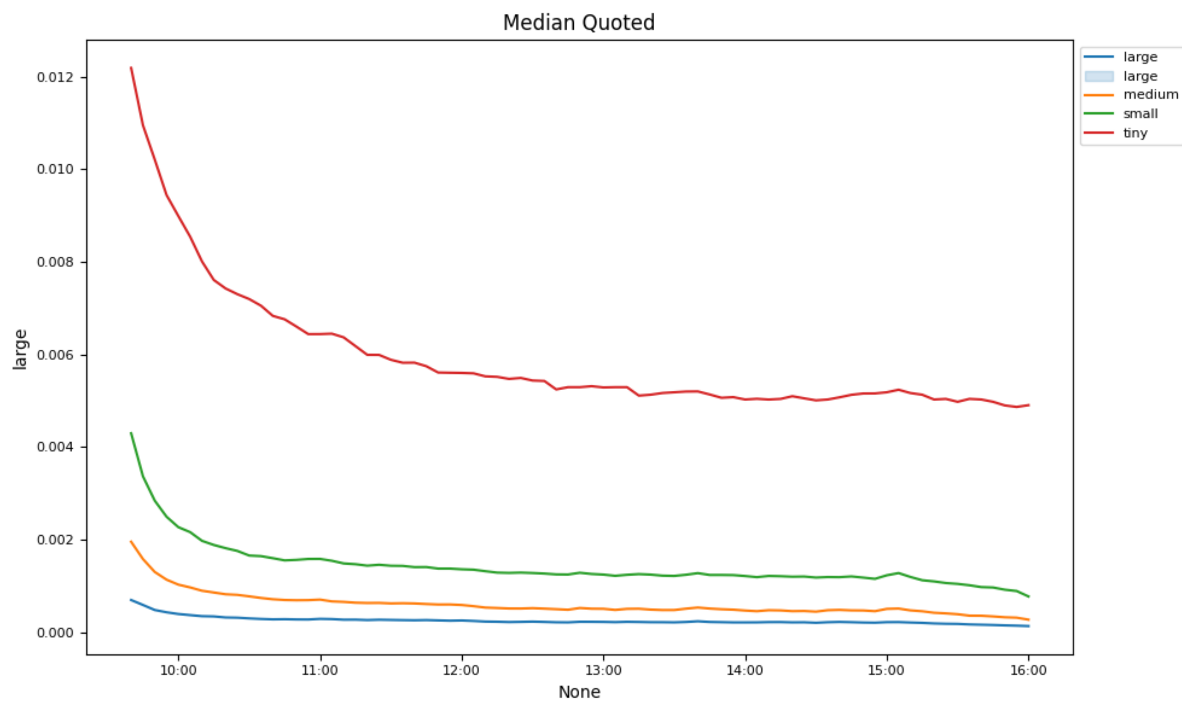
```
# Plot intraday spreads, depths and volumes
keys = ['effective', 'realized', 'impact', 'quoted',
        'volume', 'counts', 'offersize', 'bidsize']
for num, key in enumerate(keys):
    df = bins_df[key].drop(columns=['Round_Lot', 'Symbol'])
    df.index = list(zip(df['permno'], df['date']))

    # Group by market cap
    df['Size'] = pd.cut(df['decile'],
                        [0, 3.5, 6.5, 9.5, 11],
                        labels=['large', 'medium', 'small', 'tiny'])
    df = df.drop(columns=['date', 'permno', 'decile', 'exchcd', 'siccd'])\
           .dropna()\
           .groupby(['Size'], observed=False)\
           .median().T
    fig, ax = plt.subplots(1, 1, num=num+1, clear=True, figsize=(10, 6))
    plot_time(df.iloc[1:], ax=ax, fontsize=8)
    ax.legend(['large'] + list(df.columns),
              loc='upper left', bbox_to_anchor=(1.0, 1.0),
              fontsize=8)
    ax.set_title('Median ' + key.capitalize())
    plt.subplots_adjust(right=0.8)
    plt.tight_layout()
```

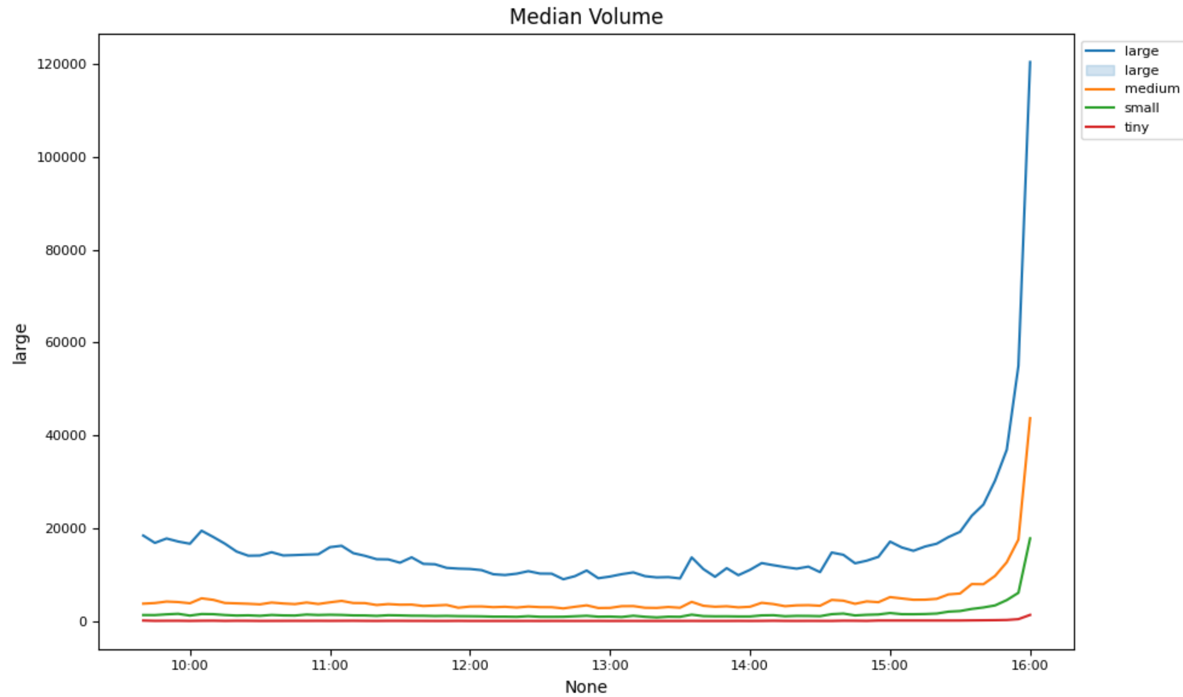


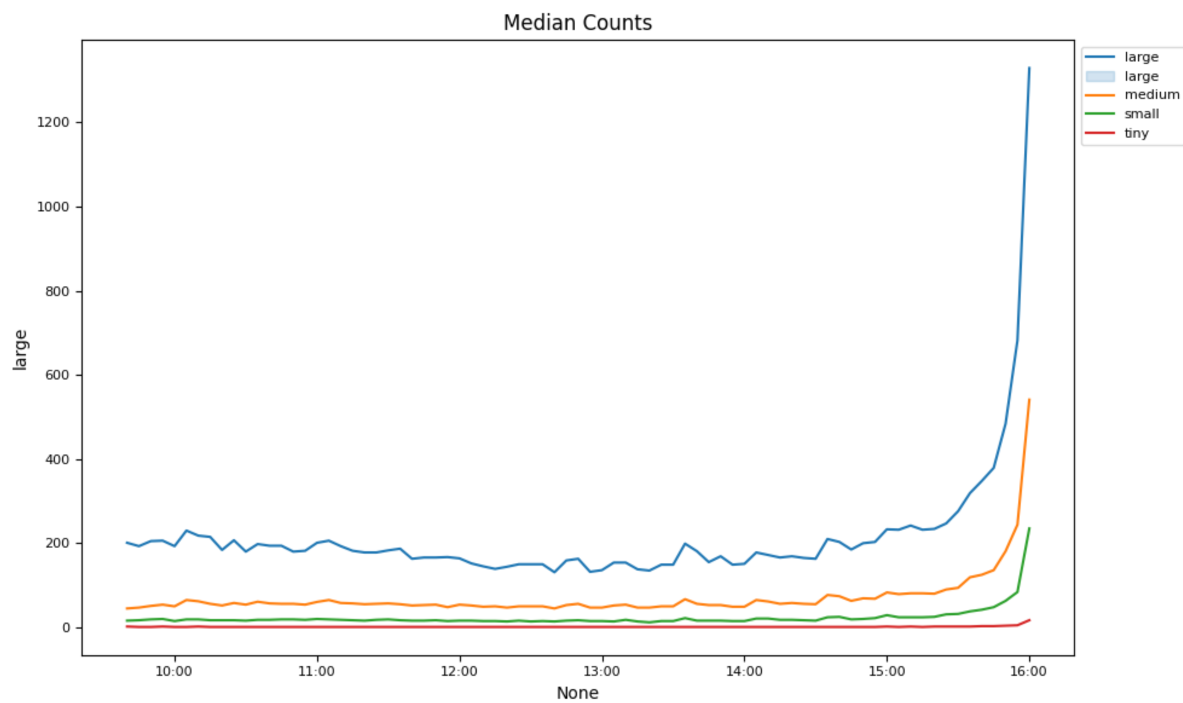
Median Effective
large
large
large
medium
small
tiny
0.0030
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
10:00
11:00
12:00
13:00
14:00
15:00
16:00
None

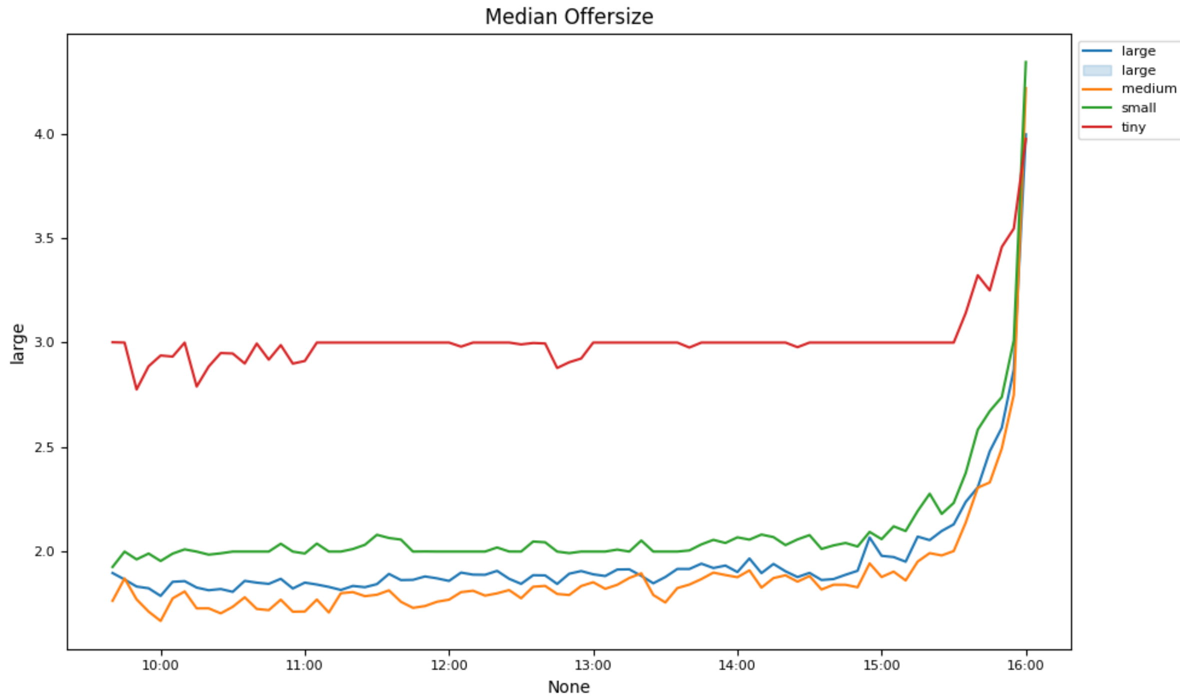
Median Realized
large
large
large
medium
small
tiny
0.00200
0.00175
0.00150
0.00125
0.00100
0.00075
0.00050
0.00025
0.00000
10:00
11:00
12:00
13:00
14:00
15:00
16:00
None

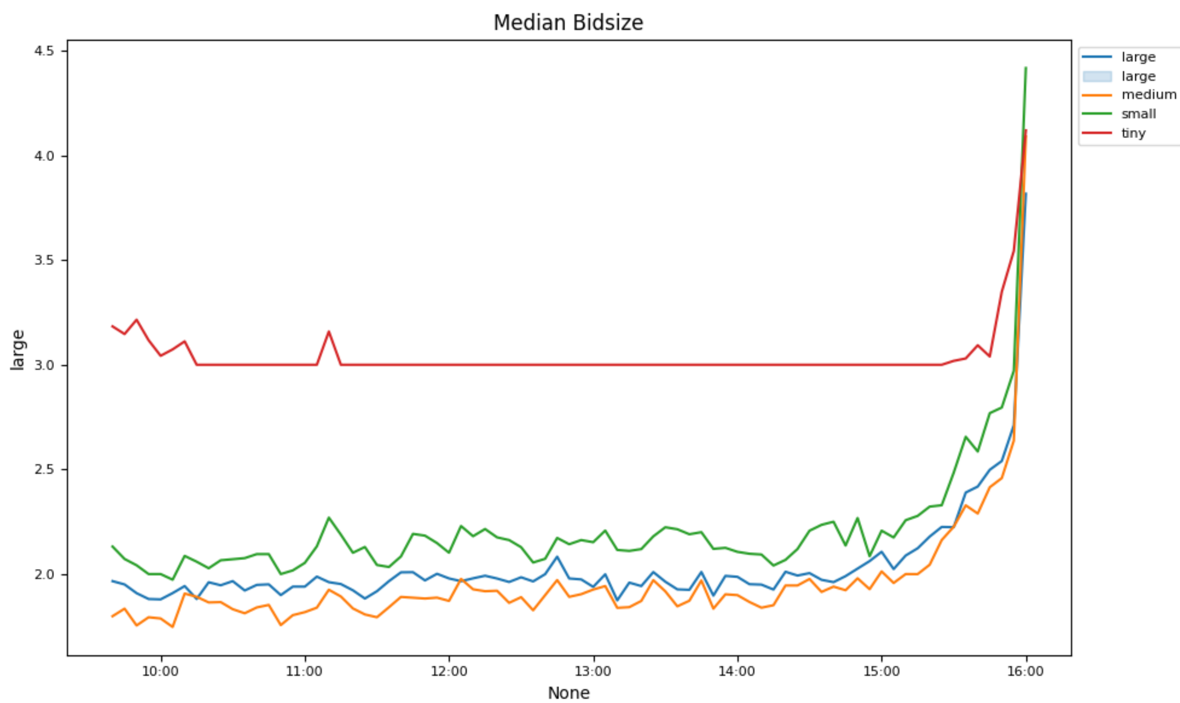

Median Impact
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
large
10:00
11:00
12:00
13:00
14:00
15:00
16:00
None
large
large
medium
small
tiny


Median Quoted
0.012
0.010
0.008
0.006
0.004
0.002
0.000
large
10:00
11:00
12:00
13:00
14:00
15:00
16:00
None
large
large
medium
small
tiny


Median Volume
120000
100000
80000
60000
40000
20000
0
large
10:00
11:00
12:00
13:00
14:00
15:00
16:00
None
large
large
medium
small
tiny


Median Counts
1200
1000
800
600
400
200
0
large
10:00
11:00
12:00
13:00
14:00
15:00
16:00
None
large
large
medium
small
tiny


Median Offersize
large
4.0
3.5
3.0
2.5
2.0
10:00
11:00
12:00
13:00
14:00
15:00
16:00
None
large
large
medium
small
tiny


Median Bidsize
large
4.5
4.0
3.5
3.0
2.5
2.0
10:00
11:00
12:00
13:00
14:00
15:00
16:00
None
large
large
medium
small
tiny

18.4 High frequency sampling

18.4.1 Variance ratio

Tick data often exhibits spurious autocorrelation due to irregularly spaced trades and quotes rather than continuous trading:

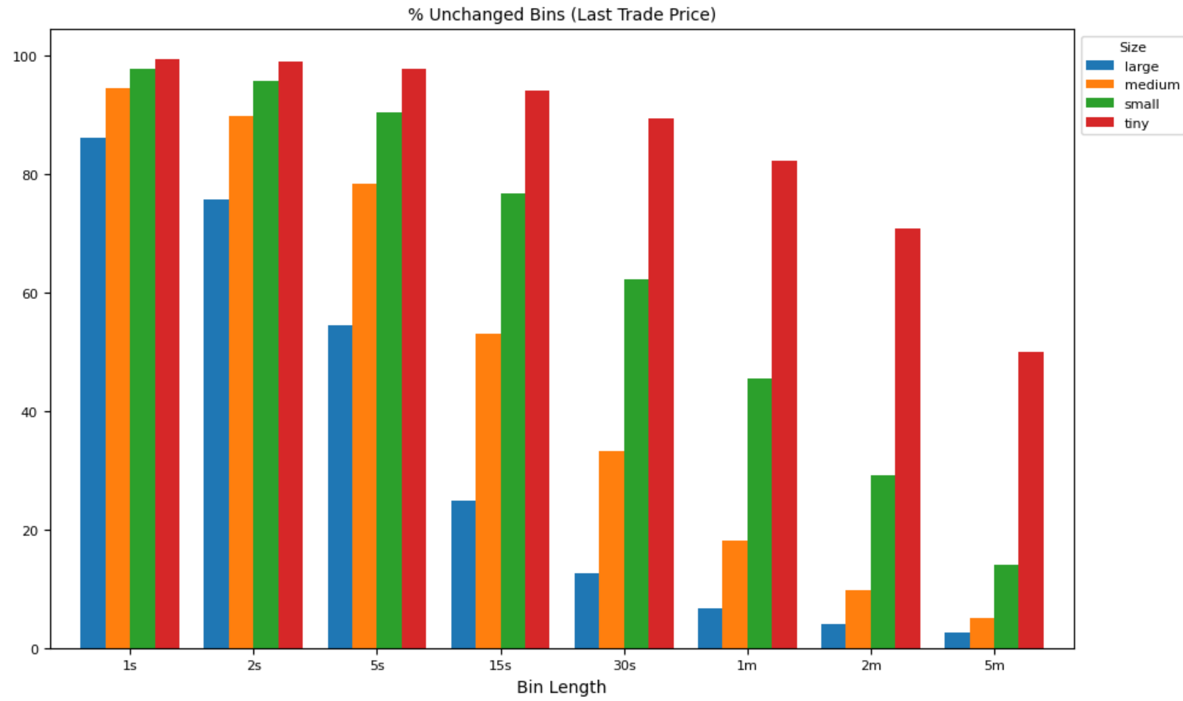
- Non-Continuous Trading: Trades and quotes occur discretely, often clustering around news or market events.
- Order Flow Clustering: Market participants submit bursts of orders, creating short-term price autocorrelation.
- Bid-Ask Bounce: Trades alternate between the bid and ask prices, creating an illusion of mean-reverting returns.

The variance ratio test (Lo & MacKinlay, 1988) checks for mean reversion or momentum by comparing **multi-period return variance to single-period return variance, scaled by the number of periods.

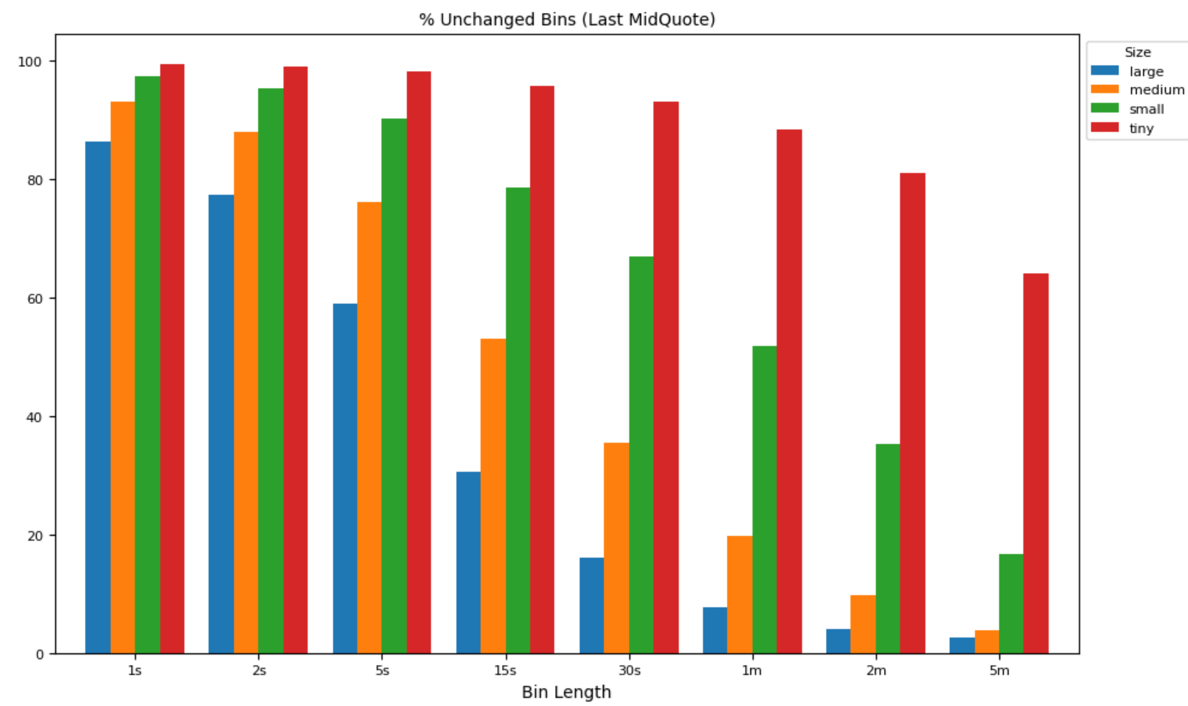
```
def plot_helper(result, xticks, keys, legend, xlabel, title, ylim=[],
                figsize=(10, 6), num=1, fontsize=8):
    """helper to plot bar graphs by sampling frequency"""
    fig, ax = plt.subplots(num=num, clear=True, figsize=figsize)
    result.plot(kind='bar',
                fontsize=fontsize,
                rot=0,
                width=0.8,
                xlabel='',
                ax=ax)
    if ylim:
        ax.set_ylim(*ylim)
    ax.set_xticklabels(xticks, fontsize=fontsize)
    ax.legend(keys, loc='upper left', bbox_to_anchor=(1.0, 1.0),
              fontsize=fontsize, title=legend, title_fontsize=8)
    ax.set_xlabel(xlabel, fontsize=fontsize + 2)
    ax.set_title(title, fontsize=fontsize + 2)
    plt.subplots_adjust(right=0.8, bottom=0.15)
    plt.tight_layout()
    return ax
```

```
xticks = [f"{v}{u}" for v, u in intervals]    # x-axis: bin lengths
keys = list(groupby.indices.keys())           # legend labels
```

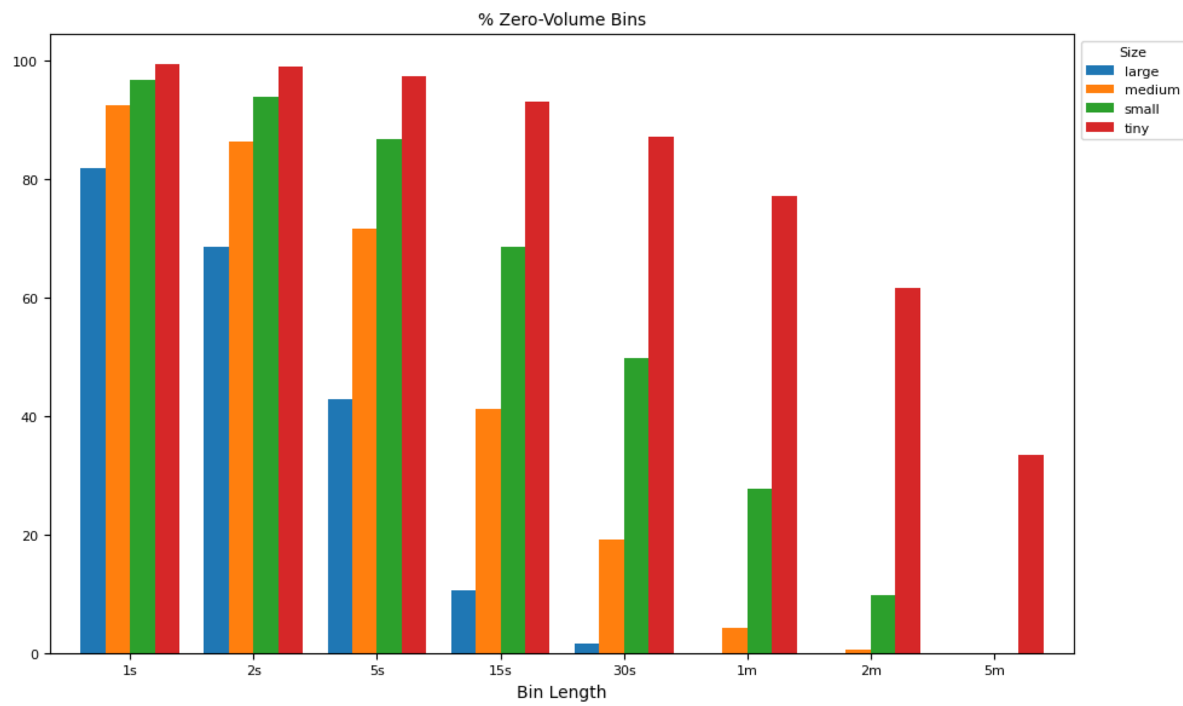
```
labels = [f"tunch{v}{u}" for v, u in intervals]
result = groupby[labels].median()*100
ax = plot_helper(result.T,
                 title="% Unchanged Bins (Last Trade Price)",
                 xticks=xticks,
                 xlabel="Bin Length",
                 keys=keys,
                 legend='Size',
                 num=1)
```



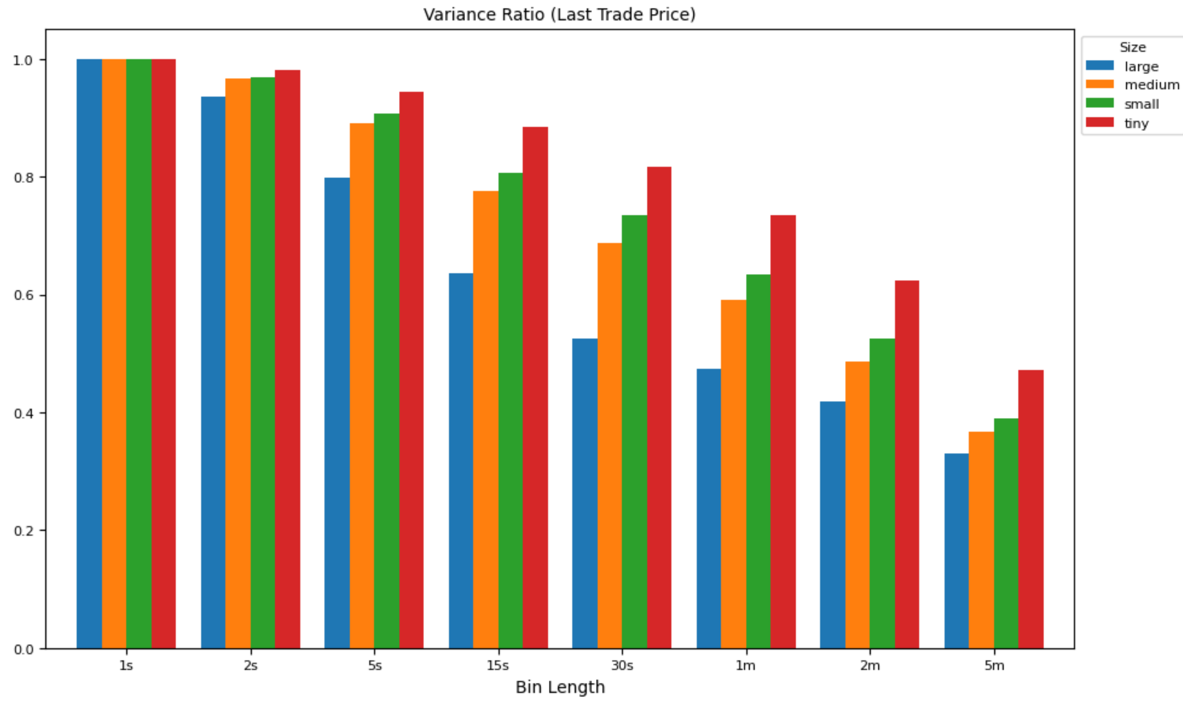

```
labels = [f"qunch{v}{u}" for v, u in intervals]
result = groupby[labels].median()*100
ax = plot_helper(result.T,
                 title="% Unchanged Bins (Last MidQuote)",
                 xticks=xticks,
                 xlabel="Bin Length",
                 keys=keys,
                 legend='Size',
                 num=2)
```



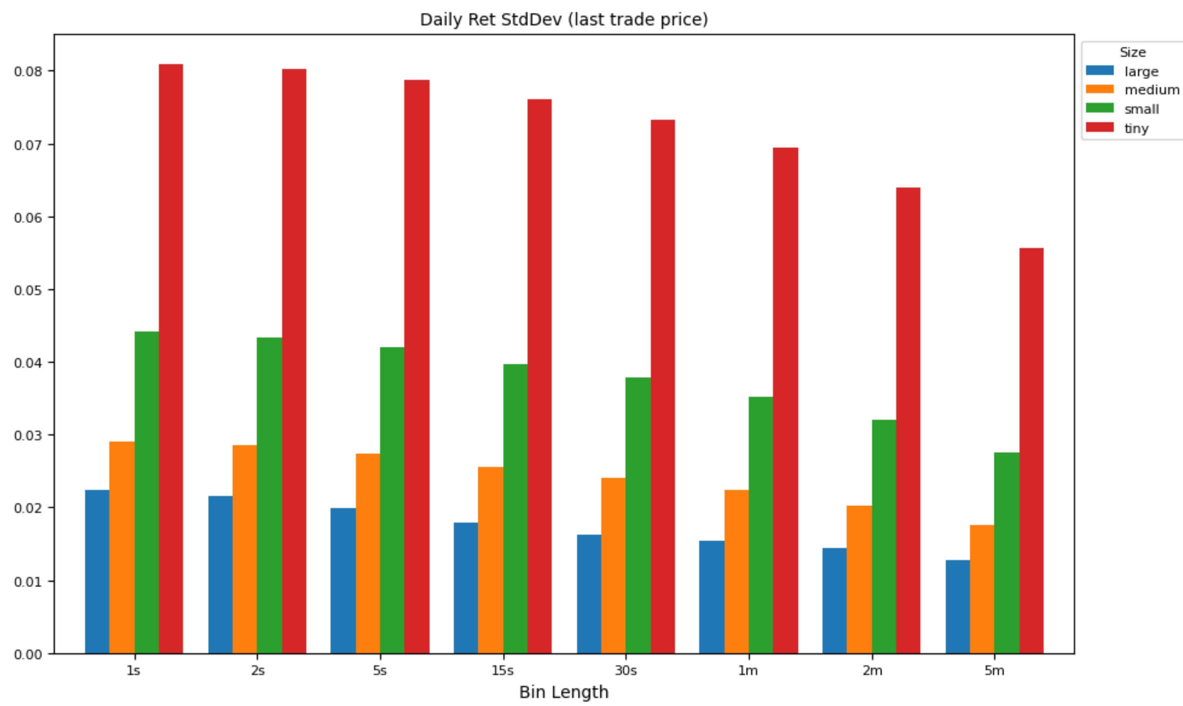

```
labels = [f"tzero{v}{u}" for v, u in intervals]
result = groupby[labels].median()*100
ax = plot_helper(result.T,
                 title="% Zero-Volume Bins",
                 xticks=xticks,
                 xlabel="Bin Length",
                 keys=keys,
                 legend='Size',
                 num=3)
```



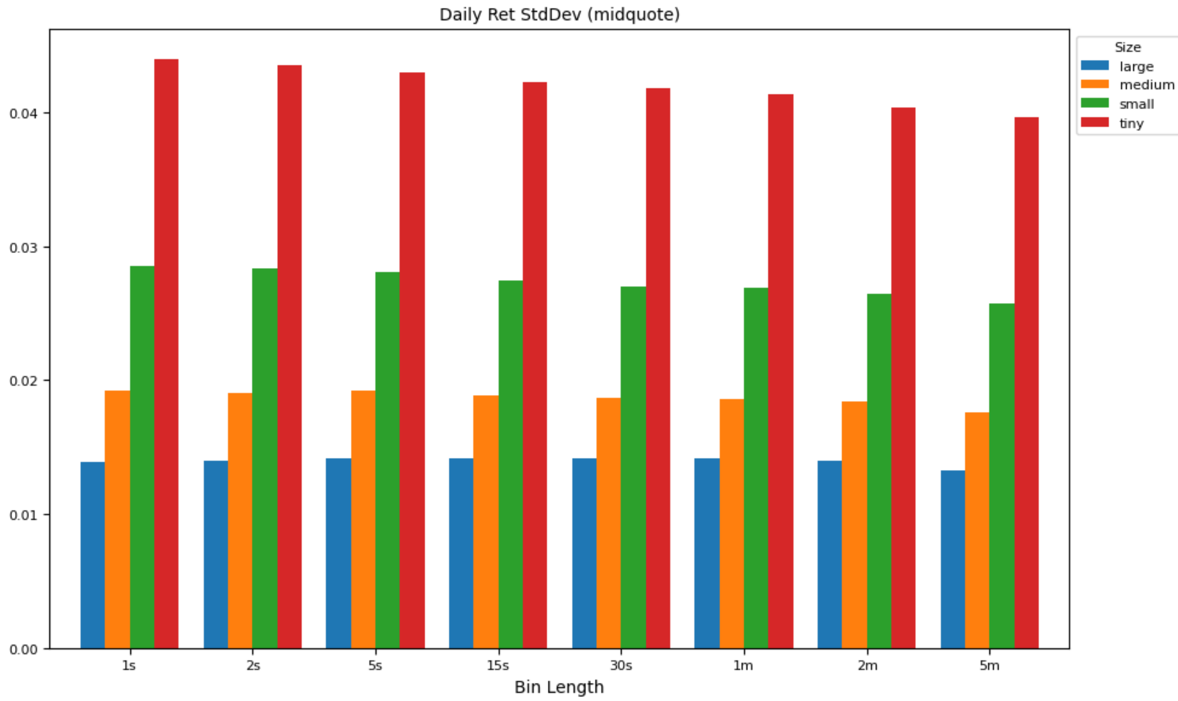

```
labels = [f"tvar{v}{u}" for v, u in intervals]
result = groupby[labels].median()
result = result.div(result['tvar1s'].values, axis=0)
ax = plot_helper(result.T,
                 title="Variance Ratio (Last Trade Price)",
                 xticks=xticks,
                 xlabel="Bin Length",
                 keys=keys,
                 legend='Size',
                 num=4)
```




```
labels = [f"tvar{v}{u}" for v, u in intervals]
result = np.sqrt(groupby[labels].median())
ax = plot_helper(result.T,
                 title="Daily Ret StdDev (last trade price)",
                 xticks=xticks,
                 xlabel="Bin Length",
                 keys=keys,
                 legend='Size',
                 num=5)
```




```
labels = [f"qvar{v}{u}" for v, u in intervals]
result = np.sqrt(groupby[labels].median())
ax = plot_helper(result.T,
                 title="Daily Ret StdDev (midquote)",
                 xticks=xticks,
                 xlabel="Bin Length",
                 keys=keys,
                 legend='Size',
                 num=6)
```



18.4.2 Volatility measures

Parkinson's (HL) Volatility estimator uses the high-low price range to estimate volatility, assuming a geometric Brownian motion without a drift term:
$\sigma_{HL} = \frac{\sqrt{\ln 2}}{\sqrt{4T \ln 2}} \times \frac{\mathrm{H-L}}{\sqrt{\Delta t}}$

This method is more efficient than close-to-close volatility but assumes continuous trading and no jumps.

The **Klass-Garman (OHLC) Volatility** estimator improves on Parkinson's by incorporating open, high, low, and close prices for better accuracy:
$\sigma^2_{OHLC} = 0.511(H-L)^2 - 0.019(C-O)(H+L-2O) - 0.383(C-O)^2$

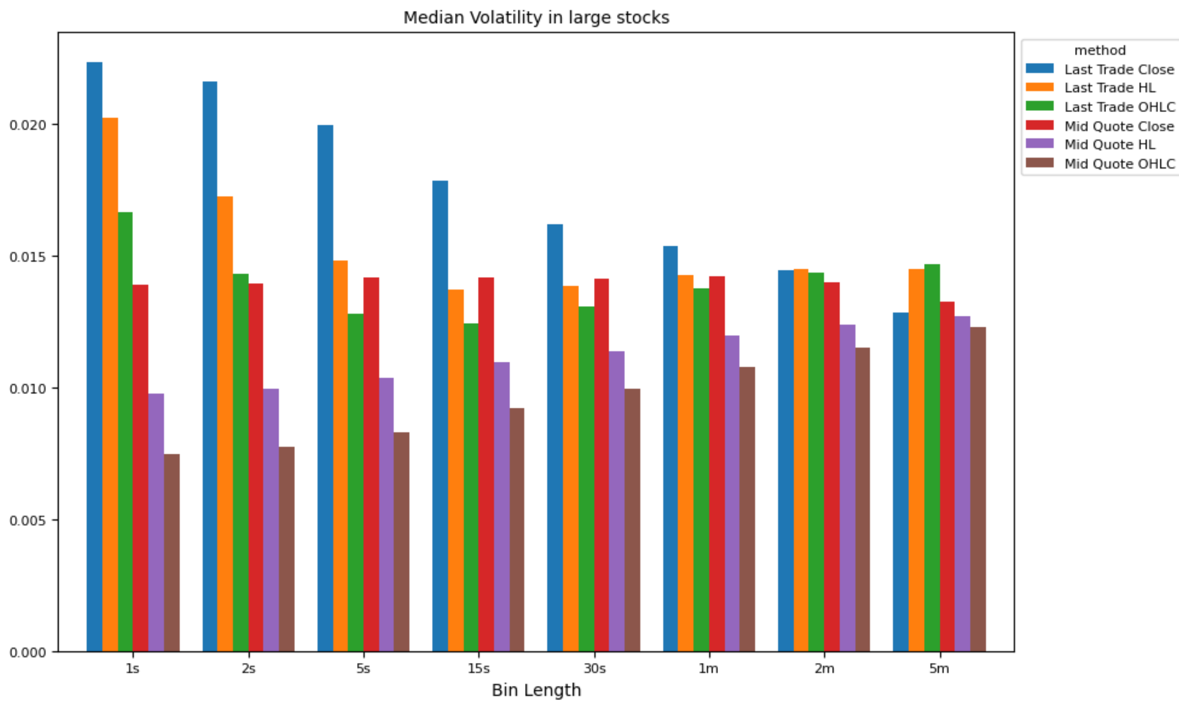
This model accounts for both overnight jumps and intra-day movements.

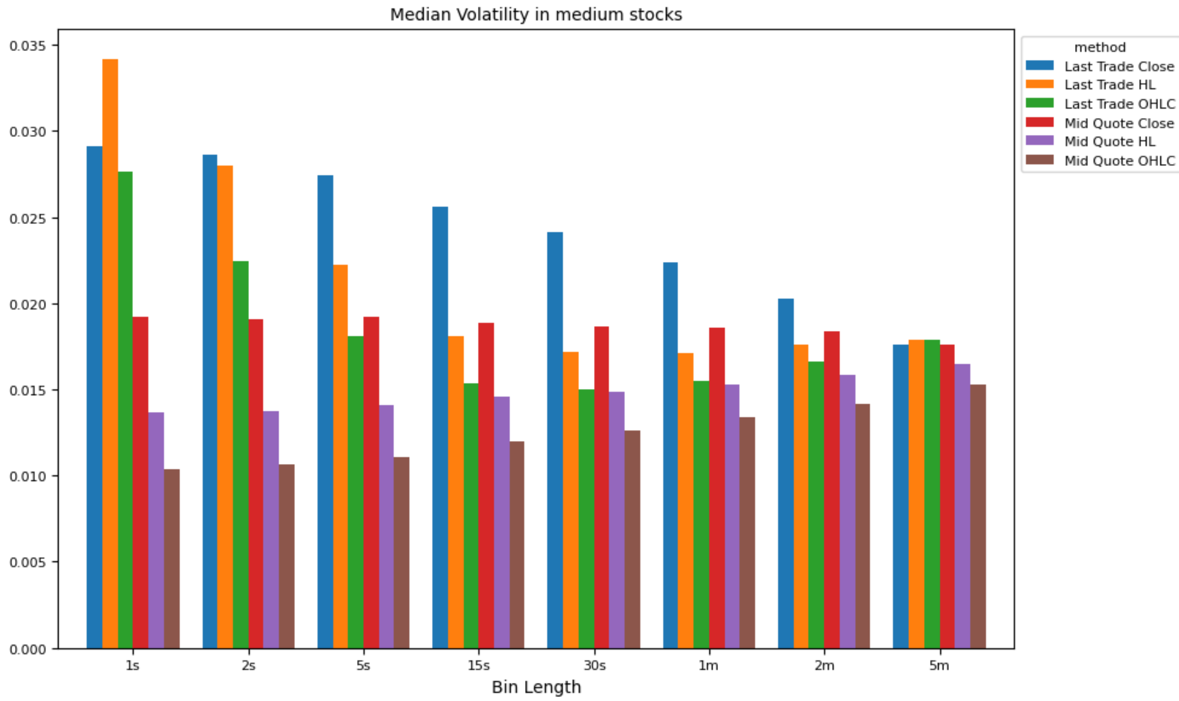
```
# Compare methods of volatility estimates, by interval and market cap
for ifig, (split_label, split_df) in enumerate(groupby):
    vol_df = np.sqrt(split_df[[c for c in daily_df.columns if "qvar" in c or "tvar"␣
↪in c]])
    result = []
    for col in [c for c in vol_df.columns if "qvar" in c or "tvar" in c]:
        if col[4] == 'H':
            m = 'HL'
        elif col[4] == 'O':
            m = 'OHLC'
        else:
            m = 'Close'
        result.append({'method': {'t': 'Last Trade', 'q': 'Mid Quote'}[col[0]] + ' '␣
↪+ m,
                       'interval': (int("".join(filter(str.isdigit, col)))
                                    * (60 if col[-1] == 'm' else 1)),
                       'val': vol_df[col].median()})
```

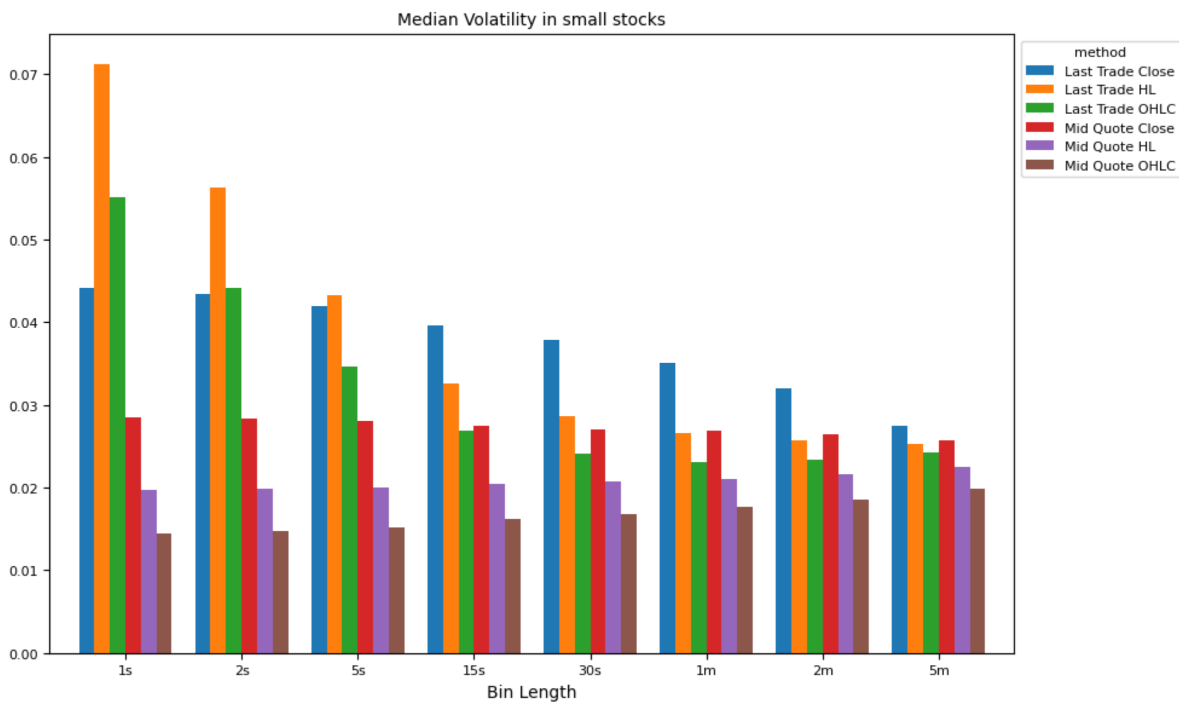
(continues on next page)

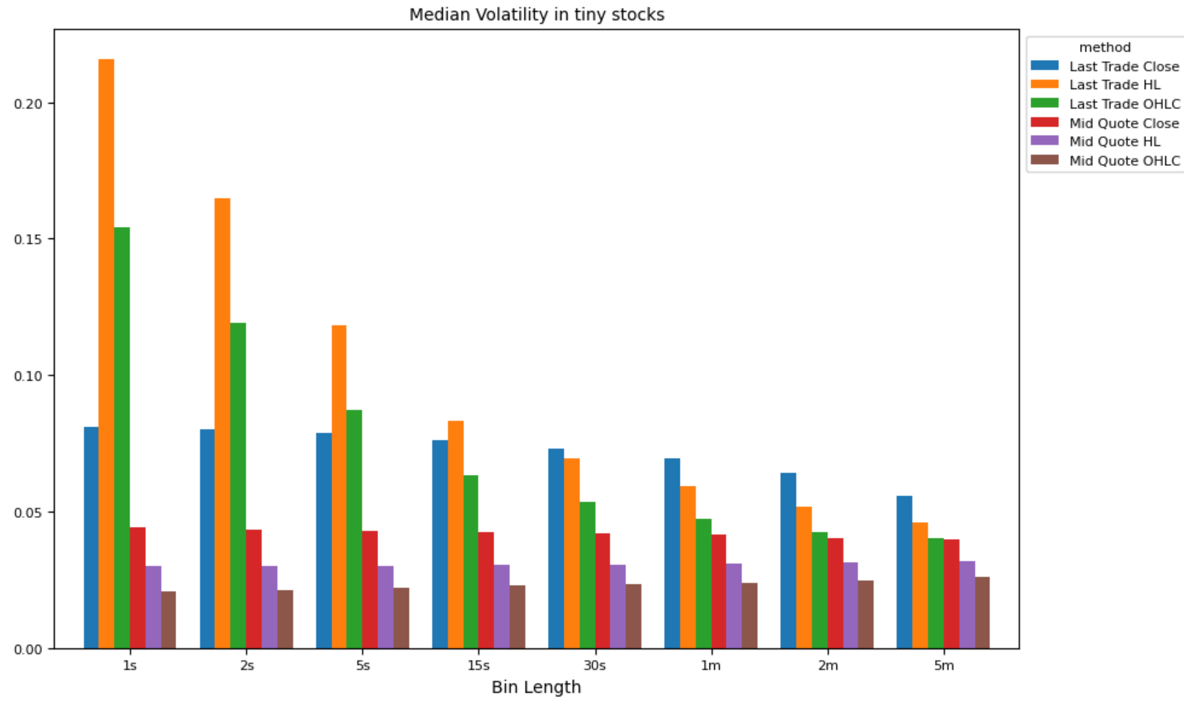
(continued from previous page)

```
    result = DataFrame.from_dict(result, orient='columns')\
        .pivot(index='interval', columns='method', values='val')\
        .sort_index()
    ax = plot_helper(result,
                     title="Median Volatility in " + " ".join(split_label) + " stocks
↪",
                     xticks=xticks,
                     xlabel="Bin Length",
                     keys=result.columns,
                     num=ifig+1,
                     legend='method')
```




Median Volatility in medium stocks
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
1s
2s
5s
15s
30s
1m
2m
5m
Bin Length
method
Last Trade Close
Last Trade HL
Last Trade OHLC
Mid Quote Close
Mid Quote HL
Mid Quote OHLC


Median Volatility in small stocks
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
1s
2s
5s
15s
30s
1m
2m
5m
Bin Length
method
Last Trade Close
Last Trade HL
Last Trade OHLC
Mid Quote Close
Mid Quote HL
Mid Quote OHLC



References:

Andrew W. Lo, A. Craig MacKinlay, Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test, The Review of Financial Studies, Volume 1, Issue 1, January 1988, Pages 41–66, https://doi.org/10.1093/rfs/1.1.41

Parkinson, M. (1980) The Extreme Value Method for Estimating the Variance of the Rate of Return. Journal of Business, 53, 61-65. http://dx.doi.org/10.1086/296071

CHAPTER

NINETEEN

EVENT RISK

Don't worry about failure; you only have to be right once —Drew Houston

Event risk refers to the potential for a sudden and significant impact on an asset's price or a portfolio's value due to an unforeseen event. These can be company-specific or broad market events that introduce volatility and uncertainty. For example, a company missing or exceeding analysts' earnings expectations can cause sharp price movements. Other types of event risk in finance include credit defaults and downgrades, mergers and acquisitions, or monetary policy changes. We explore how **Poisson regression**, an example of a **generalized linear model (GLM)** suited for count data, can be used to model the frequency of events such as earnings misses.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
from finds.database import SQL, RedisDB
from finds.structured import BusDay, SignalsFrame, CRSP, IBES
from finds.readers import Alfred
from secret import credentials, paths, CRSP_DATE
VERBOSE = 0
#%matplotlib qt
```

```
# open connections
sql = SQL(**credentials['sql'], verbose=VERBOSE)
user = SQL(**credentials['user'], verbose=VERBOSE)
rdb = RedisDB(**credentials['redis'])
bd = BusDay(sql, verbose=VERBOSE)
crsp = CRSP(sql, bd, rdb=rdb, verbose=VERBOSE)
ibes = IBES(sql, bd, verbose=VERBOSE)
LAST_DATE = CRSP_DATE
```

19.1 Earnings expectations

The consensus analyst estimate for a company's quarterly earnings is derived from the average of all the analyst forecasts covering that company. The number of analysts following a particular company typically correlates with the company's size and the liquidity of its stock. Large, well-known companies often have many analysts providing coverage, resulting in a consensus estimate derived from various opinions. In contrast, smaller or less-recognized companies may have only a few analysts covering them, or none at all.

When a company reports earnings that fall below the consensus expectation, this is referred to as an *earnings miss*. On the other hand, if the company's earnings exceed the analysts' expectations, it is called an *earnings beat*.

19.1.1 Standardized unexpected earnings (SUE)

To quantify earnings surprises, the Standardized Unexpected Earnings (SUE) is calculated. This is determined by subtracting the median of the analysts' estimates for a particular fiscal quarter from the company's reported earnings, then scaling it by the stock price.

```
# retrieve ibes Q1 where forecast period <= fiscal date, and keep latest
df = ibes.get_linked(dataset='statsum',
                     fields=['fpedats', 'medest', 'actual'],
                     date_field='statpers',
                     where=" fpi = '6' AND statpers <= fpedats")
```

```
summ = df.dropna()\
         .sort_values(['permno', 'fpedats', 'statpers'])\
         .drop_duplicates(['permno', 'fpedats'], keep='last')
summ['rebaldate'] = bd.endmo(summ['fpedats'])
summ = summ.set_index(['permno', 'statpers'])
```

```
# retrieve ibes price
df = ibes.get_linked(dataset='actpsum',
                     fields=['price'],
                     date_field='statpers')
```

```
hist = df.dropna()\
         .sort_values(['permno', 'statpers'])\
         .drop_duplicates(['permno', 'statpers'], keep='last')
hist = hist.set_index(['permno', 'statpers'])
```

```
# calculate sue with ibes surprise and price
summ = summ.join(hist[['price']], how='inner')
summ['sue'] = (summ['actual'] - summ['medest']).div(summ['price'])
```

```
# define large earnings surprises as 5% of price
START = 19841201
signals = SignalsFrame(summ.reset_index(drop=False))
rebaldates = bd.date_range(START, LAST_DATE-100, freq='quarterly')
label = 'sue'
out = []
for rebaldate in rebaldates:
    univ = crsp.get_universe(rebaldate)     # get this quarter's universe
    univ = univ[(abs(univ['prc']) >= 5.0) & (univ['decile'] < 9)]   # no small low-
↪price
    signal = signals(label=label,
                     date=rebaldate,
                     start=bd.endqr(rebaldate, quarters=-1))\
                     .reindex(univ.index)\
                     .dropna()\
                     .reset_index()
    signal['rebaldate'] = rebaldate // 100
    signal['miss'] = signal['sue'] < -0.05    # large earnings misses
    out.append(signal.set_index('permno').join(univ['decile'], how='inner')\
                  .reset_index())
out = pd.concat(out)
```

```
# median quarterly earnings surprise by firm size
fig, axes = plt.subplots(2, 2, figsize=(10, 9), sharex=True, sharey=True)
axes = axes.flatten()
for label, (ax, dec) in enumerate(zip(axes, [[1,2], [3,4], [5,6], [7,8]])):
    y = out[out['decile'].isin(dec)].groupby('rebaldate')['sue'].median()
    y.plot(ax=ax, marker='+', color="C0")
    y.rolling(4).mean().plot(ax=ax, color="C1")
    ax.set_title(f"size quintile {label+1}")
    if not label:
        ax.legend(['sue','1-year average'])
    ax.axhline(0, color='C2', ls=':')
    ax.set_xticks(y.index[::28])
    ax.minorticks_off()
plt.suptitle('Median quarterly earnings surprises by firm size')
plt.tight_layout()
plt.show()
```

Historically, the median earnings surprise was negative (i.e. estimates were too optimistic), especially for smaller stocks, until the early-1990s, after which the trend generally shifted to positive earnings surprises

19.1.2 Earnings misses

We define earnings misses to be large negative earnings surprises exceeding 5% of stock price.

```
# count number of stocks with large earnings misses
frac = out.groupby('rebaldate')['miss'].mean().rename('frac')
count = out.groupby('rebaldate')['miss'].sum().rename('count')
exposures = out['rebaldate'].value_counts().sort_index().rename('exposures')
Y = pd.concat([exposures, frac, count], axis=1)
Y.index = Y.index.astype(str)
print("Earnings misses")
Y
```

```
Earnings misses
```

```
            exposures      frac  count
rebaldate                             
198412           1327  0.051997     69
198503           1302  0.038402     50
198506           1384  0.040462     56
198509           1384  0.033960     47
198512           1398  0.063662     89
...               ...       ...    ...
202309           1845  0.008672     16
202312           1812  0.009934     18
202403           1764  0.003401      6
202406           1776  0.004505      8
202409           1737  0.005757     10

[160 rows x 3 columns]
```

19.2 Poisson regression

The Poisson distribution is often used to model count data, or events that occur in a fixed period of time or space. Poisson regression models the expected count of events as a function of certain covariates. Specifically, it is expressed as:

$$\log(E[Y|X]) = \beta X$$

which is equivalent to:

$$E[Y|X] = e^{\beta X}$$

In this model, the interpretation of the coefficient β is that an increase in X_i by one unit is associated with a multiplicative change in the expected value of Y_i by a factor of e^{β_i}. The variance of Y_i under the Poisson model is equal to its expected value, i.e., $Var[Y_i] = E[Y_i]$. Importantly, the Poisson model ensures that the fitted values are non-negative, as it only allows for counts of zero or more.

Poisson regression can also be useful for modeling rates, where the event count is divided by some measure of exposure. For example, when modeling the number of stocks with large earnings misses each quarter, the exposure would be the total number of stocks, denoted N. The log of the total number of stocks, called the *offset* variable, is included in the regression equation with a coefficient constrained to 1.

The modified equation would then look like:

$$\log\left[\frac{E[Y|X]}{\text{exposure}}\right] = X\beta - \log(\text{exposure})$$

19.2.1 Generalized Linear Models

Linear and Poisson regression models are both types of Generalized Linear Models (GLMs). GLMs are a class of models where the response variable is modeled as a function of the predictors through a link function. Specifically, the mean of the response variable Y_i is transformed using the link function, which ensures the mean of the response is related linearly to the predictors.

For linear regression, the link function is the identity function, which simply states that the transformed mean is equal to the expected value: $\nu(E[Y]) = E[Y]$. In Poisson regression, the link function is the log function, so $\nu(E[Y]) = \log(E[Y])$, which transforms the expected count to the log scale. Specifically, Poisson regression models the response as coming from the Poisson distribution (with support 0, 1, $2\cdots$,), with its canonical link function being the log function $X\beta = \log E[Y]$, and the mean function being the exponential $E[Y] = e^{X\beta}$

Both linear and Poisson regression are based on the assumption that, conditional on X, the response variable Y comes from a member of the exponential family of distributions. The exponential family includes both discrete and continuous distributions such as the Gaussian, Poisson, Bernoulli, Multinomial, Exponential, Gamma, and Negative Binomial distributions. A GLM links the transformed mean of the distribution to the predictors, enabling flexible modeling of different types of data.

```
# retrieve and transform economics series
alf = Alfred(api_key=credentials['fred']['api_key'], verbose=VERBOSE)
```

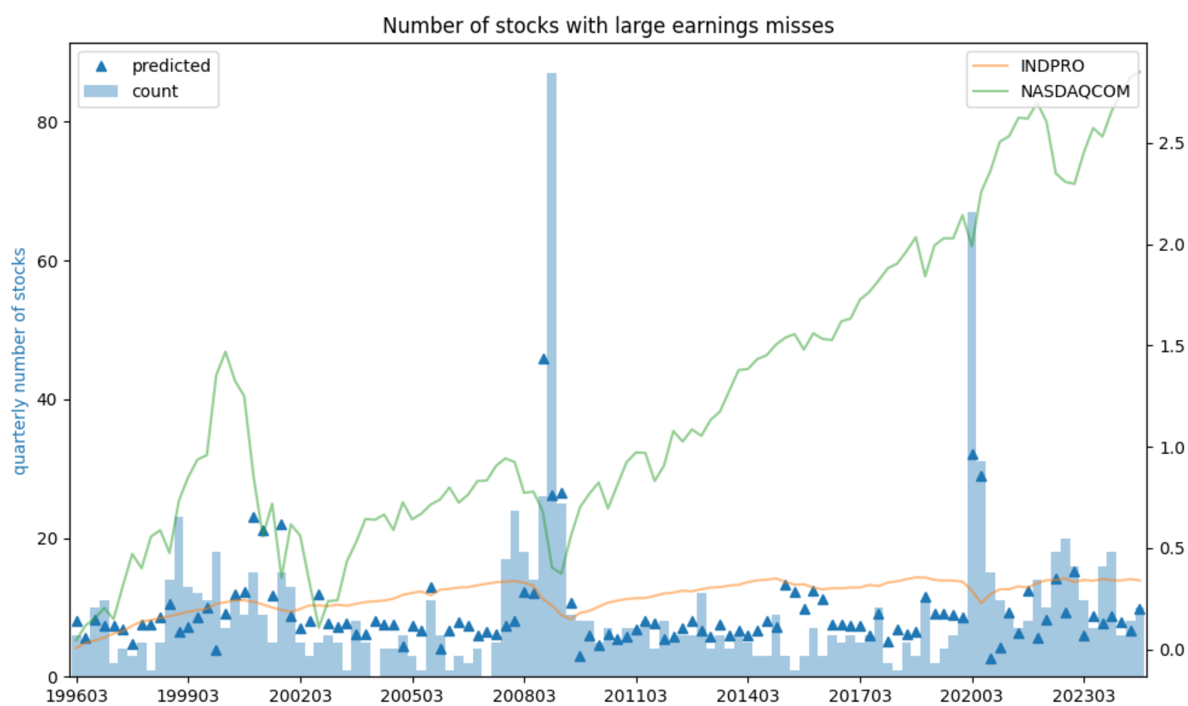
```
X = pd.concat([alf(s, log=1, diff=1, freq='Q') for s in ['INDPRO', 'NASDAQCOM']],
 ↪axis=1).dropna()
X.index = (X.index // 100).astype(str)
X
```

```
          INDPRO  NASDAQCOM
date
197106  0.014847   0.017122
197109  0.007394   0.011345
197112  0.023171   0.045627
197203  0.040696   0.115873
197206  0.012599   0.015026
...          ...        ...
202403 -0.001095   0.087222
202406  0.007142   0.079377
202409 -0.006472   0.025422
202412  0.003853   0.059838
202503  0.005111  -0.024295

[216 rows x 2 columns]
```

```
# Run GLM regression with Poisson family and Log link
Z = Y.loc[Y.index > '199512'].join(X, how='inner')
Z['const'] = 1
glm = sm.GLM(exog=Z[['const'] + list(X.columns)],
             endog=Z['count'],
             family=sm.families.Poisson(link=sm.families.links.Log()),
             exposure=Z['exposures'])\
        .fit()
y_pred = glm.predict(exog=Z[['const'] + list(X.columns)],
                     exposure=Z['exposures']).rename('predicted')
print(glm.summary())
```

```
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:                  count   No. Observations:                  115
Model:                            GLM   Df Residuals:                      112
Model Family:                 Poisson   Df Model:                            2
Link Function:                    Log   Scale:                          1.0000
Method:                          IRLS   Log-Likelihood:                -473.08
Date:                Mon, 03 Mar 2025   Deviance:                       519.46
Time:                        21:57:30   Pearson chi2:                     622.
No. Iterations:                     5   Pseudo R-squ. (CS):             0.9417
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -5.3070      0.032   -167.739      0.000      -5.369      -5.245
INDPRO       -24.3445      1.378    -17.665      0.000     -27.046     -21.643
NASDAQCOM     -1.4022      0.209     -6.724      0.000      -1.811      -0.994
==============================================================================
```

```python
# Plot predicted and predictors
fig, ax = plt.subplots(figsize=(10, 6))
bx = ax.twinx()
Z['count'].plot(kind='bar', width=1.0, alpha=0.4, color="C0", ax=ax)
y_pred.plot(ls='', marker='^', color="C0", ax=ax)
ax.set_ylabel('quarterly number of stocks', color="C0")
Z['INDPRO'].cumsum().plot(color="C1", alpha=.5, ax=bx)
Z['NASDAQCOM'].cumsum().plot(color="C2", alpha=.5, ax=bx)
ax.legend(loc='upper left')
bx.legend(loc='upper right')
ax.set_xticks(np.arange(0, len(Z), 12), Z.index[::12])
ax.set_title('Number of stocks with large earnings misses')
plt.tight_layout()
```



Credit losses

Suppose a financial institution has a portfolio N loans, and:

- **default risk** p_i is the probability of default of the i th loan
- **loss severity** or loss given default L_i is the portion of the i th loan lost in the event of default. This is often assumed known with certainty

Then Expected loss is the sum of Default Probability $\times$ Loss given default over all loans $= \sum_{i=1}^{N} p_i L_i$

The standard deviation of the expected loss on an individual loan, by applying the "Bernoulli shortcut" is $\sigma_i = \sqrt{p_i(1-p_i)} L_i$.

The standard deviation of the loss of the portfolio depends on the correlation of defaults between loands $\sigma_P = \sqrt{\sum_i \sum_j \rho_{ij} \sigma_i \sigma_j}$. For tractability, the correlations may be simplified to be constant or determined by a (Gaussian) copula, though neither assumption suffices to model real markets.

Actuarial Loss:

- Severity
- Frequency

References:

https://sites.google.com/site/zoubin019/teaching/math-5639-actuarial-loss-models

Lim, Terence, 2001, "Rationality and Analysts' Forecast Bias", Journal of Finance, Volume 56, Issue 1, Pages 369-385. https://doi.org/10.1111/0022-1082.00329

CHAPTER

TWENTY

SUPPLY CHAIN NETWORK GRAPHS

Forget about your competitors, just focus on your customers - Jack Ma

Supply chain networks comprise the interconnected relationships between suppliers and their customers. An effective way to study these relationships is by constructing graphs that visually and mathematically represent supply chain structures. We introduce key properties of graphical networks, such as degree distribution, clustering coefficients, and path lengths, and explore how to identify central and connected firms.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import math
import numpy as np
import pandas as pd
import networkx as nx
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
from finds.database import SQL
from finds.recipes import graph_draw
from secret import credentials
import warnings
# %matplotlib qt
VERBOSE = 0
if not VERBOSE:
    warnings.simplefilter(action='ignore', category=FutureWarning)
sql = SQL(**credentials['sql'], verbose=VERBOSE)
```

20.1 Principal customers

The relationships between upstream suppliers and downstream customers, collectively known as "supply chain" relationships, form a crucial component of economic linkages, representing the movement of goods, services, and financial transactions across industries and individual firms.

Principal customers are those that contribute more than 10% of a firm's total sales, as defined by Regulation S-K and SFAS 131. Under these disclosure requirements, suppliers are often significantly smaller than their corporate principal customers.

Retrieve principal customer relationships from the database:

```
# retrieve principal customers info
year = 2022
cust = sql.read_dataframe(f"select gvkey, cgvkey, stic, ctic, conm, cconm "
                          f"  from customer "
```

(continues on next page)

(continued from previous page)

```
                              f"    where srcdate >= {year}0101 "
                              f"      and srcdate <= {year}1231")
```

Construct a lookup table to map company tickers to their full names:

```
# construct Series to lookup company full name from ticker
lookup = pd.concat([Series(cust['conm'].values, cust['stic'].values),
                    Series(cust['cconm'].values, cust['ctic'].values)])\
           .drop_duplicates()
```

20.2 Graph properties

In a **graph**, **nodes** (also called vertices) represent entities, while **edges** denote relationships between them. The **degree** of a node is the number of edges directly connected to it.

Density measures the proportion of existing edges relative to the maximum possible edges:

- In undirected graphs: $\frac{2m}{n(n-1)}$
- In directed graphs: $\frac{m}{n(n-1)}$

where m is the number of edges and n is the number of nodes.

A **simple graph** has no self-loops (edges connecting a node to itself) and does not contain multiple edges between the same pair of nodes.

A **subgraph** is a subset of nodes and the edges that connect them. A **component** is a subgraph in which every node is connected to every other node by some path.

Key graph attributes include:

- **Weighted vs. Unweighted:** Indicates whether numerical values are assigned to edges.
- **Directed vs. Undirected:** Determines whether edges have a defined direction.
- **Cyclic vs. Acyclic:** A cyclic graph contains at least one cycle (a path that starts and ends at the same node), while an acyclic graph does not.
- **Connected vs. Disconnected:** A graph is connected if a path exists between any two nodes.
- **Weakly vs. Strongly Connected Components:** Weakly connected components have a path between every pair of nodes when edge direction is ignored, whereas strongly connected components require directed paths between every pair of nodes.

To model supply chains, we construct a directed graph with edges pointing from suppliers to their principal customers and analyze its structural properties.

```
# nodes are companies, with directed edges from supplier to customer
vertices = set(cust['stic']).union(cust['ctic'])
edges = cust[['stic', 'ctic']].values.tolist()  # supplier --> customer
```

```
# Populate networkx directed graph with nodes and edges
DG = nx.DiGraph()
DG.add_nodes_from(vertices)
DG.add_edges_from(edges)
```

Display graph properties:

```python
# Helper to display graph properties
def graph_info(G):
    out = dict()
    out['is_directed'] = nx.is_directed(G)
    out['num_edges'] = nx.number_of_edges(G)
    out['num_nodes'] = nx.number_of_nodes(G)
    out['num_selfloops'] = nx.number_of_selfloops(G)
    out['density'] = nx.density(G)

    out['is_weighted'] = nx.is_weighted(G)
    if nx.is_weighted(G):
        out['is_negatively_weighted'] = nx.is_negatively_weighted(G)

    # Components
    if nx.is_directed(G):
        out['is_weakly_connected'] = nx.is_weakly_connected(G)
        out['weakly_connected_components'] = nx.number_weakly_connected_components(G)
        out['size_largest_weak_component'] = len(max(
            nx.weakly_connected_components(G), key=len))
        out['is_strongly_connected'] = nx.is_strongly_connected(G)
        out['strongly_connected_components'] = nx.number_strongly_connected_
↪components(G)
        out['size_largest_strong_component'] = len(max(
            nx.strongly_connected_components(G), key=len))
    else:
        out['is_connected'] = nx.is_connected(G)
        out['connected_components'] = nx.number_connected_components(G)
        out['size_largest_component'] = len(max(
            nx.connected_components(G), key=len))
    return out
```

```python
Series(graph_info(DG)).rename('Principal Customers Graph').to_frame()
```

```
                              Principal Customers Graph
is_directed                                        True
num_edges                                          2561
num_nodes                                          1696
num_selfloops                                         5
density                                        0.000891
is_weighted                                       False
is_weakly_connected                               False
weakly_connected_components                         111
size_largest_weak_component                        1398
is_strongly_connected                             False
strongly_connected_components                      1692
size_largest_strong_component                         3
```

```python
# remove self-loops, if any
DG.remove_edges_from(nx.selfloop_edges(DG))
```

20.2.1 Clustering coefficient

The clustering coefficient measures how nodes in a graph tend to cluster together.

- A **triplet** consists of three nodes connected by two (open triplet) or three (closed triplet) edges.
- A **triangle** comprises three closed triplets.
- The **clustering coefficient** of a node is given by:

$$\frac{2 \times \text{\# of triangles}}{\text{degree} \times (\text{degree} - 1)}$$

- **Graph transitivity** is the fraction of all possible triangles that are actually present: $3 \times \frac{\text{\# of triangles}}{\text{\# of triads}}$

```
# as an undirected graph
G = nx.Graph(DG)
DataFrame({'transitivity': nx.transitivity(G),
           'average clustering': nx.average_clustering(G)},
          index=['clustering'])
```

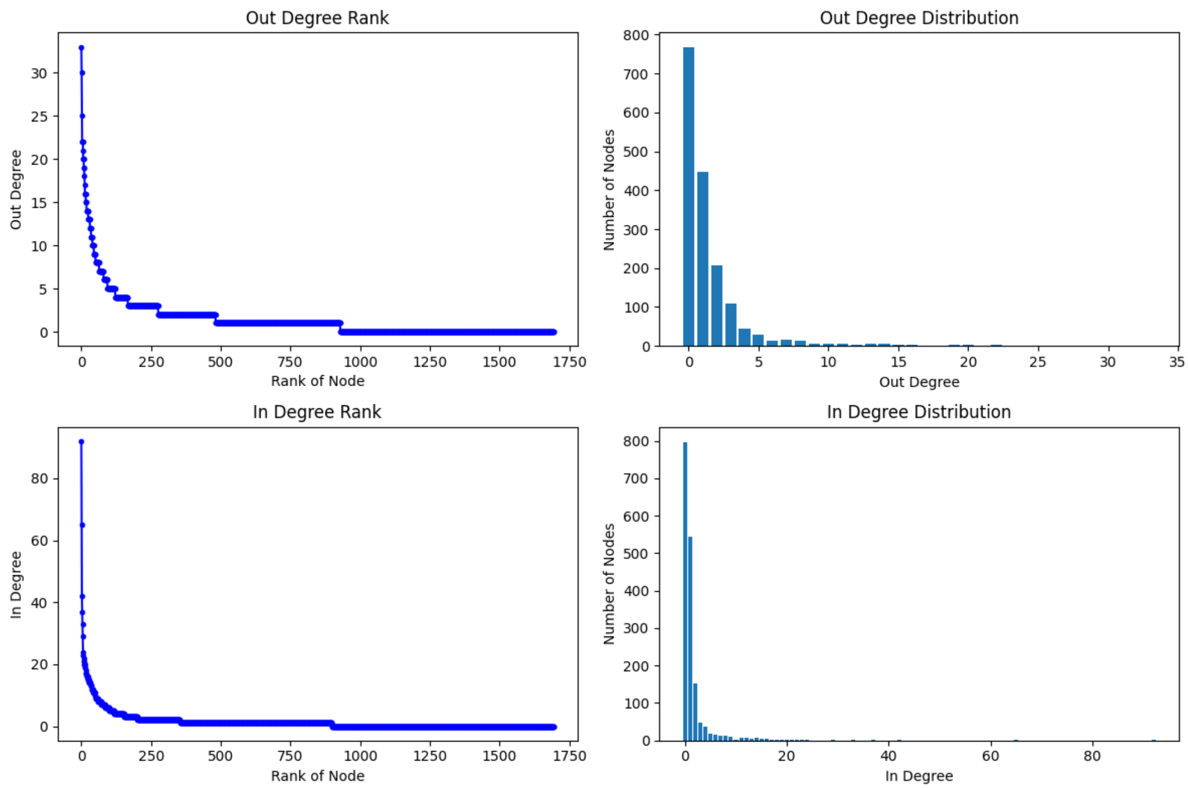
```
            transitivity  average clustering
clustering      0.007702            0.011649
```

20.2.2 Degree analysis

In directed graphs:

- **Out-degree** represents the number of edges pointing out from a node.
- **In-degree** represents the number of edges pointing into a node.

```
# Plot distribution of degrees
degrees = {'Out Degree': sorted((d for n, d in DG.out_degree()), reverse=True),
           'In Degree': sorted((d for n, d in DG.in_degree()), reverse=True)}
fig, axes = plt.subplots(nrows=2, ncols=2, figsize=(12, 8))
for ax, direction in zip(axes, degrees):
    ax[0].plot(degrees[direction], "b-", marker=".")
    ax[0].set_title(f"{direction} Rank")
    ax[0].set_ylabel(f"{direction}")
    ax[0].set_xlabel("Rank of Node")
    ax[1].bar(*np.unique(degrees[direction], return_counts=True))
    ax[1].set_title(f"{direction} Distribution")
    ax[1].set_xlabel(f"{direction}")
    ax[1].set_ylabel("Number of Nodes")
plt.tight_layout()
plt.show()
```



20.2.3 Ego network

An **ego** is a focal node in a network. Each node in a network can serve as an ego. A **neighborhood** consists of the ego and all nodes directly connected to it.

For analysis, we select the node with the highest degree as the focal node and construct its one-step neighborhood, capturing immediate suppliers and principal customers.

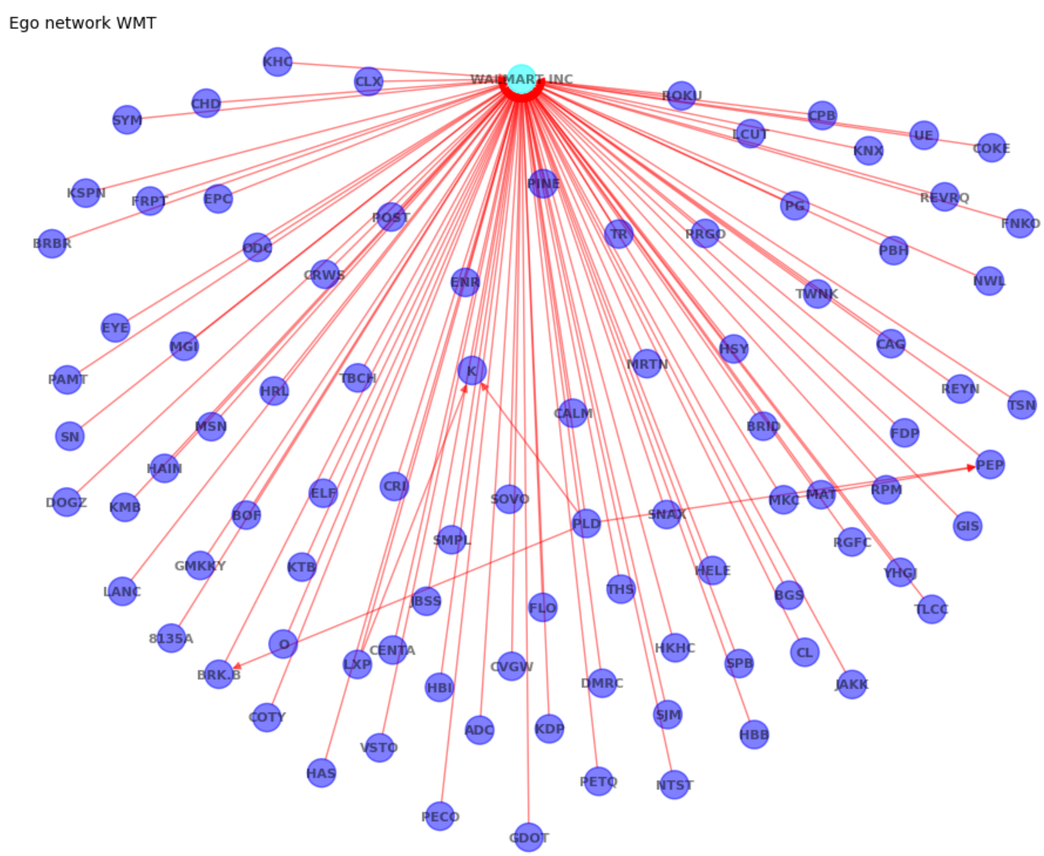
```
# find node with greatest degree
(ego, degree) = max(G.degree(), key=lambda x: x[1])

# build subgraph of ego and neighbors
all_neighbors = list(nx.all_neighbors(DG, ego))  # predecessors and successors
neighbors = list(nx.neighbors(DG, ego))           # successors only
ego_graph = DG.subgraph([ego] + all_neighbors).copy()
```

```
# Plot ego graph
node_color = (dict.fromkeys(all_neighbors, 'b')   # plot predecessor (supplier) nodes␣
↪in blue
              | dict.fromkeys(neighbors, 'g')      # plot successor (customer) nodes␣
↪in green
              | {ego: 'cyan'})

labels = ({ticker: ticker for ticker in ego_graph.nodes}
          | {ticker: lookup[ticker] for ticker in [ego] + neighbors})

graph_draw(ego_graph, figsize=(10, 8), node_size=300, seed=42,
```

(continues on next page)

(continued from previous page)

```
            width=1, node_color=node_color, labels=labels, style='-',
            title=f"Ego network {ego}")
plt.show()
```



20.2.4 Path lengths

The **distance** or **path length** between two nodes is the shortest path (fewest number of edges) between them.

- **Eccentricity** of a node is the maximum distance from that node to any other node in the graph.
- **Diameter** is the maximum eccentricity across all nodes in the graph.

To analyze path lengths, we identify the largest connected component of the graph and compute the longest undirected path within it.

```
# find component with the longest diameter
components = list(nx.weakly_connected_components(DG))
nodes = components[np.argmax([nx.diameter(G.subgraph(c)) for c in components])]
```

```
# compute all shortest path lengths, and determine the longest
best_length = 0
```

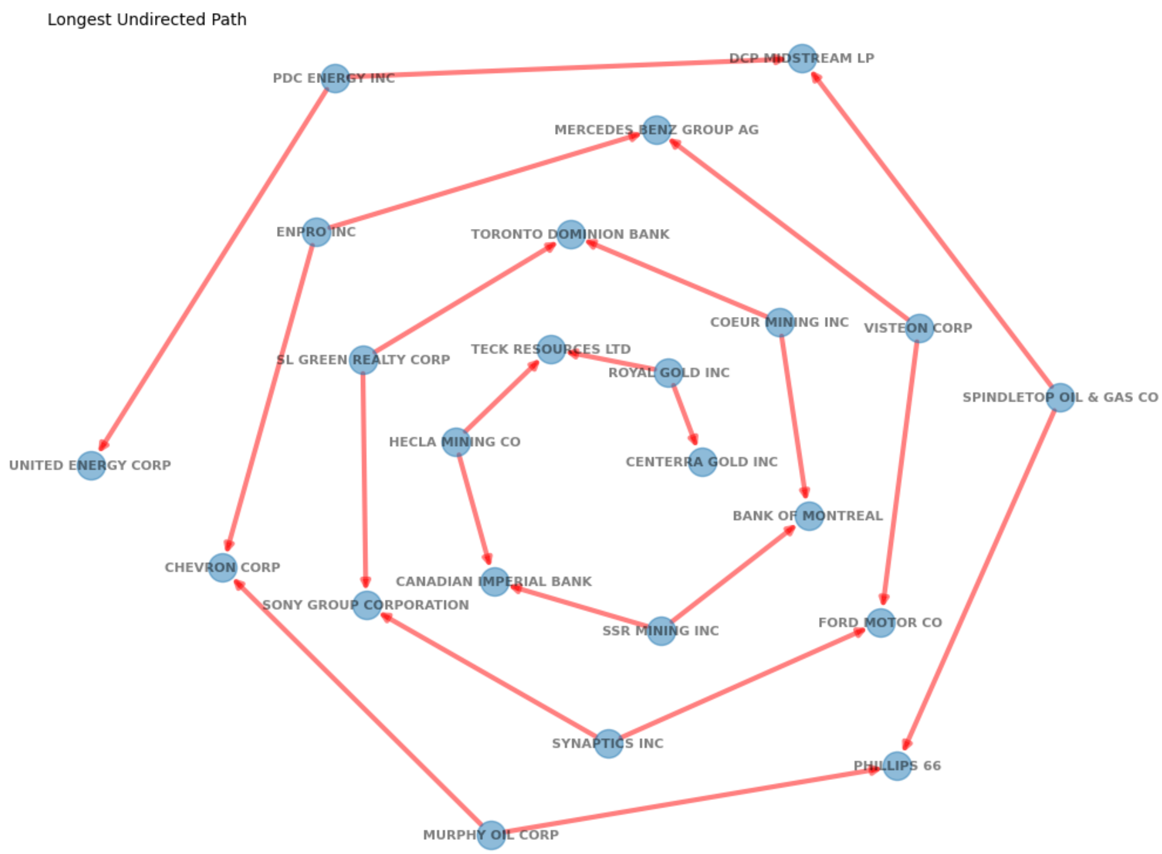
(continues on next page)

(continued from previous page)

```
for src, targets in dict(nx.shortest_path(G.subgraph(nodes))).items():
    for tgt, path in targets.items():
        length = len(path)
        if length > best_length:
            best_length = length
            best_path = path
```

```
# Archimedean spiral: r = b theta
pos = {ticker: ((n + 5) * math.cos(n), (n + 5) * math.sin(n))
       for n, ticker in enumerate(best_path)}

# Plot nodes in a spiral graph
labels = {ticker: lookup[ticker] for ticker in best_path}
graph_draw(DG.subgraph(best_path), figsize=(10, 8), node_size=300,
           width=3, labels=labels, style='-',  pos=pos,
           title=f"Longest Undirected Path")
```

```
{'CGAU': (5.0, 0.0),
 'RGLD': (3.2418138352088386, 5.048825908847379),
 'TECK': (-2.9130278558299967, 6.365081987779772),
 'HL': (-7.919939972803563, 1.1289600644789377),
 'CM': (-5.882792587772507, -6.811222457771354),
 'SSRM': (2.8366218546322624, -9.589242746631385),
 'BMO': (10.561873153154025, -3.0735704801881845),
 'CDE': (9.046827052119655, 7.883839184625469),
 'TD': (-1.8915004395119759, 12.861657206103963),
 'SLG': (-12.755823666385478, 5.769658793384592),
 'SONY': (-12.586072936146786, -8.160316663340547),
 'SYNA': (0.07081116780881257, -15.999843304811256),
 'F': (14.345517298452366, -9.121739606007393),
 'VC': (16.33404206610353, 7.563006662879537),
 'MBGYY': (2.5980071459488383, 18.821539758202537),
 'NPO': (-15.193758257176427, 13.005756803142337),
 'CVX': (-20.110849086791077, -6.045969649966372),
 'MUR': (-6.053593437135133, -21.15074482135025),
 'PSX': (15.187284289613844, -17.272706675748548),
 'SPND': (23.72891083648006, 3.597053031910856),
 'DCP': (10.202051545334799, 22.82363126819069),
 'PDCE': (-14.240960765830978, 21.75304660193746),
 'UNRG': (-26.998942312655203, -0.23898535084090466)}
```



References:

Ling Cen and Sudipto Dasgupta, 2021, The Economics and Finance of Customer-Supplier Relationships, Oxford Research Encyclopedia of Economics and Finance.

Cen, Ling, et al. "Customer–supplier relationships and corporate tax avoidance." Journal of Financial Economics 123.2 (2017): 377-394.

Cohen, Lauren, and Andrea Frazzini. "Economic links and predictable returns." The Journal of Finance 63.4 (2008): 1977-2011.

Wharton Research Data Services.

CHAPTER

TWENTYONE

INDUSTRY COMMUNITY DETECTION

Realize that everything connects to everything else - Leonardo DaVinci

Traditional industry classification systems, such as SIC and NAICS, group firms based on production processes or product similarities. Natural language processing techniques can be leveraged to analyze product descriptions and capture dynamic changes in industry structures over time, as proposed by Hoberg and Phillips (2016). Industry communities can be detected through network analysis, where firms are modeled as nodes in a graph, and connections between them are determined by similarities in their product and market descriptions.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import zipfile
import io
from itertools import chain
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import seaborn as sns
import networkx as nx
from finds.database import SQL
from finds.readers import requests_get, Sectoring
from finds.structured import BusDay, PSTAT
from finds.recipes import graph_info
from secret import credentials
# %matplotlib qt
VERBOSE = 0
```

```
sql = SQL(**credentials['sql'], verbose=VERBOSE)
bd = BusDay(sql)
pstat = PSTAT(sql, bd, verbose=VERBOSE)
```

21.1 Industry taxonomy

Industry classification, or industry taxonomy, organizes companies into groups based on shared characteristics such as production processes, product offerings, or financial market behaviors.

21.1.1 Text-based industry classification

Hoberg and Phillips (2016) developed a text-based measure of firm similarity by analyzing product descriptions in 10-K filings. They construct firm-by-firm similarity scores using word vectors, filtering out common words and focusing on nouns and proper nouns, while excluding geographic terms. The similarity between firms is quantified using cosine similarity, creating a pairwise similarity matrix across firms and years.

Since Item 101 of Regulation S-K mandates that firms accurately describe their key products in their 10-K filings, the TNIC scheme, based on textual similarity, provides a dynamic classification system that evolves with market changes. This method offers a more flexible alternative to traditional classification systems, capturing shifts in product markets over time.

Source: Hoberg and Phillips Industry Classification

The TNIC pair-wise firm similiarities are retrieved from the Hoberg and Phillips website:

```
# Retrieve TNIC scheme from Hoberg and Phillips website
tnic_scheme = 'tnic3'
root = 'https://hobergphillips.tuck.dartmouth.edu/idata/'
source = root + tnic_scheme + '_data.zip'
if source.startswith('http'):
    response = requests_get(source)
    source = io.BytesIO(response.content)

# extract the csv file from zip archive
with zipfile.ZipFile(source).open(tnic_scheme + "_data.txt") as f:
    tnic_data = pd.read_csv(f, sep='\s+')

# extract latest year of tnic as data frame
year = max(tnic_data['year']) # [1989, 1999, 2009, 2019]
tnic = tnic_data[tnic_data['year'] == year].dropna()
tnic
```

```
          year  gvkey1  gvkey2   score
26307358  2023    1004    1823  0.0127
26307359  2023    1004    4091  0.0087
26307360  2023    1004    5567  0.0063
26307361  2023    1004    9698  0.0075
26307362  2023    1004   10519  0.0191
...        ...     ...     ...     ...
26973403  2023  351038  329141  0.0684
26973404  2023  351038  331856  0.0769
26973405  2023  351038  332115  0.1036
26973406  2023  351038  347007  0.0731
26973407  2023  351038  349972  0.0871

[666050 rows x 4 columns]
```

21.1.2 Industry classification

Industry classification systems such as SIC and NAICS follow hierarchical structures to categorize firms based on their economic activities:

- **Standard Industrial Classification (SIC)**: Uses a 2-digit, 3-digit, and 4-digit hierarchy to classify industries.
- **North American Industry Classification System (NAICS)**: Expands classification granularity from 2-digit to 6-digit levels.

```
# populate dataframe of nodes with gvkey (as index), permno, sic and naics
nodes = DataFrame(index=sorted(set(tnic['gvkey1']).union(tnic['gvkey2'])))\
        .rename_axis(index='gvkey')
for code in ['lpermno', 'sic', 'naics']:
    lookup = pstat.build_lookup('gvkey', code, fillna=0)
    nodes[code] = lookup(nodes.index)
Series(np.sum(nodes > 0, axis=0)).rename('Non-missing').to_frame().T
```

```
             lpermno   sic  naics
Non-missing     3829  3829   3827
```

```
# supplement naics and sic with crosswalks in Sectoring class
naics = Sectoring(sql, 'naics', fillna=0)   # supplement from crosswalk
sic = Sectoring(sql, 'sic', fillna=0)
nodes['naics'] = nodes['naics'].where(nodes['naics'] > 0, naics[nodes['sic']])
nodes['sic'] = nodes['sic'].where(nodes['sic'] > 0, sic[nodes['naics']])
Series(np.sum(nodes > 0, axis=0)).rename('Non-missing').to_frame().T
```

```
             lpermno   sic  naics
Non-missing     3829  3829   3829
```

21.1.3 Sector groups

Industry taxonomies group detailed classifications into broader sectors for economic analysis:

- Fama and French aggregate 4-digit SIC codes into industry groups consisting of 5, 10, 12, 17, 30, 38, 48, or 49 sectors.
- The Bureau of Economic Analysis (BEA) consolidates 6-digit NAICS codes into summary-level industry groups, with updates in 1947, 1963, and 1997.

```
# include sectoring schemes
codes = {'sic': ([f"codes{c}" for c in [5, 10, 12, 17, 30, 38, 48, 49]]
                 + ['sic2', 'sic3']),
         'naics': ['bea1947', 'bea1963', 'bea1997']}
sectorings = {}   # store Sectoring objects
for key, schemes in codes.items():
    for scheme in schemes:
        if scheme not in sectorings:

            # missing value is integer 0 sic2 and sic3 shemes, else string ''
            fillna = 0 if scheme.startswith('sic') else ''

            # load the sectoring class from SQL
```

(continues on next page)

(continued from previous page)

```
            sectorings[scheme] = Sectoring(sql, scheme, fillna=fillna)

            # apply the sectoring scheme to partition the nodes
            nodes[scheme] = sectorings[scheme][nodes[key]]

        # keep nodes with non-missing data
        nodes = nodes[nodes[scheme].ne(sectorings[scheme].fillna)]
        print(len(nodes), scheme)
nodes
```

```
3845 codes5
3845 codes10
3845 codes12
3845 codes17
3845 codes30
3845 codes38
3845 codes48
3845 codes49
3829 sic2
3829 sic3
3561 bea1947
3561 bea1963
3561 bea1997
```

```
        lpermno   sic   naics codes5 codes10 codes12 codes17 codes30 codes38  \
gvkey
1004      54594  5080  423860  Cnsmr   Shops   Shops   Machn   Whlsl   Whlsl
1045      21020  4512  481111  Other   Durbl   Durbl   Trans   Trans   Trans
1050      11499  3564  333413  Manuf   Manuf   Manuf   Machn   FabPr   Machn
1076      10517  6141  522220  Other   Other   Money   Finan     Fin   Money
1078      20482  3845  334510   Hlth    Hlth    Hlth   Other    Hlth   Instr
...         ...   ...     ...    ...     ...     ...     ...     ...     ...
345980    20333  5961  455110  Cnsmr   Shops   Shops   Rtail   Rtail   Rtail
347007    15533  2836  325414   Hlth    Hlth    Hlth   Other    Hlth   Chems
349337    20867  3845  334510   Hlth    Hlth    Hlth   Other    Hlth   Instr
349972    15642  2836  325414   Hlth    Hlth    Hlth   Other    Hlth   Chems
351038    16161  2834  325412   Hlth    Hlth    Hlth   Cnsum    Hlth   Chems

       codes48 codes49  sic2  sic3 bea1947 bea1963 bea1997
gvkey
1004     Whlsl   Whlsl    50   508      42      42      42
1045     Trans   Trans    45   451      48     481     481
1050      Mach    Mach    35   356     333     333     333
1076     Banks   Banks    61   614      52   521CI   521CI
1078     MedEq   MedEq    38   384     334     334     334
...        ...     ...   ...   ...     ...     ...     ...
345980   Rtail   Rtail    59   596    44RT    44RT     4A0
347007   Drugs   Drugs    28   283     325     325     325
349337   MedEq   MedEq    38   384     334     334     334
349972   Drugs   Drugs    28   283     325     325     325
351038   Drugs   Drugs    28   283     325     325     325

[3561 rows x 16 columns]
```

21.2 Community structure

In network analysis, **community structure** refers to the clustering of nodes (firms) into partitions (groups) based on connectivity patterns. Identifying these communities helps reveal hidden industry relationships and competitive dynamics.

```
# populate undirected graph with tnic edges
edges = tnic[tnic['gvkey1'].isin(nodes.index) & tnic['gvkey2'].isin(nodes.index)]
edges = list(
    edges[['gvkey1', 'gvkey2', 'score']].itertuples(index=False, name=None))
G = nx.Graph()
G.add_weighted_edges_from(edges)
G.remove_edges_from(nx.selfloop_edges(G))  # remove self-loops: not necessary
Series(graph_info(G, fast=True)).rename(year).to_frame()
```

```
                            2023
transitivity            0.877035
average_clustering      0.575643
connected                  False
connected_components           9
size_largest_component      3523
directed                   False
weighted                    True
negatively_weighted        False
edges                     320352
nodes                       3541
selfloops                      0
density                 0.051113
```

21.2.1 Measuring partitions

The quality of graph partitions can be evaluated using modularity, a measure that assesses the strength of community structures by comparing observed connections within clusters to a random network model.

```
# evaluate modularity of sectoring schemes on TNIC graph
def community_quality(G, communities):
    """helper to measure quality of partitions"""
    out = {'communities': len(communities)}
    out['modularity'] = nx.community.modularity(G, communities)
    (out['coverage'],
     out['performance']) = nx.community.partition_quality(G, communities)
    return out
```

```
modularity = {}   # to collect measurements of each scheme
for scheme in sorted(chain(*codes.values())):
    communities = nodes.loc[list(G.nodes), scheme]\
                       .reset_index()\
                       .groupby(scheme)['gvkey']\
                       .apply(list)\
                       .to_list()    # list of lists of node labels
    modularity[scheme] = community_quality(G, communities)
df = DataFrame.from_dict(modularity, orient='index').sort_index()
print(f"Quality of sectoring schemes on TNIC graph ({year})")
df
```

```
Quality of sectoring schemes on TNIC graph (2023)
```

	communities	modularity	coverage	performance
bea1947	40	0.330481	0.779187	0.925859
bea1963	58	0.324246	0.734745	0.948296
bea1997	61	0.324169	0.734514	0.948689
codes10	10	0.335843	0.940503	0.850069
codes12	12	0.336655	0.938187	0.878268
codes17	17	0.285847	0.766719	0.794675
codes30	30	0.335544	0.934385	0.899115
codes38	36	0.333800	0.793237	0.890785
codes48	48	0.331168	0.752610	0.944559
codes49	49	0.331003	0.751526	0.951096
codes5	5	0.337074	0.945045	0.818984
sic2	67	0.327541	0.743694	0.942476
sic3	226	0.288389	0.690952	0.958297

21.2.2 Detecting partitions

Community detection in graphs can be performed using various algorithms, including:

- **Label Propagation Algorithm**: This method assigns an initial label to each node and iteratively updates labels based on the majority of its neighbors' labels, allowing communities to form dynamically. It is fast and works well for large networks but may produce different results on different runs due to randomness.
- **Louvain Method**: This hierarchical clustering algorithm optimizes modularity by iteratively merging small communities into larger ones, maximizing intra-community connections while minimizing inter-community edges.
- **Greedy Algorithm**: This algorithm builds communities by iteratively merging pairs of nodes or groups that result in the largest modularity gain, prioritizing locally optimal choices.

```python
# Run community detection algorithms
def community_detection(G):
    """Helper to run community detection algorithms on an undirected graph"""
    out = {}
    out['label'] = nx.community.label_propagation_communities(G)
    out['louvain'] = nx.community.louvain_communities(G, resolution=1)
    out['greedy'] = nx.community.greedy_modularity_communities(G, resolution=1)
    return out
```

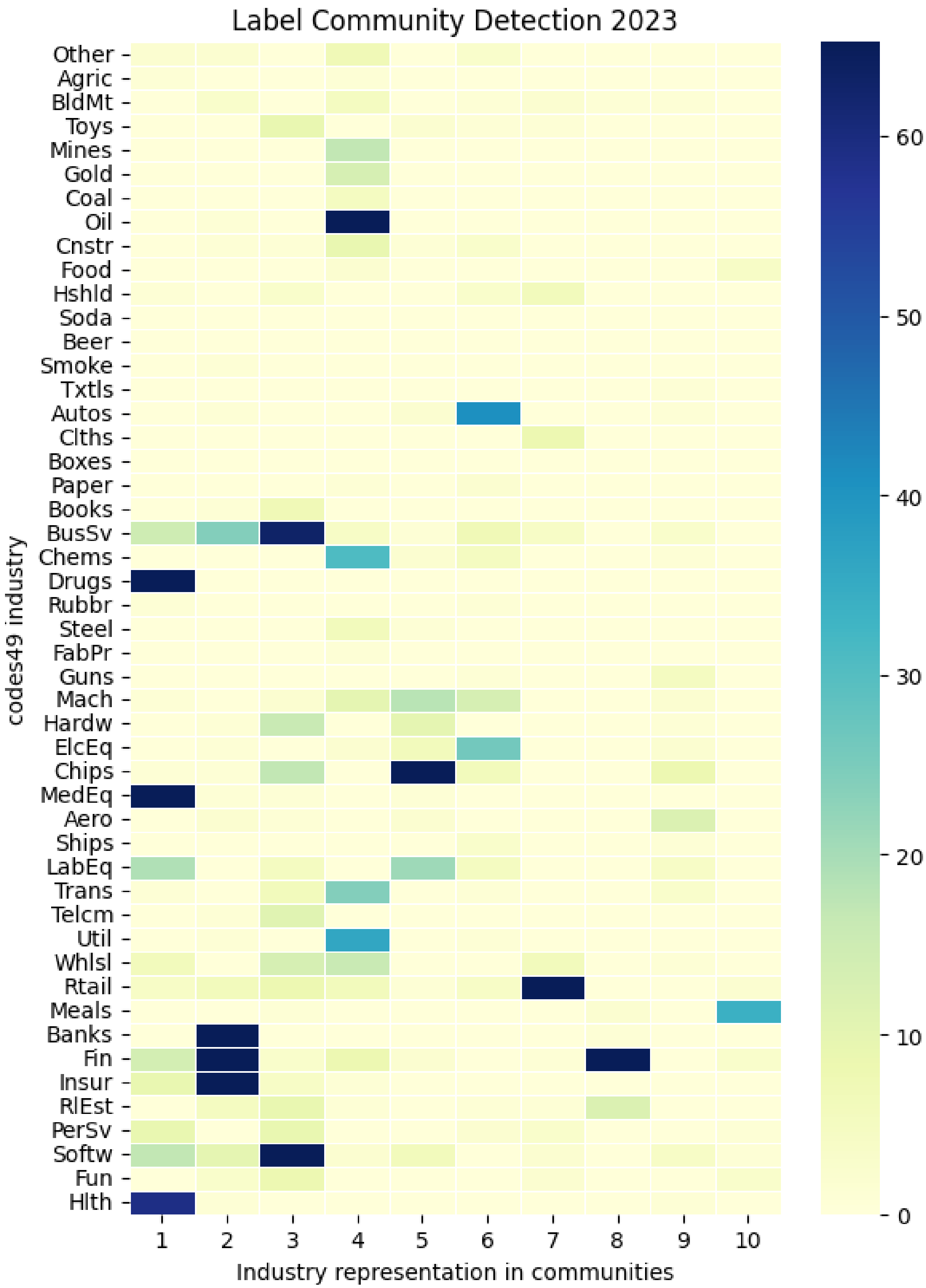
```python
communities = community_detection(G)
quality = {}
for key, community in communities.items():
    quality[key] = community_quality(G, community)
```

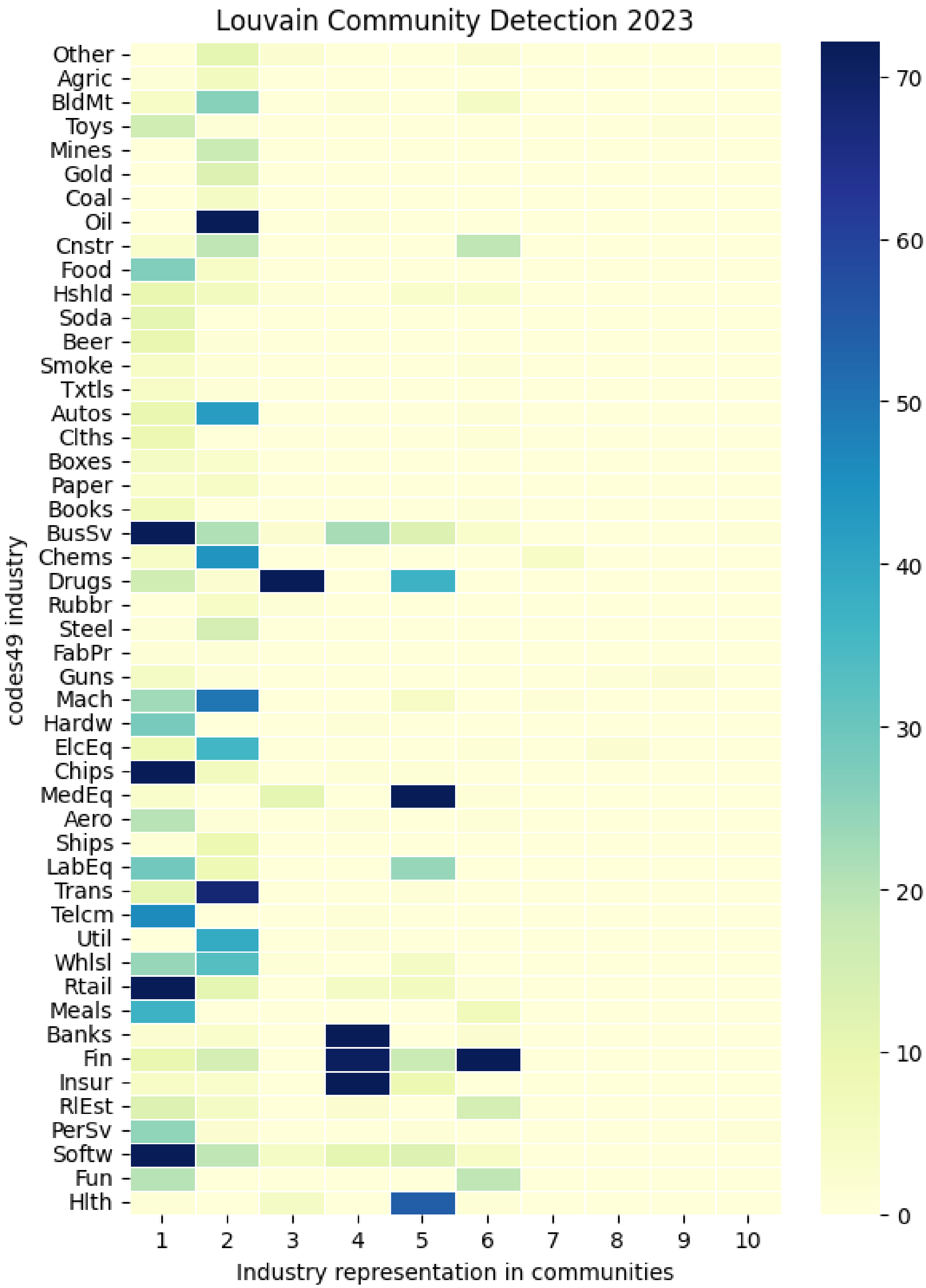
```python
df = DataFrame.from_dict(quality, orient='index').sort_index()
print(f"Modularity of community detection algorithms on TNIC graph ({year})")
df
```

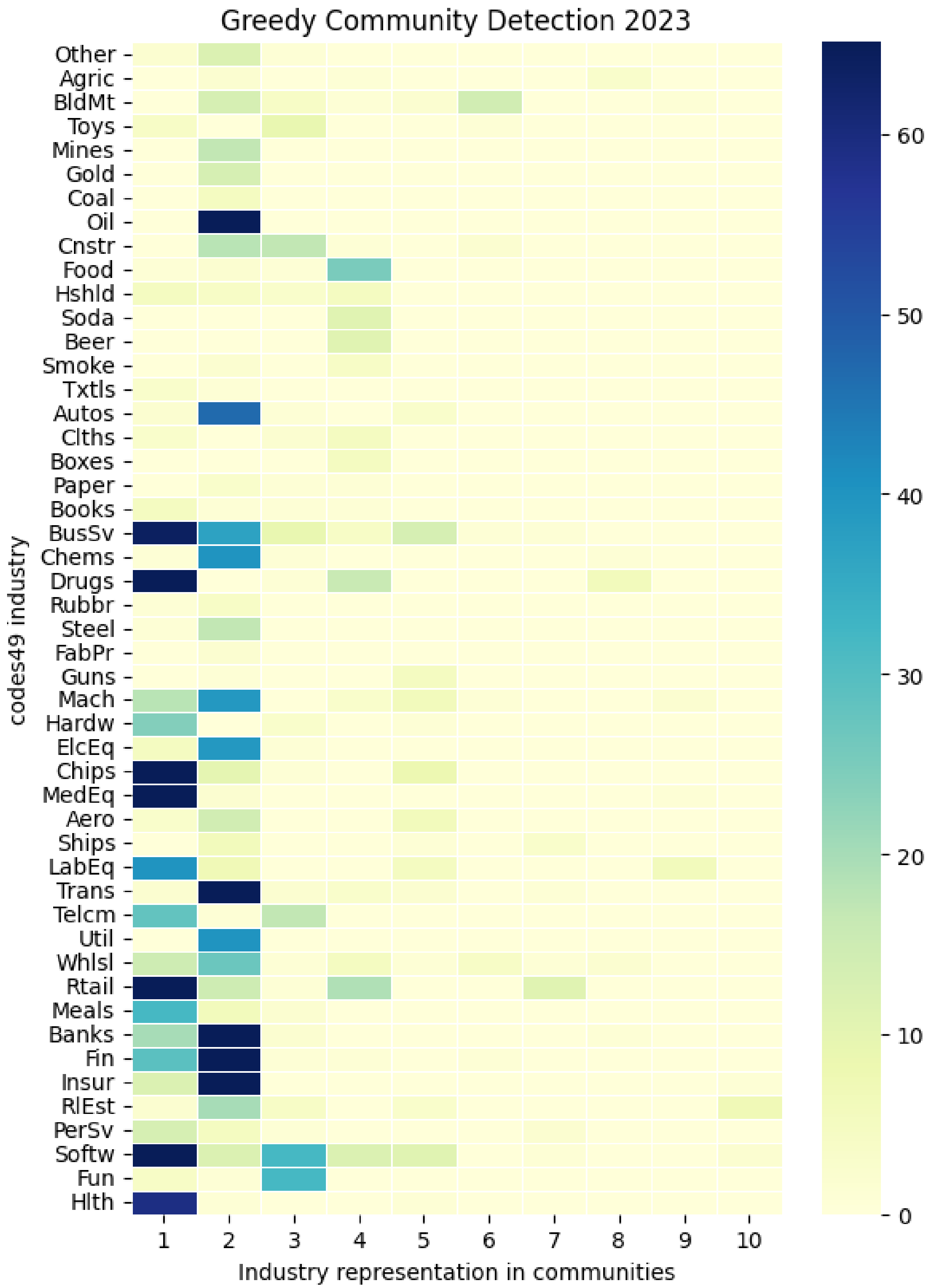
```
Modularity of community detection algorithms on TNIC graph (2023)
```

```
          communities  modularity  coverage  performance
greedy             51    0.323848  0.989711     0.689093
label             101    0.347795  0.990485     0.909486
louvain            19    0.354818  0.824855     0.838868
```

```python
# Visualize Fama-French 49-industries in the detected communities
key = 'codes49'
for ifig, detection in enumerate(communities.keys()):

    # count industries represented in each partition
    industry = []
    communities_sequence = sorted(communities[detection], key=len, reverse=True)
    for i, community in enumerate(communities_sequence):
        industry.append(nodes[key][list(community)].value_counts().rename(i+1))
    names = sectorings[key].sectors['name'].drop_duplicates(keep='first')
    df = pd.concat(industry, axis=1)\
           .dropna(axis=0, how='all')\
           .fillna(0)\
           .astype(int)\
           .reindex(names)

    # display as heatmap
    fig, ax = plt.subplots(num=ifig+1, clear=True, figsize=(6, 8))
    sns.heatmap(df.iloc[:,:10],
                square=False,
                linewidth=.5,
                ax=ax,
                yticklabels=1,
                cmap="YlGnBu",
                robust=True)
    if scheme.startswith('bea'):
        ax.set_yticklabels(Sectoring._bea_industry[df.index], size=10)
    else:
        ax.set_yticklabels(df.index, size=10)
    ax.set_title(f'{detection.capitalize()} Community Detection {year}')
    ax.set_xlabel(f"Industry representation in communities")
    ax.set_ylabel(f"{key} industry")
    fig.subplots_adjust(left=0.4)
    plt.tight_layout(pad=0)
```


Label Community Detection 2023
Other
Agric
BldMt
Toys
Mines
Gold
Coal
Oil
Cnstr
Food
Hshld
Soda
Beer
Smoke
Txtls
Autos
Clths
Boxes
Paper
Books
BusSv
Chems
Drugs
Rubbr
Steel
FabPr
Guns
Mach
Hardw
ElcEq
Chips
MedEq
Aero
Ships
LabEq
Trans
Telcm
Util
Whlsl
Rtail
Meals
Banks
Fin
Insur
RlEst
PerSv
Softw
Fun
Hlth
codes49 industry
1 2 3 4 5 6 7 8 9 10
Industry representation in communities
0 10 20 30 40 50 60


Louvain Community Detection 2023
Other
Agric
BldMt
Toys
Mines
Gold
Coal
Oil
Cnstr
Food
Hshld
Soda
Beer
Smoke
Txtls
Autos
Clths
Boxes
Paper
Books
BusSv
Chems
Drugs
Rubbr
Steel
FabPr
Guns
Mach
Hardw
ElcEq
Chips
MedEq
Aero
Ships
LabEq
Trans
Telcm
Util
Whlsl
Rtail
Meals
Banks
Fin
Insur
RlEst
PerSv
Softw
Fun
Hlth
codes49 industry
1
2
3
4
5
6
7
8
9
10
Industry representation in communities
70
60
50
40
30
20
10
0



References

Gerard Hoberg and Gordon Phillips, 2016, Text-Based Network Industries and Endogenous Product Differentiation.Journal of Political Economy 124 (5), 1423-1465.

Gerard Hoberg and Gordon Phillips, 2010, Product Market Synergies and Competition in Mergers and Acquisitions: A Text-Based Analysis. Review of Financial Studies 23 (10), 3773-3811.

CHAPTER

TWENTYTWO

INPUT-OUTPUT GRAPH CENTRALITY

And so we all matter - maybe less then a lot but always more than none - John Green

Input-output analysis in economics models the interdependencies between sectors by tracking the flow of goods and services. Graph centrality measures are valuable tools for analyzing the structure and dynamics of such networks. By representing input-output data published by the Bureau of Economic Analysis (BEA) as a directed graph (with sectors as nodes and transactions as edges from producers to consumers), we can apply centrality metrics to identify the most influential sectors driving overall economic activity.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import pandas as pd
from pandas import DataFrame, Series
import networkx as nx
from finds.database import RedisDB
from finds.readers import BEA
from finds.recipes import graph_info, graph_draw
from secret import credentials
# %matplotlib qt
VERBOSE = 0
pd.set_option('display.max_rows', None)
pd.set_option('display.max_colwidth', 200)
```

```
rdb = RedisDB(**credentials['redis'])
bea = BEA(rdb, **credentials['bea'], verbose=VERBOSE)
years = [1947, 2023]
vintages = [1997, 1963, 1947]   # when sectoring schemes were revised
vintage = min(vintages)
```

22.1 Centrality measures

Graph centrality measures help quantify the importance of nodes within a network, providing insight into their roles and influence.

- **Degree Centrality** measures the number of edges connected to a node, indicating its direct level of activity.
- **Betweenness Centrality** quantifies the extent to which a node lies on the shortest paths between other nodes, capturing its role in network connectivity.
- **Closeness Centrality** assesses how close a node is to all other nodes based on shortest path distances, highlighting its accessibility.
- **Eigenvector Centrality** evaluates a node’s importance based on the centrality of its neighbors, giving higher scores to nodes connected to other influential nodes.

- **PageRank Centrality** was originally developed to rank web pages and operates as a random walk process, estimating the long-term likelihood of visiting each node by following edges.
- **Hubs** are nodes with many outgoing edges, acting as facilitators of flow to other parts of the network.
- **Authorities** are nodes with many incoming edges, representing highly influential entities that are frequently referenced by other nodes.

```
# Helper to compute centrality measures
def nodes_centrality(G, weight='weight', cost=False, alpha=0.99):
    """Return dict of vertex centrality measures

    Args:
        G: Graph may be directed or indirected, weighted or unweighted
        weight: name of edge attribute for weights, Set to None for unweighted
        cost: If True, then weights are costs; else weights are importances
    """
    out = {}
    # Degree centrality, for directed and undirected graphs
    if nx.is_directed(G):
        out['in_degree'] = nx.in_degree_centrality(G)
        out['out_degree'] = nx.out_degree_centrality(G)
    else:
        out['degree'] = nx.degree_centrality(G)

    # Hubs and Authorities
    out['hub'], out['authority'] = nx.hits(G)

    # if weights are costs, then Eigenvector and Pagerank ignore weights
    if not cost and nx.is_weighted(G):
        out['eigenvector'] = nx.eigenvector_centrality(G, weight=weight, max_
↪iter=1000)
        out['pagerank'] = nx.pagerank(G, weight=weight, alpha=alpha)
    else:
        out['eigenvector'] = nx.eigenvector_centrality(G, max_iter=1000)
        out['pagerank'] = nx.pagerank(G, alpha=alpha)

    # if weights are importances, then Betweeness and Closeness ignore weights
    if cost and nx.is_weighted(G):
        out['betweenness'] = nx.betweenness_centrality(G, weight=weight)
        out['closeness'] = nx.closeness_centrality(G, distance=weight)
    else:
        out['betweenness'] = nx.betweenness_centrality(G)
        out['closeness'] = nx.closeness_centrality(G)
    return out
```

22.2 BEA Input-Output Use Tables

Input-output analysis is an economic modeling technique used to study interdependencies among different sectors. It involves constructing input-output tables that track the flow of goods and services between industries, quantifying how changes in one sector impact others.

```
# Read IOUse tables from BEA website
ioUses = {year: bea.read_ioUse(year=year, vintage=vintage) for year in years}
```

To analyze these relationships, we construct a directed graph from the latest BEA input-output flows, where edges flow from user sectors to make sectors. This allows us to visualize sectoral interactions and determine the most influential industries.

```python
## Direction of edges point from user industry to maker, i.e. follows the money
tail = 'colcode'    # edges follow flow of payments, from column to row
head = 'rowcode'
drop = ('F','T','U','V','Other')   # drop these codes

# Display node centrality measures from latest year
ioUse = ioUses[max(years)]
data = ioUse[(~ioUse['rowcode'].str.startswith(drop) &
              ~ioUse['colcode'].str.startswith(drop))].copy()

# extract cross data; generate and load edges (as tuples) to graph
data = data[(data['colcode'] != data['rowcode'])]
data['weights'] = data['datavalue'] / data['datavalue'].sum()
edges = data.loc[data['weights'] > 0, [tail, head, 'weights']].values.tolist()
G = nx.DiGraph()
G.add_weighted_edges_from(edges, weight='weight')

# Display graph properties
Series(graph_info(G)).rename('Properties').to_frame()
```

```
                               Properties
transitivity                     0.903811
average_clustering               0.849259
weakly_connected                     True
weakly_connected_components             1
size_largest_weak_component            47
strongly_connected                  False
strongly_connected_components           4
size_largest_strong_component          44
directed                             True
weighted                             True
negatively_weighted                 False
edges                                1689
nodes                                  47
selfloops                               0
density                          0.781221
```

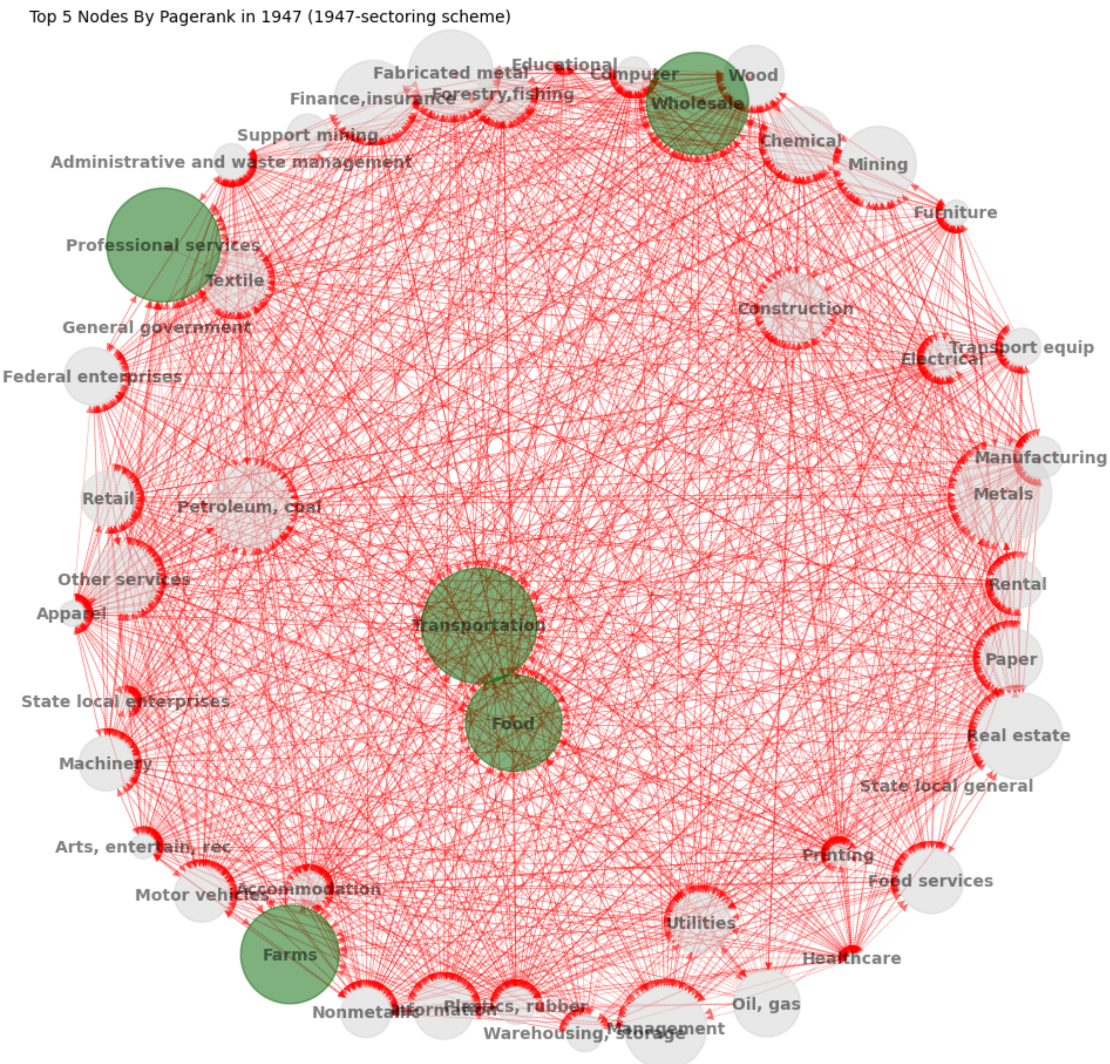
Vizualise graphs of input-output flows for the earliest and latest years available, highlighting the top five sectors with the highest PageRank centrality scores.

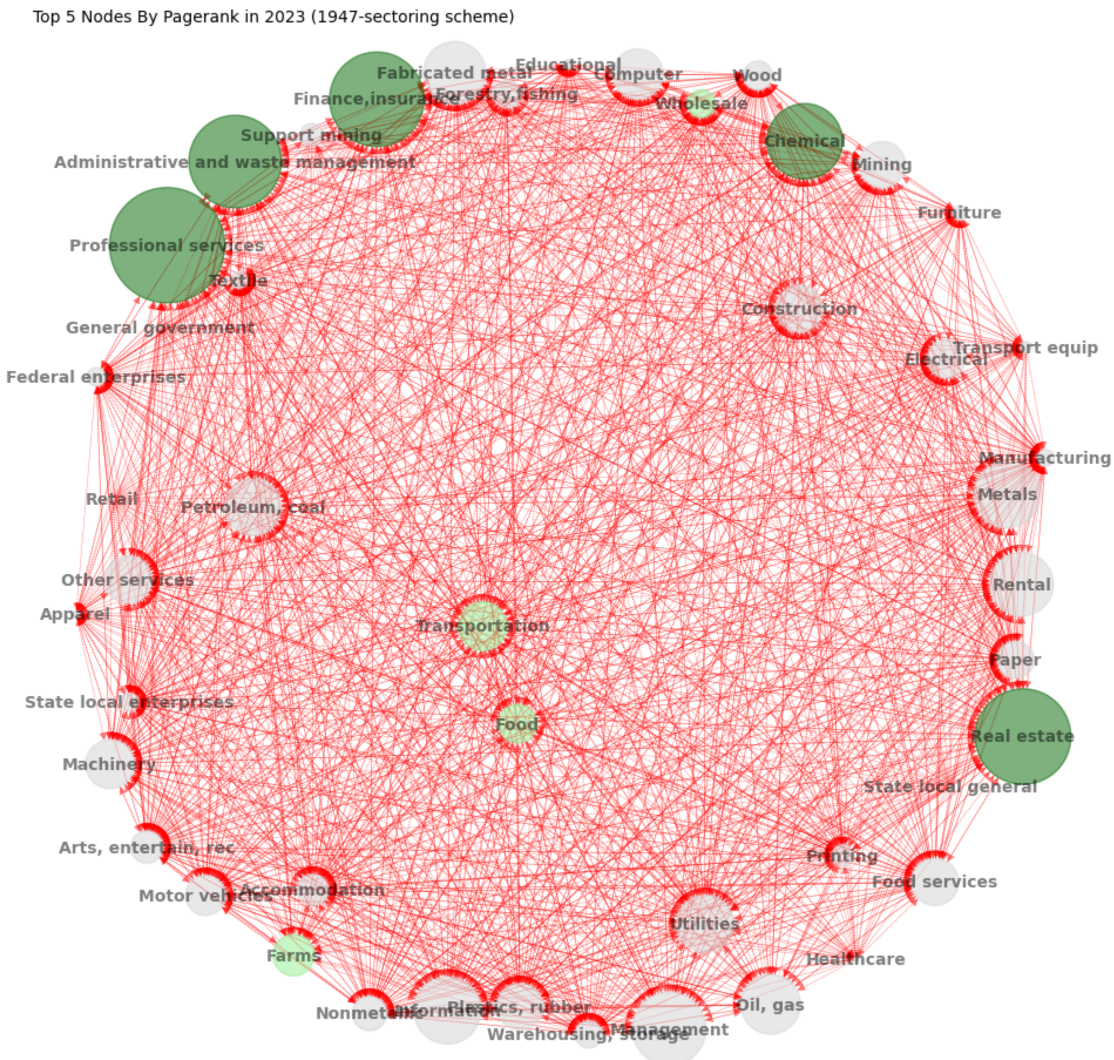
```python
colors = ['lightgrey', 'darkgreen', 'lightgreen']

# Populate and plot graph of first and last table years
for ifig, year in enumerate(years):
    # keep year, drop invalid rows
    ioUse = ioUses[year]
    data = ioUse[(~ioUse['rowcode'].str.startswith(drop) &
                  ~ioUse['colcode'].str.startswith(drop))].copy()

    # create master table of industries and measurements
    master = data[data['rowcode'] == data['colcode']][['rowcode','datavalue']]\
        .set_index('rowcode')\
```

(continues on next page)

(continued from previous page)

```
        .rename(columns={'datavalue': 'self'})

    # extract cross data; generate and load edges (as tuples) to graph
    data = data[(data['colcode'] != data['rowcode'])]
    data['weights'] = data['datavalue'] / data['datavalue'].sum()
    edges = data.loc[data['weights'] > 0, [tail, head, 'weights']]\
                .values\
                .tolist()

    G = nx.DiGraph()
    G.add_weighted_edges_from(edges, weight='weight')
    nx_labels = BEA.short_desc[list(G.nodes)].to_dict()

    # update master table industry flow values
    master = master.join(data.groupby(['colcode'])['datavalue'].sum(), how='outer')\
                   .rename(columns={'datavalue': 'user'})
    master = master.join(data.groupby(['rowcode'])['datavalue'].sum(), how='outer')\
                   .rename(columns={'datavalue': 'maker'})
    master = master.fillna(0).astype(int)

    # inweight~supply~authority~eigenvector~pagerank, outweight~demand~hub
    centrality = DataFrame(nodes_centrality(G))    # compute centrality metrics
    master = master.join(centrality, how='left')
    master['bea'] = BEA.short_desc[master.index].to_list()

    # visualize graph
    score = centrality['pagerank']
    node_size = score.to_dict()
    node_color = {node: colors[0] for node in G.nodes()}
    if ifig == 0:
        center_name = score.index[score.argmax()]
    else:
        node_color.update({k: colors[2] for k in top_color})
    top_color = list(score.index[score.argsort()[-5:]])
    node_color.update(dict.fromkeys(top_color, colors[1]))
    pos = graph_draw(G,
                     num=ifig+1,
                     figsize=(10, 10),
                     center_name=center_name,
                     node_color=node_color,
                     node_size=node_size,
                     edge_color='r',
                     k=3,
                     pos=(pos if ifig else None),
                     font_size=10,
                     font_weight='semibold',
                     labels=master['bea'].to_dict(),
                     title=f"Top 5 Nodes By Pagerank in {year} ({vintage}-sectoring␣
↪scheme)")
```


Top 5 Nodes By Pagerank in 1947 (1947-sectoring scheme)
Wholesale
Professional services
Transportation
Food
Farms
Educational
Computer
Wood
Chemical
Mining
Furniture
Construction
Transport equip
Electrical
Manufacturing
Metals
Rental
Paper
Real estate
State local general
Printing
Food services
Healthcare
Utilities
Oil, gas
Management
Warehousing, storage
Plastics, rubber
Information
Nonmetallic
Accommodation
Motor vehicles
Arts, entertain, rec
Machinery
State local enterprises
Apparel
Other services
Retail
Petroleum, coal
Federal enterprises
General government
Textile
Administrative and waste management
Support mining
Finance,insurance
Fabricated metal
Forestry,fishing



Display centrality scores for all BEA sectors based on the latest year's input-output data:

```
# show industry flow values and graph centrality measures
master = pd.concat(
    (data[data['rowcode'] == data['colcode']][['rowcode', 'datavalue']]\
     .set_index('rowcode')\
     .rename(columns={'datavalue': 'self'}),
     data.groupby(['colcode'])['datavalue'].sum().rename('user'),
     data.groupby(['rowcode'])['datavalue'].sum().rename('maker')),
    join='outer', axis=1).fillna(0).astype(int)
master = master.join(DataFrame(nodes_centrality(G)), how='left')
master['bea'] = BEA.short_desc[master.index].to_list()

print(f"Node Centrality of BEA Input-Output Use Table {year}")
master.drop(columns=['self']).round(3)
```

```
Node Centrality of BEA Input-Output Use Table 2023
```

```
            user    maker  in_degree  out_degree    hub  authority  \
111CA     183083   413260      0.478       0.783  0.005      0.008
113FF       4764    83642      0.543       0.652  0.000      0.001
211       189659   491570      0.674       0.674  0.008      0.004
212        43330   125323      0.870       0.804  0.002      0.003
213        29107    23233      0.087       0.652  0.002      0.000
22        148366   426549      1.000       0.717  0.008      0.026
23       1108514   381314      1.000       0.761  0.037      0.027
311FT     610392   378716      0.630       0.804  0.013      0.022
313TT      22726    46617      0.848       0.761  0.001      0.002
315AL       9831     7442      0.435       0.674  0.000      0.000
321        64039   187546      0.957       0.804  0.002      0.010
322        72249   177891      0.978       0.717  0.002      0.007
323        42407    84936      0.739       0.804  0.002      0.007
324       514797   537972      1.000       0.761  0.005      0.026
325       207051   659545      1.000       0.804  0.009      0.031
326       174402   358413      1.000       0.761  0.006      0.018
327        61298   227005      0.957       0.739  0.002      0.010
331        81903   335633      0.891       0.717  0.003      0.004
332       195632   501561      1.000       0.761  0.006      0.017
333       203600   266998      1.000       0.761  0.005      0.009
334        53901   373613      0.978       0.696  0.003      0.020
335        74954   201230      1.000       0.717  0.002      0.008
3361MV    333656   199393      0.978       0.783  0.008      0.010
3364OT    129087    85122      0.391       0.717  0.005      0.003
337        44645    54948      0.522       0.717  0.001      0.003
339        69034   113798      0.935       0.804  0.003      0.010
42       1134476    73996      0.848       0.848  0.085      0.003
44RT      992525        0      0.000       0.913  0.080     -0.000
48        532651   399454      1.000       0.848  0.030      0.031
493        55326   180519      0.913       0.696  0.004      0.014
51        715155   648393      1.000       0.848  0.062      0.043
52        714298  1199853      1.000       0.674  0.070      0.092
531      1209091  1565039      1.000       0.739  0.096      0.113
532RL     275723   508226      1.000       0.696  0.021      0.029
54        772709  2108318      1.000       0.935  0.045      0.152
55        279051   754756      0.891       0.848  0.025      0.051
56        496066  1206168      1.000       0.913  0.033      0.099
61        155822    31243      0.413       0.848  0.013      0.002
62       1211742    24110      0.130       0.848  0.090      0.002
71        182097   135112      1.000       0.913  0.016      0.010
721       127238   104038      0.978       0.826  0.008      0.008
722       614168   322353      1.000       0.826  0.038      0.028
81        350192   348957      1.000       0.913  0.029      0.024
GFE        40482    73578      0.761       0.739  0.002      0.006
GFG       479004        0      0.000       0.848  0.030      0.000
GSLE      296865    42462      0.891       0.783  0.016      0.003
GSLG     1162737        0      0.000       0.870  0.066     -0.000

        eigenvector  pagerank  betweenness  closeness  \
111CA         0.039     0.015        0.001      0.657
113FF         0.007     0.007        0.001      0.687
211           0.072     0.032        0.004      0.754
212           0.019     0.019        0.008      0.885
```

(continues on next page)

(continued from previous page)

```
213            0.003     0.005        0.000      0.523
22             0.085     0.029        0.002      1.000
23             0.111     0.020        0.004      1.000
311FT          0.039     0.013        0.002      0.730
313TT          0.004     0.002        0.002      0.868
315AL          0.001     0.000        0.000      0.639
321            0.037     0.007        0.003      0.958
322            0.030     0.012        0.002      0.979
323            0.022     0.004        0.003      0.793
324            0.081     0.028        0.003      1.000
325            0.140     0.051        0.003      1.000
326            0.057     0.019        0.003      1.000
327            0.046     0.010        0.002      0.958
331            0.043     0.039        0.001      0.902
332            0.090     0.034        0.002      1.000
333            0.051     0.021        0.003      1.000
334            0.076     0.022        0.001      0.979
335            0.043     0.013        0.002      1.000
3361MV         0.041     0.014        0.004      0.979
3364OT         0.004     0.001        0.000      0.622
337            0.014     0.002        0.001      0.676
339            0.009     0.003        0.004      0.939
42             0.010     0.007        0.004      0.868
44RT           0.000     0.000        0.000      0.000
48             0.097     0.021        0.005      1.000
493            0.021     0.007        0.001      0.920
51             0.212     0.040        0.005      1.000
52             0.400     0.079        0.002      1.000
531            0.368     0.081        0.003      1.000
532RL          0.147     0.036        0.001      1.000
54             0.531     0.118        0.018      1.000
55             0.172     0.048        0.009      0.902
56             0.449     0.076        0.012      1.000
61             0.002     0.001        0.001      0.630
62             0.000     0.000        0.000      0.535
71             0.053     0.009        0.012      1.000
721            0.052     0.009        0.004      0.979
722            0.118     0.020        0.004      1.000
81             0.099     0.020        0.012      1.000
GFE            0.014     0.003        0.004      0.807
GFG            0.000     0.000        0.000      0.000
GSLE           0.012     0.003        0.004      0.902
GSLG           0.000     0.000        0.000      0.000

                                      bea
111CA                               Farms
113FF                     Forestry,fishing
211                              Oil, gas
212                                Mining
213                        Support mining
22                              Utilities
23                           Construction
311FT                                Food
313TT                             Textile
315AL                             Apparel
321                                  Wood
```

(continues on next page)

(continued from previous page)

```
322                                Paper
323                             Printing
324                      Petroleum, coal
325                             Chemical
326                     Plastics, rubber
327                          Nonmetallic
331                               Metals
332                      Fabricated metal
333                            Machinery
334                             Computer
335                           Electrical
3361MV                     Motor vehicles
3364OT                    Transport equip
337                            Furniture
339                        Manufacturing
42                             Wholesale
44RT                              Retail
48                        Transportation
493                 Warehousing, storage
51                           Information
52                     Finance,insurance
531                          Real estate
532RL                             Rental
54                Professional services
55                            Management
56      Administrative and waste management
61                           Educational
62                            Healthcare
71                  Arts, entertain, rec
721                        Accommodation
722                        Food services
81                        Other services
GFE                  Federal enterprises
GFG                   General government
GSLE               State local enterprises
GSLG                 State local general
```

Compare the 1947 and 1997 sectoring schemes to examine how BEA’s industry groupings have evolved over time.

```
# Compare 1947 and 1997 sector schemes (BEA "summary"-level industry groups)
v1947 = BEA.sectoring(1947).rename(columns={'description': '1947'})
v1997 = BEA.sectoring(1997).rename(columns={'description': '1997'})
df = v1947[['title', '1947']].join(v1997['1997'])
df[df['1947'] != df['1997']]  # changes in the sectoring scheme
```

```
                                                                              ␣
↪                title  \
code
441000                                                                 Motor␣
↪vehicle and parts dealers
442000                                                                        ␣
↪     All other retail
443000                                                                        ␣
↪     All other retail
444000                                       Building material and garden␣
```

(continues on next page)

(continued from previous page)

```
↪equipment and supplies dealers
445000                                                                              ␣
↪Food and beverage stores
446000                                                                        Health␣
↪and personal care stores
447000                                                                                 ␣
↪    Gasoline stations
448000                                                           Clothing and␣
↪clothing accessories stores
451000                                                                                 ␣
↪     All other retail
452000                                                                            ␣
↪General merchandise stores
453000                                                                                 ␣
↪     All other retail
454000                                                                                 ␣
↪   Nonstore retailers
481000                                                                                 ␣
↪   Air transportation
482000                                                                                 ␣
↪  Rail transportation
483000                                                                                 ␣
↪ Water transportation
484000                                                                                 ␣
↪ Truck transportation
485000                                                        Transit and ground␣
↪passenger transportation
486000                                                                              ␣
↪Pipeline transportation
487000                                       Scenic and sightseeing transportation␣
↪and support activities
488000                                       Scenic and sightseeing transportation␣
↪and support activities
492000                                                                              ␣
↪Couriers and messengers
511000                                         Publishing industries, except␣
↪internet (includes software)
511110                                                                                 ␣
↪ Newspaper publishers
511120                                                                                 ␣
↪Periodical publishers
511130                                                                                 ␣
↪      Book publishers
511140                                                      Directory, mailing list,
↪ and other publishers
511190                                                      Directory, mailing list,
↪ and other publishers
511210                                                                                 ␣
↪  Software publishers
512000                                                      Motion picture and␣
↪sound recording industries
512100                                                                Motion␣
↪picture and video industries
512200                                                                            ␣
↪Sound recording industries
513000                                                                Broadcasting␣
```

(continues on next page)

(continued from previous page)

```
↪and telecommunications
514000                              Data processing, internet publishing, and␣
↪other information services
515100                                                                  Radio and␣
↪television broadcasting
515200                                                           Cable and other␣
↪subscription programming
517100                                                                  Wired␣
↪telecommunications carriers
517200                                             Wireless telecommunications␣
↪carriers (except satellite)
517400                         Satellite, telecommunications resellers, and all␣
↪other telecommunications
518200                                                Data processing, hosting,
↪ and related services
519110                           News syndicates, libraries, archives and all␣
↪other information services
519130                                        Internet publishing and broadcasting␣
↪and Web search portals
521000                                         Monetary authorities and depository␣
↪credit intermediation
522100                                         Monetary authorities and depository␣
↪credit intermediation
522200                                        Nondepository credit intermediation␣
↪and related activities
523000                                                      Securities, commodity␣
↪contracts, and investments
523100                                    Securities and commodity contracts␣
↪intermediation and brokerage
523900                                                          Other financial␣
↪investment activities
524000                                                      Insurance carriers␣
↪and related activities
524113                                                                  Direct␣
↪life insurance carriers
524114                                                              Insurance␣
↪carriers, except direct life
524120                                                              Insurance␣
↪carriers, except direct life
524130                                                              Insurance␣
↪carriers, except direct life
524200                                            Insurance agencies, brokerages,␣
↪and related activities
525000                                                        Funds, trusts, and␣
↪other financial vehicles
541100                                                                          ␣
↪       Legal services
541200                                  Accounting, tax preparation, bookkeeping,
↪ and payroll services
541300                                                Architectural, engineering,
↪ and related services
541400                                                                      ␣
↪Specialized design services
541500                                                        Computer systems␣
↪design and related services
541511                                                                  Custom␣
```

(continues on next page)

(continued from previous page)

```
↪computer programming services
541512                                                                    Computer␣
↪systems design services
541513                                Other computer related services, including␣
↪facilities management
541519                                Other computer related services, including␣
↪facilities management
541610                                                                        ␣
↪Management consulting services
541620                                           Environmental and other␣
↪technical consulting services
541690                                           Environmental and other␣
↪technical consulting services
541700                                                    Scientific research␣
↪and development services
541800                                             Advertising, public relations,
↪ and related services
541910                        All other miscellaneous professional, scientific,␣
↪and technical services
541920                                                                          ␣
↪Photographic services
541930                        All other miscellaneous professional, scientific,␣
↪and technical services
541940                                                                          ␣
↪  Veterinary services
541990                        All other miscellaneous professional, scientific,␣
↪and technical services
561000                                                                    ␣
↪Administrative and support services
561100                                                                    Office␣
↪administrative services
561200                                                                       ␣
↪Facilities support services
561300                                                                          ␣
↪  Employment services
561400                                                                        ␣
↪Business support services
561500                                                     Travel arrangement␣
↪and reservation services
561600                                                           Investigation␣
↪and security services
561700                                                             Services to␣
↪buildings and dwellings
561900                                                                         ␣
↪Other support services
562000                                                       Waste management␣
↪and remediation services
621000                                                                     ␣
↪Ambulatory health care services
621100                                                                          ␣
↪Offices of physicians
621200                                                                          ␣
↪  Offices of dentists
621300                                                            Offices of␣
↪other health practitioners
621400                                                                       ␣
```

(continues on next page)

(continued from previous page)

```
↪Outpatient care centers
621500                                                              Medical and␣
↪diagnostic laboratories
621600                                                                       ␣
↪Home health care services
621900                                                            Other␣
↪ambulatory health care services
622000                                                                          ␣
↪            Hospitals
623000                                                          Nursing and␣
↪residential care facilities
623100                                                            Nursing and␣
↪community care facilities
623200                  Residential mental health, substance abuse, and other␣
↪residential care facilities
623300                                                            Nursing and␣
↪community care facilities
623900                  Residential mental health, substance abuse, and other␣
↪residential care facilities
624000                                                                          ␣
↪    Social assistance
624100                                                                  ␣
↪Individual and family services
624200  Community food, housing, and other relief services, including vocational␣
↪rehabilitation services
624400                                                                        ␣
↪Child day care services
711000                                      Performing arts, spectator sports, museums,␣
↪and related activities
711100                                                                       ␣
↪Performing arts companies
711200                                                                          ␣
↪     Spectator sports
711300                             Promoters of performing arts and sports and␣
↪agents for public figures
711500                                                      Independent artists,␣
↪writers, and performers
712000                                                        Museums, historical␣
↪sites, zoos, and parks
713000                                                   Amusements, gambling, and␣
↪recreation industries
713100                                                                     ␣
↪Amusement parks and arcades
713200                                                        Gambling industries␣
↪(except casino hotels)
713900                                                         Other amusement and␣
↪recreation industries

                                       1947  \
code
441000                        RETAIL TRADE
442000                        RETAIL TRADE
443000                        RETAIL TRADE
444000                        RETAIL TRADE
445000                        RETAIL TRADE
446000                        RETAIL TRADE
```

(continues on next page)

(continued from previous page)

```
447000                          RETAIL TRADE
448000                          RETAIL TRADE
451000                          RETAIL TRADE
452000                          RETAIL TRADE
453000                          RETAIL TRADE
454000                          RETAIL TRADE
481000                        Transportation
482000                        Transportation
483000                        Transportation
484000                        Transportation
485000                        Transportation
486000                        Transportation
487000                        Transportation
488000                        Transportation
492000                        Transportation
511000                           INFORMATION
511110                           INFORMATION
511120                           INFORMATION
511130                           INFORMATION
511140                           INFORMATION
511190                           INFORMATION
511210                           INFORMATION
512000                           INFORMATION
512100                           INFORMATION
512200                           INFORMATION
513000                           INFORMATION
514000                           INFORMATION
515100                           INFORMATION
515200                           INFORMATION
517100                           INFORMATION
517200                           INFORMATION
517400                           INFORMATION
518200                           INFORMATION
519110                           INFORMATION
519130                           INFORMATION
521000                 FINANCE AND INSURANCE
522100                 FINANCE AND INSURANCE
522200                 FINANCE AND INSURANCE
523000                 FINANCE AND INSURANCE
523100                 FINANCE AND INSURANCE
523900                 FINANCE AND INSURANCE
524000                 FINANCE AND INSURANCE
524113                 FINANCE AND INSURANCE
524114                 FINANCE AND INSURANCE
524120                 FINANCE AND INSURANCE
524130                 FINANCE AND INSURANCE
524200                 FINANCE AND INSURANCE
525000                 FINANCE AND INSURANCE
541100  PROFESSIONAL AND TECHNICAL SERVICES
541200  PROFESSIONAL AND TECHNICAL SERVICES
541300  PROFESSIONAL AND TECHNICAL SERVICES
541400  PROFESSIONAL AND TECHNICAL SERVICES
541500  PROFESSIONAL AND TECHNICAL SERVICES
541511  PROFESSIONAL AND TECHNICAL SERVICES
541512  PROFESSIONAL AND TECHNICAL SERVICES
541513  PROFESSIONAL AND TECHNICAL SERVICES
```

(continues on next page)

(continued from previous page)

```
541519  PROFESSIONAL AND TECHNICAL SERVICES
541610  PROFESSIONAL AND TECHNICAL SERVICES
541620  PROFESSIONAL AND TECHNICAL SERVICES
541690  PROFESSIONAL AND TECHNICAL SERVICES
541700  PROFESSIONAL AND TECHNICAL SERVICES
541800  PROFESSIONAL AND TECHNICAL SERVICES
541910  PROFESSIONAL AND TECHNICAL SERVICES
541920  PROFESSIONAL AND TECHNICAL SERVICES
541930  PROFESSIONAL AND TECHNICAL SERVICES
541940  PROFESSIONAL AND TECHNICAL SERVICES
541990  PROFESSIONAL AND TECHNICAL SERVICES
561000    ADMINISTRATIVE AND WASTE SERVICES
561100    ADMINISTRATIVE AND WASTE SERVICES
561200    ADMINISTRATIVE AND WASTE SERVICES
561300    ADMINISTRATIVE AND WASTE SERVICES
561400    ADMINISTRATIVE AND WASTE SERVICES
561500    ADMINISTRATIVE AND WASTE SERVICES
561600    ADMINISTRATIVE AND WASTE SERVICES
561700    ADMINISTRATIVE AND WASTE SERVICES
561900    ADMINISTRATIVE AND WASTE SERVICES
562000    ADMINISTRATIVE AND WASTE SERVICES
621000    HEALTH CARE AND SOCIAL ASSISTANCE
621100    HEALTH CARE AND SOCIAL ASSISTANCE
621200    HEALTH CARE AND SOCIAL ASSISTANCE
621300    HEALTH CARE AND SOCIAL ASSISTANCE
621400    HEALTH CARE AND SOCIAL ASSISTANCE
621500    HEALTH CARE AND SOCIAL ASSISTANCE
621600    HEALTH CARE AND SOCIAL ASSISTANCE
621900    HEALTH CARE AND SOCIAL ASSISTANCE
622000    HEALTH CARE AND SOCIAL ASSISTANCE
623000    HEALTH CARE AND SOCIAL ASSISTANCE
623100    HEALTH CARE AND SOCIAL ASSISTANCE
623200    HEALTH CARE AND SOCIAL ASSISTANCE
623300    HEALTH CARE AND SOCIAL ASSISTANCE
623900    HEALTH CARE AND SOCIAL ASSISTANCE
624000    HEALTH CARE AND SOCIAL ASSISTANCE
624100    HEALTH CARE AND SOCIAL ASSISTANCE
624200    HEALTH CARE AND SOCIAL ASSISTANCE
624400    HEALTH CARE AND SOCIAL ASSISTANCE
711000  ARTS, ENTERTAINMENT, AND RECREATION
711100  ARTS, ENTERTAINMENT, AND RECREATION
711200  ARTS, ENTERTAINMENT, AND RECREATION
711300  ARTS, ENTERTAINMENT, AND RECREATION
711500  ARTS, ENTERTAINMENT, AND RECREATION
712000  ARTS, ENTERTAINMENT, AND RECREATION
713000  ARTS, ENTERTAINMENT, AND RECREATION
713100  ARTS, ENTERTAINMENT, AND RECREATION
713200  ARTS, ENTERTAINMENT, AND RECREATION
713900  ARTS, ENTERTAINMENT, AND RECREATION

                                                                  1997
code
441000                                 Motor vehicle and parts dealers
442000                                                    Other retail
443000                                                    Other retail
444000                                                    Other retail
```

(continues on next page)

(continued from previous page)

```
445000                                              Food and beverage stores
446000                                                          Other retail
447000                                                          Other retail
448000                                                          Other retail
451000                                                          Other retail
452000                                            General merchandise stores
453000                                                          Other retail
454000                                                          Other retail
481000                                                   Air transportation
482000                                                   Rail transportation
483000                                                  Water transportation
484000                                                  Truck transportation
485000                        Transit and ground passenger transportation
486000                                               Pipeline transportation
487000                          Other transportation and support activities
488000                          Other transportation and support activities
492000                          Other transportation and support activities
511000            Publishing industries, except internet (includes software)
511110            Publishing industries, except internet (includes software)
511120            Publishing industries, except internet (includes software)
511130            Publishing industries, except internet (includes software)
511140            Publishing industries, except internet (includes software)
511190            Publishing industries, except internet (includes software)
511210            Publishing industries, except internet (includes software)
512000                         Motion picture and sound recording industries
512100                         Motion picture and sound recording industries
512200                         Motion picture and sound recording industries
513000                                   Broadcasting and telecommunications
514000  Data processing, internet publishing, and other information services
515100                                   Broadcasting and telecommunications
515200                                   Broadcasting and telecommunications
517100                                   Broadcasting and telecommunications
517200                                   Broadcasting and telecommunications
517400                                   Broadcasting and telecommunications
518200  Data processing, internet publishing, and other information services
519110  Data processing, internet publishing, and other information services
519130  Data processing, internet publishing, and other information services
521000  Federal Reserve banks, credit intermediation, and related activities
522100  Federal Reserve banks, credit intermediation, and related activities
522200  Federal Reserve banks, credit intermediation, and related activities
523000                      Securities, commodity contracts, and investments
523100                      Securities, commodity contracts, and investments
523900                      Securities, commodity contracts, and investments
524000                             Insurance carriers and related activities
524113                             Insurance carriers and related activities
524114                             Insurance carriers and related activities
524120                             Insurance carriers and related activities
524130                             Insurance carriers and related activities
524200                             Insurance carriers and related activities
525000                           Funds, trusts, and other financial vehicles
541100                                                        Legal services
541200        Miscellaneous professional, scientific, and technical services
541300        Miscellaneous professional, scientific, and technical services
541400        Miscellaneous professional, scientific, and technical services
541500                             Computer systems design and related services
541511                             Computer systems design and related services
```

(continues on next page)

(continued from previous page)

```
541512                          Computer systems design and related services
541513                          Computer systems design and related services
541519                          Computer systems design and related services
541610        Miscellaneous professional, scientific, and technical services
541620        Miscellaneous professional, scientific, and technical services
541690        Miscellaneous professional, scientific, and technical services
541700        Miscellaneous professional, scientific, and technical services
541800        Miscellaneous professional, scientific, and technical services
541910        Miscellaneous professional, scientific, and technical services
541920        Miscellaneous professional, scientific, and technical services
541930        Miscellaneous professional, scientific, and technical services
541940        Miscellaneous professional, scientific, and technical services
541990        Miscellaneous professional, scientific, and technical services
561000                                   Administrative and support services
561100                                   Administrative and support services
561200                                   Administrative and support services
561300                                   Administrative and support services
561400                                   Administrative and support services
561500                                   Administrative and support services
561600                                   Administrative and support services
561700                                   Administrative and support services
561900                                   Administrative and support services
562000                             Waste management and remediation services
621000                                       Ambulatory health care services
621100                                       Ambulatory health care services
621200                                       Ambulatory health care services
621300                                       Ambulatory health care services
621400                                       Ambulatory health care services
621500                                       Ambulatory health care services
621600                                       Ambulatory health care services
621900                                       Ambulatory health care services
622000                                                             Hospitals
623000                               Nursing and residential care facilities
623100                               Nursing and residential care facilities
623200                               Nursing and residential care facilities
623300                               Nursing and residential care facilities
623900                               Nursing and residential care facilities
624000                                                     Social assistance
624100                                                     Social assistance
624200                                                     Social assistance
624400                                                     Social assistance
711000    Performing arts, spectator sports, museums, and related activities
711100    Performing arts, spectator sports, museums, and related activities
711200    Performing arts, spectator sports, museums, and related activities
711300    Performing arts, spectator sports, museums, and related activities
711500    Performing arts, spectator sports, museums, and related activities
712000    Performing arts, spectator sports, museums, and related activities
713000                         Amusements, gambling, and recreation industries
713100                         Amusements, gambling, and recreation industries
713200                         Amusements, gambling, and recreation industries
713900                         Amusements, gambling, and recreation industries
```

References:

Jason Choi & Andrew T. Foerster, 2017. "The Changing Input-Output Network Structure of the U.S. Economy," Economic Review, Federal Reserve Bank of Kansas City, issue Q II, pages 23-49

https://www.bea.gov/industry/input-output-accounts-data

https://www.bea.gov/information-updates-national-economic-accounts

CHAPTER

TWENTYTHREE

PRODUCT MARKET LINK PREDICTION

A hidden connection is stronger than an obvious one - Heraclitus

Link prediction in network analysis seeks to infer missing or future connections between nodes based on observed relationships. In the context of product markets, firms are interconnected through shared product similarities and competitive interactions. We construct firm networks from text-based product market similarity data, and explore the application of several link prediction algorithms. Additionally, accuracy metrics such as precision-recall and ROC curves are examined to assess the performance of these predictions.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import zipfile
import io
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
from sklearn import metrics
import matplotlib.pyplot as plt
import networkx as nx
from finds.database import SQL, RedisDB
from finds.structured import CRSP, BusDay, PSTAT
from finds.readers import requests_get
from finds.recipes import graph_info
from secret import credentials, paths
# %matplotlib qt
VERBOSE = 0
```

```
sql = SQL(**credentials['sql'], verbose=VERBOSE)
bd = BusDay(sql, verbose=VERBOSE)
rdb = RedisDB(**credentials['redis'])
pstat = PSTAT(sql, bd, verbose=VERBOSE)
crsp = CRSP(sql, bd, rdb, verbose=VERBOSE)
```

23.1 Product market linkages

Hoberg and Phillips (2016) developed a text-based measure of firm similarity by analyzing product descriptions in 10-K filings. Their methodology constructs firm-by-firm similarity scores using word vectors, filtering out common words and focusing on nouns and proper nouns while excluding geographic terms. This approach captures shifts in product markets over time, as revealed by their business descriptions in annual company filings.

The **TNIC-3** dataset is calibrated to align with the granularity of three-digit SIC codes, providing a structured industry classification. The **TNIC-2** dataset represents a more comprehensive version, including all firm pairs, even those with weak relationships.

Source: Hoberg and Phillips Industry Classification

The TNIC-2 and TNIC-3 datasets are retrieved from the Hoberg and Phillips website.

```
root = 'https://hobergphillips.tuck.dartmouth.edu/idata/'
tnic_data = {}
for scheme in ['tnic2', 'tnic3']:
    source = root + scheme + '_data.zip'
    if source.startswith('http'):
        response = requests_get(source)
        source = io.BytesIO(response.content)
    with zipfile.ZipFile(source).open(scheme + "_data.txt") as f:
        tnic_data[scheme] = pd.read_csv(f, sep='\s+')
for k,v in tnic_data.items():
    print(k, v.shape)
```

```
tnic2 (52812348, 4)
tnic3 (27161830, 4)
```

```
# extract last year of both tnic schemes, merge in permno, and require in univ
year = max(tnic_data['tnic2']['year'])
capsize = 10  # large cap (large than NYSE median)
univ = crsp.get_universe(bd.endyr(year))
univ = univ[univ['decile'] <= capsize]
lookup = pstat.build_lookup('gvkey', 'lpermno', fillna=0)
nodes = {}
tnic = {}
edges = {}
for scheme in ['tnic2', 'tnic3']:
    tnic[scheme] = tnic_data[scheme][tnic_data[scheme].year == year].dropna()
    gvkeys = sorted(set(tnic[scheme]['gvkey1']).union(tnic[scheme]['gvkey2']))
    df = DataFrame(index=gvkeys, data=lookup(gvkeys), columns=['permno'])
    nodes[scheme] = df[df['permno'].gt(0)
                       & df['permno'].isin(univ.index)].drop_duplicates()
nodes['tnic2'] = nodes['tnic2'][nodes['tnic2'].index.isin(nodes['tnic3'].index)]
nodes['tnic3'] = nodes['tnic3'][nodes['tnic3'].index.isin(nodes['tnic2'].index)]
```

Using the TNIC-2 and TNIC-3 datasets, undirected graphs are constructed where nodes represent firms and edges indicate product market similarities based on the chosen granularity of the classification schemes.

```
# create graphs of tnic2 (denser graph) and tnic3 (sparser graph) schemes
for scheme in ['tnic2', 'tnic3']:
    e = tnic[scheme][tnic[scheme]['gvkey1'].isin(nodes[scheme].index) &
                     tnic[scheme]['gvkey2'].isin(nodes[scheme].index)]
    edges[scheme] = list(e[['gvkey1', 'gvkey2', 'score']]\
                         .itertuples(index=False, name=None))
```

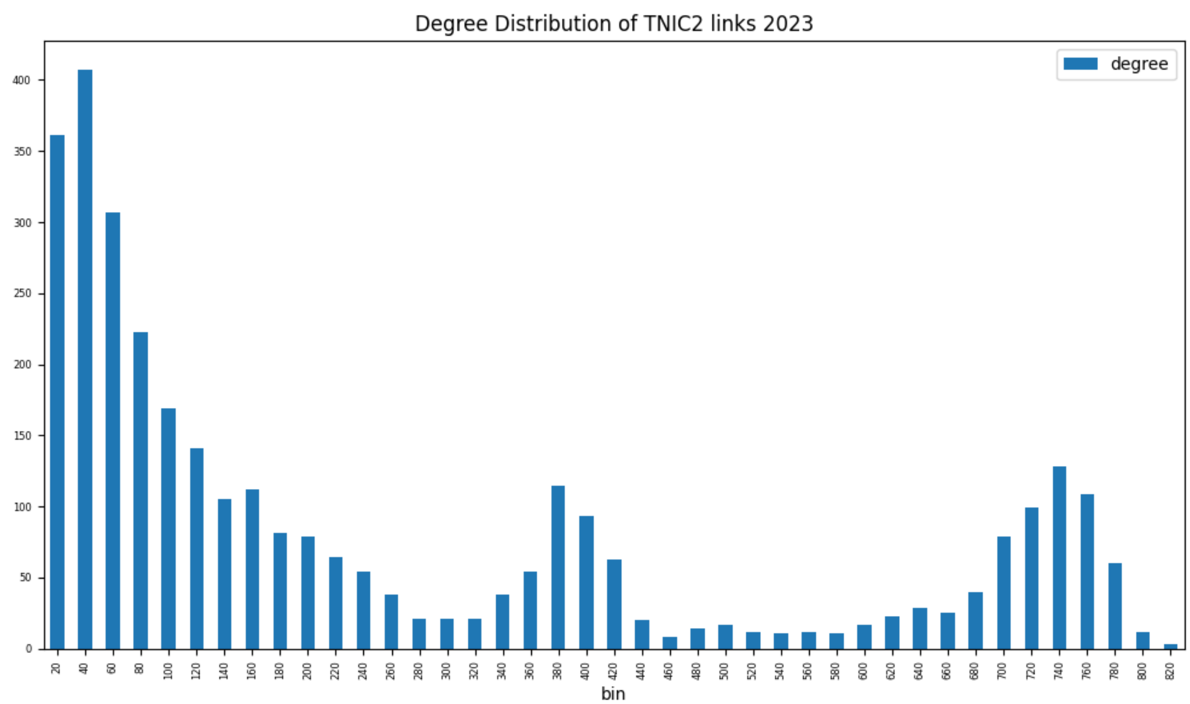
```
results = {}
G = {}
for (scheme, node), (_, edge) in zip(nodes.items(), edges.items()):
    print(scheme, 'nodes =', len(node), 'edges =', len(edge))

    # populate graph
    g = nx.Graph()
    g.add_nodes_from(node.index)
```

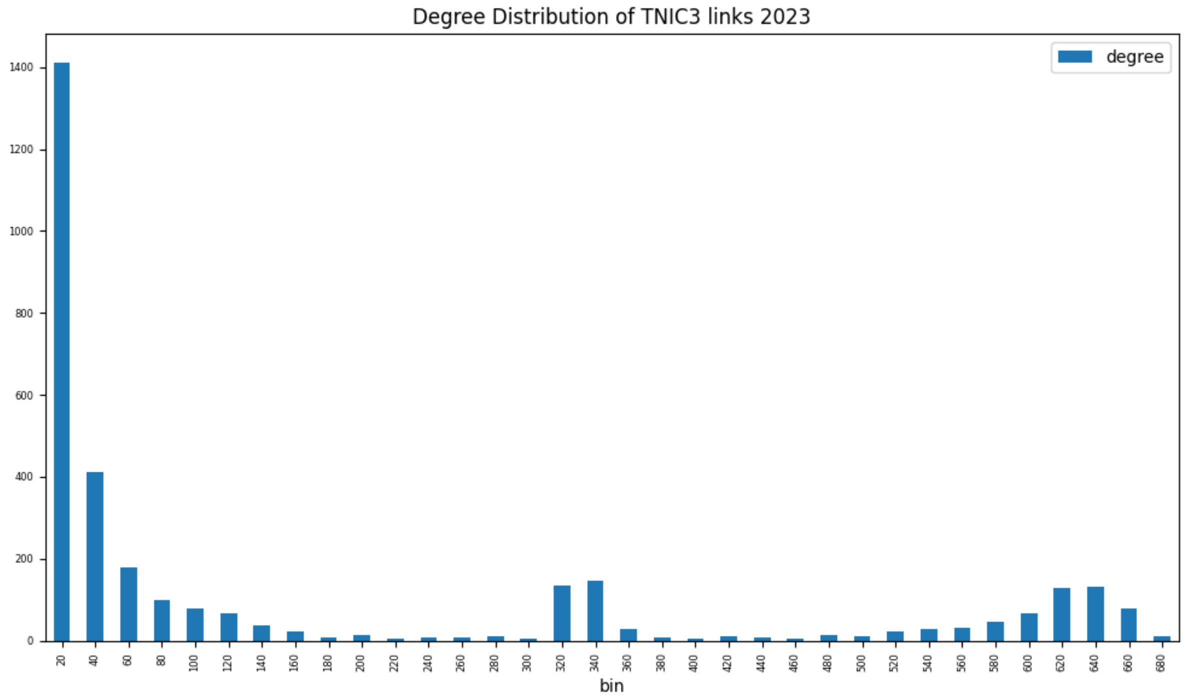
(continues on next page)

(continued from previous page)

```
    g.add_weighted_edges_from(edge)

    # remove self-loops: not necessary
    g.remove_edges_from(nx.selfloop_edges(g))

    # graph info
    results[scheme] = Series(graph_info(g, fast=True))

    # Plot degree distribution
    fig, ax = plt.subplots(clear=True, figsize=(10, 6))
    degree = nx.degree_histogram(g)
    degree = DataFrame(data={'degree': degree[1:]},   # exclude degree 0
                       index=np.arange(1, len(degree)))
    degree['bin'] = (degree.index // (2*capsize) + 1) * (2*capsize)
    degree.groupby('bin').sum().plot(kind='bar', ax=ax, fontsize=6)
    ax.set_title(f'Degree Distribution of {scheme.upper()} links {year}')
    plt.tight_layout()

    G[scheme] = g
```

```
tnic2 nodes = 3296 edges = 819984
tnic3 nodes = 3296 edges = 528012
```






```
print(f"Graph properties of TNIC schemes {year}")
DataFrame(results)
```

```
Graph properties of TNIC schemes 2023
```

```
                           tnic2     tnic3
transitivity            0.834713  0.881587
average_clustering      0.592383  0.571318
connected                   True     False
connected_components           1        25
size_largest_component      3296      3255
directed                   False     False
weighted                    True      True
negatively_weighted        False     False
edges                     409992    264006
nodes                       3296      3296
selfloops                      0         0
density                 0.075503  0.048618
```

23.2 Link prediction algorithms

Link prediction aims to identify missing or future connections between nodes in a network. Given a partially observed network, these algorithms infer which links are most likely to be added or missing based on existing connections and network structure.

Common link prediction algorithms include:

- **Jaccard Coefficient**: Measures the similarity between two nodes by comparing their shared neighbors relative to their total number of neighbors.

- **Resource Allocation**: Assigns a higher likelihood of connection between nodes that share many common neighbors, emphasizing smaller-degree nodes.
- **Adamic-Adar**: Enhances the Resource Allocation approach by weighting common neighbors based on their overall connectivity.
- **Preferential Attachment**: Predicts new links based on the idea that nodes with higher degrees are more likely to form new connections.

```
# helper to call link prediction algorithms
def link_prediction(G):
    """Predict link scores for all nonexistent edges in graph"""

    def links(links):
        """returns list of edge-score 3-tuples sorted by highest score"""
        return sorted(links, key=lambda x: x[2], reverse=True)

    resource = links(nx.resource_allocation_index(G))
    jaccard = links(nx.jaccard_coefficient(G))
    adamic = links(nx.adamic_adar_index(G))
    preferential = links(nx.preferential_attachment(G))
    return {'resource_allocation': resource,
            'jaccard_coefficient': jaccard,
            'adamic_adar': adamic,
            'preferential_attachment': preferential}
```

```
links = link_prediction(G['tnic3'])
```

23.3 Accuracy metrics

Common metrics to assess the accuracy of link prediction (or any binary classification) models:

- **Precision**: The fraction of predicted links that are actual links.
- **Recall**: The fraction of actual links that were correctly predicted.
- **Accuracy**: The overall correctness of predictions.
- **Confusion Matrix**: A summary of prediction outcomes.
- **F1 Score**: A balanced measure between precision and recall, useful for imbalanced datasets. It is calculated as:

$$2\frac{precision \cdot recall}{precision + recall} = \frac{TP}{TP + (FP + FN)/2}$$

where

- False Positive (FP): Incorrectly predicted links.
- False Negative (FN): Missed actual links.
- True Positive (TP): Correctly predicted links.
- True Negative (TN): Correctly identified non-links.

23.3.1 ROC Curve

The **Receiver Operating Characteristic (ROC) curve** and the **Area Under the Curve (AUC)** evaluates model performance by measuring the trade-off between true positive and false positive rates at various classification score thresholds.

The ROC curve plots the following metrics on the two axes:

- **True Positive Rate (TPR)** (or **Sensitivity**) $= \frac{TP}{TP+FN}$: Measures how many actual positives are correctly identified.
- **False Positive Rate (FPR)** $= \frac{FP}{FP+TN}$: Measures how many negative cases are incorrectly classified as positive.

The AUC measures the area under the ROC curve and provides a single number to quantify model performance. A higher AUC means the model is better at distinguishing between positive and negative classes.

```python
def make_sample(prediction, edges):
    """helper to extract predicted scores and gold labels"""
    names = [e[:2] for e in prediction]          # node-pairs of nonexistent edges
    scores = [e[-1] for e in prediction]         # predicted scores (ordered)
    gold = [e[:2] in edges for e in prediction] # gold labels of nonexistent edges
    return gold, scores, names  # actual, predicted score, names
```

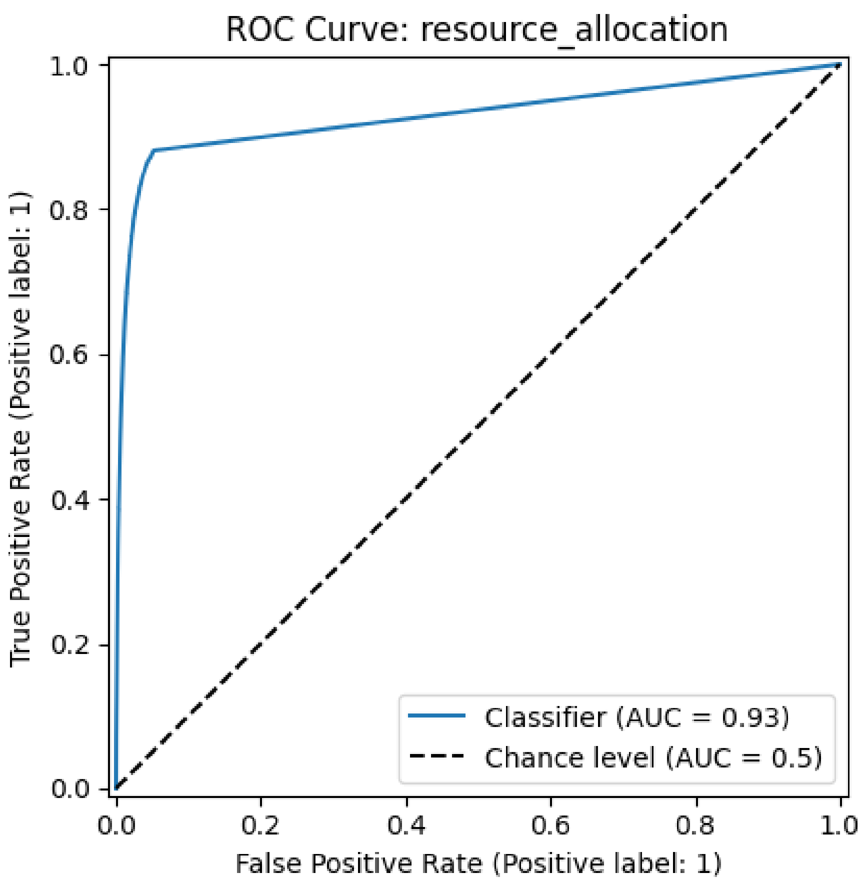
```python
report = {}
for ifig, (method, pred) in enumerate(links.items()):

    # extract predicted scores and gold labels of nonexistent edges
#    y, scores, names = make_sample(pred, G['tnic2'].edges)

    names = [e[:2] for e in pred]                    # node-pairs of nonexistent edges
    scores = [e[-1] for e in pred]                   # predicted scores of edge
    y = [e[:2] in G['tnic2'].edges for e in pred]  # gold labels of nonexistent edges

    # plot roc curve
    metrics.RocCurveDisplay.from_predictions(y_true=y, y_pred=scores,
                                             plot_chance_level=True)
    plt.title(f"ROC Curve: {method}")
    plt.tight_layout()

    # set classification threshold at class proportion
    thresh = scores[sum(y)]
    y_pred = [score >= thresh for score in scores]

    # generate and plot confusion matrix
    cm = metrics.confusion_matrix(y_true=y, y_pred=y_pred, normalize='all')
    disp = metrics.ConfusionMatrixDisplay(confusion_matrix=cm)
    disp.plot()
    plt.title(f"Confusion Matrix: {method}")
    plt.tight_layout()

    # generate classification report
    report[method] = metrics.classification_report(y_true=y, y_pred=y_pred)
    print(f"Classification Report: {method}")
    print(report[method])
```

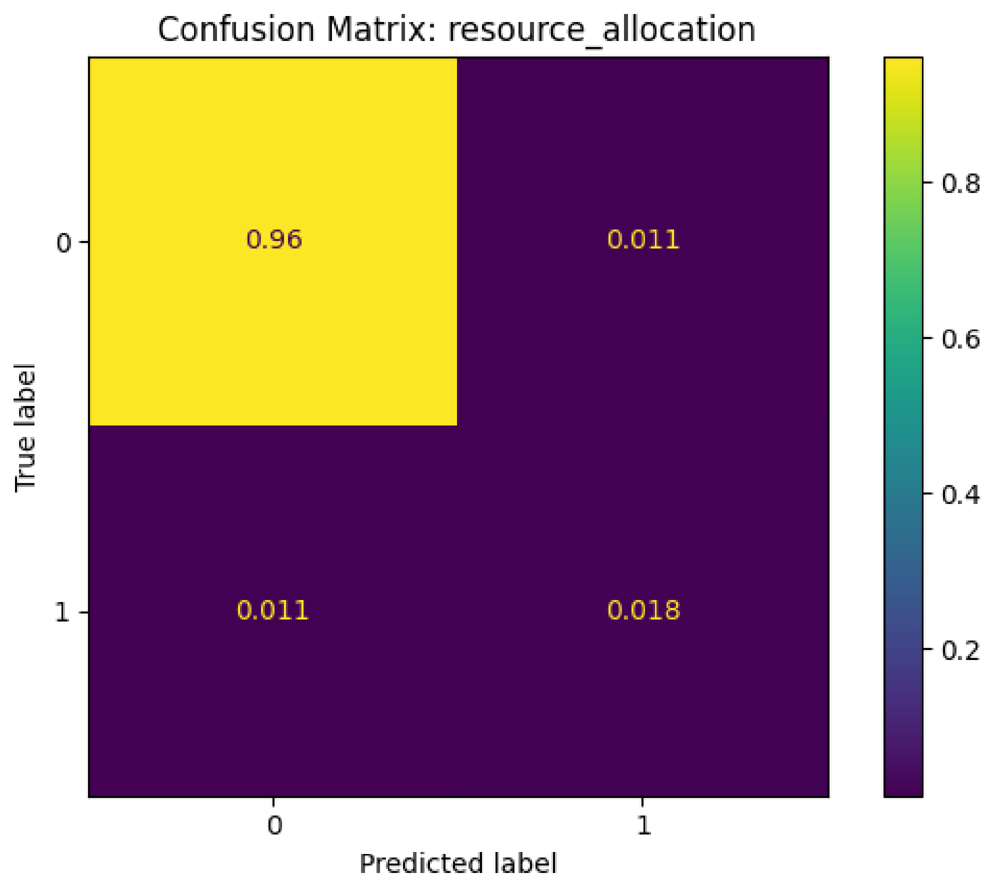
```
Classification Report: resource_allocation
              precision    recall  f1-score   support
```

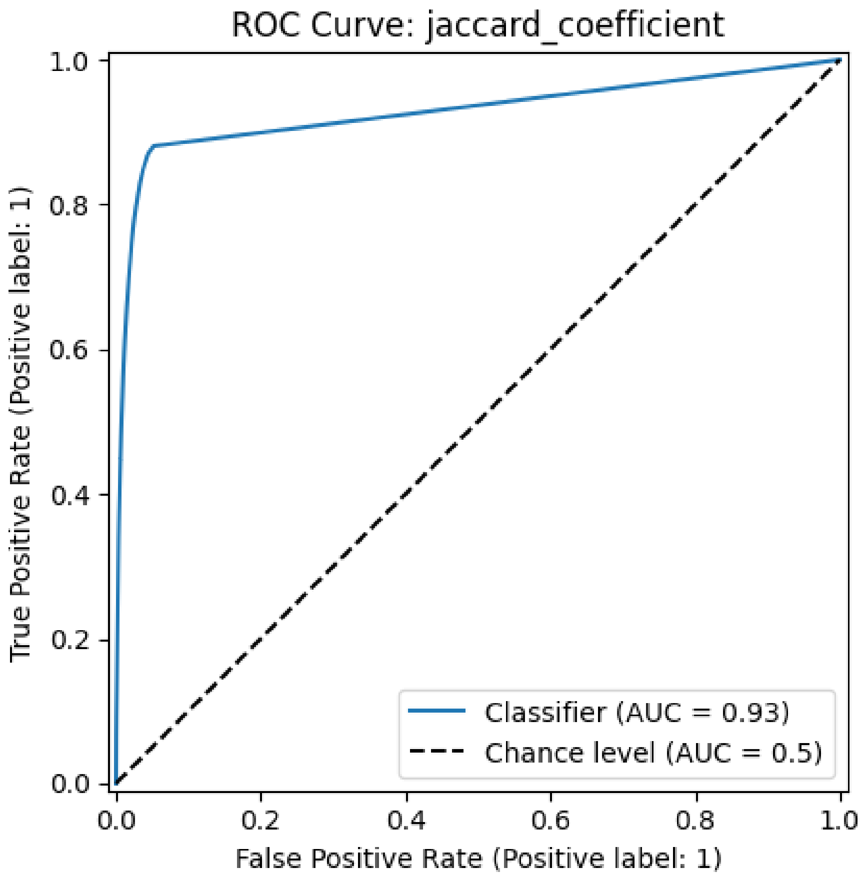
(continues on next page)

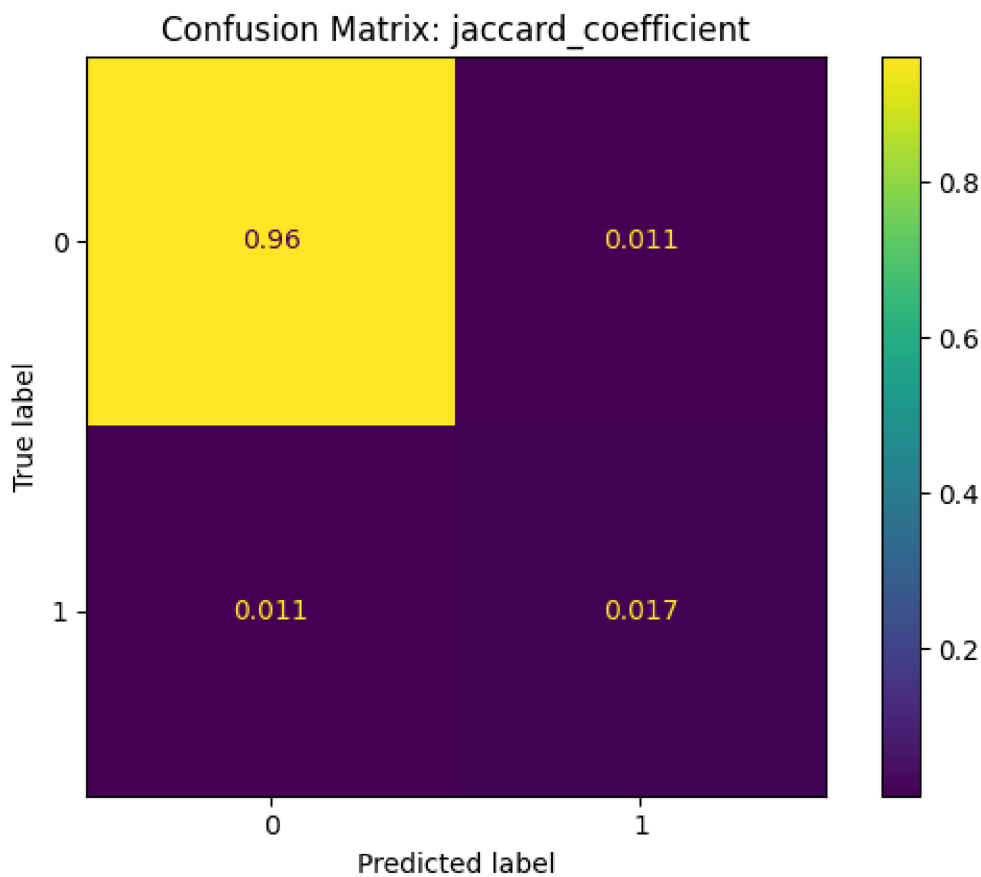
(continued from previous page)

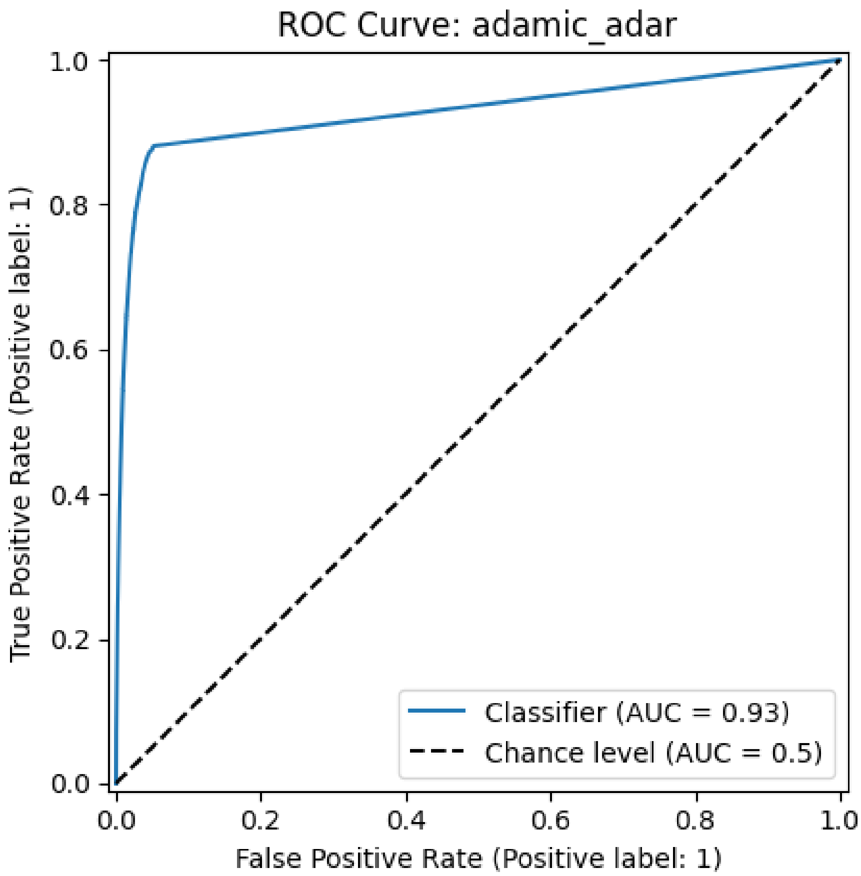
```
       False       0.99      0.99      0.99   5020168
        True       0.62      0.62      0.62    145986

    accuracy                           0.98   5166154
   macro avg       0.80      0.80      0.80   5166154
weighted avg       0.98      0.98      0.98   5166154

Classification Report: jaccard_coefficient
              precision    recall  f1-score   support

       False       0.99      0.99      0.99   5020168
        True       0.60      0.60      0.60    145986

    accuracy                           0.98   5166154
   macro avg       0.79      0.79      0.79   5166154
weighted avg       0.98      0.98      0.98   5166154

Classification Report: adamic_adar
              precision    recall  f1-score   support

       False       0.99      0.99      0.99   5020168
        True       0.60      0.60      0.60    145986

    accuracy                           0.98   5166154
   macro avg       0.79      0.79      0.79   5166154
weighted avg       0.98      0.98      0.98   5166154

Classification Report: preferential_attachment
              precision    recall  f1-score   support

       False       0.97      0.97      0.97   5020168
        True       0.09      0.09      0.09    145986

    accuracy                           0.95   5166154
   macro avg       0.53      0.53      0.53   5166154
weighted avg       0.95      0.95      0.95   5166154
```

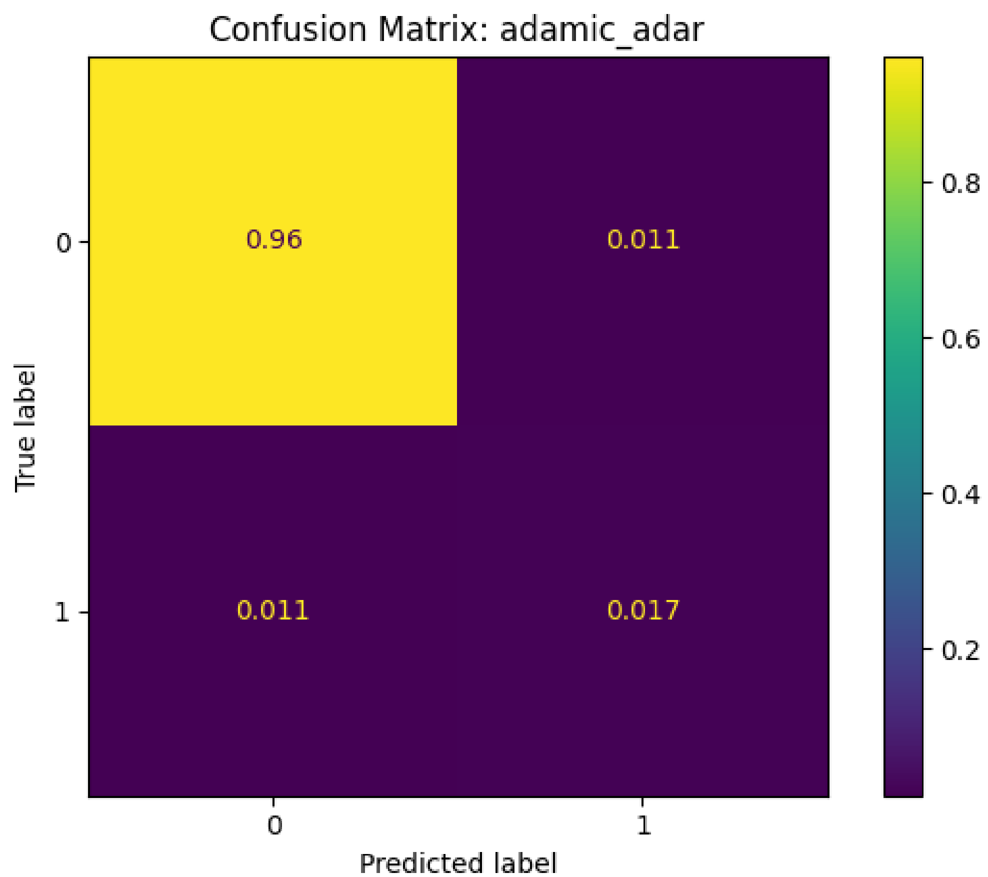

ROC Curve: resource_allocation
True Positive Rate (Positive label: 1)
False Positive Rate (Positive label: 1)
Classifier (AUC = 0.93)
Chance level (AUC = 0.5)
0.0
0.2
0.4
0.6
0.8
1.0

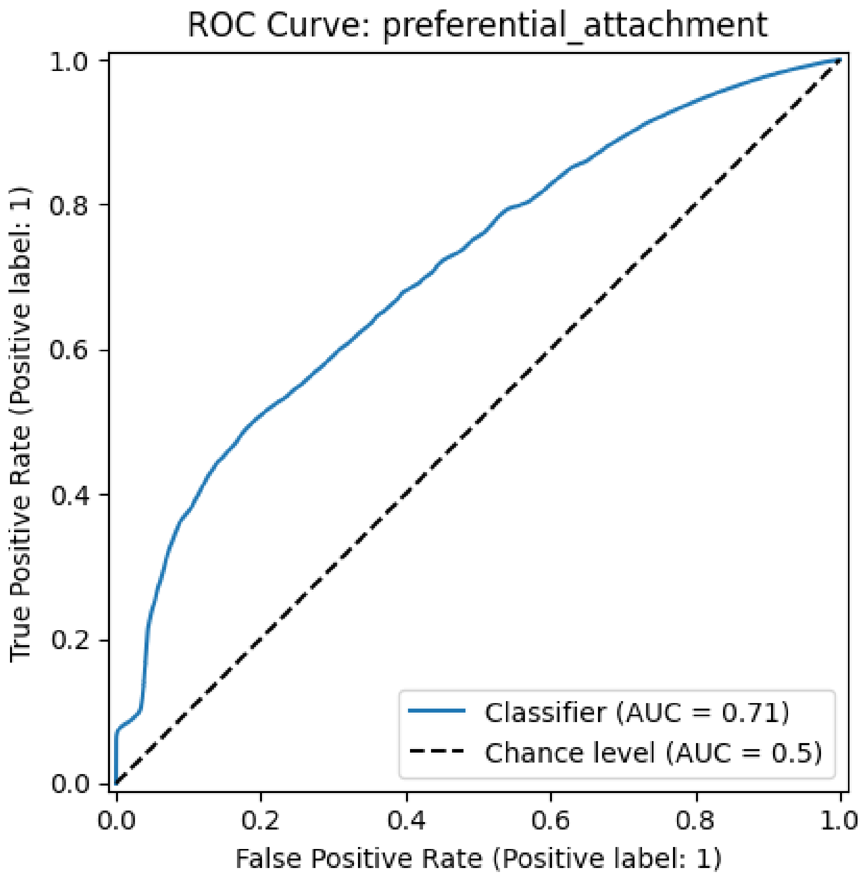

Confusion Matrix: resource_allocation
0.96
0.011
0.011
0.018
True label
Predicted label
0
1
0
1
0.8
0.6
0.4
0.2

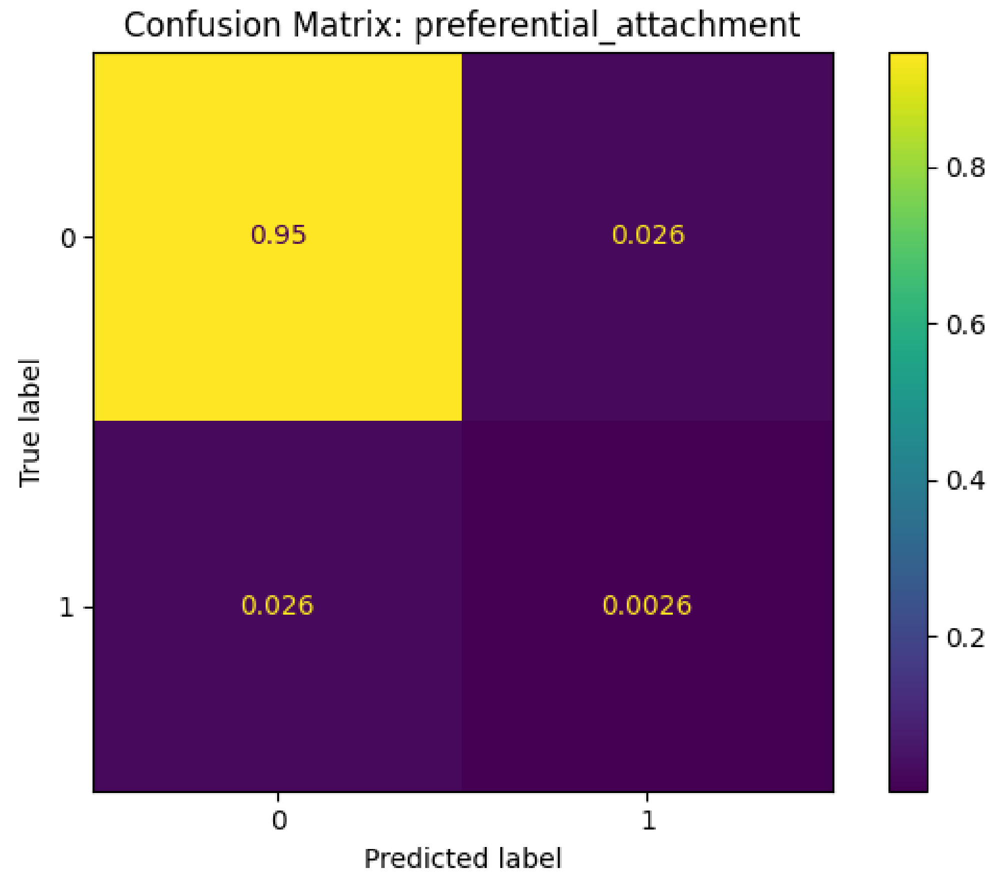

ROC Curve: jaccard_coefficient
1.0
0.8
0.6
0.4
0.2
0.0
True Positive Rate (Positive label: 1)
Classifier (AUC = 0.93)
Chance level (AUC = 0.5)
0.0
0.2
0.4
0.6
0.8
1.0
False Positive Rate (Positive label: 1)


Confusion Matrix: jaccard_coefficient
0.96
0.011
0.011
0.017
0
1
True label
0
1
Predicted label
0.8
0.6
0.4
0.2


ROC Curve: adamic_adar
True Positive Rate (Positive label: 1)
False Positive Rate (Positive label: 1)
Classifier (AUC = 0.93)
Chance level (AUC = 0.5)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


Confusion Matrix: adamic_adar
0.96
0.011
0.011
0.017
True label
Predicted label
0
1
0
1
0.8
0.6
0.4
0.2


ROC Curve: preferential_attachment
True Positive Rate (Positive label: 1)
False Positive Rate (Positive label: 1)
Classifier (AUC = 0.71)
Chance level (AUC = 0.5)
0.0
0.2
0.4
0.6
0.8
1.0



References:

Gerard Hoberg and Gordon Phillips, 2016, Text-Based Network Industries and Endogenous Product Differentiation.Journal of Political Economy 124 (5), 1423-1465.

Gerard Hoberg and Gordon Phillips, 2010, Product Market Synergies and Competition in Mergers and Acquisitions: A Text-Based Analysis. Review of Financial Studies 23 (10), 3773-3811.

CHAPTER

TWENTYFOUR

EARNINGS SPATIAL REGRESSION

Everything is related to everything else. But near things are more related than distant things - Waldo Tobler

Earnings surprises occur when a company reports earnings that are significantly different from what analysts had predicted. Spatial regression models help analyze how these surprises propagate through interconnected firms, particularly when firms operate within related industries. By using text-based firm similarity scores as spatial weights, we examine how earnings surprises cluster and spread across firms with similar product market characteristics.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
from pandas import DataFrame, Series
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from tqdm import tqdm
import io
import zipfile
from libpysal.weights import W
from esda.moran import Moran
import spreg
import networkx as nx
import statsmodels.formula.api as smf
import warnings
import requests
from finds.database import SQL, RedisDB
from finds.structured import BusDay, CRSP, PSTAT
from finds.readers import Alfred
from finds.recipes import remove_outliers
from finds.utils import Store
from secret import credentials, paths, CRSP_DATE
#pd.set_option('display.max_rows', 50)
VERBOSE = 0
#%matplotlib qt
```

```
# open connections
sql = SQL(**credentials['sql'], verbose=VERBOSE)
user = SQL(**credentials['user'], verbose=VERBOSE)
rdb = RedisDB(**credentials['redis'])
bd = BusDay(sql, verbose=VERBOSE)
crsp = CRSP(sql, bd, rdb=rdb, verbose=VERBOSE)
pstat = PSTAT(sql, bd, verbose=VERBOSE)
store = Store(paths['scratch'])
```

```
LAST_DATE = CRSP_DATE
scheme = 'tnic3'
# fetch NBER recession dates
alf = Alfred(api_key=credentials['fred']['api_key'], verbose=VERBOSE)
vspans = alf.date_spans('USREC')  # to indicate recession periods in the plots
```

```
# create table to lookup gvkey by permno
lookup = pstat.build_lookup(source='lpermno', target='gvkey', fillna=0)
```

24.1 Earnings surprise

An earnings surprise, or unexpected earnings, is the difference between the reported earnings and expected earnings. The unexpected earnings per share when scaled by the stock price, at the fiscal end date, is more comparable across stocks of different market size and price levels.

```
# require and get actual quarterly earnings, reindex by (permno, rebaldate)
df = pstat.get_linked(dataset='quarterly',
                      fields=['prccq', 'cshoq', 'ibq'],
                      date_field='datadate',
                      where=f"datadate <= {LAST_DATE}")
# store['fund'] = df
```

```
fund = df.dropna(subset=['ibq'])\
         .sort_values(['permno', 'datadate', 'cshoq'])\
         .drop_duplicates(['permno', 'datadate'])\
         .reset_index()
fund['rebaldate'] = bd.endmo(fund['datadate'])

# calculate sue with lag(4) difference in compustat quarterly and price
lag = fund.shift(4, fill_value=0)
keep = ((lag['permno'] == fund['permno']) &
        (fund['prccq'] > 5)).values
fund.loc[keep, 'sue'] = ((fund.loc[keep, 'ibq'] - lag.loc[keep, 'ibq']) /
                         abs(fund.loc[keep, 'prccq'] * fund.loc[keep, 'cshoq']))
print('with pstat earnings', np.sum(~fund['sue'].isna()))
```

```
with pstat earnings 740370
```

```
sue = fund.loc[~fund['sue'].isna(), ['permno', 'rebaldate', 'sue']]\
          .reset_index(drop=True)
sue['rebaldate'] //= 100
```

24.2 Spatial dependence models

24.2.1 Moran's I

Moran's I is a statistical measure that quantifies spatial autocorrelation, assessing whether earnings surprises exhibit clustering across firms. The expected value of Moran's I under a random distribution is:

$$E(I) = \frac{-1}{N-1}$$

A positive Moran's I value suggests earnings surprises tend to cluster among similar firms, while a negative value indicates dispersion. This test provides a conservative benchmark by comparing results to a null hypothesis of zero spatial dependence, which helps identify significant deviations from randomness.

More details: Moran's I (Wikipedia)

24.2.2 Spatial lag model

The **spatial lag model** extends traditional regression analysis by incorporating the influence of neighboring observations. The model is expressed as:

The **spatial lag model** extends traditional regression analysis by incorporating the influence of neighboring observations. The model is expressed as:

$$Y_i = \beta X_i + \rho W Y_j + \epsilon_i$$

where:

- Y_i is the dependent variable (e.g., earnings surprise) for firm i,
- Y_j represents the corresponding values from other firms,
- W is a matrix of spatial weights that assigns higher values to firms that are more closely related to firm i,
- ho captures the strength of spatial dependence, and
- ϵ_i is the error term.

Further reading:

- Spatial Lag Model (Lost Stats)
- spreg.ML_Lag (PySAL)

We use text-based firm similarity scores as spatial weights: the TNIC-3 scores, developed by Hoberg and Phillips (2016), are derived from product descriptions in 10-K filings. Their methodology constructs firm-by-firm similarity scores using word vectors while filtering out common words and focusing on nouns and proper nouns, excluding geographic terms.

```
# Retrieve TNIC linkages from Hoberg and Phillips website
root = 'https://hobergphillips.tuck.dartmouth.edu/idata/'
source = root + scheme + '_data.zip'
if source.startswith('http'):
    response = requests.get(source)
    source = io.BytesIO(response.content)
with zipfile.ZipFile(source).open(scheme + "_data.txt") as f:
    tnic_df = pd.read_csv(f, sep='\s+')
store['spatial'] = (fund, tnic_df)
tnic_df['year'].value_counts().sort_index().to_frame()
```

```
        count
year
1988   311616
1989   640221
1990   663746
1991   699152
1992   788029
1993   940368
1994  1027776
1995  1095721
1996  1197991
1997  1138937
1998  1093821
1999  1174065
2000  1121918
2001   956591
2002   843735
2003   781521
2004   719787
2005   699340
2006   709914
2007   683662
2008   615445
2009   532159
2010   509755
2011   499887
2012   484441
2013   513159
2014   579649
2015   597923
2016   580988
2017   591097
2018   644292
2019   669597
2020   700725
2021   861057
2022   823596
2023   670149
```

```
# Compute quarterly spatial regression coefficients
out = dict(moran={}, rho={}, beta={})
years = range(1988, 2024)
for year in tqdm(years):
    tnic = tnic_df[tnic_df.year == year].dropna()  # extract the year's tnic links

    # populate graph from tnic edges, with gvkey as nodes
    graph = nx.Graph()
    graph.add_edges_from(tnic[['gvkey1', 'gvkey2']].values)
    graph.remove_edges_from(nx.selfloop_edges(graph))   # not necessary

    for qtr in [12, 103, 106, 109]:   # loop over next four quarters of sue
        rebaldate = year*100 + qtr

        # extract sue's with in this fiscal quarter
        y = sue[sue['rebaldate'] == rebaldate].set_index('permno')[['sue']]
```

(continues on next page)

(continued from previous page)

```
        # lookup gvkeys
        y['gvkey'] = lookup(y.index, date=bd.endmo(rebaldate))

        # merge in stock returns in the quarter
        y = y.join(crsp.get_ret(bd.begmo(rebaldate, months=-2),
                                bd.endmo(rebaldate)),
                   how='left')\
             .set_index('gvkey')

        # require available and not outlier
        y = remove_outliers(y[~y.index.duplicated() & (y.index > 0)]).dropna()

        # extract subgraph, its nodes and their neighbors
        G = graph.subgraph(y.index)
        G = G.subgraph(max(nx.connected_components(G), key=len))
        neighbors = {node: list(G.neighbors(node)) for node in G.nodes()}
        w = W(neighbors)
        y = y.loc[sorted(neighbors.keys())]

        # compute Moran's I and spatial lag model of sue on past stock returns
        mi = Moran(y['sue'].values,  w)
        with warnings.catch_warnings():
            warnings.simplefilter("ignore")
            spatial = spreg.ML_Lag(y=y['sue'].values, x=y[['ret']].values, w=w,
                                   name_x=['ret'], name_y='sue', name_w=scheme,
                                   name_ds=str(rebaldate))
        out['moran'][rebaldate] = mi.I
        out['rho'][rebaldate] = float(spatial.rho)
        out['beta'][rebaldate] = float(spatial.betas[1][0])
```

```
100%|██████████| 36/36 [17:49<00:00, 29.70s/it]
```

```
# Show latest quarter's results
print(spatial.summary)
```

```
REGRESSION
----------
SUMMARY OF OUTPUT: MAXIMUM LIKELIHOOD SPATIAL LAG (METHOD = FULL)
-----------------------------------------------------------------
Data set            :      202409
Weights matrix      :       tnic3
Dependent Variable  :         sue                Number of Observations:       ␣
 ↪2177
Mean dependent var  :     -0.0003                Number of Variables   :          ␣
 ↪3
S.D. dependent var  :      0.0242                Degrees of Freedom    :       ␣
 ↪2174
Pseudo R-squared    :      0.0204
Spatial Pseudo R-squared:  0.0040
Sigma-square ML     :       0.001                Log likelihood        :    5026.
 ↪680
S.E of regression   :       0.024                Akaike info criterion :  -10047.
 ↪360
                                                 Schwarz criterion     :  -10030.
```

(continues on next page)

(continued from previous page)

```
↪303

--------------------------------------------------------------------------------
↪-
            Variable     Coefficient       Std.Error     z-Statistic     ␣
↪Probability
--------------------------------------------------------------------------------
↪-
            CONSTANT      -0.0008998       0.0005767      -1.5602720       0.
↪1186956
                 ret       0.0067306       0.0022802       2.9516958       0.
↪0031603
               W_sue       0.2247160       0.0440540       5.1009164       0.
↪0000003
--------------------------------------------------------------------------------
↪-
================================ END OF REPORT␣
↪=====================================
```

```
# Show spatial regression results
ols = spreg.OLS(y=y['sue'].values, x=y[['ret']].values, w=w,
                robust='white', spat_diag=True, moran=True,
                name_x=['ret'], name_y='sue', name_w=scheme,
                name_ds=str(rebaldate))
print(ols.summary)
```

```
REGRESSION
----------
SUMMARY OF OUTPUT: ORDINARY LEAST SQUARES
-----------------------------------------
Data set            :      202409
Weights matrix      :       tnic3
Dependent Variable  :         sue                Number of Observations:       ␣
↪2177
Mean dependent var  :     -0.0003                Number of Variables   :          ␣
↪2
S.D. dependent var  :      0.0242                Degrees of Freedom    :       ␣
↪2175
R-squared           :      0.0042
Adjusted R-squared  :      0.0037
Sum squared residual:       1.273                F-statistic           :      9.
↪0806
Sigma-square        :       0.001                Prob(F-statistic)     :    0.
↪002613
S.E. of regression  :       0.024                Log likelihood        :    5014.
↪003
Sigma-square ML     :       0.001                Akaike info criterion :   -10024.
↪005
S.E of regression ML:      0.0242                Schwarz criterion     :   -10012.
↪634

White Standard Errors
--------------------------------------------------------------------------------
↪-
            Variable     Coefficient       Std.Error     t-Statistic     ␣
```

(continues on next page)

(continued from previous page)

```
↪Probability
--------------------------------------------------------------------------------
↪-
            CONSTANT      -0.0011401       0.0006542      -1.7428829       0.
↪0814954
                 ret       0.0069264       0.0032943       2.1025231       0.
↪0356221
--------------------------------------------------------------------------------
↪-

REGRESSION DIAGNOSTICS
MULTICOLLINEARITY CONDITION NUMBER            1.628

TEST ON NORMALITY OF ERRORS
TEST                             DF        VALUE           PROB
Jarque-Bera                       2        4787.738           0.0000

DIAGNOSTICS FOR HETEROSKEDASTICITY
RANDOM COEFFICIENTS
TEST                             DF        VALUE           PROB
Breusch-Pagan test                1          63.692           0.0000
Koenker-Bassett test              1          13.760           0.0002

DIAGNOSTICS FOR SPATIAL DEPENDENCE
TEST                           MI/DF       VALUE           PROB
Moran's I (error)              0.0516         5.283           0.0000
Lagrange Multiplier (lag)         1          26.684           0.0000
Robust LM (lag)                   1           0.627           0.4285
Lagrange Multiplier (error)       1          27.229           0.0000
Robust LM (error)                 1           1.172           0.2790
Lagrange Multiplier (SARMA)       2          27.856           0.0000

================================ END OF REPORT␣
↪=====================================
```

```python
# Show Moran's I
print(f"Quarter: {rebaldate}")
DataFrame({"Moran's I": mi.I, "Expected": mi.EI, "p_norm": mi.p_norm},
          index=["under assumption of normality"])
```

```
Quarter: 202409
```

```
                              Moran's I  Expected        p_norm
under assumption of normality   0.051519  -0.00046  1.401822e-07
```

```python
# Show ordinary regression results
results = DataFrame(out)
results.index = bd.to_datetime(bd.endmo(results.index))
summary = dict()
for col in results.columns:
    model = smf.ols(f'{col} ~ 1', data=results).fit()
    robust = smf.ols(f'{col} ~ 1', data=results)\
                .fit(cov_type='HAC', cov_kwds={'maxlags': 6})
```

(continues on next page)

(continued from previous page)

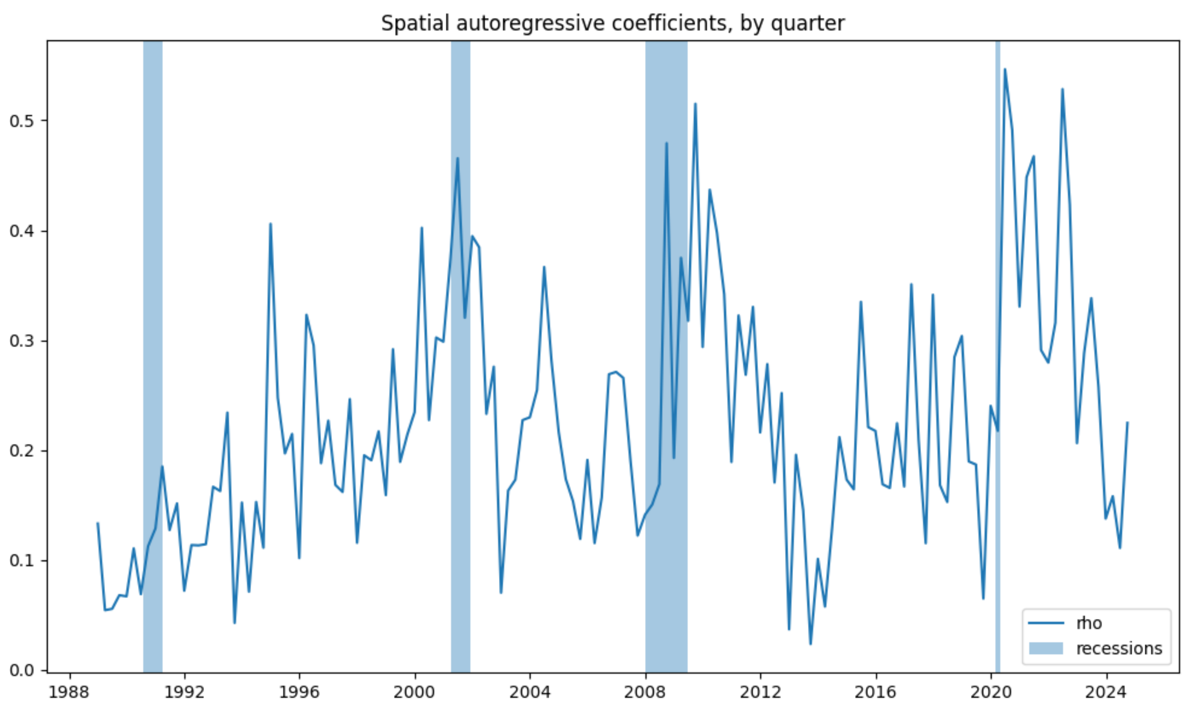
```
    summary[col] = dict(mean=float(model.params.iloc[0]),
                        stderr=float(model.bse.iloc[0]),
                        t=float(model.tvalues.iloc[0]),
                        nw_stderr=float(robust.bse.iloc[0]),
                        nw_t=float(robust.tvalues.iloc[0]))
print("Tests of statistical significance")
DataFrame(summary)
```

```
Tests of statistical significance
```

	moran	rho	beta
mean	0.053146	0.224189	0.011726
stderr	0.002577	0.009331	0.000690
t	20.623401	24.027336	17.006094
nw_stderr	0.004606	0.017202	0.000952
nw_t	11.538987	13.032706	12.323118

To track changes over time, we plot the strength of spatial coefficients on a quarterly basis. Additionally, we highlight U.S. recession periods in blue, as times of economic stress may amplify the strength of linkages.

```
# Visualize autoregressive coefficients by quarter
fig, ax = plt.subplots(figsize=(10, 6))
ax.plot(results['rho'])
ax.set_title("Spatial autoregressive coefficients, by quarter")
for a,b in vspans:
    if a >= min(results.index):
        ax.axvspan(a, min(b, max(results.index)), alpha=0.4)
plt.legend(['rho', 'recessions'])
plt.tight_layout()
plt.show()
```



CHAPTER

TWENTYFIVE

FOMC TOPIC MODELING

Our discussions of the economy may sometimes ring in the ears of the public with more certainty than is appropriate - Jerome Powell

The Federal Open Market Committee (FOMC) meeting minutes may reveal the Federal Reserve's economic outlook, policy decisions, and potential future actions. Analyzing these minutes can help identify key topics discussed over time, shedding light on trends in monetary policy and their implications for financial markets. We review methods for accessing and pre-processing textual data, including the use MongoDB for storing and retrieving unstructured data. Several topic modeling algorithms, including Latent Semantic Analysis (LSA), Latent Dirichlet Allocation (LDA), Non-negative Matrix Factorization (NMF), and Probabilistic Latent Semantic Indexing (PLSI), are applied to uncover patterns within the minutes.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
from pandas import Series, DataFrame
import os
import sklearn.feature_extraction, sklearn.decomposition
from sklearn.decomposition import TruncatedSVD, LatentDirichletAllocation, NMF
from sklearn.cluster import KMeans
from scipy.special import softmax
from wordcloud import WordCloud
import wordcloud
import matplotlib.pyplot as plt
from finds.database import MongoDB
from finds.unstructured import Unstructured
from finds.utils import Store
from finds.readers import FOMCReader, Alfred
from pprint import pprint
from secret import credentials, paths
# %matplotlib qt
VERBOSE = 0
```

```
## retrieve recessions dates for plotting
alf = Alfred(api_key=credentials['fred']['api_key'])
vspans = alf.date_spans(series_id='USREC')
```

25.1 FOMC meeting minutes

The FOMC holds eight scheduled meetings per year, with additional meetings as necessary. The minutes from these meetings are released three weeks after the policy decision, offering crucial insights into the Federal Reserve's economic stance and possible future monetary policy actions. These insights can influence interest rates, inflation expectations, and overall market conditions.

For official meeting schedules and minutes, visit: Federal Reserve FOMC Calendar.

25.1.1 FinDS `fomcreader` module

The `fomcreader` module in the FinDS packages offers functions for searching and retrieving meeting minutes from the Federal Reserve website.

```
# Helper class to scrape FOMC meeting minutes from Federal Reserve website
minutes = FOMCReader()

# Show number and date range of minutes available from website
DataFrame({'dates': len(minutes), 'start': min(minutes), 'end': max(minutes)},
          index=['FOMC minutes'])
```

```
100%|██████████| 27/27 [00:16<00:00,  1.66it/s]
```

```
              dates     start       end
FOMC minutes    256  19930203  20250129
```

25.1.2 MongoDB

MongoDB is a NoSQL ("not only SQL") document-oriented database designed to efficiently manage unstructured and semi-structured data. Unlike traditional relational databases, MongoDB does not enforce a fixed schema, allowing for greater flexibility in handling diverse data formats. Each document (record) within a collection (table equivalent) can contain different fields and structures, including key-value pairs, arrays, and embedded subdocuments.

```
# store unstructured minutes text data in MongoDB
from pprint import pprint
mongodb = MongoDB(**credentials['mongodb'], verbose=VERBOSE)
print('uptime:', mongodb.client.admin.command("serverStatus")['uptime'])
fomc = Unstructured(mongodb, 'FOMC')
```

```
177.0
```

```
# retrieve keys (dates) of minutes previously retrieved and stored locally
dates = fomc['minutes'].distinct('date')
```

```
# fetch new minutes from FOMC site
docs = {d: minutes[d] for d in minutes if d not in dates}
print("New minutes:")
pprint([f"{k}: {len(v)} chars" for k,v in docs.items()])
```

```
New minutes:
['20250129: 54324 chars',
 '20240612: 43860 chars',
 '20240731: 44839 chars',
 '20240918: 44372 chars',
 '20241107: 49178 chars',
 '20241218: 45712 chars']
```

Preprocessing Meeting Minutes

1. Extract relevant sections of the minutes, starting from:
   - "Review of Monetary Policy Strategy, Tools, and Communications"
   - "Developments in Financial Markets"
   - "Discussion of Guidelines for Policy Normalization"
   - "Financial Developments and Open Market Operations"
   - "Discussion of the Economic Outlook"
   - "The information reviewed at this meeting"
   - "The staff presented several briefings"
2. Remove text following the adjournment line or the date scheduled for the next meeting. Exclude:
   - Notation votes, approvals of minutes, signatures, and footnotes
   - Intermeeting conference call discussions

Once extracted, store and retrieve all meeting minutes from MongoDB for further processing.

```python
# Helper function to trim minutes text
def edit(text: str) -> str:
    """helper to spawn editor and write/edit/read to tempfile"""
    import subprocess
    import tempfile
    with tempfile.NamedTemporaryFile(suffix=".tmp") as f: # save temp file
        f.write(text.encode("utf-8"))
        f.flush()
        subprocess.call([os.environ.get('EDITOR','emacs'), "-nw", f.name])
        f.seek(0)
        return f.read().decode("utf-8")         # keep edited text
```

```python
if docs:
    # to edit out head and tail of each document
    results = list()
    for date, initial_message in docs.items():
        edited_text = edit(initial_message)
        results.append({'date': date, 'text' : edited_text})
    results = sorted(results, key = lambda x: x['date'])    # sort by date

    # save edited docs
    Store(paths['scratch'] / 'fomc', ext='gz').dump(results, f"{max(docs.keys())}.json
↪")
    for doc in results: # store docs for new dates
        fomc.insert('minutes', doc, keys=['date'])
```

Retrieve all minutes that were stored locally in MongoDB

```
docs = Series({doc['date']: doc['text'] for doc in fomc.select('minutes')},
              name='minutes').sort_index()
DataFrame(docs)
```

```
                                                          minutes
19930203  The Manager of the System Open Market Account ...
19930323  The Deputy Manager for Domestic Operations rep...
19930518  The Manager of the System Open Market Account ...
19930707  The Deputy Manager for Domestic Operations rep...
19930817  The Deputy Manager for Domestic Operations rep...
...                                                   ...
20240731  Developments in Financial Markets and Open Mar...
20240918  Developments in Financial Markets and Open Mar...
20241107  Developments in Financial Markets and Open Mar...
20241218  Developments in Financial Markets and Open Mar...
20250129  Developments in Financial Markets and Open Mar...

[256 rows x 1 columns]
```

25.2 Text pre-processing

Text preprocessing involves cleaning and preparing raw text data by removing noise, standardizing formats, and converting text into a structured form suitable for analysis. This process typically includes tokenization, removing stop words, and vectorizing the text.

25.2.1 Tokenization

Tokenization splits text into smaller units called **tokens**, typically words, though they may also be phrases or subwords. The pattern of tokenization is often defined using regular expressions.

The original **casing** of words may be preserved if it carries meaningful information, such as distinguishing proper nouns, acronyms, or sentence boundaries. Some models may interpret all uppercase words as organizations.

25.2.2 Regular expressions

Regular expressions (**regex**) define search patterns for text processing, often used in tokenization to find word boundaries and remove unwanted characters.

Basic Regular Expressions

Expression	Description
`\d`	Matches a digit (0-9)
`\w`	Matches a word character (ASCII letter, digit, or underscore)
`\s`	Matches a whitespace character (space, tab, newline)
`\D`	Matches a non-digit character
`\W`	Matches a non-word character
`\S`	Matches a non-whitespace character
`[ ... ]`	Matches one of the characters in the brackets
`[a-zA-Z]`	Matches any letter (uppercase or lowercase)
`[^a]`	Matches any character except `a`
`\b`	Matches a word boundary
`.`	Matches any character except a line break
`\.`	Matches a period character
`^`	Matches the start of a string
`$`	Matches the end of a string
`+`	Matches one or more occurrences
`*`	Matches zero or more occurrences
`?`	Matches zero or one occurrence
`|`	Acts as an OR operator
`( ... )`	Defines a capturing group

25.2.3 Stopwords

Common, uninformative words (e.g., "the," "is," "and") can be removed to focus on meaningful content in the analysis. Corpus-specific uninformative words (such as calendar-related terms in FOMC minutes) can also be excluded.

```
# ignore these stop words
StopWords = [w for w in set(wordcloud.STOPWORDS) if "'" not in w]
StopWords += ['january', 'february', 'march', 'april', 'may', 'june',
              'july', 'august', 'september', 'october', 'november',
              'december', 'first', 'second', 'third', 'fourth', 'twelve',
              'participants', 'members', 'meeting']
```

25.2.4 Vectorization

Vectorization transforms text into numerical representations suitable for analysis.

- **Bag-of-Words (BoW):** Represents a document by the frequency of its words, ignoring grammar and word order.
- **N-grams:** Treats consecutive words as distinct items rather than isolated words.
- Indexing: Assigns unique integer indexes to words in the corpus.
- **TF-IDF (Term Frequency-Inverse Document Frequency):** Measures word importance based on its frequency in a document relative to its occurrence across all documents.

- The `scikit-learn` package provides `TfidfVectorizer` for TF-IDF vectorization and `CountVectorizer` for raw word counts.

```
# To vectorize the input words
#ngram_range = (1, 1)    # unigrams
#ngram_range = (2, 2)    # bigrams
ngram_range = (1, 2)    # unigrams and bigrams
max_df, min_df, max_features = 0.5, 6, 5000 # some reasonable constraints
tfidf_vectorizer = sklearn.feature_extraction.text.TfidfVectorizer(
    strip_accents='unicode',
    lowercase=True,
    stop_words=StopWords,
    ngram_range=ngram_range,
    max_df=max_df,
    min_df=min_df,
    max_features=max_features,
    token_pattern=r"\b[^\d\W][^\d\W][^\d\W]+\b") #r'\b[^\d\W]+\b'
tf_vectorizer = sklearn.feature_extraction.text.CountVectorizer(
    strip_accents='unicode',
    lowercase=True,
    stop_words=StopWords,
    ngram_range=ngram_range,        # (2, 2) for bigrams
    max_df=max_df,
    min_df=min_df,
    max_features=max_features,
    token_pattern=r"\b[^\d\W][^\d\W][^\d\W]+\b")
```

25.3 Topic Modeling

Topic modeling applies statistical techniques to discover latent topics within a collection of documents.

25.3.1 Latent Semantic Analysis (LSA)

LSA uses singular value decomposition (SVD) to analyze relationships between terms and documents. By reducing dimensionality, it groups terms and documents that frequently co-occur in a lower-dimensional space, revealing underlying topics.

25.3.2 Latent Dirichlet Allocation (LDA)

LDA assumes each document is a mixture of topics, and each topic is a mixture of words. It iteratively assigns words to topics based on probability distributions, refining topic assignments over multiple iterations.

25.3.3 Non-negative Matrix Factorization (NMF)

NMF decomposes a term-document matrix into two non-negative matrices:

- A basis matrix representing topics
- A coefficient matrix representing the distribution of topics in documents

By optimizing these matrices, NMF extracts meaningful topics from the text.

25.3.4 Probabilistic Latent Semantic Indexing (PLSI)

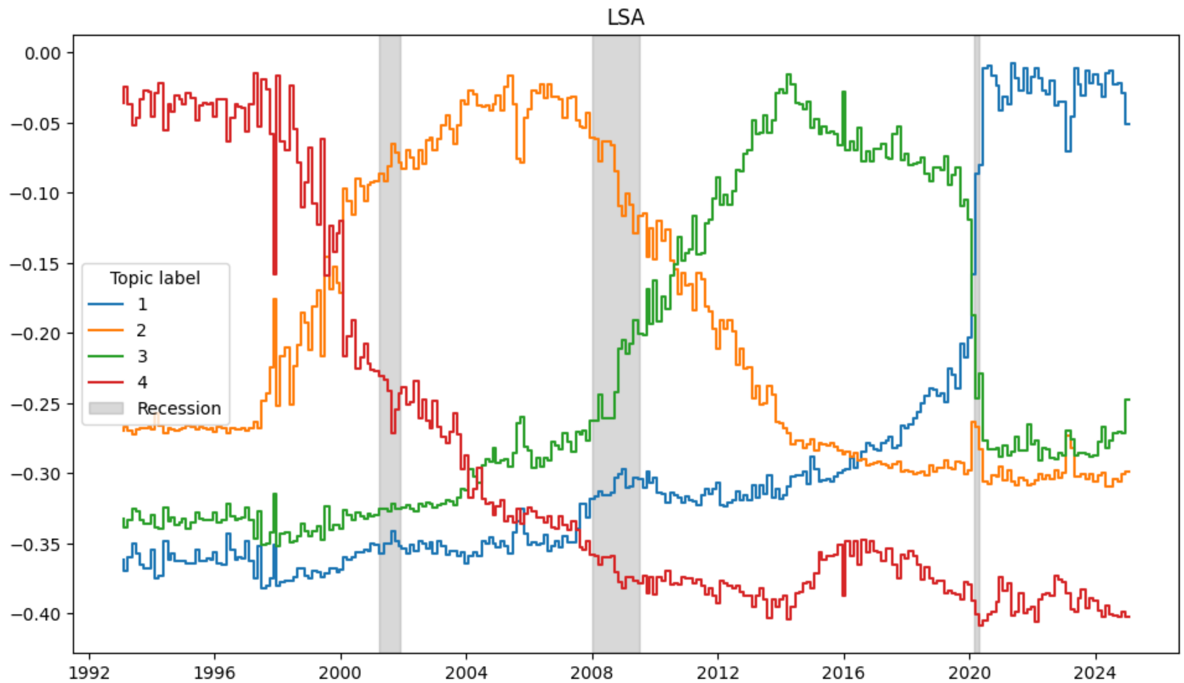
PLSI models documents as mixtures of topics, estimating probability distributions of words and topics iteratively until convergence. It can be implemented using NMF with generalized Kullback-Leibler divergence as the loss function.

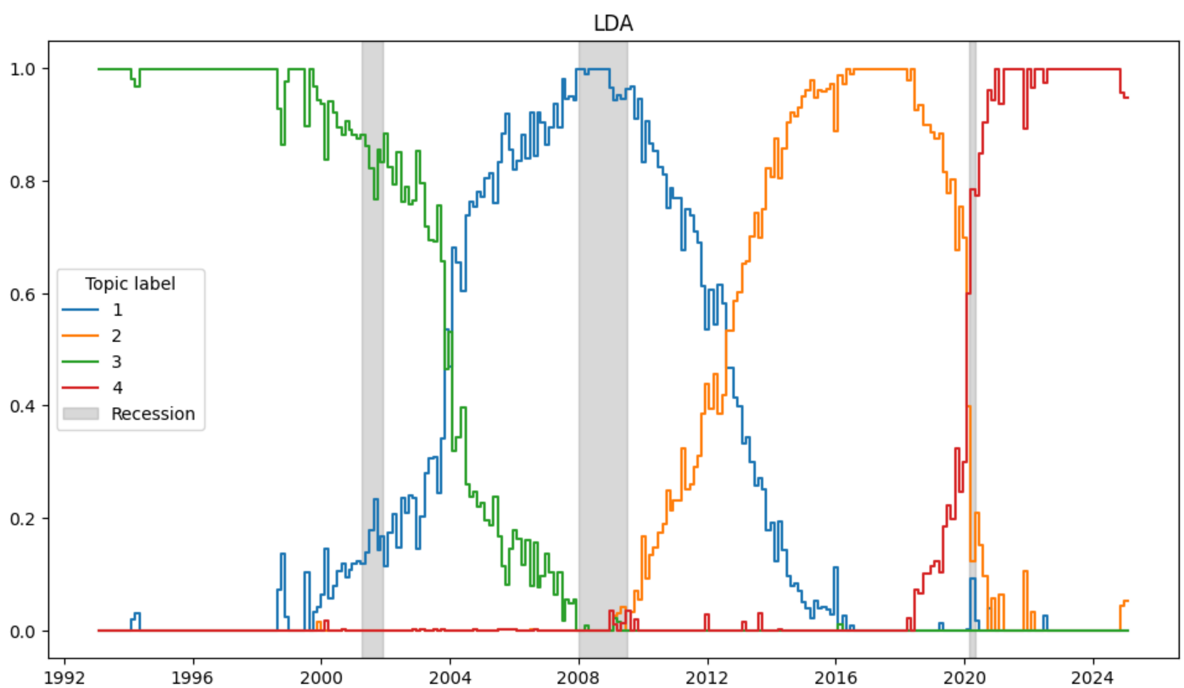
- scikit-learn decomposition documentation
- Example of topic extraction with NMF & LDA

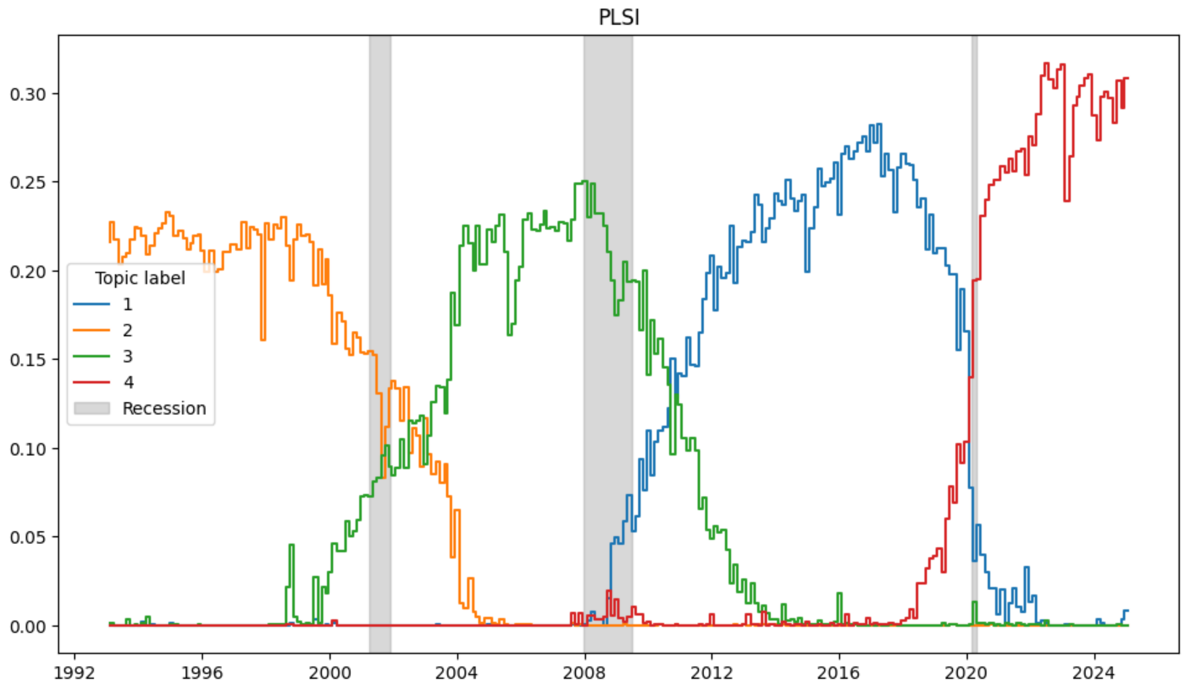
```
# Define models
n_components = 4   # fix number of latent topics
algos = {
    'LSA': (TruncatedSVD(n_components=n_components),
            tfidf_vectorizer),
    'LDA': (LatentDirichletAllocation(n_components=n_components,
                                      learning_method='batch', #'online',
                                      # learning_offset = 50.0,
                                      max_iter = 40,
                                      random_state = 42),
            tf_vectorizer),
    'PLSI': (NMF(n_components=n_components,
                 beta_loss='kullback-leibler',
                 solver='mu',
                 alpha_W=0.00005,
                 alpha_H=0.00005,
                 l1_ratio=0.5,
                 max_iter=1000,
                 random_state = 42),
             tfidf_vectorizer),
    'NMF': (NMF(n_components=n_components,
                random_state=42,
                beta_loss='frobenius',
                alpha_W=0.00005,
                alpha_H=0.00005,
                l1_ratio=0.5),
            tfidf_vectorizer)}
```

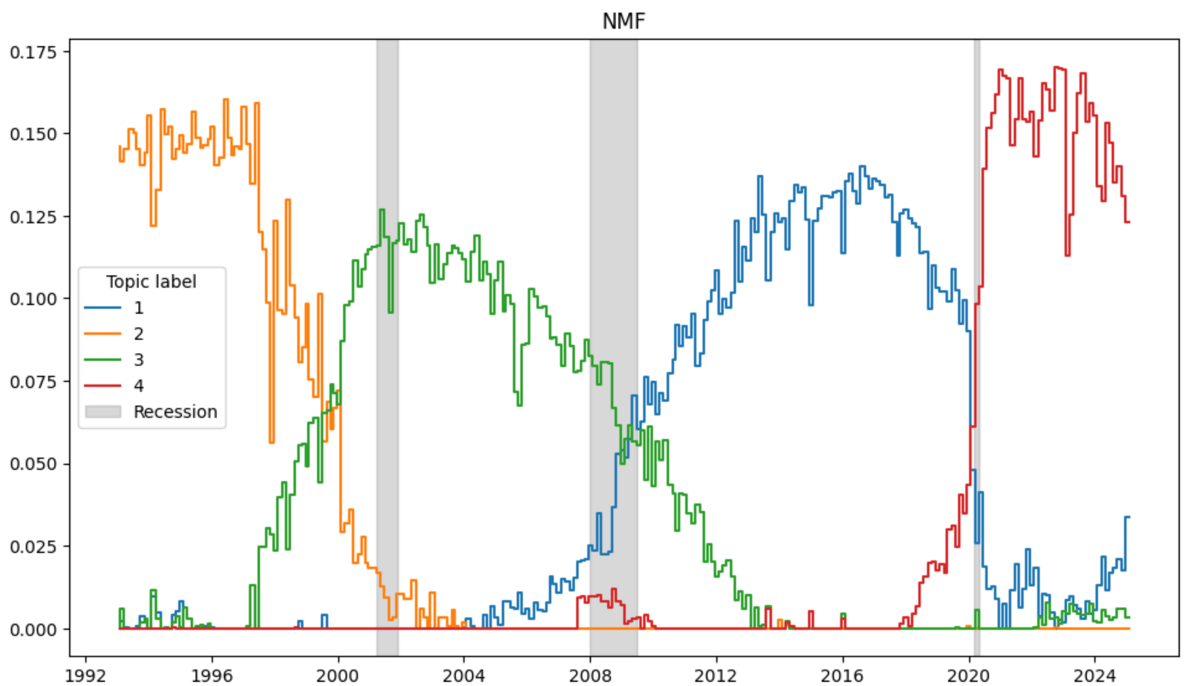
Once topics are extracted, they can be visualized over time to observe trends in FOMC discussions.

```
# Fit and plot models
scores = dict()   # to save model coefficients
topics = dict()   # to save dates of primary topic
for ifig, (name, (base, vectorizer)) in enumerate(algos.items()):

    # vectorize the input words
    vectorized = vectorizer.fit_transform(docs.to_list())
    feature_names = vectorizer.get_feature_names_out()

    # fit model and transform inputs
    model = base.fit(vectorized)
    transformed = model.transform(vectorized)

    # save the model fitted coefficients
    if name == 'LSA':  # Additional step for LSA
        kmeans = KMeans(n_clusters=n_components, n_init=5, random_state=37)\
            .fit(transformed)    # find centroids of the latent factors
        transformed = kmeans.transform(transformed)  # distance to centroid
        transformed = -(transformed / transformed.sum(axis=1, keepdims=True))
        scores[name] = softmax(model.components_, axis=1)  # scale word scores
    else:
        scores[name] = model.components_

    # plot topic scores over time
    fig, ax = plt.subplots(num=1 + ifig, clear=True, figsize=(10, 6))
    dates = pd.DatetimeIndex(docs.index.astype(str))
    ax.step(dates, transformed, where='pre')
    for a,b in vspans:
        if b >= min(dates):
            ax.axvspan(a, min(b, max(dates)), alpha=0.3, color='grey')
    ax.set_title(name)
    ax.legend([f"{i+1}" for i in range(n_components)] + ['Recession'],
              loc='center left', title='Topic label')
    plt.tight_layout(pad=2)

    # save dates of primary topic
    for topic in range(transformed.shape[1]):
        arg = DataFrame({'t': np.argmax(transformed, axis=1),
                         'dt': docs.index})
        dates = (arg!=arg.shift()).cumsum().groupby('t').agg(['first', 'last']) - 1
        dates['topic'] = arg.loc[dates.iloc[:,1], 't'].values
        topics[name] = {topic: [(arg['dt'].iloc[row[0]], arg['dt'].iloc[row[1]])
                                for row in dates.itertuples(index=False, name=None)
                                if row[2] == topic]
                        for topic in range(transformed.shape[1])}
```


LSA
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
−0.35
−0.40
Topic label
1
2
3
4
Recession
1992
1996
2000
2004
2008
2012
2016
2020
2024


LDA
1.0
0.8
0.6
0.4
0.2
0.0
Topic label
1
2
3
4
Recession
1992
1996
2000
2004
2008
2012
2016
2020
2024

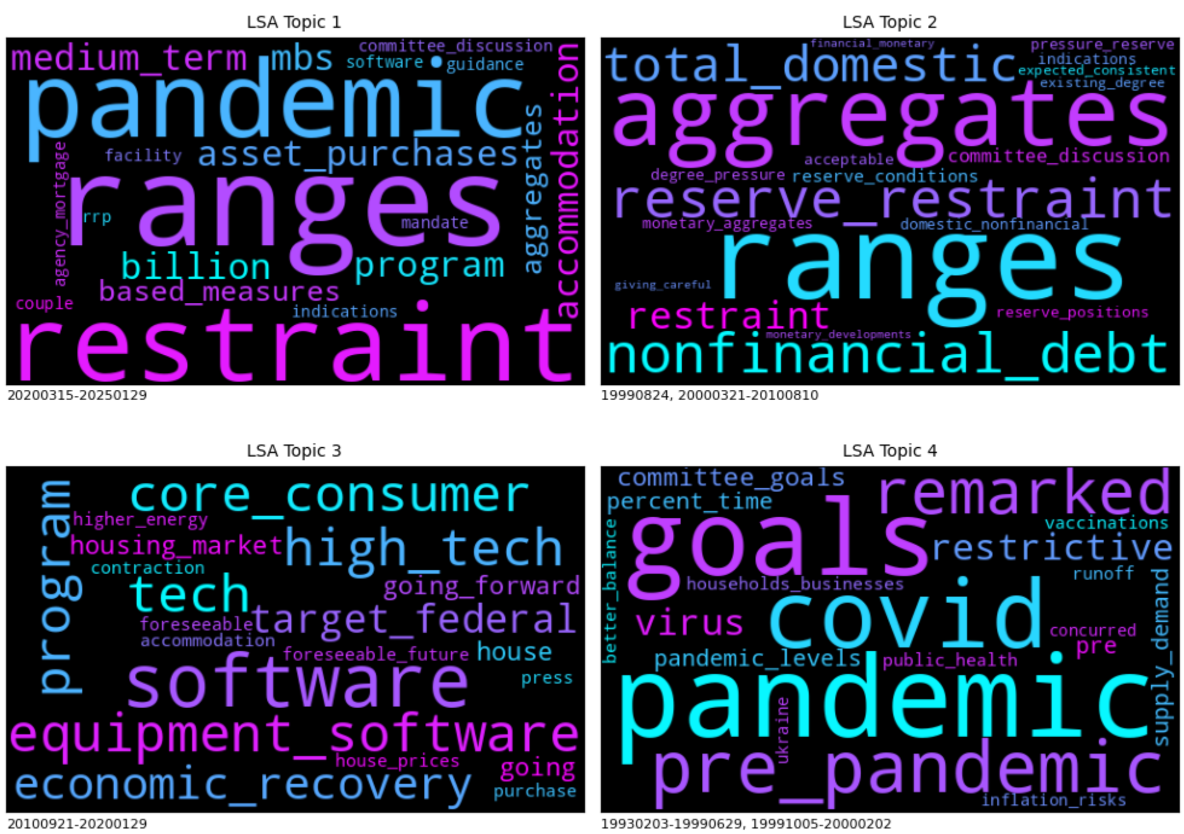

PLSI
Topic label
1
2
3
4
Recession
0.30
0.25
0.20
0.15
0.10
0.05
0.00
1992
1996
2000
2004
2008
2012
2016
2020
2024


NMF
Topic label
1
2
3
4
Recession
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
1992
1996
2000
2004
2008
2012
2016
2020
2024

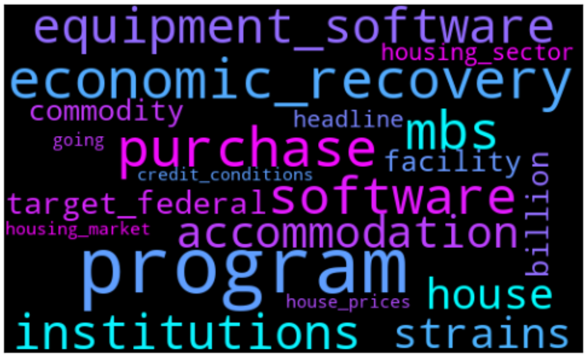
25.3.5 Feature importance

The most significant words for each topic are identified and visualized using the `WordCloud` package, providing an intuitive representation of key themes.

```
# Display word cloud of top n features, by model and topic
figsize = (10, 8)
for ifig, (name, score) in enumerate(scores.items()):
    wc = WordCloud(height=300, width=500, colormap='cool')
    top_n = 20
    fig, axes = plt.subplots(2, 2, num=ifig+5, figsize=figsize, clear=True)
    for topic, components in enumerate(score):
        words = {feature_names[i].replace(' ','_') : components[i]
                 for i in components.argsort()[:-top_n - 1:-1]}
        #print("Topic", topic+1, topics[name])
        #print(list(words.keys()))
        ax = axes[topic//2, topic % 2]
        ax.imshow(wc.generate_from_frequencies(words))
        ax.axes.yaxis.set_visible(False)   # make axes ticks invisible
        ax.xaxis.set_ticks([])
        ax.xaxis.set_ticklabels([])
        ax.set_title(f"{name} Topic {topic+1}", fontsize=10)
        regime = ", ".join([f"{d[0]}-{d[1]}" if d[0] != d[1] else f"{d[0]}"
                            for d in topics[name][topic]])
        ax.set_xlabel(regime if len(regime) < 75 else '-- many --',
                      fontsize=8,
                      loc='left')
        plt.tight_layout()
```

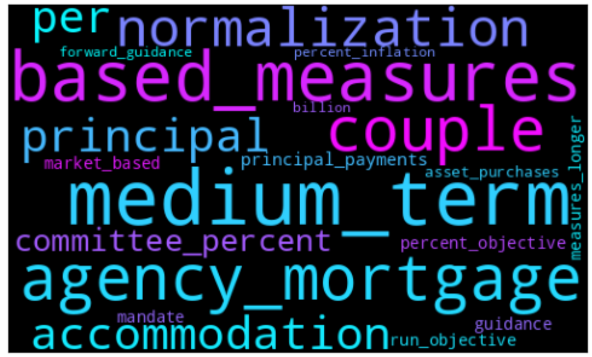


LDA Topic 1



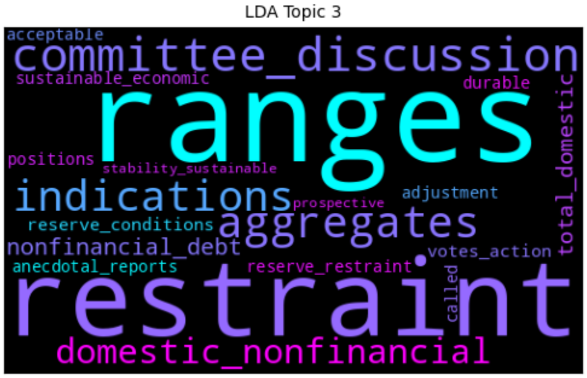
20031209, 20040316-20120801

LDA Topic 2



20120913-20200129

LDA Topic 3

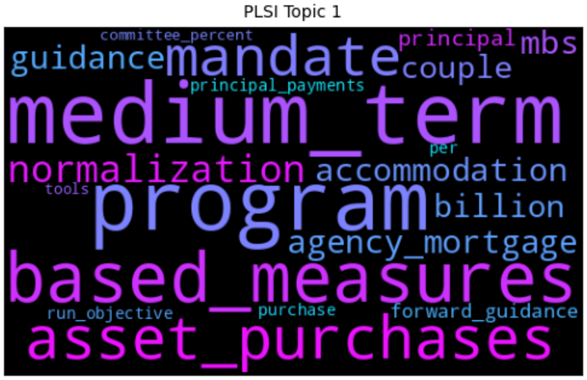


19930203-20031028, 20040128

LDA Topic 4

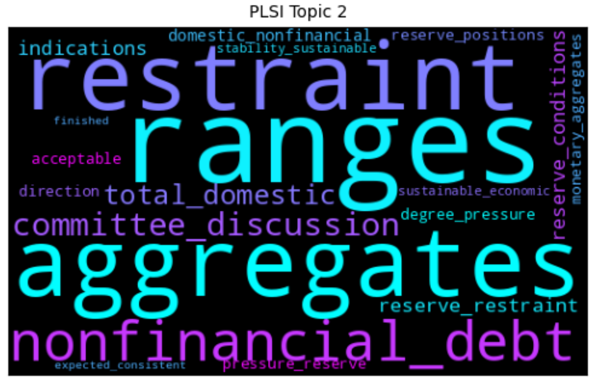
20200315-20250129

PLSI Topic 1



20101103, 20110126-20200129

PLSI Topic 2

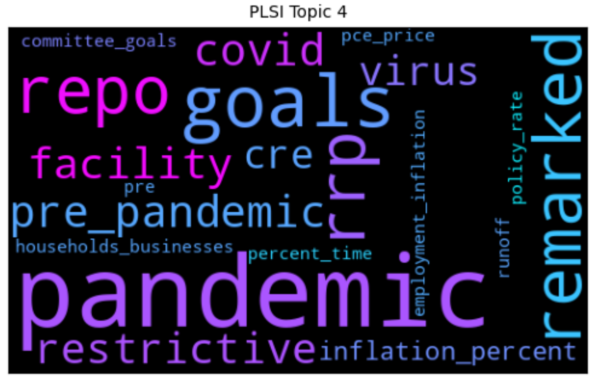


19930203-20010821, 20011106-20020626, 20030129

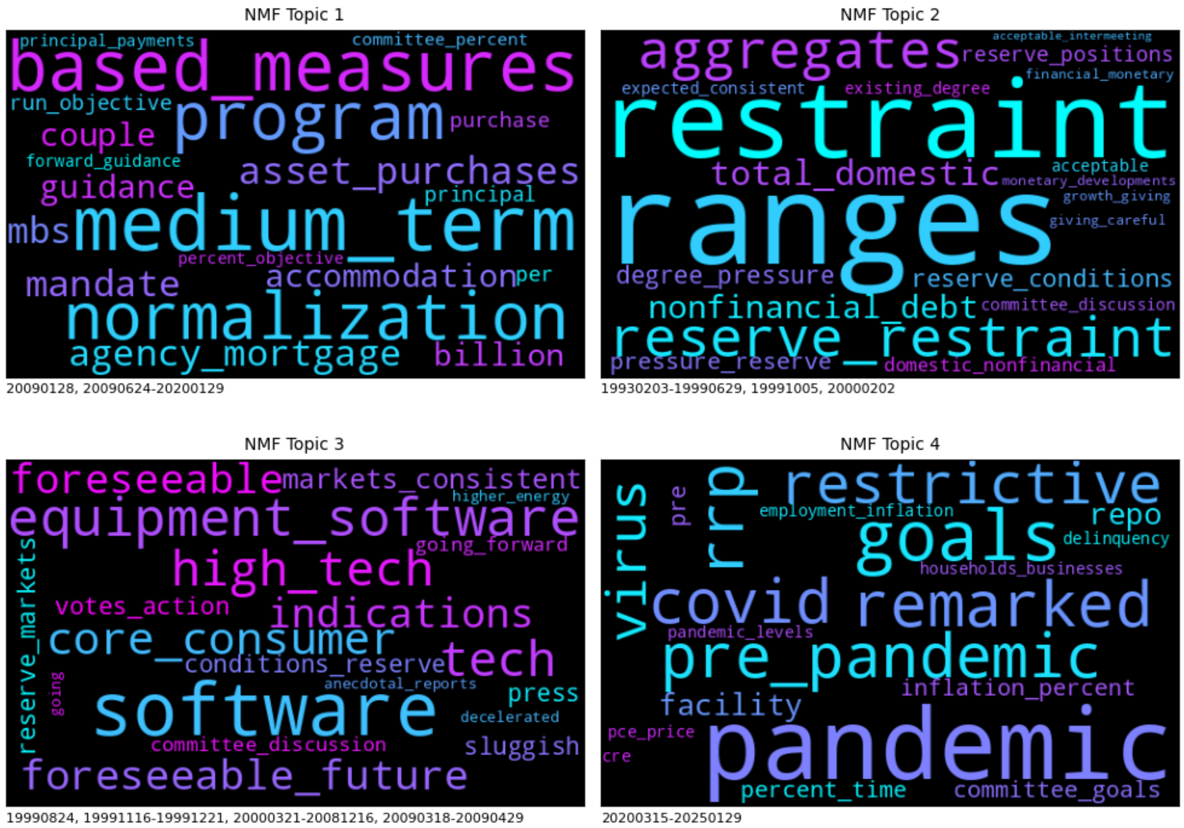
PLSI Topic 3

20011002, 20020813-20021210, 20030318-20100921, 20101214

PLSI Topic 4



20200315-20250129


NMF Topic 1
principal_payments
committee_percent
based_measures
run_objective
program
purchase
couple
forward_guidance
asset_purchases
guidance
principal
mbs
medium_term
percent_objective
mandate
accommodation
per
normalization
agency_mortgage
billion
20090128, 20090624-20200129
NMF Topic 2
aggregates
acceptable_intermeeting
reserve_positions
financial_monetary
expected_consistent
existing_degree
restraint
total_domestic
acceptable
monetary_developments
growth_giving
ranges
giving_careful
degree_pressure
reserve_conditions
nonfinancial_debt
committee_discussion
reserve_restraint
pressure_reserve
domestic_nonfinancial
19930203-19990629, 19991005, 20000202
NMF Topic 3
foreseeable
markets_consistent
higher_energy
equipment_software
going_forward
high_tech
reserve_markets
votes_action
indications
core_consumer
conditions_reserve
tech
anecdotal_reports
going
press
software
decelerated
committee_discussion
sluggish
foreseeable_future
19990824, 19991116-19991221, 20000321-20081216, 20090318-20090429
NMF Topic 4
virus
pre
rrp
restrictive
employment_inflation
repo
goals
delinquency
households_businesses
covid
remarked
pandemic_levels
pre_pandemic
inflation_percent
facility
pce_price
cre
pandemic
percent_time
committee_goals
20200315-20250129

CHAPTER TWENTYSIX

MANAGEMENT SENTIMENT ANALYSIS

Life is a math equation. In order to gain the most, you have to know how to convert negatives into positives - Anonymous

Sentiment analysis helps quantify the tone of financial disclosures, revealing whether a company's management expresses optimism, caution, or concern. This analysis can be conducted using dictionary-based methods, which rely on predefined sentiment word lists, or more sophisticated machine learning models, such as large language models (LLMs). We explore the application of sentiment analysis in financial documents, focusing on the Loughran-MacDonald dictionary and the SEC's EDGAR system for retrieving company filings. Specifically, we analyze sentiment trends in 10-K Management Discussion and Analysis (MD&A) sections and their relationship with stock returns.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
from sklearn.feature_extraction.text import CountVectorizer
from tqdm import tqdm
from finds.database import SQL, RedisDB
from finds.structured import BusDay, CRSP, Signals
from finds.unstructured import Edgar
from finds.readers import Alfred
from finds.recipes import weighted_average, fractile_split
from finds.utils import Store
from secret import credentials, paths, CRSP_DATE
# %matplotlib qt
VERBOSE = 0
LAST_YEAR = CRSP_DATE // 10000
```

```
sql = SQL(**credentials['sql'], verbose=VERBOSE)
user = SQL(**credentials['user'], verbose=VERBOSE)
bd = BusDay(sql)
rdb = RedisDB(**credentials['redis'])
crsp = CRSP(sql=sql, bd=bd, rdb=rdb, verbose=VERBOSE)
signals = Signals(user)
store = Store(paths['scratch'], ext='pkl')
```

26.1 Sentiment analysis

Dictionary-based (or lexicon-based) sentiment analysis relies on predefined word lists, where words are assigned sentiment scores along with intensity levels. While this approach is straightforward and computationally efficient, it struggles with complex linguistic phenomena such as negations, irony, and sarcasm, which can lead to reduced accuracy. On the other hand, large language models excel at understanding context, syntax, and semantics, making them more effective for sentiment analysis of intricate financial texts. However, despite their limitations, dictionary-based methods remain popular due to their ease of implementation and transparent results.

26.1.1 Loughran-MacDonald dictionary

Loughran and McDonald (2011) found that general-purpose sentiment lexicons were not well-suited for financial statement analysis. To address this, they developed a dictionary based on company 10-K filings, categorizing words into seven sentiment-related groups relevant to finance: "negative," "positive," "litigious," "uncertainty," "constraining," and "superfluous." This domain-specific approach provides a more accurate reflection of sentiment in financial disclosures.

- Loughran-MacDonald Master Dictionary

The dictionary includes:

- A list of positive sentiment words
- A list of negative sentiment words
- A set of stop words to ignore

```
# to download from google drive, need this prefix and the source file id
_prefix = "https://drive.google.com/uc?export=download&id="
source =  _prefix + '1ptUgGVeeUGhCbaKL14Ri3Xi5xOKkPkUD'
# source = "Loughran-McDonald_MasterDictionary_1993-2023.csv"
df = pd.read_csv(source, sep=',')

# sets of positive and negative sentiment words
words = {'positive': set(df.loc[df['Positive'] != 0, 'Word'].str.lower()),
         'negative': set(df.loc[df['Negative'] != 0, 'Word'].str.lower())}
```

Positive sentiment words

```
DataFrame(words['positive'], columns=['Positive Words'])
```

```
     Positive Words
0        outperform
1         rebounded
2        efficiency
3           honored
4        leadership
..              ...
349        improves
350  collaborations
351      innovators
352    advantageous
353        honoring

[354 rows x 1 columns]
```

Negative sentiment words

```
DataFrame(words['negative'], columns=['Negative Words'])
```

```
      Negative Words
0        threatening
1           immature
2     falsifications
3       uncontrolled
4              panic
...              ...
2350      objections
2351        coercion
2352     extenuating
2353           lying
2354   contradiction

[2355 rows x 1 columns]
```

List of stop words to ignore

```
# stopwords
generic = ['ME', 'MY', 'MYSELF', 'WE', 'OUR', 'OURS', 'OURSELVES',
           'YOU', 'YOUR', 'YOURS', 'YOURSELF', 'YOURSELVES',
           'HE', 'HIM', 'HIS', 'HIMSELF', 'SHE', 'HER', 'HERS',
           'HERSELF', 'IT', 'ITS', 'ITSELF', 'THEY', 'THEM',
           'THEIR', 'THEIRS', 'THEMSELVES', 'WHAT', 'WHICH',
           'WHO', 'WHOM', 'THIS', 'THAT', 'THESE', 'THOSE',
           'AM', 'IS', 'ARE', 'WAS', 'WERE', 'BE', 'BEEN',
           'BEING', 'HAVE', 'HAS', 'HAD', 'HAVING', 'DO',
           'DOES', 'DID', 'DOING', 'AN', 'THE', 'AND', 'BUT',
           'IF', 'OR', 'BECAUSE', 'AS', 'UNTIL', 'WHILE', 'OF',
           'AT', 'BY', 'FOR', 'WITH', 'ABOUT', 'BETWEEN',
           'INTO', 'THROUGH', 'DURING', 'BEFORE', 'AFTER',
           'ABOVE', 'BELOW', 'TO', 'FROM', 'UP', 'DOWN', 'IN',
           'OUT', 'ON', 'OFF', 'OVER', 'UNDER', 'AGAIN',
           'FURTHER', 'THEN', 'ONCE', 'HERE', 'THERE', 'WHEN',
           'WHERE', 'WHY', 'HOW', 'ALL', 'ANY', 'BOTH', 'EACH',
           'FEW', 'MORE', 'MOST', 'OTHER', 'SOME', 'SUCH',
           'NO', 'NOR', 'NOT', 'ONLY', 'OWN', 'SAME', 'SO',
           'THAN', 'TOO', 'VERY', 'CAN', 'JUST', 'SHOULD',
           'NOW', 'AMONG']
```

```
# CountVectorizer to tokenize and code counts of inputs words
vectorizer = CountVectorizer(strip_accents='unicode',
                             lowercase=True,
                             stop_words=generic,
                             token_pattern=r"\b[^\d\W][^\d\W][^\d\W]+\b")
```

```
# vectorizer to encode sentiment dictionary words
sentiment_vectorizer = CountVectorizer(strip_accents='unicode',
                                       lowercase=True,
                                       token_pattern=r"\b[^\d\W][^\d\W][^\d\W]+\b")
sentiment_vectorizer.fit([" ".join(words['positive'].union(words['negative']))])

# create a lookup Series and a score vector for computing net sentiment
```

(continues on next page)

(continued from previous page)

```
features = Series(sentiment_vectorizer.get_feature_names_out())
sentiment_points = (features.isin(words['positive']).astype(int).values
                    - features.isin(words['negative']).astype(int).values)
```

26.2 10-K Management discussion and analysis

A 10-K is a comprehensive annual report that publicly traded companies must file with the SEC. It provides an in-depth view of the company' s financial health and operational performance. Key sections include the business description, management discussion and analysis (**MD&A**), risk factors, and financial statements.

26.2.1 SEC Edgar website

The SEC EDGAR (Electronic Data Gathering, Analysis, and Retrieval) system is an online database managed by the U.S. Securities and Exchange Commission (SEC). It provides public access to financial filings, including annual and quarterly reports (Forms 10-K and 10-Q), significant event disclosures (Form 8-K), beneficial ownership reports (Schedule 13D), insider sales reports (Form 144), proxy statements, and registration statements (S-1).

- SEC EDGAR Search and Access

Additionally, the SEC has recently released an EDGAR Application Programming Interface (API): EDGAR API

Loughran and MacDonald have also made SEC EDGAR data files and other textual resources available in their research repository: SEC EDGAR Data

26.2.2 FinDS `edgar` module

The `edgar` module in the FinDS package provides functions for

- Searching and retrieving company filings from the SEC Edgar website
- Storing and indexing data locally
- Identifying and extracting specific sections of text, such as MD&A

Retrieve MD&A section text from 10-K' s:

```
# open 10-K archive of MD&A text
ed = Edgar(paths['10X'], zipped=True, verbose=VERBOSE)
item, form = 'mda10K', '10-K'
rows = DataFrame(ed.open(form=form, item=item))
Series((rows['date'] // 10000).astype(int)).value_counts().sort_index().to_frame()
```

```
      count
date
1993      2
1994   1362
1995   1674
1996   2960
1997   4461
1998   4501
1999   4364
2000   4287
```

(continues on next page)

(continued from previous page)

```
2001   4168
2002   4400
2003   5478
2004   5203
2005   5090
2006   4999
2007   4954
2008   5033
2009   5273
2010   5050
2011   4863
2012   4730
2013   4617
2014   4643
2015   4720
2016   4612
2017   4488
2018   4431
2019   4417
2020   4388
2021   4615
2022   4766
2023   4641
2024   4501
```

For all investment universe stocks between 1993 through the present, compute the average sentiment, and change in sentiment, of 10-K’s

```
# retrieve usual universe at end of each year
univs = {year: crsp.get_universe(bd.endyr(year-1)).assign(year=year)
         for year in range(1992, LAST_YEAR + 1)}
permnos = rows['permno'].unique().astype(int)
DataFrame({'permnos': len(permnos),
           'documents': len(rows),
           'first': min(rows['date']),
           'last': max(rows['date'])},
          index=['10K-mdas'])
```

```
          permnos  documents     first      last
10K-mdas    14696     137691  19931129  20241231
```

We assume up to a 3-month lag when the 10-Ks are made available. For example, filings submitted between January and March of 2024 are assigned to fiscal year 2023.

```
# Compute average sentiment and change in sentiment for all companies and years
results = []
for permno in tqdm(permnos):  # Loop over all permnos

    # retrieve all valid mda's for this permno by year
    mdas, dates = {}, {}   # to collect mdas and doc dates for this permno
    docs = rows[rows['permno'].eq(permno)].to_dict('records')
    for doc in docs:
        year = bd.endmo(doc['date'], -3) // 10000   # assign fiscal year
        if (year in univs and (year not in mdas or doc['date'] < dates[year])):
            tokens = ed[doc['pathname']]
```

(continues on next page)

(continued from previous page)

```python
            if len(tokens):
                mdas[year] = tokens
                dates[year] = doc['date']

    # compute sentiment as net sentiment word counts divided by doc length
    if len(mdas):
        X = sentiment_vectorizer.transform(list(mdas.values()))\
                                .dot(sentiment_points)
        X = np.divide(X, vectorizer.fit_transform(list(mdas.values())).sum())
        sentiment = {k:x for k,x in zip(mdas.keys(), X)}

    # derive sentiment change and similarity scores by year
    for year in sorted(mdas.keys()):
        result = {'year': year, 'permno': permno, 'date': dates[year]}
        result['mdasent'] = sentiment[year]
        result['currlen'] = len(mdas[year])
        if year-1 in mdas:
            result['prevlen'] = len(mdas[year-1])
            result['mdachg'] = sentiment[year] - sentiment[year-1]

            corpus = [mdas[year], mdas[year-1]]
        results.append(result)
```

```
  0%|          | 0/14696 [00:00<?, ?it/s]/home/terence/env3.11/lib/python3.11/site-
↪packages/sklearn/feature_extraction/text.py:408: UserWarning: Your stop_words␣
↪may be inconsistent with your preprocessing. Tokenizing the stop words generated␣
↪tokens ['about', 'above', 'after', 'again', 'all', 'among', 'and', 'any', 'are',
↪'because', 'been', 'before', 'being', 'below', 'between', 'both', 'but', 'can',
↪'did', 'does', 'doing', 'down', 'during', 'each', 'few', 'for', 'from', 'further
↪', 'had', 'has', 'have', 'having', 'her', 'here', 'hers', 'herself', 'him',
↪'himself', 'his', 'how', 'into', 'its', 'itself', 'just', 'more', 'most', 'myself
↪', 'nor', 'not', 'now', 'off', 'once', 'only', 'other', 'our', 'ours', 'ourselves
↪', 'out', 'over', 'own', 'same', 'she', 'should', 'some', 'such', 'than', 'that',
↪ 'the', 'their', 'theirs', 'them', 'themselves', 'then', 'there', 'these', 'they
↪', 'this', 'those', 'through', 'too', 'under', 'until', 'very', 'was', 'were',
↪'what', 'when', 'where', 'which', 'while', 'who', 'whom', 'why', 'with', 'you',
↪'your', 'yours', 'yourself', 'yourselves'] not in stop_words.
  warnings.warn(
100%|██████████| 14696/14696 [13:49<00:00, 17.71it/s]
```

```python
# save in signals database
data = DataFrame.from_records(results)
data['rebaldate'] = bd.offset(data['date'])
print(signals.write(data, 'mdasent', overwrite=True),
      signals.write(data, 'mdachg', overwrite=True))
```

```
131045 107665
```

```python
# right join data with univ, to identify universe companies with missing mda
data = pd.concat([data[data['year']==year]\
                  .drop(columns=['year'])\
                  .set_index('permno')\
                  .join(univ[['year']], how='right')\
```

(continues on next page)

(continued from previous page)

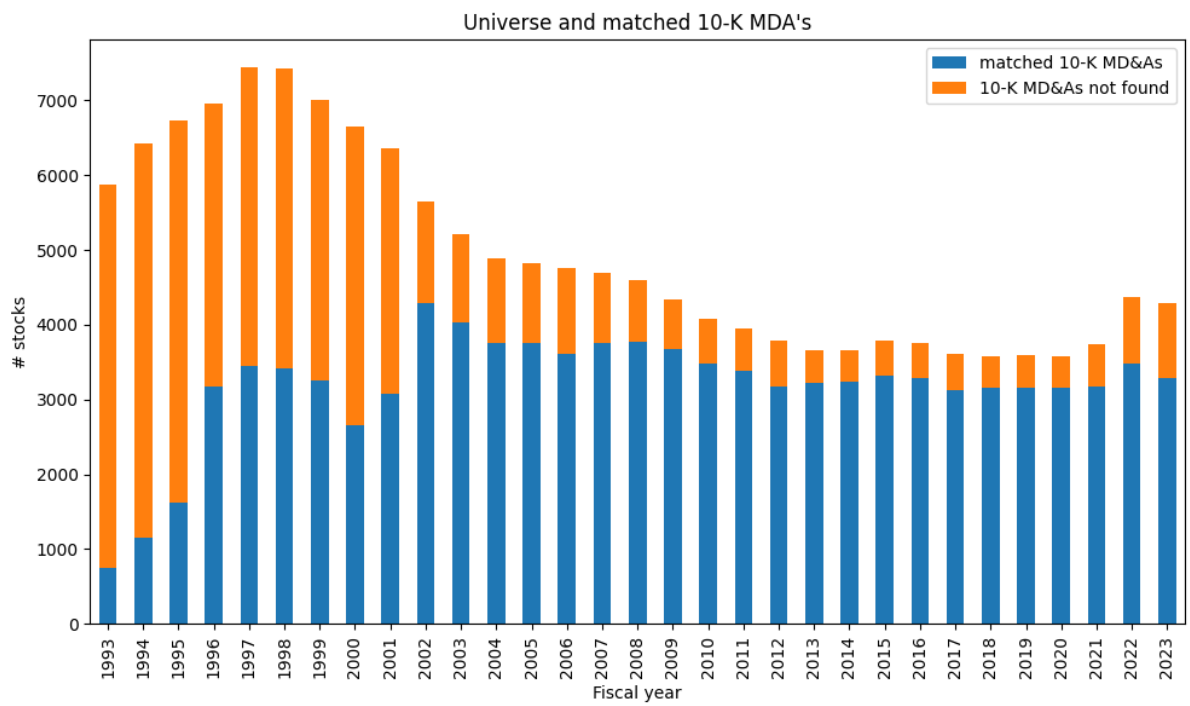
```
                 .reset_index()
                 for year, univ in univs.items() if year < LAST_YEAR],
                ignore_index=True)
# store temporary
store['sentiment'] = data
```

```
data = store['sentiment']
```

Exploratory analysis

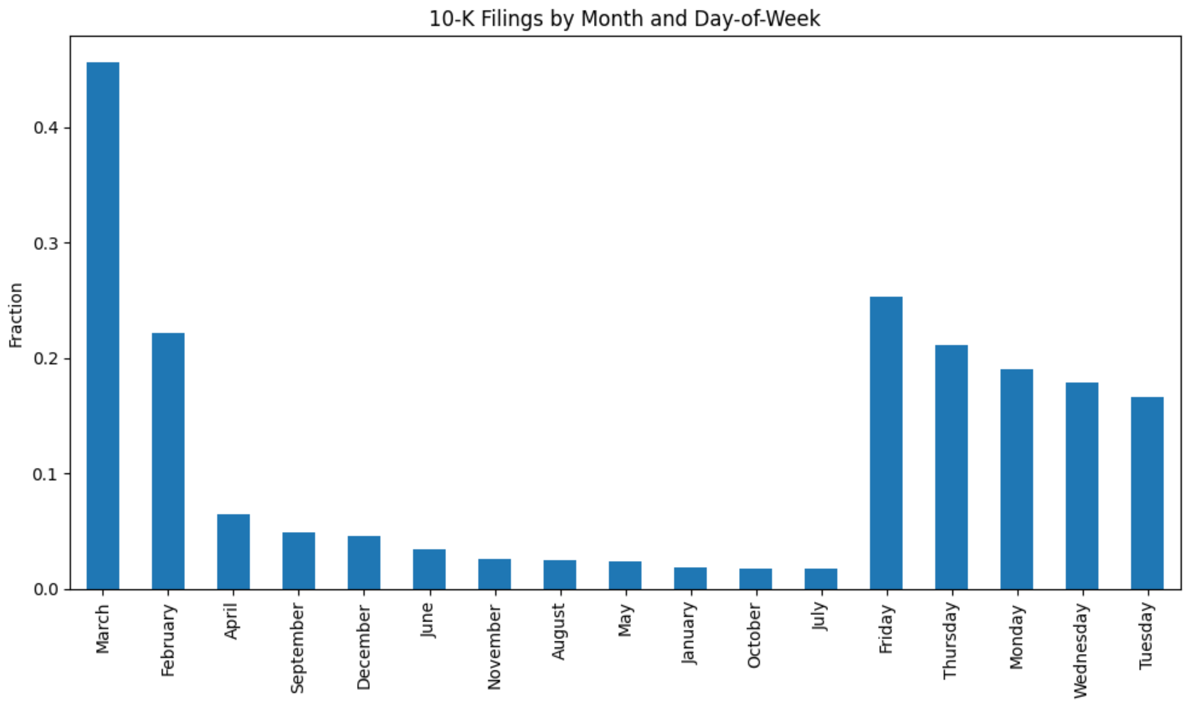
Universe coverage by year:

```
# Stacked Bar Plot of universe coverage by year
y1 = data[data['mdasent'].notna()].groupby('year')['permno'].count()
y0 = data[data['mdasent'].isna()].groupby('year')['permno']\
                                 .count()\
                                 .reindex(y1.index)
fig, ax = plt.subplots(1, 1, clear=True, figsize=(10, 6))
y1.plot(kind='bar', label='matched 10-K MD&As', color='C0', ax=ax, rot=90)
y0.plot(kind='bar', label='10-K MD&As not found', color='C1', ax=ax, rot=90,␣
↪bottom=y1)
ax.set_ylabel('# stocks')
ax.set_xlabel('Fiscal year')
ax.set_title("Universe and matched 10-K MDA's")
ax.legend()
plt.tight_layout()
```



When are 10-K’s filed?

```
# Stacked Bar Plot of filings date, by month and day-of-week
y = DataFrame.from_records([{'date': int(d),
                             'day': bd.datetime(int(d)).strftime('%A'),
                             'month': bd.datetime(int(d)).strftime('%B')}
                            for d in data.loc[data['mdasent'].notna(), 'date']])
z = pd.concat([y['month'].value_counts()/len(y),
               y['day'].value_counts()/len(y)])
fig, ax = plt.subplots(1, 1, clear=True, figsize=(10, 6))
z.plot(kind='bar', color='C0', ax=ax, rot=90)
ax.set_ylabel('Fraction')
ax.set_title("10-K Filings by Month and Day-of-Week")
plt.tight_layout()
```



Median sentiment and change in sentiment vs total corporate profits (of all US companies), by year:

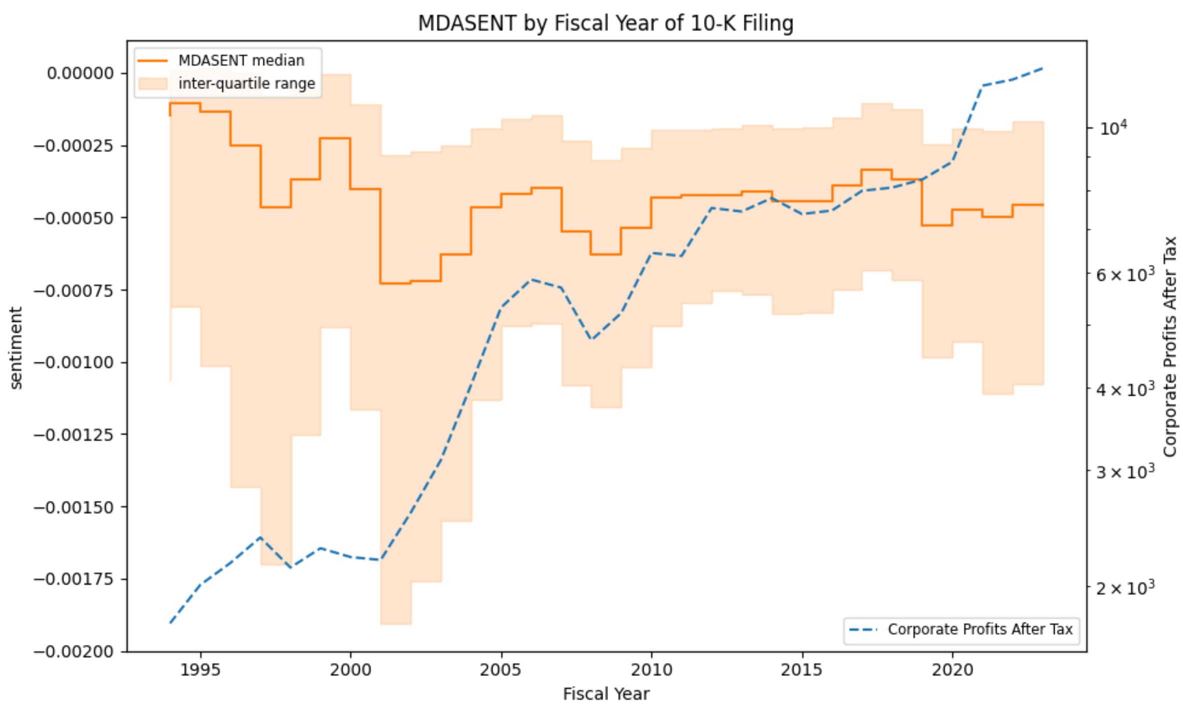
```
alf = Alfred(api_key=credentials['fred']['api_key'], verbose=VERBOSE)
series_id = 'CP'  # Corporate Profits
econ = alf(series_id).to_frame()
econ = econ.assign(year=econ.index // 10000).groupby('year').sum()

for sent, ylab in {'mdasent': 'sentiment', 'mdachg': 'sentiment change'}.items():
    print(sent, ylab)
    g = data[data['currlen'].gt(500)].dropna().groupby('year')
    iq1, iq2, iq3 = [g[sent].quantile(p) for p in [.25, .5, .75]]
    y = iq2.index.astype(int)
    fig, ax = plt.subplots(1, 1, clear=True, figsize=(10, 6))
    ax.step(y, iq2, ls='-', color='C1', where='pre')
    ax.fill_between(y, iq1, iq3, alpha=0.2, color='C1', step='pre')
    ax.set_title(f"{sent.upper()} by Fiscal Year of 10-K Filing")
    ax.set_xlabel("Fiscal Year")
    ax.set_ylabel(ylab)
    ax.legend([f"{sent.upper()} median", f"inter-quartile range"],
```

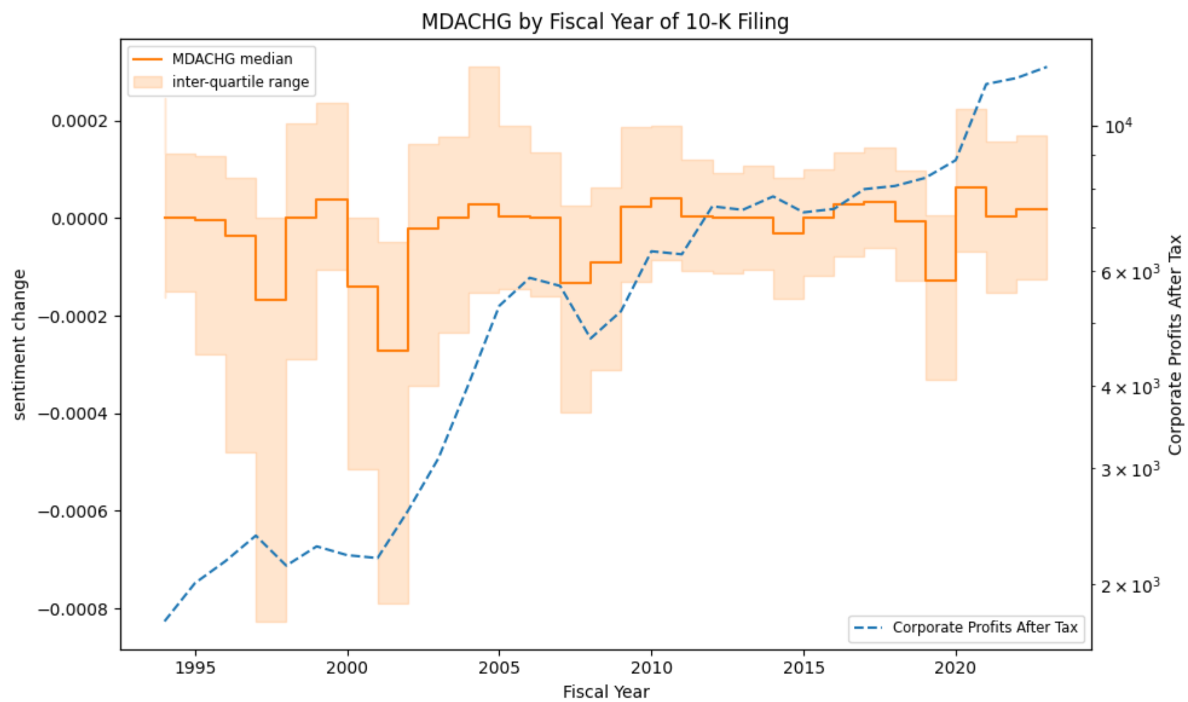
(continues on next page)

(continued from previous page)

```
            fontsize='small', loc='upper left')

    bx = ax.twinx()
    econ[(econ.index >= min(y)) & (econ.index <= max(y))].plot(ls='--', ax=bx)
    bx.legend([alf.header(series_id)[:27]], fontsize='small', loc='lower right')
    bx.set_ylabel(alf.header(series_id)[:27])
    bx.set_yscale('log')
    plt.tight_layout()
```

```
mdasent sentiment
mdachg sentiment change
```





26.2.3 Management sentiment and stock returns

We analyze the relationship between sentiment in 10-K filings and subsequent stock returns by constructing cap-weighted decile-spread returns. These are calculated for two periods:

1. The same calendar year (January–December) to examine contemporaneous relationships between sentiment and stock performance.
2. The following year (April–March) to explore an investable strategy based on sentiment changes after the release of 10-K reports for the prior fiscal year.

```
for ifig, key in enumerate(['mdasent', 'mdachg']):
    ret1 = {}   # to collect year-ahead spread returns
    ret0 = {}   # to collect current-year spread returns
    for year in tqdm(range(1999, max(data['year'])+1)):  # loop over years

        # compute current year average spread returns
        begyr = bd.begyr(year)
        endyr = bd.endyr(year)
        univ = data[data['year'] == year]\
                   .dropna(subset=[key])\
                   .set_index('permno')\
                   .join(crsp.get_cap(bd.offset(begyr, -1)), how='inner')\
                   .join(crsp.get_ret(begyr, endyr), how='left')
        if len(univ):
            sub = fractile_split(univ[key], [10, 90])
            pos = weighted_average(univ.loc[sub==1, ['cap', 'ret']], 'cap')['ret']
            neg = weighted_average(univ.loc[sub==3, ['cap', 'ret']], 'cap')['ret']
            ret0[endyr] = {'ret': pos-neg, 'npos': sum(sub==1), 'nneg': sum(sub==3)}

        # compute year ahead average spread returns
```

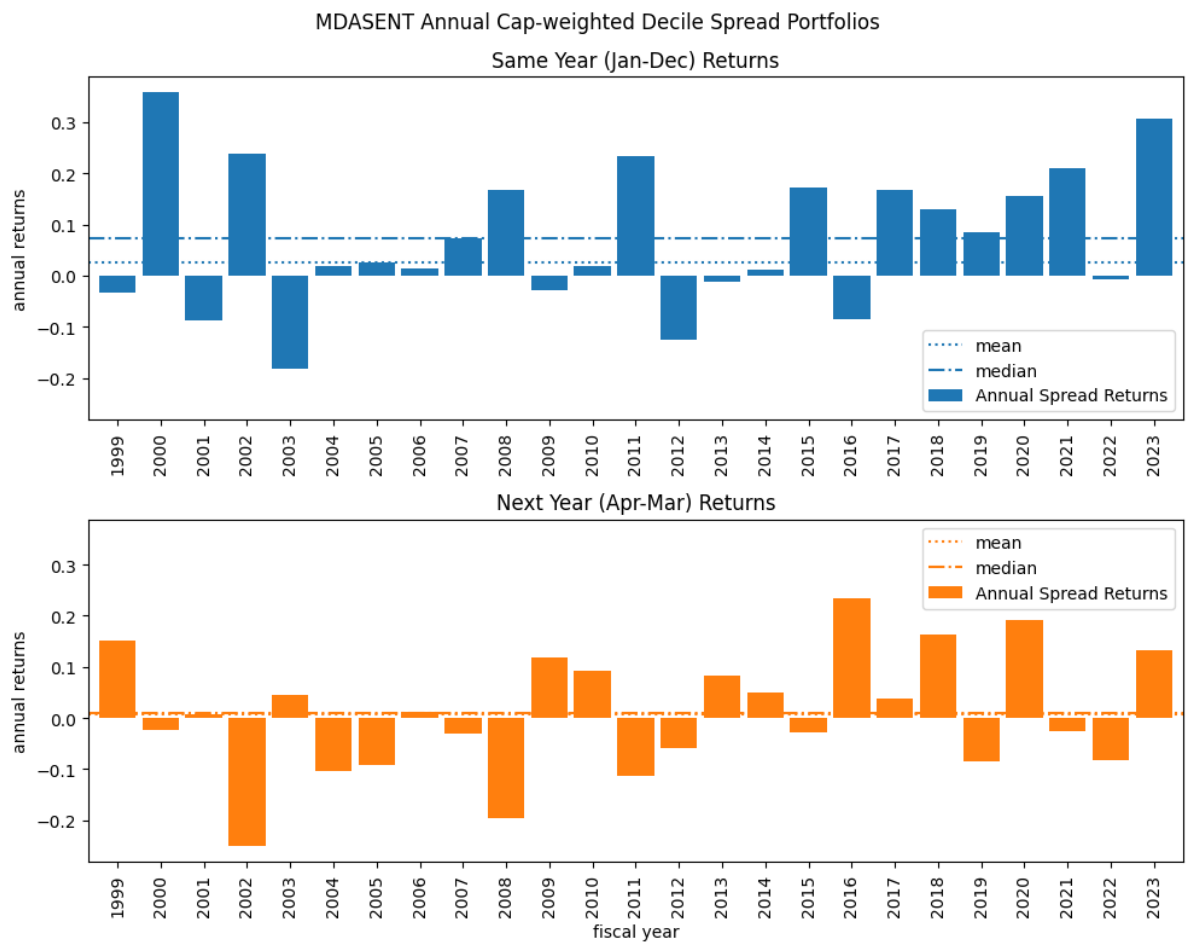
(continues on next page)

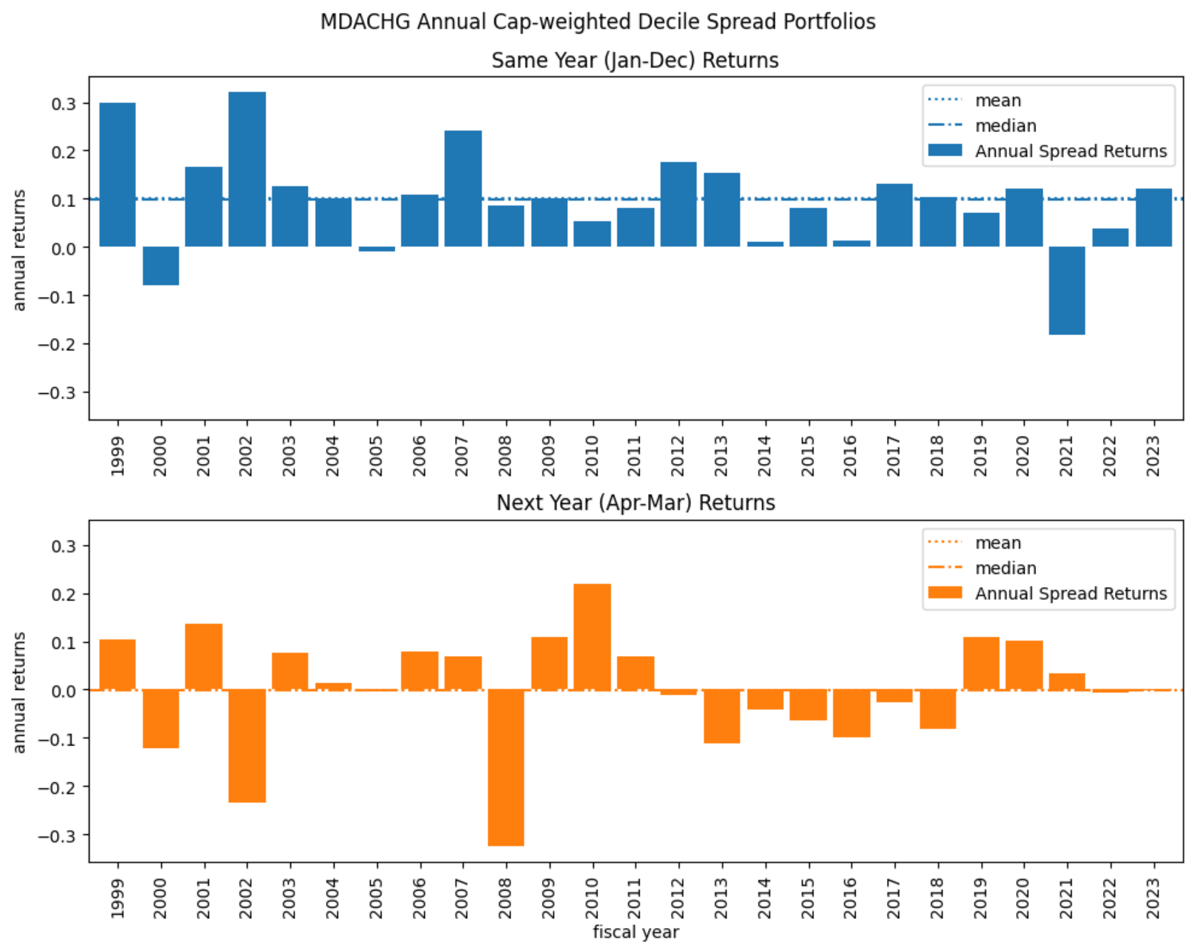
(continued from previous page)

```
        beg = bd.begmo(endyr, 4)
        end = bd.endmo(endyr, 15)
        univ = data[data['year'] == year]\
                    .dropna(subset=[key])\
                    .set_index('permno')\
                    .join(crsp.get_cap(bd.offset(beg, -1)), how='inner')\
                    .join(crsp.get_ret(beg, end), how='left')
        if len(univ):
            sub = fractile_split(univ[key], [10, 90])
            pos = weighted_average(univ.loc[sub==1, ['cap', 'ret']], 'cap')['ret']
            neg = weighted_average(univ.loc[sub==3, ['cap', 'ret']], 'cap')['ret']
            ret1[end] = {'ret': pos-neg, 'npos': sum(sub==1), 'nneg': sum(sub==3)}

    # collect same-year and year-ahead average spread returns
    r0 = DataFrame.from_dict(ret0, orient='index').sort_index()
    r0.index = r0.index // 10000
    r1 = DataFrame.from_dict(ret1, orient='index').sort_index()
    r1.index = (r1.index // 10000) - 2

    # plot same-year average spread returns
    fig, ax = plt.subplots(nrows=2, clear=True, figsize=(10, 8), sharey=True)
    r0['ret'].plot(kind='bar', ax=ax[0], width=.85, color="C0")
    ax[0].axhline(r0['ret'].median(), linestyle=':', color='C0')
    ax[0].axhline(r0['ret'].mean(), linestyle='-.', color='C0')
    ax[0].set_title(f"Same Year (Jan-Dec) Returns")
    ax[0].set_ylabel('annual returns')
    ax[0].legend(['mean', 'median', 'Annual Spread Returns'])

    # plot year-ahead average spread returns
    r1['ret'].plot(kind='bar', ax=ax[1], width=.85, color="C1")
    ax[1].axhline(r1['ret'].median(), linestyle=':', color='C1')
    ax[1].axhline(r1['ret'].mean(), linestyle='-.', color='C1')
    ax[1].set_title(f"Next Year (Apr-Mar) Returns")
    ax[1].set_ylabel('annual returns')
    ax[1].legend(['mean', 'median', 'Annual Spread Returns'])
    ax[1].set_xlabel('fiscal year')

    plt.suptitle(f"{key.upper()} Annual Cap-weighted Decile Spread Portfolios")
    plt.tight_layout()
```

```
  0%|          | 0/25 [00:00<?, ?it/s]
```

```
100%|██████████| 25/25 [06:21<00:00, 15.26s/it]
100%|██████████| 25/25 [00:48<00:00,  1.92s/it]
```


MDASENT Annual Cap-weighted Decile Spread Portfolios
Same Year (Jan-Dec) Returns
annual returns
mean
median
Annual Spread Returns
Next Year (Apr-Mar) Returns
fiscal year



References:

Tim Loughran and Bill McDonald, 2011, When is a Liability not a Liability? Textual Analysis, Dictionaries, and 10-Ks, Journal of Finance, 66:1, 35-65.

Cohen, Malloy and Nguyen (2020), Lazy Prices, Journal of Finance, Volume 75, Issue3, June 2020, Pages 1371-1415

CHAPTER TWENTYSEVEN

BUSINESS TEXTUAL ANALYSIS

You shall know a word by the company it keeps - J. R. Firth

Text mining techniques allow new insights to be extracted from unstructured text documents. We retrieve business descriptions text from 10-K filings, and use the **Spacy** NLP package for syntactic analysis, including part-of-speech tagging, named entity recognition and dependency parsing. Additionally, we explore dimentionality reduction techniques to visualize and cluster companies based on the relationships between business descriptions, represented as word embeddings, in a lower-dimensional space.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import re
import numpy as np
from scipy import spatial
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import spacy
from sklearn import cluster
from sklearn.decomposition import PCA
from tqdm import tqdm
from finds.database import SQL, RedisDB
from finds.structured import CRSP, BusDay
from finds.unstructured import Edgar
from finds.utils import Store, Finder, ColorMap
from secret import credentials, paths
# %matplotlib qt
VERBOSE = 0
```

```
sql = SQL(**credentials['sql'], verbose=VERBOSE)
user = SQL(**credentials['user'], verbose=VERBOSE)
bd = BusDay(sql)
rdb = RedisDB(**credentials['redis'])
crsp = CRSP(sql, bd, rdb, verbose=VERBOSE)
ed = Edgar(paths['10X'], zipped=True, verbose=VERBOSE)
store = Store(paths['scratch'])
find = Finder(sql)

begdate, enddate = 20240101, 20241231
```

Retrieve the usual investment universe and retain only the largest size decile (based on NYSE market cap breakpoints).

```
# Retrieve universe of stocks
univ = crsp.get_universe(bd.endmo(begdate, -1))
```

(continues on next page)

(continued from previous page)

```
comnam = crsp.build_lookup('permno', 'comnam', fillna="")  # company name
univ['comnam'] = comnam(univ.index)
ticker = crsp.build_lookup('permno', 'ticker', fillna="")  # tickers
univ['ticker'] = ticker(univ.index)
```

Extract Business Description text from 10-K filings.

```
# retrieve business decriptions from 10K's
item, form = 'bus10K', '10-K'
rows = DataFrame(ed.open(form=form, item=item))
found = rows[rows['permno'].isin(univ.index[univ.decile <= 1])  # largest decile only
             & rows['date'].between(begdate, enddate)]\
             .drop_duplicates(subset=['permno'], keep='last')\
             .set_index('permno')
```

27.1 Syntactic analysis

Syntactic analysis examines the roles of words in sentences and how they combine to form phrases and larger linguistic structures. This process helps model relationships such as subject-verb-object dependencies, which are fundamental for NLP tasks like dependency and constituent parsing.

27.1.1 SpaCy

spaCy is a widely used open-source Python library for advanced NLP tasks, including POS tagging, named entity recognition (NER), and dependency parsing. It provides pre-trained models for various languages and domains, as well as customizable pipelines for processing text data.

- spaCy Models

```
# ! python -m spacy download en_core_web_sm
nlp = spacy.load("en_core_web_lg")
```

27.1.2 Lemmatization

Lemmatization reduces a word to its base or dictionary form (lemma), representing its morphological root.

```
# Using spaCy pipeline to tokenize NVIDIA's 10-K business description text
nvidia = find('NVIDIA')['permno'].iloc[0]
doc = nlp(ed[found.loc[nvidia, 'pathname']][:nlp.max_length].lower())
tokens = DataFrame.from_records([{'text': token.text,
                                  'lemma': token.lemma_,
                                  'alpha': token.is_alpha,
                                  'stop': token.is_stop,
                                  'punct': token.is_punct}
                                 for token in doc], index=range(len(doc)))
tokens.head(30)
```

```
              text           lemma  alpha   stop  punct
0             item            item   True  False  False
1                1               1  False  False  False
2                .               .  False  False   True
3         business        business   True  False  False
4             \n\n            \n\n  False  False  False
5              our             our   True   True  False
6          company         company   True  False  False
7             \n\n            \n\n  False  False  False
8           nvidia          nvidia   True  False  False
9        pioneered         pioneer   True  False  False
10     accelerated      accelerate   True  False  False
11       computing       computing   True  False  False
12              to              to   True   True  False
13            help            help   True  False  False
14           solve           solve   True  False  False
15             the             the   True   True  False
16            most            most   True   True  False
17     challenging     challenging   True  False  False
18   computational   computational   True  False  False
19        problems         problem   True  False  False
20               .               .  False  False   True
21          nvidia          nvidia   True  False  False
22              is              be   True   True  False
23             now             now   True   True  False
24               a               a   True   True  False
25            full            full   True   True  False
26               -               -  False  False   True
27           stack           stack   True  False  False
28       computing       computing   True  False  False
29  infrastructure  infrastructure   True  False  False
```

27.1.3 Part-of-speech

Part-of-speech (POS) tagging assigns grammatical categories (e.g., noun, verb, adjective) to words in a text corpus. This aids in understanding sentence structure and extracting meaning by identifying the roles of words within sentences.

```
tags = DataFrame.from_records([{'text': token.text,
                                'pos': token.pos_,
                                'tag': token.tag_,
                                'dep': token.dep_}
                                 for token in doc], index=range(len(doc)))
tags.head(30)
```

```
              text    pos   tag       dep
0             item   NOUN    NN      ROOT
1                1    NUM    CD    nummod
2                .  PUNCT     .     punct
3         business   NOUN    NN     nsubj
4             \n\n  SPACE   _SP       dep
5              our   PRON  PRP$      poss
6          company   NOUN    NN     appos
7             \n\n  SPACE   _SP       dep
8           nvidia  PROPN   NNP     appos
```

(continues on next page)

(continued from previous page)

```
9        pioneered   VERB   VBD       ROOT
10     accelerated   VERB   VBD      xcomp
11       computing   NOUN    NN       dobj
12              to   PART    TO        aux
13            help   VERB    VB      advcl
14           solve   VERB    VB      xcomp
15             the    DET    DT        det
16            most    ADV   RBS     advmod
17     challenging    ADJ    JJ       amod
18   computational    ADJ    JJ       amod
19        problems   NOUN   NNS       dobj
20               .  PUNCT     .      punct
21          nvidia  PROPN   NNP      nsubj
22              is    AUX   VBZ       ROOT
23             now    ADV    RB     advmod
24               a    DET    DT        det
25            full    ADJ    JJ       amod
26               -  PUNCT  HYPH      punct
27           stack   NOUN    NN   compound
28       computing   NOUN    NN   compound
29  infrastructure   NOUN    NN   compound
```

27.1.4 Named entity recognition

Named Entity Recognition (NER) identifies and categorizes named entities (e.g., people, organizations, locations, dates) in text. This process helps classify textual data into meaningful categories.

```
ents = DataFrame.from_records([{'text': ent.text,
                                'label': ent.label_,
                                'start': ent.start_char,
                                'end': ent.end_char}
                               for ent in doc.ents], index=range(len(doc.ents)))
ents.head(20)
```

```
                      text     label  start   end
0                        1  CARDINAL      5     6
1                   nvidia    PERSON    133   139
2     as well as hundreds  CARDINAL    352   371
3               healthcare       ORG    853   863
4       tens of thousands  CARDINAL   1012  1029
5                      gpu       ORG   1033  1036
6                      gpu       ORG   1239  1242
7                    today      DATE   1347  1352
8                thousands  CARDINAL   1498  1507
9                      gpu       ORG   1771  1774
10               thousands  CARDINAL   1819  1828
11                    gpus       GPE   2594  2598
12     multi-billion-dollar     MONEY   2708  2728
13                   third   ORDINAL   2849  2854
14        over 45.3 billion     MONEY   3100  3117
15                     gpu       ORG   3248  3251
16                    1999      DATE   3255  3259
17                    2006      DATE   3391  3395
```

(continues on next page)

(continued from previous page)

```
18                    gpu       ORG   3451  3454
19                   2012      DATE   3557  3561
```

```
# Entity Visualizer
from spacy import displacy
displacy.render(doc[:300], style="ent", jupyter=True)
```

```
<IPython.core.display.HTML object>
```

27.1.5 Dependency parsing

Dependency parsing determines grammatical relationships between words in a sentence, representing these relationships as a tree structure where each word (except the root) depends on another word (its head). This technique helps identify syntactic roles, such as subjects, objects, and modifiers.

Transition-based parsing algorithms uses a set of transition operations (e.g. shift, reduce) to incrementally build a dependency tree from an input sentence.

Unlike dependency parsing, **constituent parsing** focuses on identifying and representing the hierarchical structure of phrases in a sentence based on formal grammar rules. It groups words into nested syntactic units (e.noun phrases and verb phrases) and represents them in a tree structure.

The **CKY (Cocke-Kasami-Younger) algorithm** is a dynamic programming technique used for parsing sentences and constructing parse trees.

Probabilistic Context-Free Grammar (PCFG) extends standard Context-Free Grammar (CFG) by assigning probabilities to production rules, indicating the likelihood of specific grammatical structures. Each rule defines how non-terminal symbols (e.g., NP for noun phrase) expand into words or other non-terminals, guiding sentence generation and parsing.

Models for automatic tagging and parsing rely on labeled datasets known as **treebanks**, which contain syntactically annotated sentences. The **Penn Treebank** is a widely used treebank for English, providing annotations for POS tags and parse trees.

```
sentence_spans = list(doc.sents)
displacy.render(sentence_spans[2:4], style="dep", jupyter=True,
                options=dict(compact=False, distance=175))
```

```
<IPython.core.display.HTML object>
```

27.2 Semantic similarity

27.2.1 Word vectors

Word vectors are numerical representations of words in a multidimensional space, learned from their co-occurrence patterns in large text corpora. Words with similar syntactic and semantic meanings tend to have vector representations that are close together in this space.

For example, spaCy’s `en_core_web_lg` model represents over 500,000 words using 300-dimensional vectors.

Extract lemmatized noun forms from business descriptions using spaCy’s POS tagger:

```
# Extract nouns
bus = {}
for permno in tqdm(found.index):
    doc = nlp(ed[found.loc[permno, 'pathname']][:nlp.max_length].lower())
    nouns = " ".join([re.sub("[^a-zA-Z]+", "", token.lemma_) for token in doc
                      if token.pos_ in ['NOUN'] and len(token.lemma_) > 2])
    if len(nouns) > 100:
        bus[permno] = nouns
store['business'] = bus
```

```
100%|██████████| 192/192 [03:12<00:00,  1.00s/it]
```

```
bus = store.load('business')
permnos = list(bus.keys())
tickers = univ.loc[permnos, 'ticker'].to_list()
```

Compute the average word vector for NVIDIA’ s business description text:

```
# example of word vector
vec1 = nlp(bus[nvidia]).vector
vec1
```

```
array([-0.5495549 ,  0.0522468 , -0.70095205,  1.1134154 ,  2.659689  ,
        0.29627848,  1.3154105 ,  3.8272932 , -2.232087  , -1.3053178 ,
        6.069174  ,  2.0604212 , -4.542866  ,  2.3177896 , -1.1287518 ,
        2.3917935 ,  3.1968606 ,  1.5996909 , -2.3438275 ,  0.03434967,
        0.22686806,  1.7824569 , -2.384547  ,  0.8608239 , -1.2319311 ,
       -1.774604  , -1.8425854 , -1.7403452 , -0.7102895 ,  1.0869901 ,
        1.2046682 ,  1.2530138 , -1.1417824 , -0.4984767 ,  0.34321743,
       -0.37546915,  1.4804035 ,  0.8114897 ,  1.3119912 ,  0.38791072,
        0.25189775, -0.1770816 ,  0.1785395 ,  1.0146813 , -1.4704382 ,
        1.6199547 ,  2.021769  , -1.9505422 ,  0.4602281 , -1.281002  ,
        0.07107421,  2.3507724 , -0.18837918, -3.80177   , -0.54604673,
        0.5786306 , -1.7812697 ,  1.5003949 ,  0.40720284, -1.5742034 ,
        2.2474368 ,  1.2563457 , -2.2915537 , -1.2388986 ,  2.408017  ,
        1.9807013 , -2.2583349 , -3.3942797 ,  0.5241013 ,  3.0477126 ,
       -0.97571445,  0.7010974 , -1.2003129 ,  0.38448045, -0.30499592,
        1.3931054 , -1.3777359 ,  0.99693114, -1.8231292 , -0.1508515 ,
       -2.595402  , -0.6554817 ,  0.96918464,  1.4329529 , -0.2420673 ,
       -0.08297056, -1.2713333 , -1.8489853 ,  0.77105236, -0.235054  ,
       -1.2015461 ,  1.0783287 ,  1.5740684 , -2.2493582 ,  0.43573543,
       -0.56152374,  0.58803385, -0.5924455 ,  0.8756682 ,  1.2502667 ,
        3.0838132 ,  0.3415912 ,  1.887193  ,  1.8639498 ,  0.2804779 ,
        4.218261  , -1.1121116 , -1.805579  , -0.22270828, -2.56832   ,
        2.4099169 , -0.22422643, -1.1543158 ,  0.06765282,  0.83664316,
        1.7835015 , -2.6237123 , -1.3570241 , -0.46247697, -2.476458  ,
       -1.9124763 , -2.6474092 ,  0.44025388,  1.1519567 , -0.42653838,
       -2.8462713 ,  0.2980864 , -2.9798195 ,  2.9588706 , -1.819657  ,
       -2.464192  ,  0.32522804,  3.3010955 ,  0.7937097 , -0.15216802,
       -0.42828757, -0.9942988 , -0.44921628,  1.9312432 ,  0.28458852,
       -0.6386989 , -0.87969756, -0.13129689,  0.85792863,  1.7823339 ,
        0.16759728, -3.5592077 , -0.2112106 ,  0.06164274,  3.368905  ,
       -0.3132938 ,  1.2434231 ,  0.21065742,  1.2389091 , -0.66589624,
        0.5398899 ,  2.8091311 ,  1.7358444 , -0.91453594, -2.6195803 ,
```

(continues on next page)

(continued from previous page)

```
        -0.9402572 , -1.2330531 ,  0.71588945,  1.699086  , -1.72446   ,
        -1.1087135 , -2.044233  ,  0.5116665 ,  0.7511035 , -1.510701  ,
        -1.5667175 , -0.22543517, -0.37966043,  0.95050013,  2.100138  ,
         1.966492  ,  0.6108724 , -0.31808442, -1.9397378 , -1.2919445 ,
        -1.7783433 ,  1.7884035 ,  1.1175009 , -1.9068832 , -1.0304377 ,
         0.6533309 , -0.94094557, -1.4831365 ,  1.5252017 ,  1.73394   ,
         0.05441582, -1.1678139 ,  0.10588835, -1.8394533 ,  1.820474  ,
         0.70712596, -2.8049684 , -0.1084957 ,  0.5009497 , -0.05614741,
        -0.74415725, -1.0815455 , -0.28300503, -1.0312603 ,  3.5577624 ,
         0.81285644, -3.4548876 ,  1.3007482 , -0.0527516 , -1.5823797 ,
         0.94375974,  0.01696876, -1.1761798 ,  2.141124  ,  0.8728857 ,
         1.7190704 ,  2.6591263 , -4.227103  , -0.5748424 ,  0.36368647,
        -1.838823  ,  1.3353976 , -1.5363356 , -0.98404247, -0.64337295,
        -2.6795921 ,  0.4494206 ,  2.0296626 ,  1.1337993 , -0.15482494,
         2.2946403 , -2.6738138 , -1.2725773 ,  1.763216  ,  2.8063855 ,
         0.46778396, -0.36578366, -0.26262623,  0.9403954 ,  1.0015386 ,
        -2.0167701 , -1.1006184 , -0.1618742 ,  0.9444055 , -0.27051136,
         0.33339593, -1.7367964 ,  1.3408182 ,  0.32765946,  1.1382856 ,
         0.6616231 , -1.8980618 , -3.5863128 , -2.049999  ,  0.11113743,
        -2.0100012 ,  0.91014284, -1.1377766 ,  0.30864185,  0.8823487 ,
        -1.5351313 ,  4.9388237 ,  1.6379622 ,  1.251249  ,  1.8976227 ,
        -0.77638566, -0.17837228,  1.8998926 , -0.8797592 , -0.77389985,
        -0.19354557, -0.14190447,  0.15318388, -1.0070832 ,  0.3741701 ,
        -2.8941076 ,  0.9670155 , -1.7984045 , -1.1186454 ,  1.0722593 ,
         3.4040382 ,  0.38004097,  1.4921545 , -0.0391783 ,  3.8353264 ,
         0.2084171 ,  1.266672  ,  1.8079621 , -2.3702457 , -0.04794558,
         0.3776366 , -0.42777508, -0.809167  ,  0.7592459 , -1.5167016 ,
         0.2553154 ,  1.1878173 , -2.2171407 , -0.76328766,  2.2422533 ],
       dtype=float32)
```

Compute the average word vector for all companies' business descriptions:

```
# Compute sentence vectors
vecs = np.array([nlp(bus[permno]).vector for permno in bus.keys()])
store['vectors'] = vecs
```

```
vecs = store['vectors']
```

```
# Distance matrix
n = len(bus)
distances = np.zeros((n, n))
for row in range(n):
    for col in range(row, n):
        distances[row, col] = spatial.distance.cosine(vecs[row], vecs[col])
        distances[col, row] = distances[row, col]
```

Identify companies with the most similar business descriptions:

```
def most_similar(p):
    dist = distances[permnos.index(p)]
    dist[permnos.index(p)] = max(dist)   # to ignore own distance
    return univ.loc[permnos[np.argmin(dist)]]
for name in ['NVIDIA', 'APPLE COMPUTER', 'JNJ', 'EXXON MOBIL', 'AMERICAN EXPRESS']:
    p = find(name)['permno'].iloc[-1]
```

(continues on next page)

(continued from previous page)

```
    print(f"{most_similar(p)['comnam']}' is most similar to '{name}'")
```

```
QUALCOMM INC' is most similar to 'NVIDIA'
SALESFORCE INC' is most similar to 'APPLE COMPUTER'
PFIZER INC' is most similar to 'JNJ'
PIONEER NATURAL RESOURCES CO' is most similar to 'EXXON MOBIL'
U S BANCORP DEL' is most similar to 'AMERICAN EXPRESS'
```

27.3 Dimensionality reduction

27.3.1 t-SNE visualization

T-distributed Stochastic Neighbor Embedding (t-SNE) visualizes high-dimensional data by converting similarities between points into joint probabilities and minimizing the Kullback-Leibler divergence between the high-dimensional and lower-dimensional representations. t-SNE preserves local structures, making it effective for clustering and uncovering hidden patterns in business descriptions.

t-SNE in scikit-learn

```
from sklearn.manifold import TSNE
Z = TSNE(n_components=2, perplexity=10, random_state=42)\
    .fit_transform(vecs)
```

Reduce business description vectors to 2D using t-SNE and label points with ticker symbols:

```
fig, ax = plt.subplots(figsize=(10, 8))
ax.scatter(Z[:, 0], Z[:, 1], color="C0", alpha=.3)
for text, x, y in zip(tickers, Z[:, 0], Z[:, 1]):
    ax.annotate(text=text, xy=(x, y), fontsize='small')
ax.set_title(f"t-SNE visualization of largest decile stocks ({enddate//10000})")
plt.tight_layout()
```

27.3.2 DBSCAN clustering

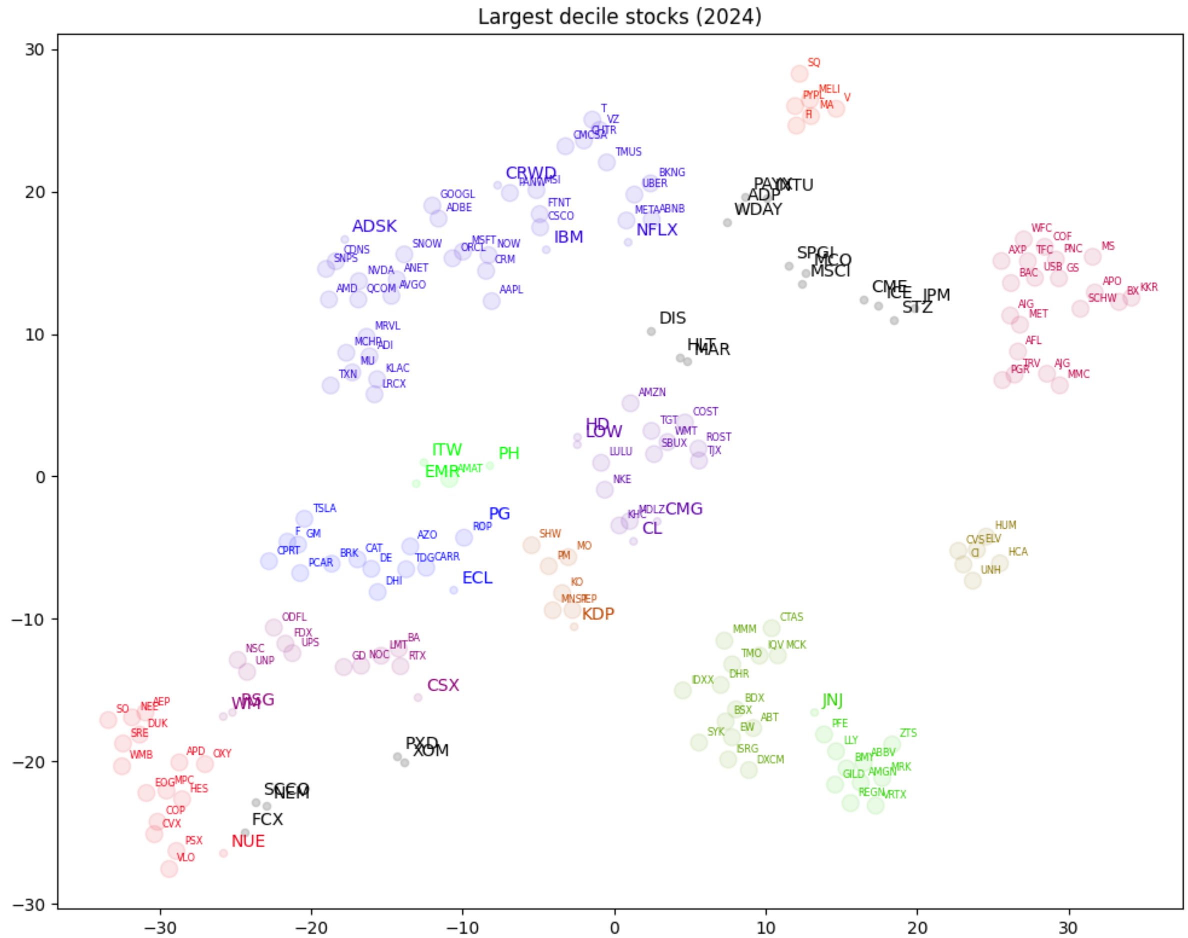
DBSCAN (Density-Based Spatial Clustering of Applications with Noise) is an unsupervised clustering algorithm that detects clusters of varying densities and identifies outliers. Unlike k-means, it does not require a predefined number of clusters. Instead, it uses two parameters: epsilon (ε) and the minimum number of points required to form a dense region.

```
# eps is the most important parameter for DBSCAN
eps = 4
db = cluster.DBSCAN(eps=eps)    # default eps

db.fit(Z)
n_clusters = len(set(db.labels_).difference({-1}))
n_noise = np.sum(db.labels_ == -1)
DataFrame(dict(clusters=n_clusters, noise=n_noise, eps=eps),  index=['DBSCAN'])
```

	clusters	noise	eps
DBSCAN	12	19	4

Visualize DBSCAN clusters in 2D space. Display outlier ticker symbols with larger font sizes:

```
cmap = ColorMap(n_clusters)
fig, ax = plt.subplots(figsize=(10, 8))
# plot core samples with larger marker size
ax.scatter(Z[db.core_sample_indices_, 0],
           Z[db.core_sample_indices_, 1],
           c=cmap[db.labels_[db.core_sample_indices_]],
           alpha=.1, s=100, edgecolors=None)
# plot non-core samples with smaller marker size
non_core = np.ones_like(db.labels_, dtype=bool)
non_core[db.core_sample_indices_] = False
non_core[db.labels_ < 0] = False
ax.scatter(Z[non_core, 0], Z[non_core, 1], c=cmap[db.labels_[non_core]],
           alpha=.1, s=20, edgecolors=None)
# plot noise samples
ax.scatter(Z[db.labels_ < 0, 0], Z[db.labels_ < 0, 1], c="darkgrey",
           alpha=.5, s=20, edgecolors=None)

# annotate with tickers not in core samples
for i, (t, c, xy) in enumerate(zip(tickers, db.labels_, Z)):
    if i in db.core_sample_indices_:
        ax.annotate(text=t, xy=xy+.5, color=cmap[c], fontsize='xx-small')
    elif c == -1:
        ax.annotate(text=t, xy=xy+.5, color='black', fontsize='medium')
    else:
        ax.annotate(text=t, xy=xy+.5, color=cmap[c], fontsize='medium')
ax.set_title(f"Largest decile stocks ({enddate//10000})")
plt.tight_layout()
```



List companies tagged as noisy samples:

```python
print("Samples tagged as noise:")
univ.loc[np.array(permnos)[db.labels_ < 0]].sort_values('naics')
```

```
Samples tagged as noise:
```

permno	cap	capco	decile	nyse	siccd	prc	naics	\
75241	5.246653e+07	5.246653e+07	1	True	1311	224.88	211120	
21207	4.770164e+07	4.770164e+07	1	True	1041	41.39	212220	
81774	6.104440e+07	6.104440e+07	1	True	1021	42.57	212230	
82800	6.654158e+07	6.654158e+07	1	True	1021	86.07	212230	
69796	4.440053e+07	4.440053e+07	1	True	2084	241.75	312130	
11850	4.005332e+08	4.005332e+08	1	True	2911	99.98	324110	
78975	1.749684e+08	1.749684e+08	1	False	7370	625.03	513210	
26403	1.652592e+08	1.652592e+08	1	True	4833	90.29	516120	
44644	9.568078e+07	9.568078e+07	1	False	7374	232.97	518210	
47896	4.917605e+08	4.917605e+08	1	True	6021	170.10	522110	
90993	7.350871e+07	7.350871e+07	1	True	6231	128.43	523210	
89626	7.565405e+07	7.565405e+07	1	False	6200	210.60	523210	
17478	1.395567e+08	1.395567e+08	1	True	6282	440.52	523930	
61621	4.285840e+07	4.285840e+07	1	False	8700	119.11	541219	

(continues on next page)

(continued from previous page)

```
13628   5.769654e+07  5.769654e+07       1  False   7372  276.06  541511
48506   7.147248e+07  7.147248e+07       1   True   7323  390.56  561450
92402   4.473782e+07  4.473782e+07       1   True   7389  565.65  561499
85913   6.551968e+07  6.551968e+07       1  False   7011  225.51  721110
14338   4.669516e+07  4.669516e+07       1   True   7011  182.09  721110

                                  comnam ticker
permno
75241        PIONEER NATURAL RESOURCES CO    PXD
21207                        NEWMONT CORP    NEM
81774                FREEPORT MCMORAN INC    FCX
82800                SOUTHERN COPPER CORP   SCCO
69796            CONSTELLATION BRANDS INC    STZ
11850                    EXXON MOBIL CORP    XOM
78975                          INTUIT INC   INTU
26403                      DISNEY WALT CO    DIS
44644       AUTOMATIC DATA PROCESSING INC    ADP
47896                  JPMORGAN CHASE & CO    JPM
90993   INTERCONTINENTALEXCHANGE GRP INC    ICE
89626                    C M E GROUP INC    CME
17478                    S & P GLOBAL INC   SPGI
61621                         PAYCHEX INC   PAYX
13628                         WORKDAY INC   WDAY
48506                         MOODYS CORP    MCO
92402                         M S C I INC   MSCI
85913      MARRIOTT INTERNATIONAL INC NEW    MAR
14338       HILTON WORLDWIDE HOLDINGS INC    HLT
```

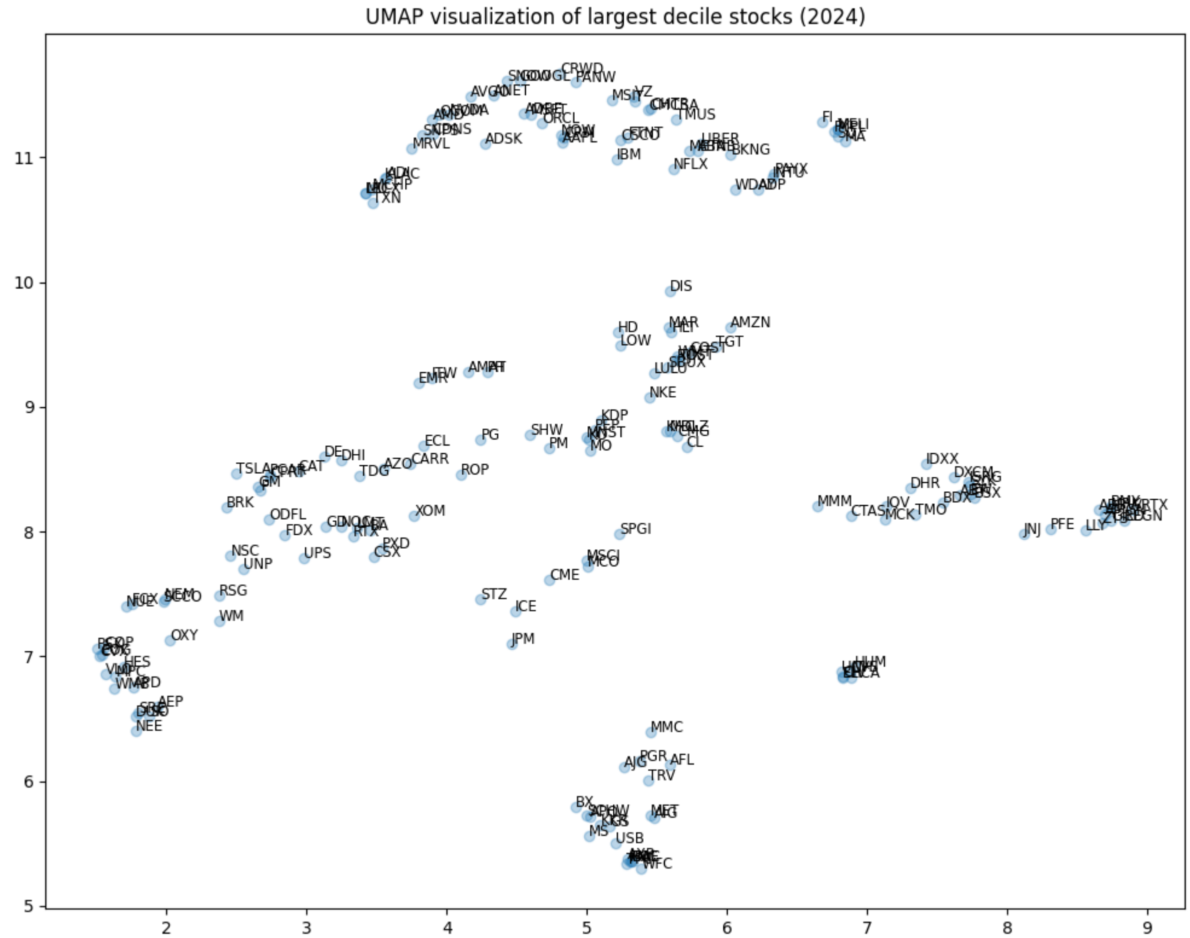
27.3.3 UMAP vizualization

UMAP (Uniform Manifold Approximation and Projection) is a dimensionality reduction technique that constructs a high-dimensional graph of data points and optimizes a lower-dimensional representation while preserving essential relationships. Compared to t-SNE, UMAP is faster, scales better for large datasets, and retains more global structure.

UMAP Documentation

```python
import umap
Z = umap.UMAP(n_components=2, n_jobs=1, min_dist=0.0, random_state=42)\
        .fit_transform(vecs)
```

```python
fig, ax = plt.subplots(figsize=(10, 8))
ax.scatter(Z[:, 0], Z[:, 1], color="C0", alpha=.3)
for text, x, y in zip(tickers, Z[:, 0], Z[:, 1]):
    ax.annotate(text=text, xy=(x, y), fontsize='small')
ax.set_title(f"UMAP visualization of largest decile stocks ({enddate//10000})")
plt.tight_layout()
```



27.3.4 HDBSCAN clustering

HDBSCAN (Hierarchical Density-Based Spatial Clustering of Applications with Noise) extends DBSCAN by varying the epsilon parameter and optimizing cluster stability. This makes it more robust to variations in density and parameter selection.

```
# eps is the most important parameter for DBSCAN
hdb = cluster.HDBSCAN()
hdb.fit(Z)
n_clusters = len(set(hdb.labels_).difference({-1}))
n_noise = np.sum(hdb.labels_ == -1)
DataFrame(dict(clusters=n_clusters, noise=n_noise),  index=['HDBSCAN'])
```

```
         clusters  noise
HDBSCAN        14     32
```

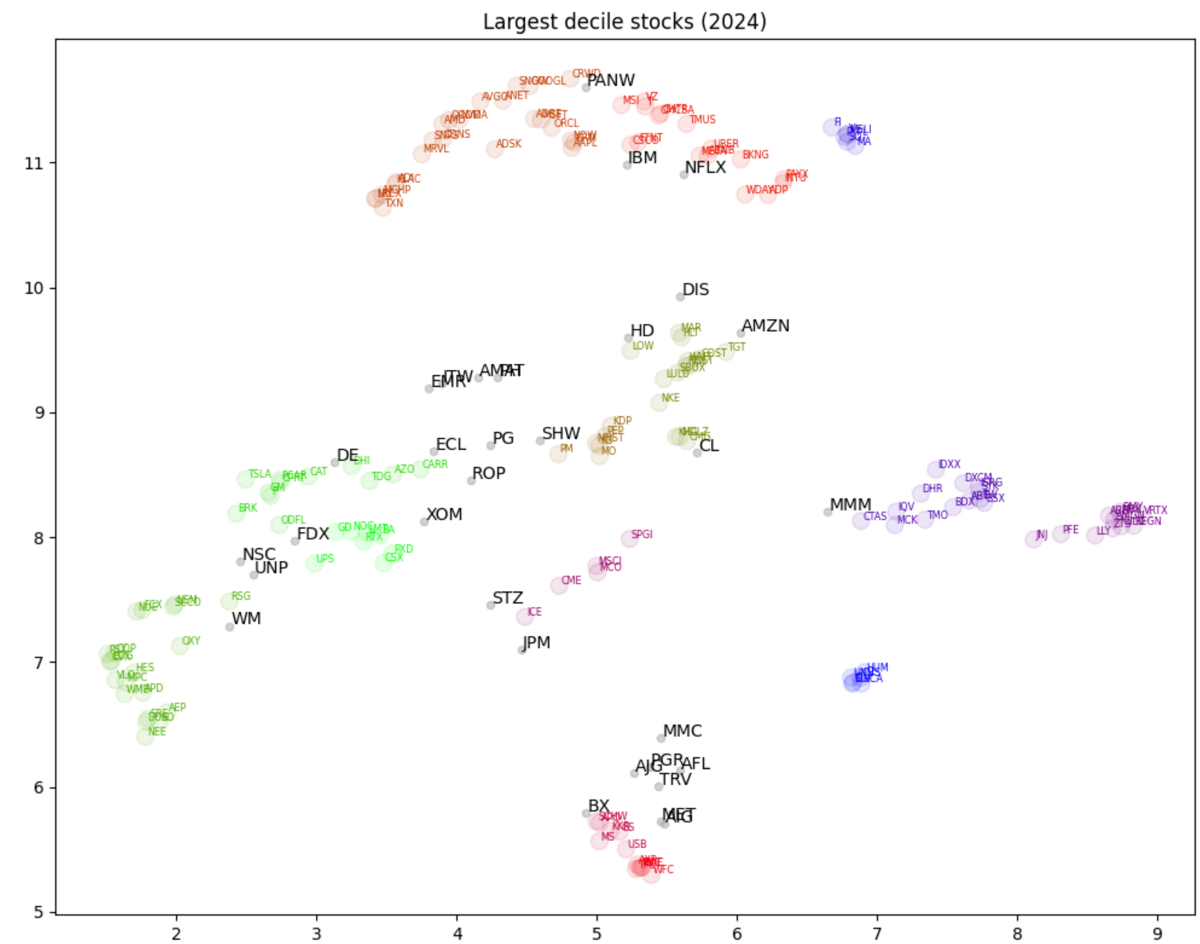
Visualize HDBSCAN clusters in 2D space. Display outlier ticker symbols with larger font sizes.

```
cmap = ColorMap(n_clusters)
fig, ax = plt.subplots(figsize=(10, 8))
# plot core samples with larger marker size
```

(continues on next page)

(continued from previous page)

```
ax.scatter(Z[hdb.labels_ >= 0, 0],
           Z[hdb.labels_ >= 0, 1],
           c=cmap[hdb.labels_[hdb.labels_ >= 0]],
           alpha=.1, s=100, edgecolors=None)
# plot noise samples
ax.scatter(Z[hdb.labels_ < 0, 0], Z[hdb.labels_ < 0, 1], c="darkgrey",
           alpha=.5, s=20, edgecolors=None)

# annotate with tickers not in core samples
for i, (t, c, xy) in enumerate(zip(tickers, hdb.labels_, Z)):
    if c >= 0:
        ax.annotate(text=t, xy=xy+.01, color=cmap[c], fontsize='xx-small')
    else:
        ax.annotate(text=t, xy=xy+.01, color="black", fontsize='medium')
ax.set_title(f"Largest decile stocks ({enddate//10000})")
plt.tight_layout()
```



List companies tagged as noisy samples

```
print("Samples tagged as noise:")
univ.loc[np.array(permnos)[hdb.labels_ < 0]].sort_values('naics')
```

```
Samples tagged as noise:
```

```
                cap         capco  decile   nyse  siccd     prc   naics  \
permno
69796   4.440053e+07  4.440053e+07       1   True   2084  241.75  312130
11850   4.005332e+08  4.005332e+08       1   True   2911   99.98  324110
36468   7.983580e+07  7.983580e+07       1   True   2851  311.90  325510
70578   5.655752e+07  5.655752e+07       1   True   2841  198.35  325611
18163   3.453781e+08  3.453781e+08       1   True   2844  146.54  325620
18729   6.563098e+07  6.563098e+07       1   True   2844   79.71  325620
22103   5.548783e+07  5.548783e+07       1   True   3491   97.33  332911
19350   1.120656e+08  1.120656e+08       1   True   3523  399.87  333111
14702   1.345954e+08  1.345954e+08       1  False   3550  162.07  333248
41355   5.918889e+07  5.918889e+07       1   True   3593  460.70  333995
56573   7.881408e+07  7.881408e+07       1   True   3569  261.94  333999
12490   1.493406e+08  1.493406e+08       1   True   3571  163.55  334111
77338   5.827867e+07  5.827867e+07       1  False   3823  545.17  334513
22592   6.037929e+07  6.037929e+07       1   True   3841  109.32  339112
66181   3.449080e+08  3.449080e+08       1   True   5211  346.55  444110
84788   1.556169e+09  1.556169e+09       1  False   7370  151.94  454110
48725   1.497292e+08  1.497292e+08       1   True   4011  245.62  482111
64311   5.345403e+07  5.345403e+07       1   True   4731  236.38  488510
60628   6.321543e+07  6.321543e+07       1   True   4513  252.97  492110
26403   1.652592e+08  1.652592e+08       1   True   4833   90.29  516120
47896   4.917605e+08  4.917605e+08       1   True   6021  170.10  522110
92108   9.302455e+07  9.302455e+07       1   True   6282  130.92  523940
87842   4.894876e+07  4.894876e+07       1   True   6311   66.13  524113
57904   4.821135e+07  4.821135e+07       1   True   6321   82.50  524114
59459   4.350773e+07  4.350773e+07       1   True   6331  190.49  524126
64390   9.318533e+07  9.318533e+07       1   True   6331  159.28  524126
66800   4.756321e+07  4.756321e+07       1   True   6331   67.75  524126
38093   4.855159e+07  4.855159e+07       1   True   6411  224.88  524210
45751   9.342235e+07  9.342235e+07       1   True   6411  189.47  524210
89393   2.107022e+08  2.107022e+08       1  False   7841  486.88  532282
13511   9.297566e+07  9.297566e+07       1  False   7371  294.88  541511
11955   7.213700e+07  7.213700e+07       1   True   4953  179.10  562219

                                 comnam ticker
permno
69796          CONSTELLATION BRANDS INC    STZ
11850                  EXXON MOBIL CORP    XOM
36468               SHERWIN WILLIAMS CO    SHW
70578                       ECOLAB INC    ECL
18163               PROCTER & GAMBLE CO     PG
18729              COLGATE PALMOLIVE CO     CL
22103               EMERSON ELECTRIC CO    EMR
19350                       DEERE & CO     DE
14702             APPLIED MATERIALS INC   AMAT
41355              PARKER HANNIFIN CORP     PH
56573            ILLINOIS TOOL WORKS INC    ITW
12490   INTERNATIONAL BUSINESS MACHS COR    IBM
77338            ROPER TECHNOLOGIES INC    ROP
22592                            3M CO    MMM
66181                   HOME DEPOT INC     HD
84788                   AMAZON COM INC   AMZN
48725                UNION PACIFIC CORP    UNP
```

(continues on next page)

(continued from previous page)

```
64311              NORFOLK SOUTHERN CORP    NSC
60628                         FEDEX CORP    FDX
26403                     DISNEY WALT CO    DIS
47896                JPMORGAN CHASE & CO    JPM
92108                    BLACKSTONE INC      BX
87842                       METLIFE INC     MET
57904                         AFLAC INC     AFL
59459            TRAVELERS COMPANIES INC    TRV
64390                PROGRESSIVE CORP OH    PGR
66800   AMERICAN INTERNATIONAL GROUP INC    AIG
38093            GALLAGHER ARTHUR J & CO    AJG
45751          MARSH & MCLENNAN COS INC     MMC
89393                       NETFLIX INC    NFLX
13511             PALO ALTO NETWORKS INC   PANW
11955          WASTE MANAGEMENT INC DEL      WM
```

References:

Greg Durrett, 2023, "CS388 Natural Language Processing course materisl", retrieved from https://www.cs.utexas.edu/~gdurrett/courses/online-course/materials.html

Text-Based Network Industries and Endogenous Product Differentiation. Gerard Hoberg and Gordon Phillips, 2016, Journal of Political Economy 124 (5), 1423-1465.

Product Market Synergies and Competition in Mergers and Acquisitions: A Text-Based Analysis. Gerard Hoberg and Gordon Phillips, 2010, Review of Financial Studies 23 (10), 3773-3811.

CHAPTER

TWENTYEIGHT

MACHINE LEARNING: CLASSIFICATION

When you come to a fork in the road, take it - Yogi Berra

We apply supervised learning models to a text classification task, using natural language processing (NLP) techniques to analyze business descriptions from the latest 10-K filings. Our goal is to predict firms' industry classifications, evaluating models such as Naive Bayes, Perceptron, Support Vector Machine (SVM), and Logistic Regression. We assess model performance using metrics like the confusion matrix and examine interpretability by visualizing feature importances.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import time
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import seaborn as sns
from wordcloud import WordCloud
from sklearn.model_selection import train_test_split
from sklearn.metrics import confusion_matrix
from sklearn.feature_extraction import text
from sklearn.linear_model import LogisticRegression, Perceptron
from sklearn.svm import LinearSVC
from sklearn.naive_bayes import MultinomialNB
from sklearn import metrics
from nltk.tag import pos_tag
from nltk.tokenize import word_tokenize
from nltk.stem import WordNetLemmatizer
from tqdm import tqdm
from finds.database import SQL, RedisDB
from finds.unstructured import Edgar
from finds.structured import BusDay, CRSP, PSTAT
from finds.readers import Sectoring
from finds.utils import Store
from secret import paths, credentials, CRSP_DATE
# %matplotlib qt
VERBOSE = 0
store = Store('assets', ext='pkl')
```

```
sql = SQL(**credentials['sql'], verbose=VERBOSE)
user = SQL(**credentials['user'], verbose=VERBOSE)
bd = BusDay(sql)
rdb = RedisDB(**credentials['redis'])
crsp = CRSP(sql, bd, rdb, verbose=VERBOSE)
pstat = PSTAT(sql, bd, verbose=VERBOSE)
ed = Edgar(paths['10X'], zipped=True, verbose=VERBOSE)
```

28.1 Text classification

Hoberg and Phillips (2016) proposed a system for classifying firms based on their business descriptions in 10-K filings, using these descriptions to measure firm similarity. We extend their analysis for text-based industry classification, focusing on U.S.-domiciled common stocks. The text data for each firm is drawn from the most recent year' s Business Description section of their 10-K filings.

```
# Retrieve universe of stocks, as of beginning of latest year
univ = crsp.get_universe(bd.endyr(CRSP_DATE, -1))
```

```
# Construct table to lookup company names
comnam = crsp.build_lookup(source='permno', target='comnam', fillna="")
univ['comnam'] = comnam(univ.index)
```

```
# Construct table to lookup ticker symbols
ticker = crsp.build_lookup(source='permno', target='ticker', fillna="")
univ['ticker'] = ticker(univ.index)
```

```
# Construct table to lookup sic codes from Compustat, and map to FF 10-sector code
sic = pstat.build_lookup(source='lpermno', target='sic', fillna=0)
industry = Series(sic[univ.index], index=univ.index)
industry = industry.where(industry > 0, univ['siccd'])
sectors = Sectoring(sql, scheme='codes10', fillna='')   # supplement from crosswalk
univ['sector'] = sectors[industry]
```

28.1.1 Text pre-processing

The pre-processing step involves several key operations to clean and prepare the text for analysis. We begin by extracting the Business Description text from the most recent 10-K filings of all stocks in our dataset.

```
# retrieve latest year's bus10K's
item, form = 'bus10K', '10-K'
rows = DataFrame(ed.open(form=form, item=item))
found = rows[rows['date'].between(bd.begyr(CRSP_DATE), bd.endyr(CRSP_DATE))]\
             .drop_duplicates(subset=['permno'], keep='last')\
             .set_index('permno')
```

- The text is then **lemmatized** using WordNet' s built-in morphy function, which reduces words to their base or root form. This step helps standardize the text by consolidating variations of words into their common form.

```
# !nltk.download('averaged_perceptron_tagger')
lemmatizer = WordNetLemmatizer()
```

```
100%|██████████| 4488/4488 [11:25<00:00,  6.55it/s]
```

- Next, we apply the **part-of-speech (POS) tagger** from the `nltk` library, retaining only nouns, which are the most informative about for industry classification tasks.

```
bus = {}
for permno in tqdm(found.index):
```

(continues on next page)

(continued from previous page)

```
    if permno not in univ.index:
        continue
    doc = word_tokenize(ed[found.loc[permno, 'pathname']].lower())
    tags = pos_tag(doc)
    nouns = [lemmatizer.lemmatize(w[0]) for w in tags
             if w[1] in ['NN', 'NNS'] and w[0].isalpha() and len(w[0]) > 2]
    if len(nouns) > 100:
        bus[permno] = nouns
store['nouns'] = bus
```

```
bus = store.load('nouns')
permnos = list(bus.keys())
labels = univ.loc[permnos, 'sector']
data = [" ".join(list(nouns)) for nouns in bus.values()]
classes = sorted(np.unique(labels))
```

- Finally, we split the corpus into training and testing samples, stratifying the data is to maintain the distribution of class labels in both sets.

```
test_size = 0.2
X_train, X_test, y_train, y_test = train_test_split(
    data, labels, test_size=test_size, random_state=42, stratify=labels)
summary = Series(y_train).value_counts().rename('n_train').to_frame()
summary['n_test'] = Series(y_test).value_counts()
summary['frac_train'] = summary['n_train'] / summary['n_train'].sum()
summary['frac_test'] = summary['n_test'] / summary['n_test'].sum()
print('Stratified Train/Test Split by Event')
summary.sort_values('n_train', ascending=False).round(2)
```

```
Stratified Train/Test Split by Event
```

```
        n_train  n_test  frac_train  frac_test
sector
Hlth        657     164        0.24       0.24
Other       612     153        0.22       0.22
HiTec       554     139        0.20       0.20
Manuf       275      69        0.10       0.10
Shops       246      62        0.09       0.09
Durbl       131      33        0.05       0.05
NoDur       114      28        0.04       0.04
Enrgy        81      20        0.03       0.03
Utils        72      18        0.03       0.03
Telcm        37       9        0.01       0.01
```

28.1.2 Text vectorization

After pre-processing the text, we convert the textual data into numerical features that can be fed into machine learning models. This is achieved through the **Term Frequency-Inverse Document Frequency (TF-IDF)** method. TF-IDF weights terms based on their importance in a document relative to their frequency in a collection of documents (corpus).

- **Term Frequency (TF)** measures how often a word appears in a document. A higher frequency suggests the word is more important within that document.
- **Inverse Document Frequency (IDF)** adjusts the weight of a term based on its rarity across the entire corpus. Terms that appear frequently across many documents are given less weight, while terms that appear less frequently are given greater importance.

To focus on the most relevant and informative words, we filter out extremely common words that appear in more than 50% of the documents (using `max_df=0.5`), exclude rare words that appear in fewer than 200 documents (using `min_df=200`), and limist the vocabulary to the 10,000 most frequent remaining terms (`max_features=10000`).

```
# Tfidf vectorizor
max_df, min_df, max_features = 0.5, 10, 20000
tfidf_vectorizer = text.TfidfVectorizer(
    encoding='latin-1',
    strip_accents='unicode',
    lowercase=True,
    #stop_words=stop_words,
    max_df=max_df,
    min_df=min_df,
    max_features=max_features,
    token_pattern=r'\b[a-z_]+\b',
)
x_train = tfidf_vectorizer.fit_transform(X_train)   # sparse array
x_test = tfidf_vectorizer.transform(X_test)
feature_names = tfidf_vectorizer.get_feature_names_out()
print("n_sample x n_features")
DataFrame([[x_train.shape, x_test.shape]],
          index=['data shape:'],
          columns=['train', 'test'])
```

```
n_sample x n_features
```

```
                      train          test
data shape:  (2779, 10060)  (695, 10060)
```

28.2 Classification models

In machine learning, there are generally two types of approaches for classification tasks:

- **Generative models** estimate a probability distribution and define the classifier based on these estimates. An example of this is the Naive Bayes classifier.
- **Discriminative models** directly define a decision boundary between classes. Examples of discriminative models include Logistic Regression, Perceptron, and Support Vector Machines (SVM).

Define a helper function to compute and save accuracy scores for both the training and testing samples.

```
results = dict()
```

```
def update_results(name, clf, elapsed):
    """helper to update results dict with train and test accuracy"""
    tic = time.time()
    test_score = clf.score(x_test, y_test)
    toc = time.time() - tic
    results[name] = dict(model=clf,
                         train_score=clf.score(x_train, y_train),
                         test_score=test_score,
                         test_time=toc,
                         train_time=elapsed)
    #store['classification'] = results
    print('Accuracy')
    return DataFrame.from_dict(results, orient='index').iloc[:, 1:]
```

28.2.1 Naive Bayes

The Naive Bayes classifier is a simple yet effective method for text classification. It assumes that the features (words) are conditionally independent given the class, meaning that the occurrence of one word in a document does not affect the occurrence of another. Despite this strong assumption, Naive Bayes performs surprisingly well on many real-world classification tasks.

- **Binomial Naive Bayes** is used for binary classification problems, where each document is assigned to one of two classes.
- **Multinomial Naive Bayes** is typically used for multi-class classification, where documents can belong to more than two classes.

Since Naive Bayes relies on the multiplication of probabilities for each feature, it can encounter problems when a feature has a zero probability in the training set. This is addressed through **Laplace smoothing**, which adds a small constant to all feature counts to avoid zero probabilities.

The basic formula for Naive Bayes classification is: $P(f_1, f_2, ..., f_n|c) = P(f_1|c) \cdot P(f_2|c) \cdot ... \cdot P(f_n|c)$

where $P(f_i|c)$ is the probability of feature f_i occurring in class c. The classification decision is made by selecting the class that maximizes the likelihood of the observed features, i.e.: $\hat{c} = \arg\max_c \left(\log P(c) + \sum_{i=1}^n \log P(f_i|c)\right)$

where the maximum likelihood estimate of the probability of frequency of word w_i is $P(w_i|c) = \frac{\text{count}(w_i,c)}{\sum_{w\in V}\text{count}(w,c)}$

```
clf = MultinomialNB(alpha=1.0)
tic = time.time()
clf.fit(x_train, y_train)
toc = time.time() - tic
update_results('naivebayes', clf, toc)
```

```
Accuracy
```

	train_score	test_score	test_time	train_time
naivebayes	0.751709	0.738129	0.002441	0.013695

28.2.2 Perceptron

The Perceptron is a linear classifier that updates weights based on classification errors. It uses a 0-1 loss function, which assigns a loss of zero for correct classifications and a loss of one for incorrect ones. The update rule for the Perceptron is as follows:

- If the predicted label $\hat{y}$ is different from the true label y, the weight vector w is updated: \$$w \leftarrow w + \alpha x(y - \hat{y}) Where$ \alpha $is the learning rate and$ x \$ is the feature vector of the document.

In multiclass classification, the **One-vs-Rest (OVR)** approach is used, where the Perceptron is trained to distinguish between each class and the rest of the classes.

```
clf = Perceptron(penalty='elasticnet',
                 #n_jobs=4, # -1
                 random_state=0,
                 verbose=VERBOSE)
tic = time.time()
clf.fit(x_train, y_train)
toc = time.time() - tic
update_results('perceptron', clf, toc)
```

```
Accuracy
```

```
            train_score  test_score  test_time  train_time
naivebayes     0.751709    0.738129   0.002441    0.013695
perceptron     0.949982    0.761151   0.004099    0.866820
```

28.2.3 Support Vector Machine

Support Vector Machines (SVM) are powerful classifiers that work by finding the decision boundary that maximizes the margin between classes. The decision boundary is chosen to minimize classification errors, and SVM uses **hinge loss** to penalize misclassifications: \$Loss $= \max(0, 1 - y(w \cdot x))$\$

SVM can handle both linear and non-linear decision boundaries by using different kernel functions. In this analysis, we use the **LinearSVC** kernel, which optimizes the classification with a linear decision boundary. For multiclass classification, SVM uses the One-vs-Rest (OVR) method.

```
clf = LinearSVC(multi_class='ovr',
                penalty='l2',
                verbose=VERBOSE)
tic = time.time()
clf.fit(x_train, y_train)
toc = time.time() - tic
update_results('linearsvc', clf, toc)
```

```
/home/terence/env3.11/lib/python3.11/site-packages/sklearn/svm/_classes.py:31:␣
↪FutureWarning: The default value of `dual` will change from `True` to `'auto'`␣
↪in 1.5. Set the value of `dual` explicitly to suppress the warning.
  warnings.warn(
```

```
Accuracy
```

```
             train_score  test_score  test_time  train_time
naivebayes      0.751709    0.738129   0.002441    0.013695
perceptron      0.949982    0.761151   0.004099    0.866820
linearsvc       0.990284    0.831655   0.002280    0.348787
```

28.2.4 Logistic Regression

Logistic Regression is a widely used model for binary and multi-class classification tasks. It uses **cross-entropy loss** to measure the difference between predicted probabilities and actual class labels. The logistic regression model updates weights iteratively based on the gradient of the loss function:

- For a binary classification problem, the probability of class 1 is given by: $$P(y = 1|x) = \frac{1}{1+e^{-wx}}$$

The update step is:

- $P(y = 1|x) \leftarrow 1/(1 + e^{-wx})$
- $w \leftarrow w + \alpha\, x\, (1 - P(y = 1|x))$ if $y = 1$
- $w \leftarrow w - \alpha\, x\, (1 - P(y = 0|x))$ if $y = 0$

$\Leftrightarrow w \leftarrow w + \alpha\, x\, (y - P(y = 1|x))$ where $y \in \{0,\ 1\}$

In multiclass problems, **softmax** is used to generalize logistic regression, providing a probability distribution over all classes. The model' s weights are updated using gradient descent to minimize the cross-entropy loss.

```
clf = LogisticRegression(verbose=VERBOSE,
                         penalty='l2',
                         multi_class='multinomial',
                         # n_jobs=-1,      # when multi_class='ovr'
                         max_iter=1000)
tic = time.time()
clf.fit(x_train, y_train)
toc = time.time() - tic
update_results('logistic', clf, toc)
```

```
Accuracy
```

```
             train_score  test_score  test_time  train_time
naivebayes      0.751709    0.738129   0.002441    0.013695
perceptron      0.949982    0.761151   0.004099    0.866820
linearsvc       0.990284    0.831655   0.002280    0.348787
logistic        0.896366    0.833094   0.005731    3.705192
```

28.3 Evaluation

28.3.1 Overfitting

Evidence of overfitting can be observed when a model performs well on the training data but poorly on the test data. In such cases, the model may have learned to memorize the training data instead of generalizing to new examples. Overfitting can be mitigated by using techniques like cross-validation and regularization.

28.3.2 Accuracy Metrics

To evaluate model performance, we consider several metrics:

- **Precision**: The proportion of true positive predictions among all positive predictions.
- **Recall**: The proportion of true positive predictions among all actual positive cases.
- **F1 Score**: The harmonic mean of precision and recall, which balances the two metrics and provides a single score for model performance.

In multiclass and multilabel scenarios, the F1 score is computed as a weighted average of the F1 scores for each class.

```
# Compute precision, recall, f1 and confusion matrix
res = DataFrame.from_dict(results, orient='index')
models = {k: v['model'] for k,v in results.items()}

scores, cf_test, cf_train = {}, {}, {}
for ifig, (name, clf) in enumerate(models.items()):
    train_pred = clf.predict(x_train)
    test_pred = clf.predict(x_test)
    scores[name] = {
        'train': metrics.precision_recall_fscore_support(
            y_train, train_pred, average='macro')[:3],
        'test': metrics.precision_recall_fscore_support(
            y_test, test_pred, average='macro')[:3]
    }
    cf = DataFrame(confusion_matrix(y_train, train_pred, labels=classes),
                   index=pd.MultiIndex.from_product([['Actual'], classes]),
                   columns=pd.MultiIndex.from_product([['Predicted'], classes]))
    cf_train[name] = cf
    cf = DataFrame(confusion_matrix(y_test, test_pred, labels=classes),
                   index=pd.MultiIndex.from_product([['Actual'], classes]),
                   columns=pd.MultiIndex.from_product([['Predicted'], classes]))
    cf_test[name] = cf
```

```
/home/terence/env3.11/lib/python3.11/site-packages/sklearn/metrics/_classification.
↪py:1509: UndefinedMetricWarning: Precision is ill-defined and being set to 0.0␣
↪in labels with no predicted samples. Use `zero_division` parameter to control␣
↪this behavior.
  _warn_prf(average, modifier, f"{metric.capitalize()} is", len(result))
/home/terence/env3.11/lib/python3.11/site-packages/sklearn/metrics/_classification.
↪py:1509: UndefinedMetricWarning: Precision is ill-defined and being set to 0.0␣
↪in labels with no predicted samples. Use `zero_division` parameter to control␣
↪this behavior.
  _warn_prf(average, modifier, f"{metric.capitalize()} is", len(result))
```

```
caption="Model Accuracy"
DataFrame({(metric, sample): [score[sample][i] for score in scores.values()]
           for i, metric in enumerate(['Precision', 'Recall', 'F1-score'])
           for sample in ['train', 'test']},
          index=scores.keys())
```

```
            Precision           Recall            F1-score
                train      test     train      test     train      test
naivebayes   0.758517  0.747060  0.522357  0.498908  0.541932  0.513666
```

(continues on next page)

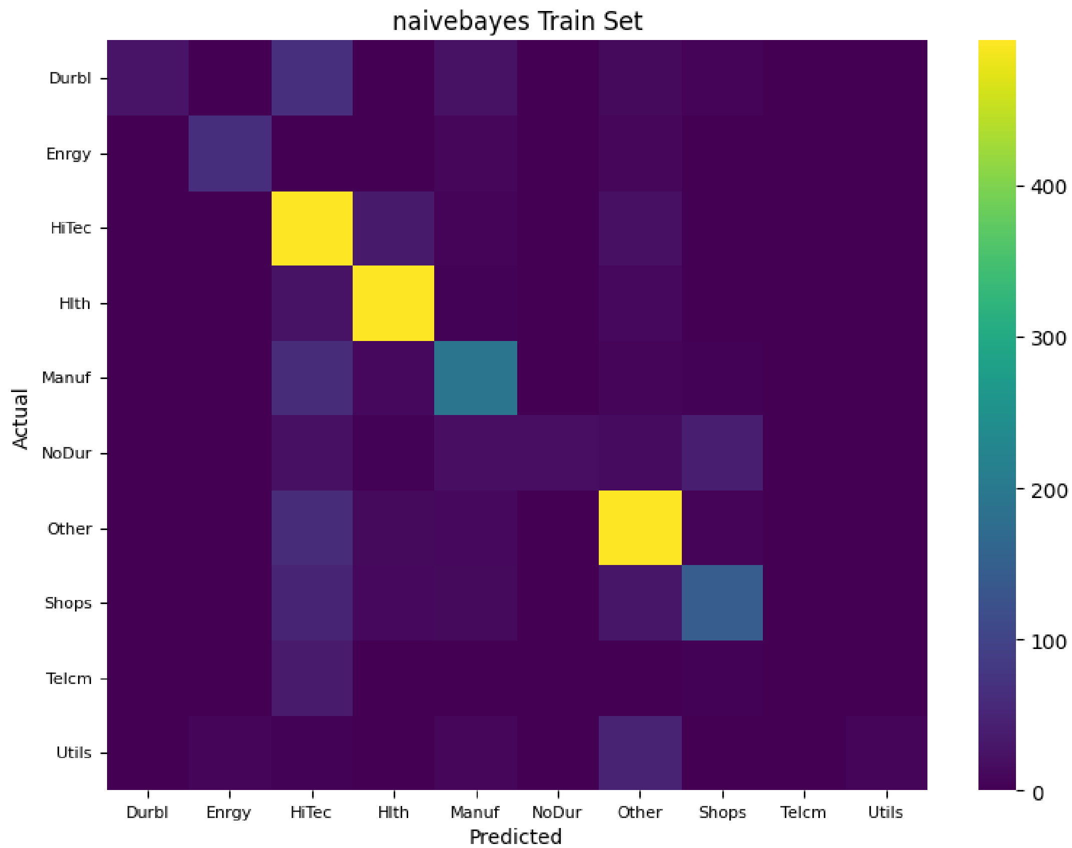
(continued from previous page)

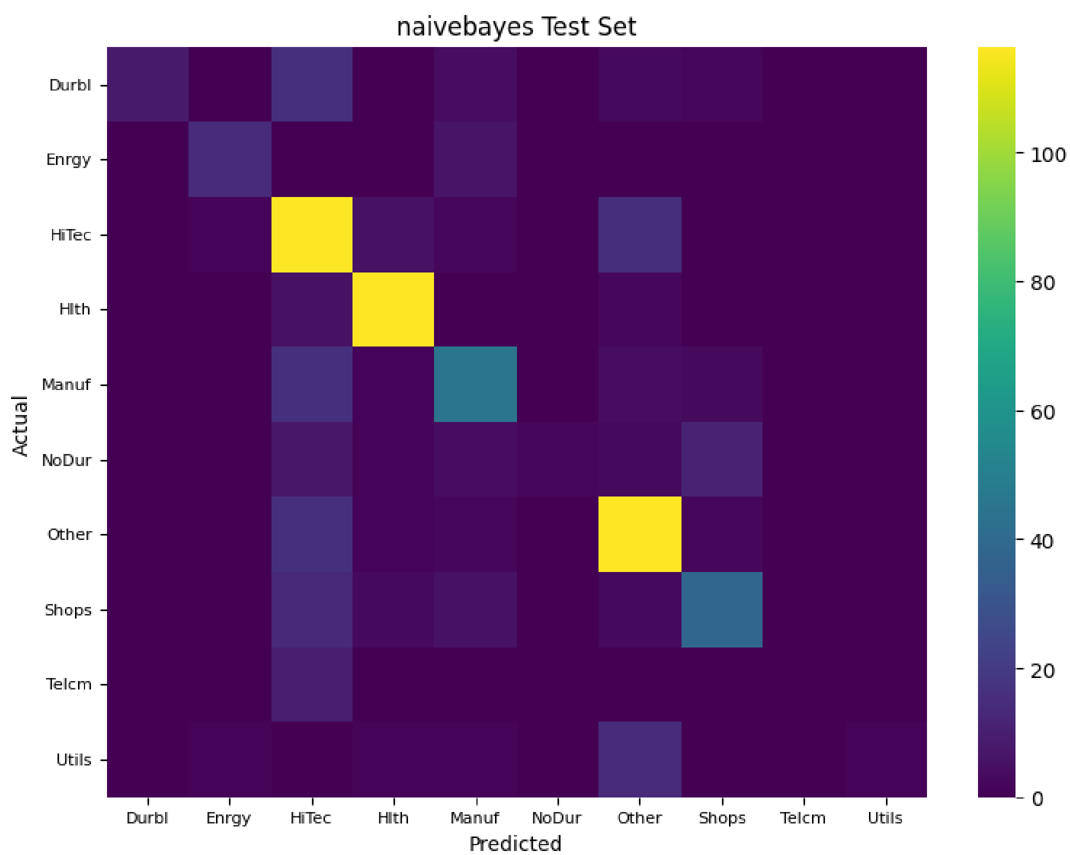
```
perceptron  0.942079  0.732010  0.953285  0.765879  0.946463  0.741730
linearsvc   0.990133  0.809224  0.993392  0.811619  0.991729  0.806204
logistic    0.901001  0.823311  0.844640  0.793319  0.867906  0.805434
```

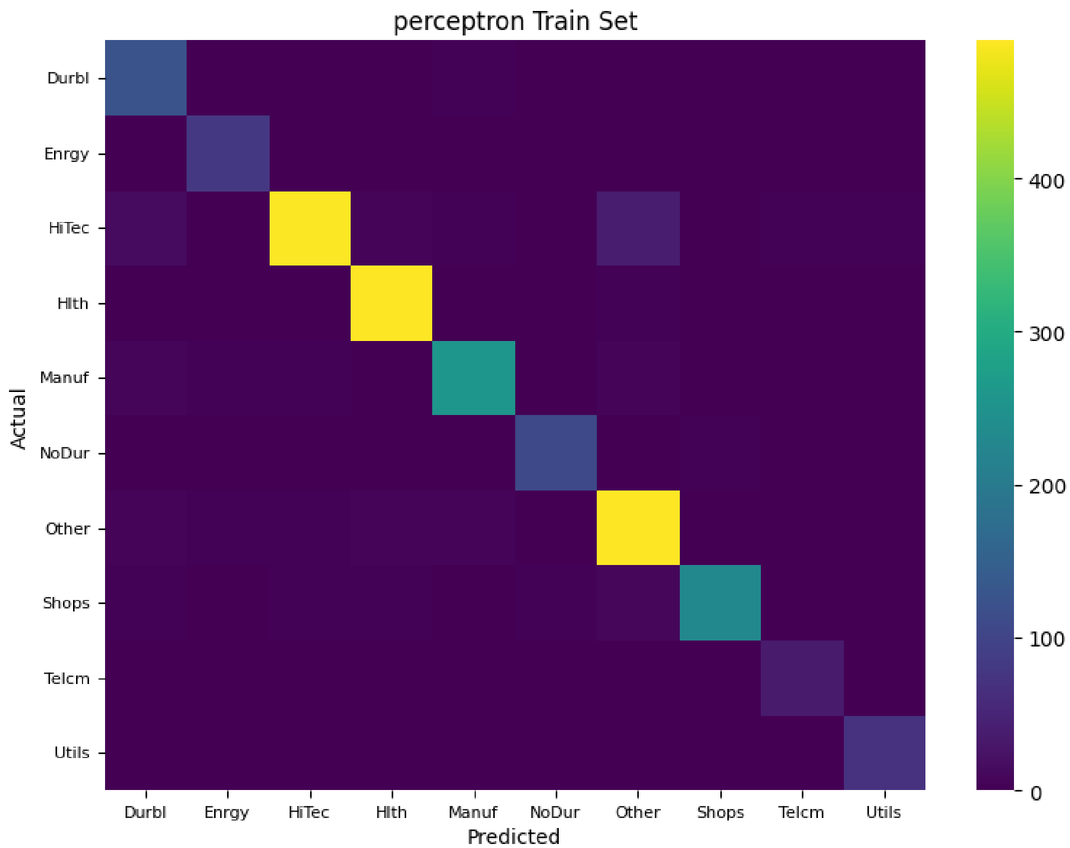
28.3.3 Confusion Matrix

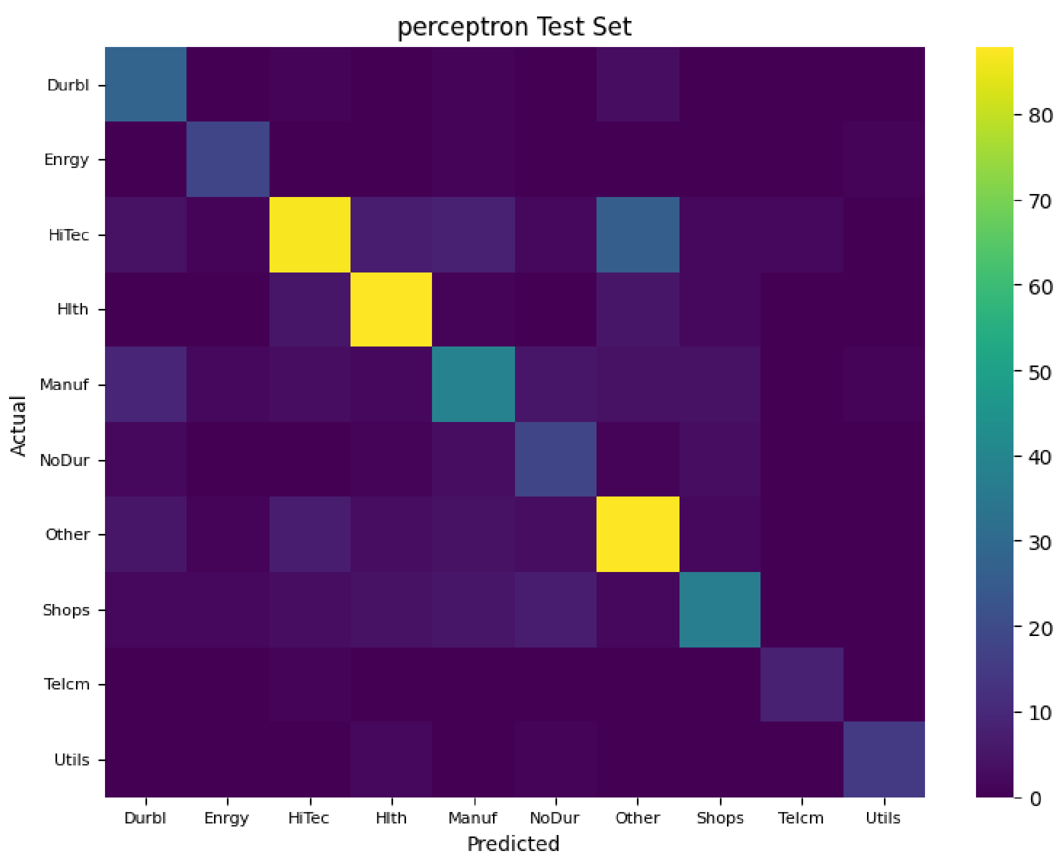
A confusion matrix provides a detailed breakdown of the classification results, showing the true positive, false positive, true negative, and false negative counts for each class.

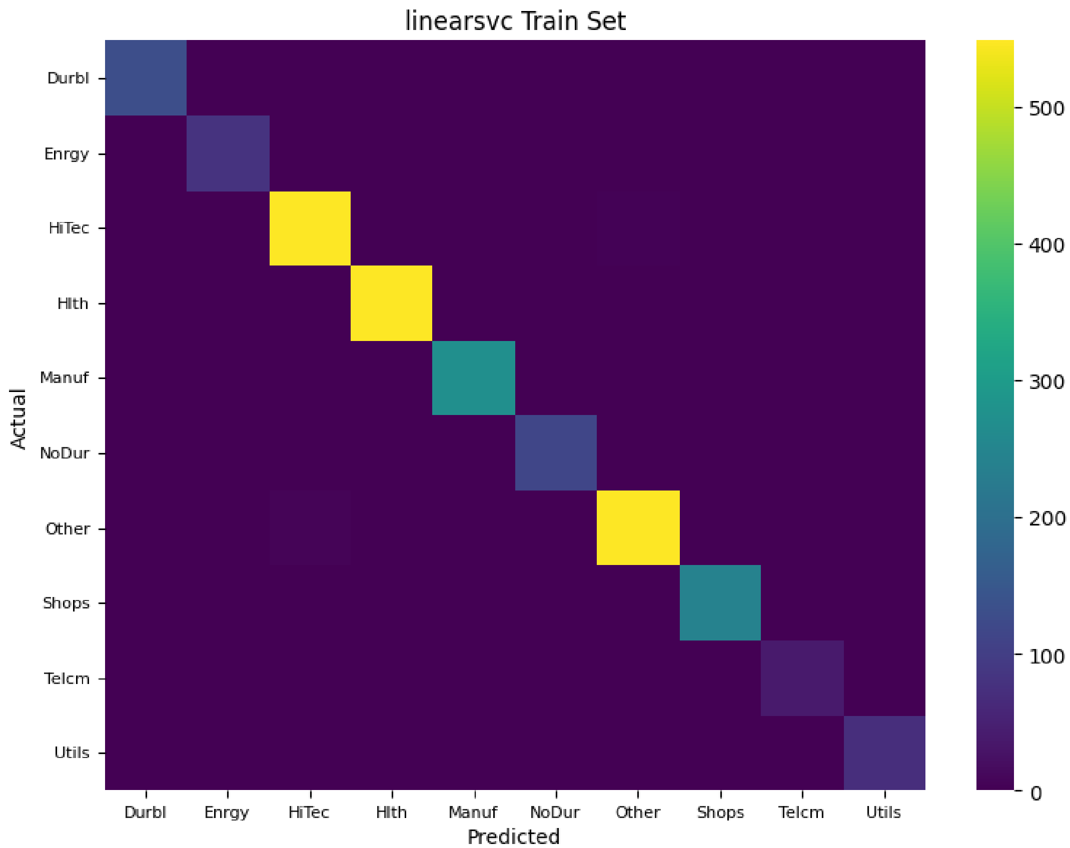
```
for name, model in models.items():
    fig, axes = plt.subplots(nrows=2, figsize=(8, 12))
    for label, cf, ax in [('Train Set', cf_train[name], axes[0]),
                          ('Test Set', cf_test[name], axes[1])]:
        sns.heatmap(cf, ax=ax, annot= False, fmt='d', cmap='viridis', robust=True,
                    yticklabels=model.classes_,
                    xticklabels=model.classes_)
        ax.set_title(f"{name} {label}")
        ax.set_xlabel('Predicted')
        ax.set_ylabel('Actual')
        ax.yaxis.set_tick_params(labelsize=8, rotation=0)
        ax.xaxis.set_tick_params(labelsize=8, rotation=0)
        #plt.subplots_adjust(left=0.35, bottom=0.25)
        plt.tight_layout()
plt.close()
```

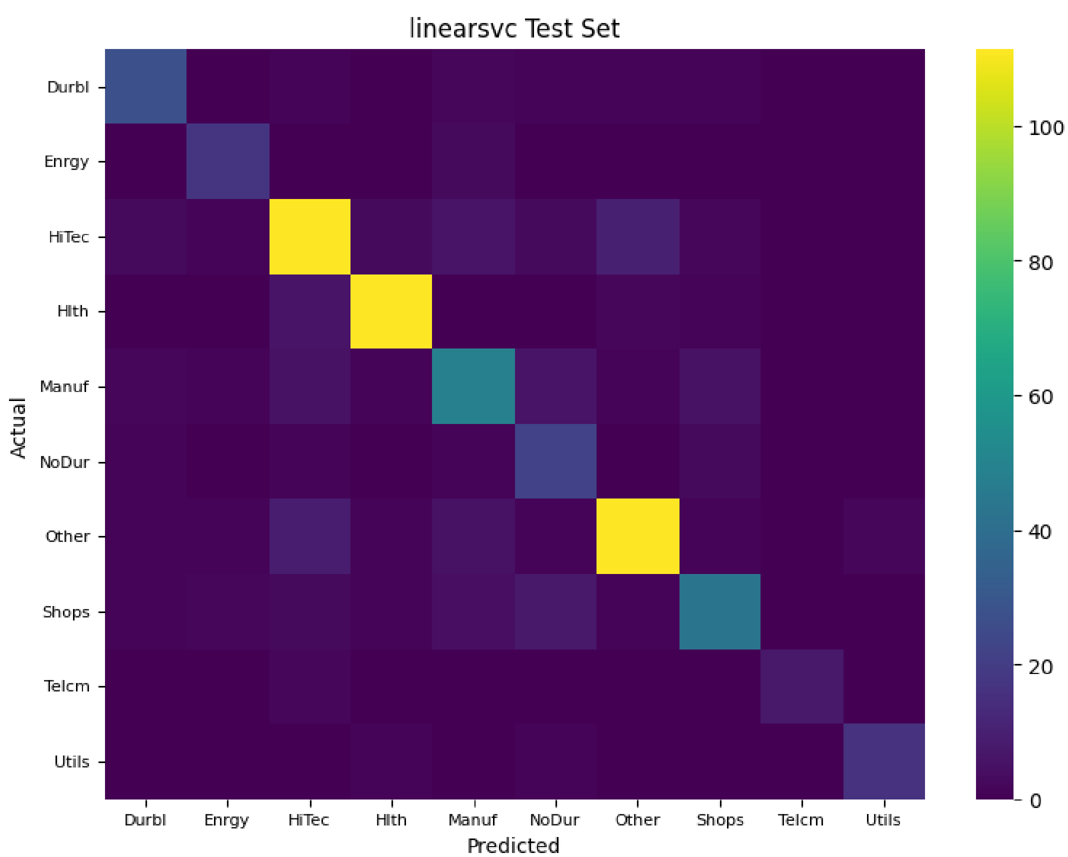

naivebayes Train Set
Actual
Durbl
Enrgy
HiTec
Hlth
Manuf
NoDur
Other
Shops
Telcm
Utils
Predicted
400
300
200
100
0


naivebayes Test Set
Actual
Durbl
Enrgy
HiTec
Hlth
Manuf
NoDur
Other
Shops
Telcm
Utils
Predicted
100
80
60
40
20
0


perceptron Train Set
Durbl
Enrgy
HiTec
Hlth
Manuf
NoDur
Other
Shops
Telcm
Utils
Actual
Predicted
0
100
200
300
400


perceptron Test Set
Durbl
Enrgy
HiTec
Hlth
Manuf
NoDur
Other
Shops
Telcm
Utils
Actual
Predicted
0
10
20
30
40
50
60
70
80


linearsvc Train Set
Durbl
Enrgy
HiTec
Hlth
Manuf
NoDur
Other
Shops
Telcm
Utils
Actual
Predicted
0
100
200
300
400
500

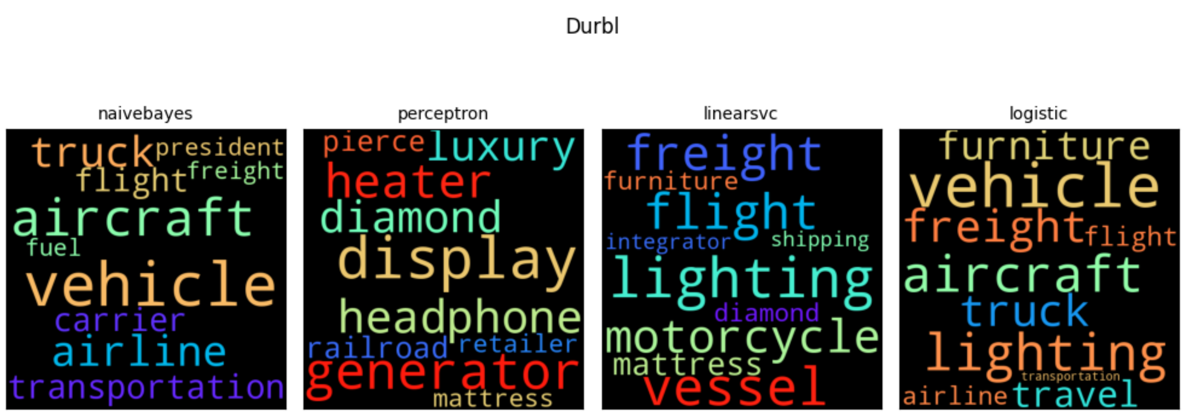

linearsvc Test Set
Durbl
Enrgy
HiTec
Hlth
Manuf
NoDur
Other
Shops
Telcm
Utils
Actual
Predicted
0
20
40
60
80
100

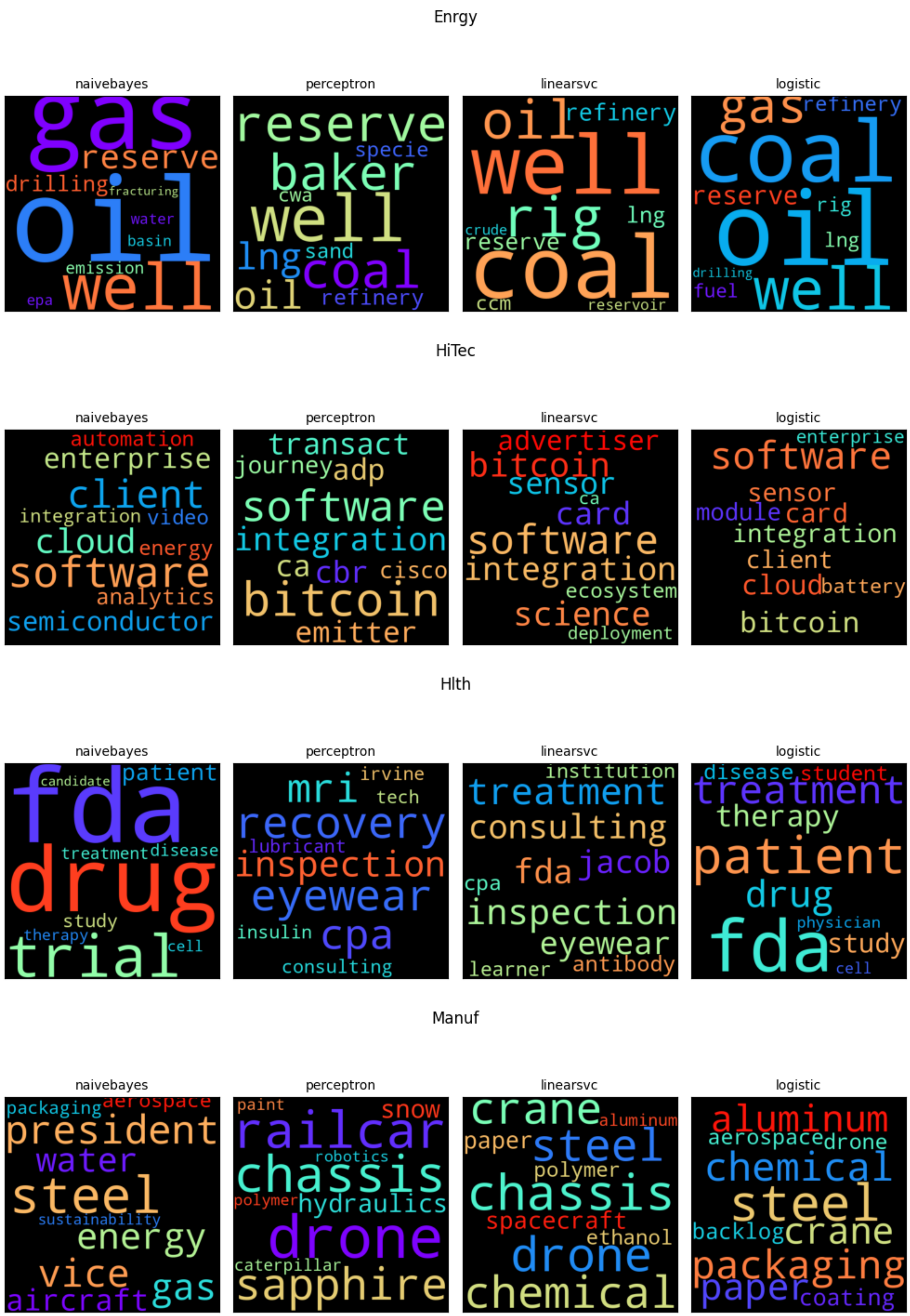
28.3.4 Feature importance

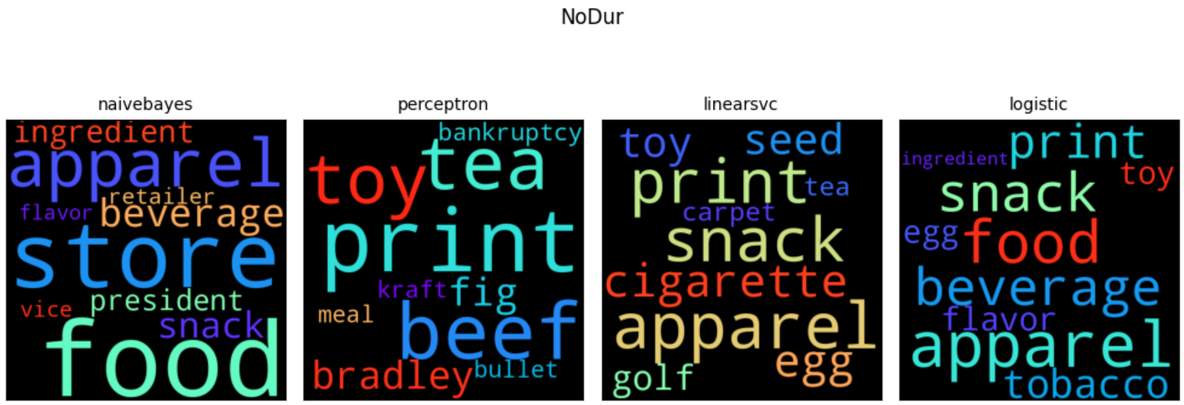
Feature importance can be assessed by examining the weights or probabilities assigned to each term in the model, allowing us to visualize the most important terms for each class. For Naive Bayes, the weights can be exponentiated to probabilities

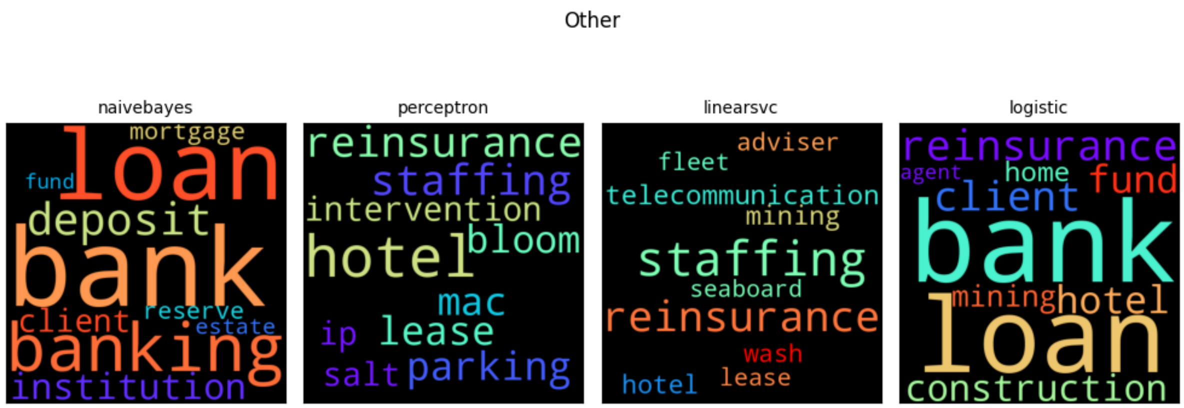
Word clouds, facilitated by packages such as `WordCloud`, can help visualize the most frequent and important words in the dataset, highlighting the key terms that influence classification decisions.

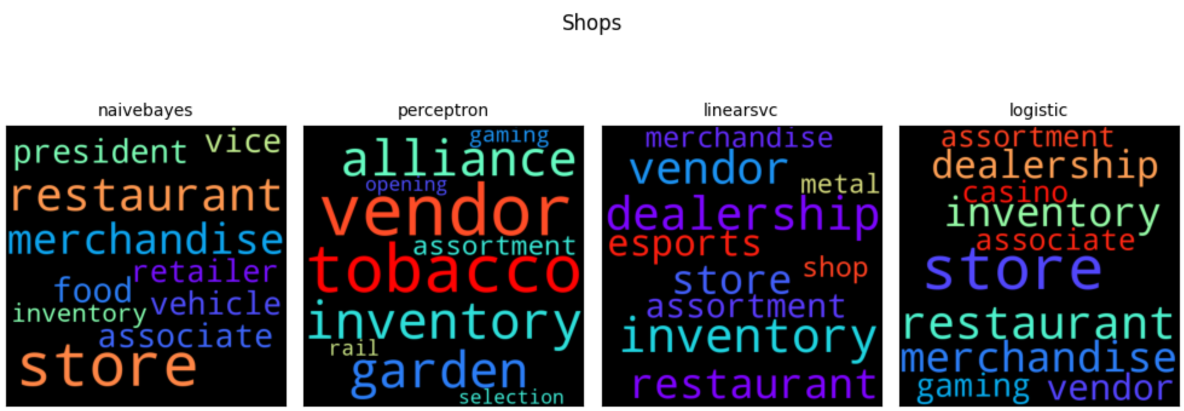
```
wc = WordCloud(height=500, width=500, prefer_horizontal=1.0, colormap='rainbow')

top_n = 10
for topic in classes:   # loop over classes
    fig, axes = plt.subplots(ncols=len(models), nrows=1, figsize=(10, 4))
    fig.suptitle(topic)
    for imodel, (ax, name, clf) in enumerate(zip(
            axes, models.keys(), models.values())):
        assert hasattr(clf, 'coef_') or hasattr(clf, 'feature_log_prob_')
        k = clf.classes_.tolist().index(topic)
        #print("Event %d %s:" % (topic, events_[clf.classes_[topic]]))
        if hasattr(clf, 'coef_'):
            importance = clf.coef_[k, :]
        else:
            importance = np.exp(clf.feature_log_prob_[k, :])
        words = {feature_names[i]: importance[i]
                 for i in importance.argsort()[-top_n:]}
        ax.imshow(wc.generate_from_frequencies(words))
        #Series(words).plot(kind='barh', color=f"C{imodel}", ax=ax)
        #ax.yaxis.set_tick_params(labelsize=7)
        ax.axes.yaxis.set_visible(False)   # make axes ticks invisible
        ax.xaxis.set_ticks([])
        ax.xaxis.set_ticklabels([])
        ax.set_title(name, fontdict={'fontsize':10})
    fig.tight_layout()
plt.close()
```

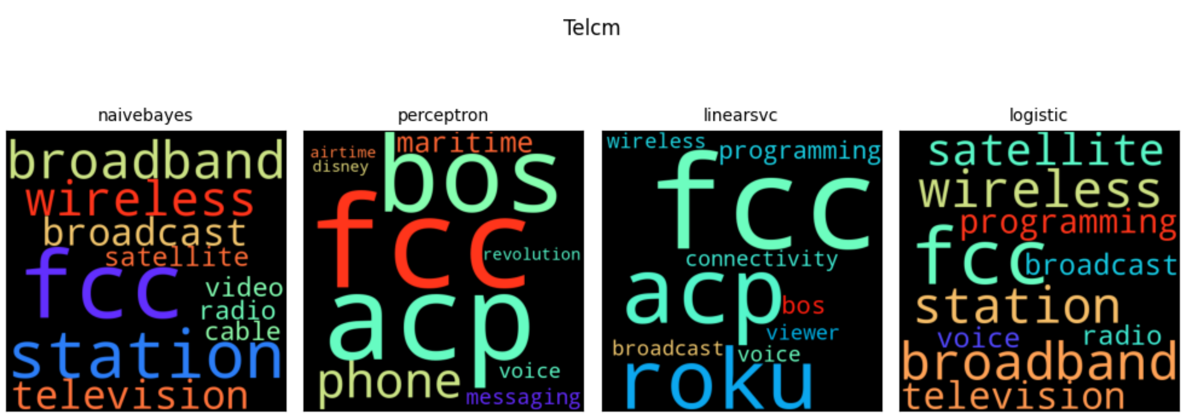



Enrgy
naivebayes
gas
reserve
drilling
fracturing
oil
water
basin
emission
epa
well
perceptron
reserve
specie
baker
cwa
well
lng
sand
coal
oil
refinery
linearsvc
oil
refinery
well
rig
lng
crude
reserve
coal
ccm
reservoir
logistic
gas
refinery
coal
reserve
rig
oil
lng
drilling
fuel
well
HiTec
naivebayes
automation
enterprise
client
integration
video
cloud
energy
software
analytics
semiconductor
perceptron
transact
journey
adp
software
integration
ca
cbr
cisco
bitcoin
emitter
linearsvc
advertiser
bitcoin
sensor
ca
card
software
integration
ecosystem
science
deployment
logistic
enterprise
software
sensor
module
card
integration
client
cloud
battery
bitcoin
Hlth
naivebayes
fda
candidate
patient
treatment
disease
drug
study
therapy
trial
cell
perceptron
irvine
mri
tech
recovery
lubricant
inspection
eyewear
insulin
cpa
consulting
linearsvc
institution
treatment
consulting
cpa
fda
jacob
inspection
eyewear
learner
antibody
logistic
disease
student
treatment
therapy
patient
drug
physician
fda
study
cell
Manuf
naivebayes
packaging
aerospace
president
water
steel
sustainability
energy
vice
aircraft
gas
perceptron
paint
snow
railcar
robotics
chassis
polymer
hydraulics
drone
caterpillar
sapphire
linearsvc
crane
aluminum
paper
steel
polymer
chassis
spacecraft
ethanol
drone
chemical
logistic
aluminum
aerospace
drone
chemical
steel
backlog
crane
packaging
paper
coating


NoDur
naivebayes
ingredient apparel flavor retailer beverage store vice president snack food
perceptron
bankruptcy toy tea print kraft fig meal beef bradley bullet
linearsvc
toy seed print tea carpet snack cigarette apparel golf egg
logistic
ingredient print toy snack egg food beverage flavor apparel tobacco


Other
naivebayes
mortgage fund loan deposit bank client reserve estate banking institution
perceptron
reinsurance staffing intervention hotel bloom mac ip lease salt parking
linearsvc
adviser fleet telecommunication mining staffing seaboard reinsurance wash hotel lease
logistic
reinsurance agent home fund client bank mining hotel loan construction


Shops
naivebayes
president vice restaurant merchandise retailer food inventory vehicle associate store
perceptron
gaming alliance opening vendor assortment tobacco inventory rail garden selection
linearsvc
merchandise vendor metal dealership esports shop store assortment inventory restaurant
logistic
assortment dealership casino inventory associate store restaurant merchandise gaming vendor


Telcm
naivebayes
broadband wireless broadcast satellite fcc video radio cable station television
perceptron
airtime disney maritime bos fcc revolution acp phone voice messaging
linearsvc
wireless programming fcc connectivity acp bos viewer broadcast voice roku
logistic
satellite wireless programming fcc broadcast station voice radio broadband television

References:

Gareth James, Daniela Witten, Trevor Hastie, Robert Tibshirani. "An Introduction to Statistical Learning with Applications in R". New York, Springer, 2013.

Gerard Hoberg and Gordon Phillips, 2016, Text-Based Network Industries and Endogenous Product Differentiation.Journal of Political Economy 124 (5), 1423-1465.

Gerard Hoberg and Gordon Phillips, 2010, Product Market Synergies and Competition in Mergers and Acquisitions: A Text-Based Analysis. Review of Financial Studies 23 (10), 3773-3811.

CHAPTER

TWENTYNINE

MACHINE LEARNING: REGRESSION

Whereof what' s past is prologue - Shakespeare

We explore supervised machine learning and regression models for macroeconomic forecasting, with a focus on predicting the industrial production index (INDPRO) using a wide range of economic indicators. To improve generalization and predictive performance, we investigate methods such as subset selection, penalized regression, decision trees, and ensemble learning, aiming to strike a balance between model complexity and accuracy.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import statsmodels.api as sm
from sklearn.metrics import mean_squared_error
from finds.readers.alfred import Alfred, fred_md
from finds.utils import plot_date
from secret import credentials
# %matplotlib qt
VERBOSE = 0
```

29.1 Macroeconomic forecasting

For the candidate regression models, the independent variables consist of up to three lags of each economic indicator. The objective is to predict the rate of change in the industrial production index (INDPRO) for the following month while minimizing the **mean squared error (MSE)** loss function, which is appropriate for continuous-valued targets.

Monthly macroeconomic data is retrieved from FRED-MD, with any missing recent values supplemented from other public sources. Suggested transformations are applied to each time series to ensure stationarity. The pre-2023 period is used for training, while the post-2023 period is reserved for testing.

```
alf = Alfred(api_key=credentials['fred']['api_key'], verbose=-1)
```

```
# Get latest FRED-MD data
freq = 'M'
beg = 19640701  # 19620701
df, t = fred_md()   #
transforms = t['transform']
end = df.index[-2]
split_date = end - 20000
print("Train/test date ranges:", beg, split_date, end)
```

```
FRED-MD vintage: monthly/current.csv
Train/test date ranges: 19640701 20221231 20241231
```

```
# Splice in common updates: source of PE ratio, Commercial Paper
for col in ['S&P PE ratio']:
    df[col] = alf.splice(col)
df['COMPAPFF'] = df['COMPAPFF'].ffill()  # forward fill 20200430
df['CP3M'] = df['CP3M'].ffill()          # forward fill 20200430
```

```
# Apply time series transformations according to FRED-MD
transformed = []
for col in df.columns:
    transformed.append(alf.transform(df[col], tcode=transforms[col], freq=freq))
data = pd.concat(transformed, axis=1).iloc[2:]
c = list(data.columns)
data = data.loc[(data.index >= beg) & (data.index <= end)]
```

```
# Drop columns with missing data
missing = []
for series_id in df.columns:
    g = data[series_id].notna()
    missing.extend([(date, series_id) for date in data.index[~g]])
missing_per_row = data.isna().sum(axis=1)
missing = DataFrame.from_records(missing, columns=['date', 'series_id'])
print('original:', data.shape, 'dropna:', data.dropna(axis=1).shape)
data = data.dropna(axis=1)   # drop columns where missing values
print(missing['series_id'].value_counts())
data
```

```
original: (726, 126) dropna: (726, 122)
series_id
ACOGNO          332
UMCSENT         163
TWEXAFEGSMTH    103
ANDENO           44
Name: count, dtype: int64
```

```
                RPI   W875RX1  DPCERA3M086SBEA  CMRMTSPL    RETAIL    INDPRO  \
19640731  0.005422  0.005404         0.007343  0.019986  0.004947  0.006552
19640831  0.005644  0.006061         0.005992 -0.020139  0.013974  0.006506
19640930  0.004123  0.004205        -0.004164  0.027173  0.009372  0.003703
19641031  0.000555  0.000650         0.006499 -0.013866 -0.039421 -0.013947
19641130  0.006703  0.007352        -0.009111 -0.009745  0.009335  0.030429
...            ...       ...              ...       ...       ...       ...
20240831  0.000066 -0.000117         0.000711  0.000375 -0.001144  0.004869
20240930  0.001458  0.000240         0.005257  0.007155  0.008903 -0.004205
20241031  0.003690  0.003488         0.001757 -0.001714  0.005575 -0.004547
20241130  0.002184  0.002756         0.004346  0.003993  0.006484 -0.001453
20241231  0.001234  0.001391         0.005441  0.006665  0.007175  0.009853

            IPFPNSS   IPFINAL   IPCONGD  IPDCONGD  ...  DDURRG3M086SBEA  \
19640731  0.010342  0.011313  0.014079  0.016438  ...         0.000667
19640831 -0.000936 -0.000939 -0.001865  0.007219  ...        -0.002031
```

(continues on next page)

(continued from previous page)

```
19640930 -0.004690 -0.003759 -0.011267 -0.023656  ...        0.000728
19641031 -0.010404 -0.013269 -0.020030 -0.109875  ...       -0.001712
19641130  0.029043  0.031929  0.036884  0.128121  ...        0.004137
...           ...       ...       ...       ...  ...             ...
20240831  0.004515  0.007094  0.009218  0.047547  ...        0.000600
20240930 -0.005809 -0.008928 -0.002317 -0.001441  ...        0.005721
20241031 -0.007220 -0.010970 -0.006158 -0.028598  ...       -0.002965
20241130 -0.001180 -0.000155 -0.004246  0.018794  ...       -0.000986
20241231  0.007300  0.006569  0.003359 -0.013311  ...       -0.004215

          DNDGRG3M086SBEA  DSERRG3M086SBEA  CES0600000008  CES2000000008  \
19640731        -0.000369        -0.000090      -0.000016       0.003231
19640831        -0.002398         0.001406      -0.000016      -0.003263
19640930         0.003749        -0.001674      -0.000015      -0.006462
19641031        -0.002344         0.000525      -0.011757       0.016093
19641130         0.000985        -0.000352       0.011772      -0.019272
...                   ...              ...            ...            ...
20240831        -0.002222        -0.000061      -0.002580       0.001376
20240930        -0.002598         0.000553       0.004137       0.001906
20241031         0.003609         0.000788      -0.003839      -0.005006
20241130         0.000117        -0.002176      -0.000639      -0.002760
20241231         0.004218         0.002086       0.002521       0.005776

          CES3000000008  DTCOLNVHFNM  DTCTHFNM    INVEST   VIXCLS
19640731      -0.004158    -0.003798 -0.002430 -0.002831  11.2238
19640831       0.004141    -0.005958 -0.001818  0.011673  13.6898
19640930       0.004090    -0.011142 -0.003802  0.010582  10.5167
19641031      -0.024760     0.005974 -0.000704 -0.003579  11.0924
19641130       0.024828    -0.009946 -0.002806 -0.002694  12.0087
...                 ...          ...       ...       ...      ...
20240831      -0.006110    -0.001974 -0.000766  0.001445  19.6750
20240930       0.004652    -0.002368 -0.001586 -0.001644  17.6597
20241031      -0.002154    -0.000146  0.003227 -0.000473  19.9478
20241130       0.003190    -0.000872 -0.001679 -0.011753  15.9822
20241231      -0.001439     0.002963  0.002748  0.002421  15.6997

[726 rows x 122 columns]
```

Data beyond the split date is excluded from model training to ensure a proper evaluation of generalization performance.

```python
# Split time series train and test set
def ts_split(X, Y, end=split_date):
    """helper to split train/test time series"""
    return X[Y.index<=end], X[Y.index>end], Y[Y.index<=end], Y[Y.index>end]
```

```python
def columns_map(columns, lag):
    """helper to create lagged column names"""
    return {col: col + '_' + str(lag) for col in columns}
```

```python
def columns_unmap(columns):
    """helper to extract lagged column names"""
    cols = [col.split('_') for col in columns]
    return [col[0] for col in cols], [int(col[1]) for col in cols]
```

```
target_id = 'INDPRO'
lags = 3   # Include up to 3 lags of exogs
Y = data[target_id].iloc[lags:]
X = pd.concat([data.shift(lag).iloc[lags:]\
                .rename(columns=columns_map(data.columns, lag))
                for lag in range(1, lags+1)],
               axis=1)
```

```
# new table to collect final fitted models
test = Series(name='test', dtype=float)    # collect test and train errors
train = Series(name='train', dtype=float)
final_models = {}
```

29.2 Regression models

Complex regression models with numerous features or parameters risk overfitting, capturing noise rather than true patterns. Several techniques help mitigate overfitting and improve model generalization:

- **Cross-validation**: Hyperparameter tuning using cross-validation helps ensure models generalize well across different data subsets. **K-fold Cross-Validation (K-fold CV)** divides the dataset into K parts, training the model on $K-1$ sections and testing it on the remaining part, iterating through all folds and averaging results. **Leave-One-Out Cross-Validation (LOOCV)** is a special case where each data point serves as the test set once, helping refine hyperparameter selection.
- **Information criteria penalty**: The **Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC)** aid model selection by balancing goodness of fit and complexity. AIC, defined as $AIC = 2k - 2\ln(L)$, where k is the number of parameters and L is the likelihood, favors models that explain the data with minimal complexity. BIC, given by $BIC = k\ln(n) - 2\ln(L)$, introduces a stronger penalty for sample size n, preferring simpler models for large datasets. Lower AIC or BIC values indicate better models.
- **Ensemble learning**: By combining multiple weak learners, ensemble methods enhance model robustness. Bagging averages predictions from multiple models trained on different data subsets, reducing variance. Boosting sequentially corrects model errors, improving overall accuracy.

29.2.1 Forward Subset Selection

Forward Subset Regression is a stepwise variable selection method used in regression modeling to identify the best subset of predictors. It begins with no variables in the model and iteratively adds the most significant predictor at each step, based on a chosen criterion, and stops when adding more variables does not significantly improve model performance. Penalized selection criteria, such as AIC or BIC, helps avoid overfitting. Forward selection is computationally more efficient than exhaustive search methods but may miss the optimal subset if an initially excluded variable becomes relevant later in combination with others.

```
def forward_select(Y, X, selected, ic='aic'):
    """helper to forward select next regressor, using sm.OLS"""
    remaining = [x for x in X.columns if x not in selected]
    results = []
    for x in remaining:
        r = sm.OLS(Y, X[selected + [x]]).fit()
        results.append({'select': x,
```

(continues on next page)

(continued from previous page)

```
                        'aic': r.aic,
                        'bic': r.bic,
                        'rsquared': r.rsquared,
                        'rsquared_adj': r.rsquared_adj})
    return DataFrame(results).sort_values(by=ic).iloc[0].to_dict()
```

```
# find best bic, and show selection criteria scores
ic = 'bic'    # select by information criterion
X_train, X_test, Y_train, Y_test = ts_split(X, Y)
selected = []
models = {}
for i in range(1, 32):
    select = forward_select(Y_train,
                            X_train,
                            selected,
                            ic=ic)
    models[i] = select
    selected.append(select['select'])
selected = DataFrame.from_dict(models, orient='index')
best = selected[[ic]].iloc[selected[ic].argmin()]
subset = selected.loc[:best.name].round(3)
subset.index = [alf.header(s.split('_')[0]) for s in subset['select']]
print('Forward Selection Subset')
subset
```

```
Forward Selection Subset
```

```
                                                            select       aic  \
Initial Claims                                            CLAIMS_1 -4772.162
Real M2 Money Stock                                       M2REAL_2 -4814.414
All Employees, Wholesale Trade                          USWTRADE_2 -4843.063
All Employees, Service-Providing                          SRVPRD_2 -4864.114
5-Year Treasury Constant Maturity Minus Federal...        T5YFFM_3 -4881.249
Total Business: Inventories to Sales Ratio               ISRATIO_2 -4895.191
All Employees, Service-Providing                          SRVPRD_1 -4911.828
All Employees, Financial Activities                       USFIRE_2 -4928.888
All Employees, Trade, Transportation, and Utili...         USTPU_1 -4945.576
Consumer Motor Vehicle Loans Owned by Finance C...   DTCOLNVHFNM_3 -4952.630
Industrial Production: Nondurable Energy Consum...     IPB51222S_3 -4960.975
Industrial Production: Nondurable Goods Materials         IPNMAT_3 -4974.770
Canadian Dollars to U.S. Dollar Spot Exchange Rate        EXCAUS_3 -4983.489
M1                                                          M1SL_3 -4993.678
Nonrevolving consumer credit to Personal Income           CONSPI_2 -5001.564
Market Yield on U.S. Treasury Securities at 10-...          GS10_1 -5008.976
Capacity Utilization: Manufacturing (SIC)                 CUMFNS_2 -5014.366
Industrial Production: Manufacturing (SIC)             IPMANSICS_2 -5041.837
All Employees, Nondurable Goods                         NDMANEMP_1 -5056.048
Average Hourly Earnings of Production and Nonsu...  CES2000000008_1 -5065.393
Average Weekly Overtime Hours of Production and...       AWOTMAN_2 -5075.474
Industrial Production: Equipment: Business Equi...       IPBUSEQ_2 -5083.967
Industrial Production: Total Index                        INDPRO_2 -5091.928
Real Estate Loans, All Commercial Banks                   REALLN_1 -5096.759
All Employees, Mining, Quarrying, and Oil and G...  CES1021000001_2 -5102.029
Moody's Seasoned Baa Corporate Bond Yield                    BAA_1 -5106.126
```

(continues on next page)

(continued from previous page)

```
Moody's Seasoned Aaa Corporate Bond Yield                    AAA_1 -5110.792
All Employees, Mining, Quarrying, and Oil and G...  CES1021000001_1 -5116.399
Industrial Production: Materials                           IPMAT_1 -5120.986

                                                         bic  rsquared  \
Initial Claims                                     -4767.612     0.355
Real M2 Money Stock                                -4805.315     0.394
All Employees, Wholesale Trade                     -4829.414     0.420
All Employees, Service-Providing                   -4845.916     0.439
5-Year Treasury Constant Maturity Minus Federal... -4858.500     0.454
Total Business: Inventories to Sales Ratio         -4867.893     0.467
All Employees, Service-Providing                   -4879.981     0.481
All Employees, Financial Activities                -4892.490     0.495
All Employees, Trade, Transportation, and Utili... -4904.629     0.508
Consumer Motor Vehicle Loans Owned by Finance C... -4907.133     0.514
Industrial Production: Nondurable Energy Consum... -4910.928     0.521
Industrial Production: Nondurable Goods Materials  -4920.175     0.532
Canadian Dollars to U.S. Dollar Spot Exchange Rate -4924.343     0.539
M1                                                 -4929.983     0.547
Nonrevolving consumer credit to Personal Income    -4933.319     0.554
Market Yield on U.S. Treasury Securities at 10-... -4936.181     0.560
Capacity Utilization: Manufacturing (SIC)          -4937.022     0.564
Industrial Production: Manufacturing (SIC)         -4959.943     0.582
All Employees, Nondurable Goods                    -4969.605     0.592
Average Hourly Earnings of Production and Nonsu... -4974.400     0.598
Average Weekly Overtime Hours of Production and... -4979.931     0.605
Industrial Production: Equipment: Business Equi... -4983.875     0.611
Industrial Production: Total Index                 -4987.286     0.617
Real Estate Loans, All Commercial Banks            -4987.568     0.620
All Employees, Mining, Quarrying, and Oil and G... -4988.288     0.624
Moody's Seasoned Baa Corporate Bond Yield          -4987.835     0.627
Moody's Seasoned Aaa Corporate Bond Yield          -4987.951     0.631
All Employees, Mining, Quarrying, and Oil and G... -4989.008     0.635
Industrial Production: Materials                   -4989.046     0.638

                                                    rsquared_adj
Initial Claims                                             0.354
Real M2 Money Stock                                        0.393
All Employees, Wholesale Trade                             0.418
All Employees, Service-Providing                           0.436
5-Year Treasury Constant Maturity Minus Federal...         0.450
Total Business: Inventories to Sales Ratio                 0.462
All Employees, Service-Providing                           0.475
All Employees, Financial Activities                        0.489
All Employees, Trade, Transportation, and Utili...         0.502
Consumer Motor Vehicle Loans Owned by Finance C...         0.507
Industrial Production: Nondurable Energy Consum...         0.514
Industrial Production: Nondurable Goods Materials          0.524
Canadian Dollars to U.S. Dollar Spot Exchange Rate         0.531
M1                                                         0.538
Nonrevolving consumer credit to Personal Income            0.544
Market Yield on U.S. Treasury Securities at 10-...         0.549
Capacity Utilization: Manufacturing (SIC)                  0.553
Industrial Production: Manufacturing (SIC)                 0.571
All Employees, Nondurable Goods                            0.580
Average Hourly Earnings of Production and Nonsu...         0.586
```

(continues on next page)

(continued from previous page)

```
Average Weekly Overtime Hours of Production and...          0.593
Industrial Production: Equipment: Business Equi...          0.598
Industrial Production: Total Index                          0.604
Real Estate Loans, All Commercial Banks                     0.607
All Employees, Mining, Quarrying, and Oil and G...          0.610
Moody's Seasoned Baa Corporate Bond Yield                   0.613
Moody's Seasoned Aaa Corporate Bond Yield                   0.616
All Employees, Mining, Quarrying, and Oil and G...          0.620
Industrial Production: Materials                            0.623
```

```
DataFrame.from_dict({n: {'series_id': s.split('_')[0],
                         'lag' : s.split('_')[1],
                         'description': alf.header(s.split('_')[0])}
                     for n, s in selected.loc[:best.name, 'select'].items()},
                    orient='index').set_index('series_id')
```

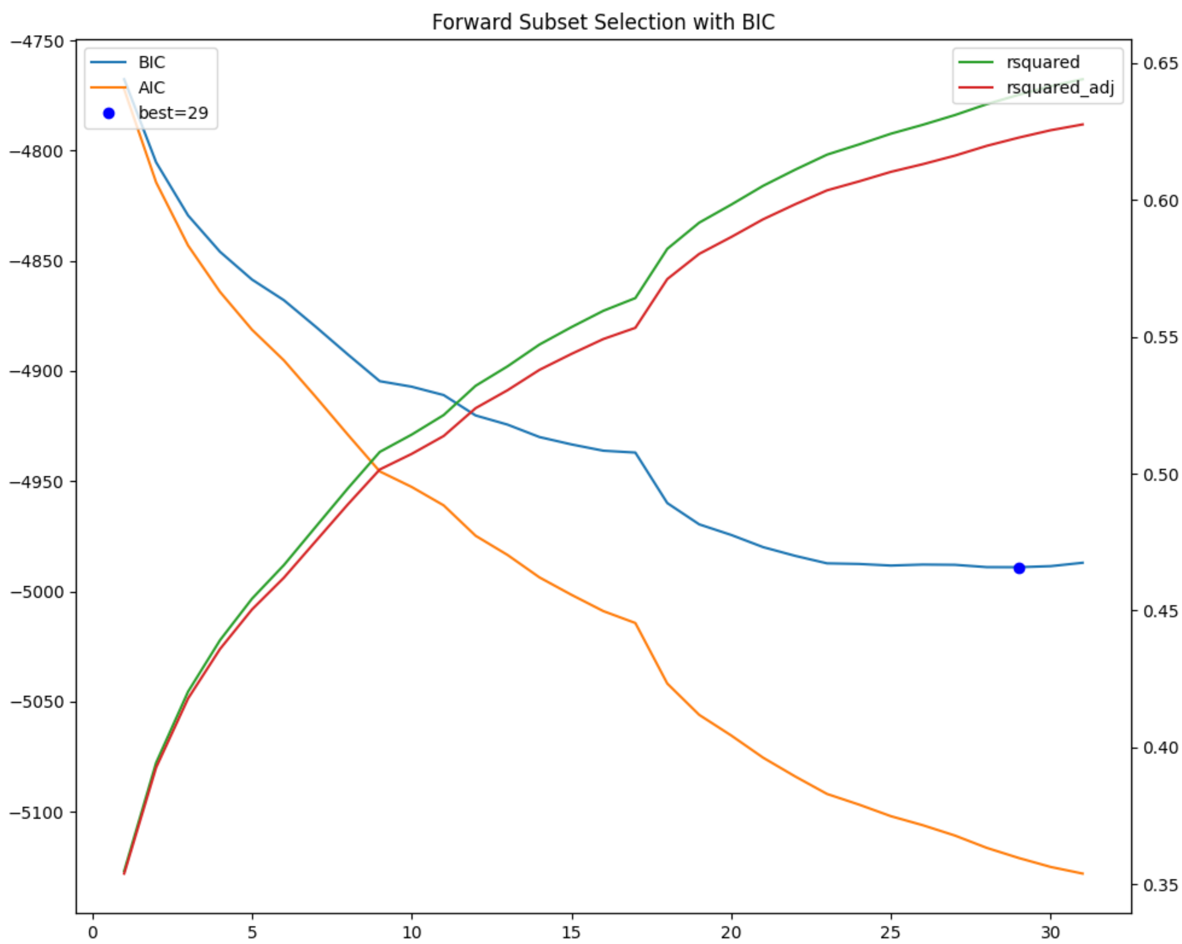
```
              lag                                        description
series_id
CLAIMS          1                                     Initial Claims
M2REAL          2                                Real M2 Money Stock
USWTRADE        2                     All Employees, Wholesale Trade
SRVPRD          2                   All Employees, Service-Providing
T5YFFM          3  5-Year Treasury Constant Maturity Minus Federa...
ISRATIO         2         Total Business: Inventories to Sales Ratio
SRVPRD          1                   All Employees, Service-Providing
USFIRE          2                All Employees, Financial Activities
USTPU           1  All Employees, Trade, Transportation, and Util...
DTCOLNVHFNM     3  Consumer Motor Vehicle Loans Owned by Finance ...
IPB51222S       3  Industrial Production: Nondurable Energy Consu...
IPNMAT          3  Industrial Production: Nondurable Goods Materials
EXCAUS          3  Canadian Dollars to U.S. Dollar Spot Exchange ...
M1SL            3                                                 M1
CONSPI          2    Nonrevolving consumer credit to Personal Income
GS10            1  Market Yield on U.S. Treasury Securities at 10...
CUMFNS          2          Capacity Utilization: Manufacturing (SIC)
IPMANSICS       2         Industrial Production: Manufacturing (SIC)
NDMANEMP        1                    All Employees, Nondurable Goods
CES2000000008   1  Average Hourly Earnings of Production and Nons...
AWOTMAN         2  Average Weekly Overtime Hours of Production an...
IPBUSEQ         2  Industrial Production: Equipment: Business Equ...
INDPRO          2                 Industrial Production: Total Index
REALLN          1            Real Estate Loans, All Commercial Banks
CES1021000001   2  All Employees, Mining, Quarrying, and Oil and ...
BAA             1          Moody's Seasoned Baa Corporate Bond Yield
AAA             1          Moody's Seasoned Aaa Corporate Bond Yield
CES1021000001   1  All Employees, Mining, Quarrying, and Oil and ...
IPMAT           1                   Industrial Production: Materials
```

```
# Plot BIC vs number selected
fig, ax = plt.subplots(num=1, figsize=(10, 8))
selected['bic'].plot(ax=ax, c='C0')
selected['aic'].plot(ax=ax, c='C1')
ax.plot(best.name, float(best.iloc[0]), "ob")
ax.legend(['BIC', 'AIC', f"best={best.name}"], loc='upper left')
```

(continues on next page)

(continued from previous page)

```
ax.set_title(f"Forward Subset Selection with {ic.upper()}")
bx = ax.twinx()
selected['rsquared'].plot(ax=bx, c='C2')
selected['rsquared_adj'].plot(ax=bx, c='C3')
bx.legend(['rsquared', 'rsquared_adj'], loc='upper right')
bx.set_xlabel('# Predictors')
plt.tight_layout()
```




```
# evaluate train and test mse
X_subset = X_train[subset['select']]
model = sm.OLS(Y_train, X_subset).fit()
name = f"Forward Subset Regression (k={len(subset)})"
Y_pred = model.predict(X_test[subset['select']])
test[name] = mean_squared_error(Y_test, Y_pred)
train[name] = mean_squared_error(Y_train, model.predict(X_subset))
final_models[name] = model
```

```
fig, ax = plt.subplots(figsize=(10, 8))
plot_date(pd.concat([Y_test.cumsum(), Y_pred.cumsum()], axis=1),
          legend1=[target_id, 'Cumulative Test Prediction'],
          title=name,
          xlabel='Test Period',
```

(continues on next page)

(continued from previous page)

```
          fontsize=8,
          ax=ax)
plt.tight_layout()
```


```
DataFrame({'name': name,
           'train': np.sqrt(train[name]),
           'test': np.sqrt(test[name])}, index=['RMSE'])
```

```
                                    name     train      test
RMSE  Forward Subset Regression (k=29)  0.005955  0.007674
```

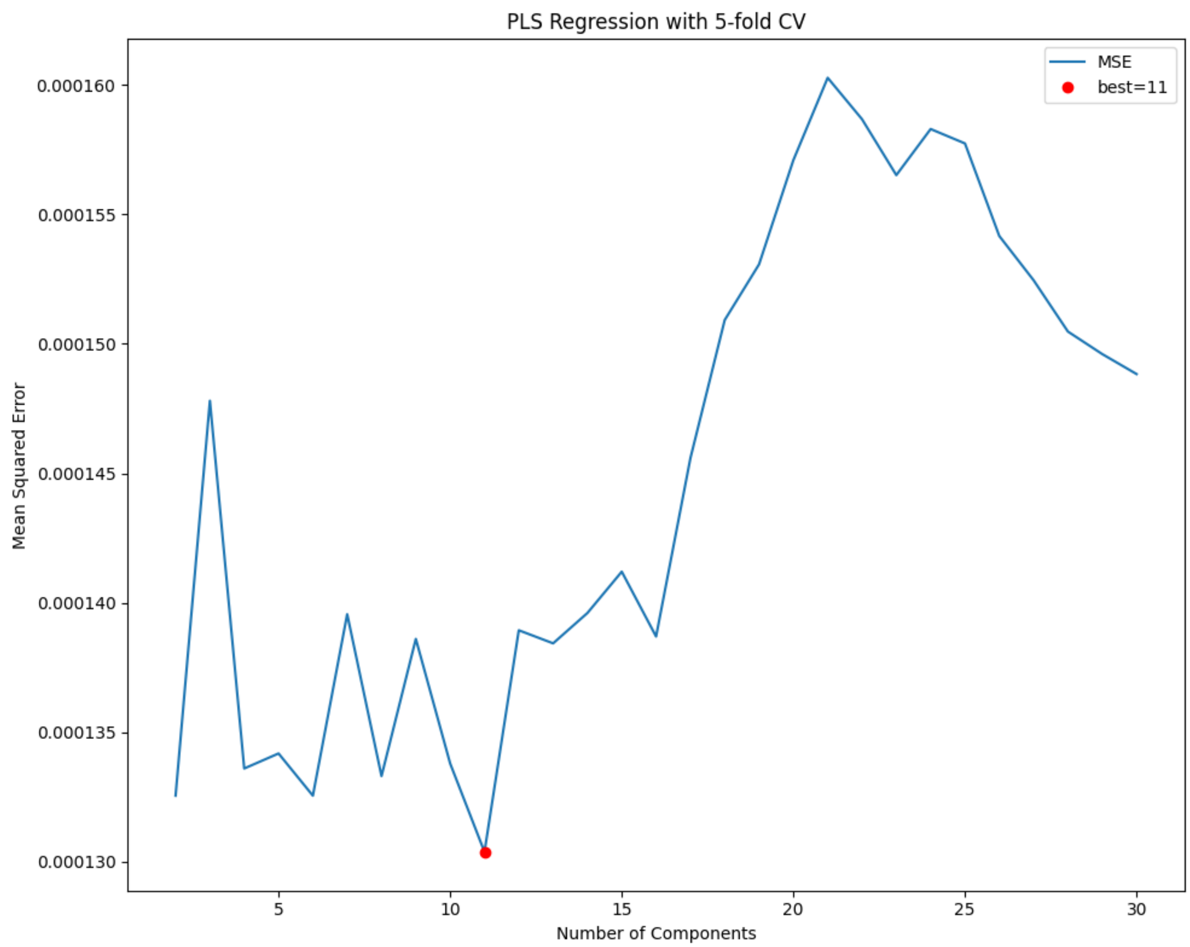
29.2.2 Partial Least Squares Regression

Partial Least Squares (PLS) Regression combines features of principal component analysis (PCA) and multiple linear regression. PLS constructs new latent variables (components) as linear combinations of the original predictors that maximize covariance with the response variable. The number of components (`n_components`) parameter controls the dimensionality reduction and must be chosen carefully to balance bias-variance tradeoff. This method is particularly useful when datasets have more predictors than observations and strong multicollinearity.

```python
# split train and test, fit standard scaling using train set
from sklearn.preprocessing import StandardScaler
from sklearn.cross_decomposition import PLSRegression
from sklearn.model_selection import cross_val_score, KFold
X_train, X_test, Y_train, Y_test = ts_split(X, Y)
scale = StandardScaler().fit(X_train)
X_train = scale.transform(X_train)
X_test = scale.transform(X_test)
```

```python
# fit with 5-fold CV to choose n_components
n_splits=5
kf = KFold(n_splits=n_splits, shuffle=True, random_state=0)
mse = Series(dtype=float)
for i in np.arange(2, 31):
    pls = PLSRegression(n_components=i)
    score = cross_val_score(estimator=pls,
                            X=X_train,
                            y=Y_train,
                            n_jobs=5,
                            verbose=VERBOSE,
                            cv=kf,
                            scoring='neg_mean_squared_error').mean()
    mse.loc[i] = -score
```

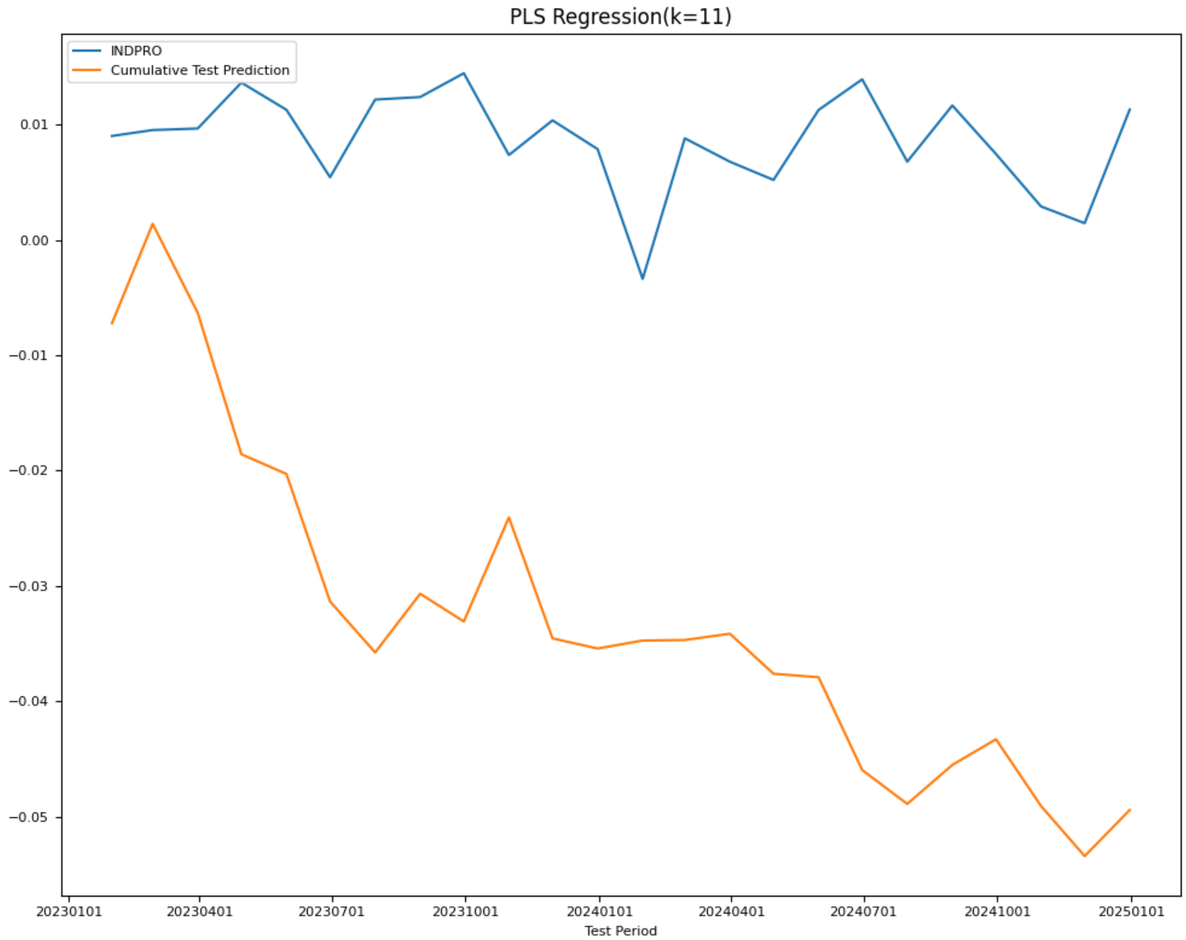
```python
# show cross-validation results and best model
fig, ax = plt.subplots(figsize=(10, 8))
mse.plot(ylabel='Mean Squared Error',
         xlabel='Number of Components',
         title=f"PLS Regression with {n_splits}-fold CV",
         ax=ax)
best = mse.index[mse.argmin()]
ax.plot(best, mse.loc[best], "or")
ax.legend(['MSE', f"best={best}"])
plt.tight_layout()
```




```
### evaluate train and test mse
model = PLSRegression(n_components=best).fit(X_train, Y_train)
name = f"PLS Regression(k={best})"
Y_pred = Series(index=Y_test.index,
                data=model.predict(X_test).reshape((-1,)))
```

```
test[name] = mean_squared_error(Y_test, Y_pred)
train[name] = mean_squared_error(Y_train, model.predict(X_train))
final_models[name] = model
```

```
fig, ax = plt.subplots(figsize=(10, 8))
plot_date(pd.concat([Y_test.cumsum(), Y_pred.cumsum()], axis=1),
          legend1=[target_id, 'Cumulative Test Prediction'],
          title=name,
          xlabel='Test Period',
          fontsize=8,
          ax=ax)
plt.tight_layout()
```




```
DataFrame({'name': name,
           'train': np.sqrt(train[name]),
           'test': np.sqrt(test[name])}, index=['RMSE'])
```

```
                     name     train      test
RMSE  PLS Regression(k=11)  0.005306  0.008791
```

29.2.3 Ridge Regression

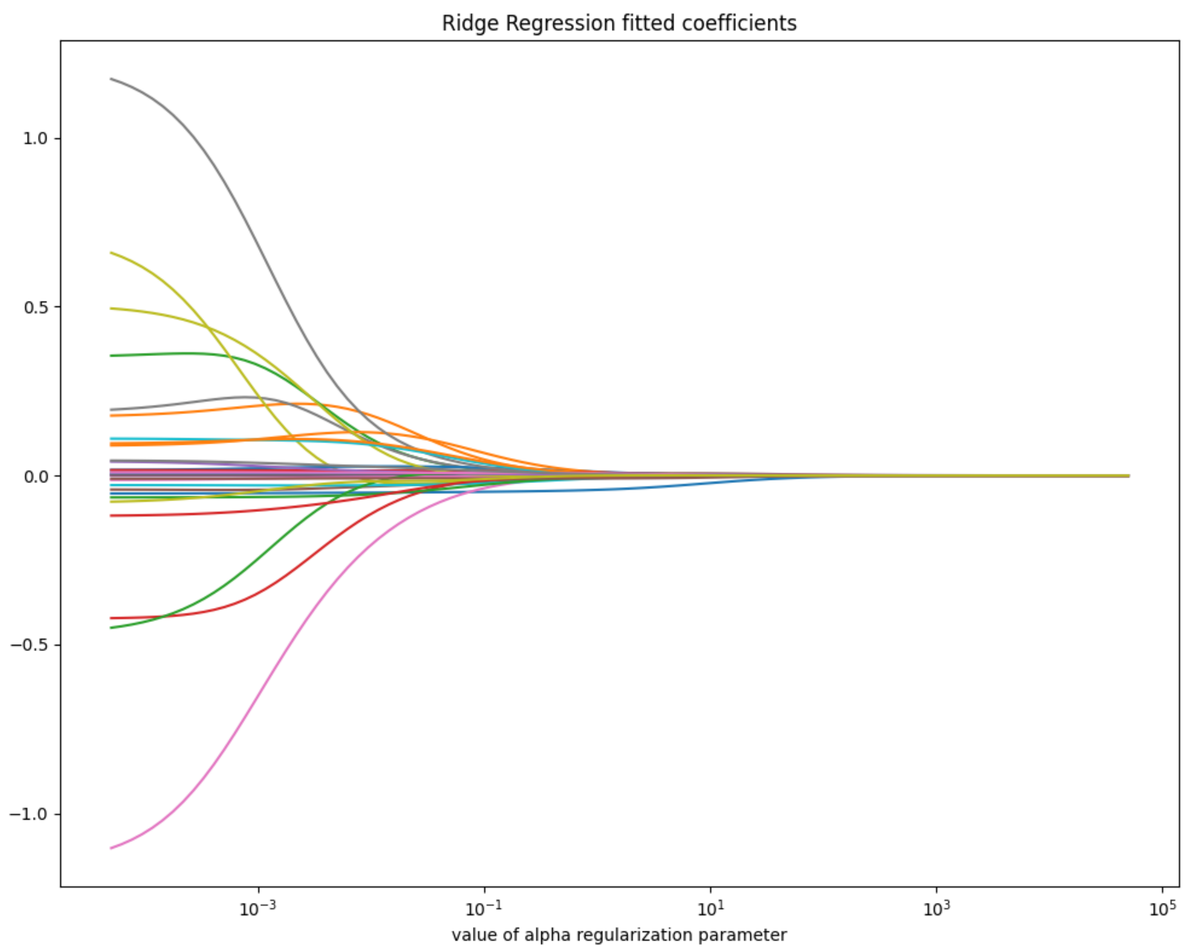
Ridge Regression is a linear regression model that includes L2 regularization, which helps prevent overfitting by adding a penalty term to the loss function. The objective function is modified as follows:

$$\min_{w} ||y - Xw||^2 + \lambda ||w||^2$$

where $||y - Xw||^2$ is the standard least squares error, and $\lambda||w||^2$ is the penalty term that shrinks the model coefficients w towards zero. The λ parameter (`alpha` in `sklearn`' s `Ridge`) controls the strength of regularization: larger values reduce model complexity by forcing coefficients to be smaller, while smaller values make Ridge behave like standard linear regression. Ridge retains all features but reduces their impact, which is useful when dealing with multicollinearity, without completely eliminating any predictor.

```
from sklearn.linear_model import Ridge, RidgeCV
alphas = 10**np.linspace(5, -4, 100)*0.5  # for parameter tuning
X_train, X_test, Y_train, Y_test = ts_split(X, Y)
scale = StandardScaler().fit(X_train)
X_train = scale.transform(X_train)
X_test = scale.transform(X_test)
np.random.seed(42)
```

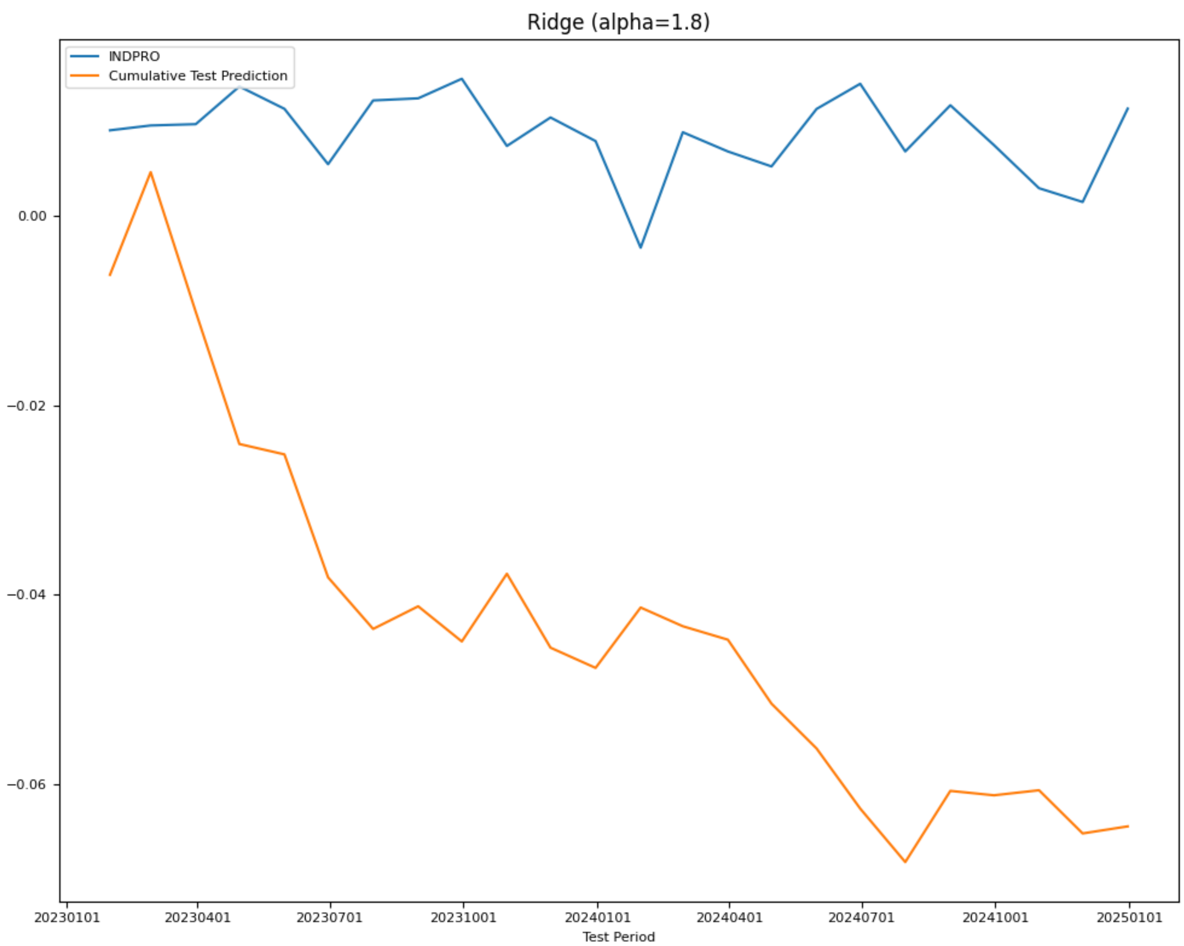
```
# Plot fitted coefficients vs regularization alpha
coefs = [Ridge(alpha, fit_intercept=False)\
         .fit(X_subset, Y_train).coef_ for alpha in alphas]
fig, ax = plt.subplots(figsize=(10, 8))
ax.plot(alphas, coefs)
ax.set_xscale('log')
ax.set_xlabel('value of alpha regularization parameter')
ax.set_title('Ridge Regression fitted coefficients')
plt.tight_layout()
```




```
# RidgeCV LOOCV
model = RidgeCV(alphas=alphas,
                scoring='neg_mean_squared_error',
                cv=None,  # to use Leave-One-Out cross validation
                store_cv_values=True).fit(X_train, Y_train)
```

```
name = f"Ridge (alpha={model.alpha_:.1f})"
Y_pred = Series(index=Y_test.index,
                data=model.predict(X_test).reshape((-1,)))
test[name] = mean_squared_error(Y_test, Y_pred)
train[name] = mean_squared_error(Y_train, model.predict(X_train))
final_models[name] = model
```

```
fig, ax = plt.subplots(figsize=(10, 8))
plot_date(pd.concat([Y_test.cumsum(), Y_pred.cumsum()], axis=1),
          legend1=[target_id, 'Cumulative Test Prediction'],
          title=name,
          xlabel='Test Period',
          fontsize=8,
          ax=ax)
plt.tight_layout()
```




```
DataFrame({'name': name,
           'train': np.sqrt(train[name]),
           'test': np.sqrt(test[name])}, index=['RMSE'])
```

```
                   name     train      test
RMSE  Ridge (alpha=1.8)  0.004305  0.009825
```

29.2.4 Lasso Regression

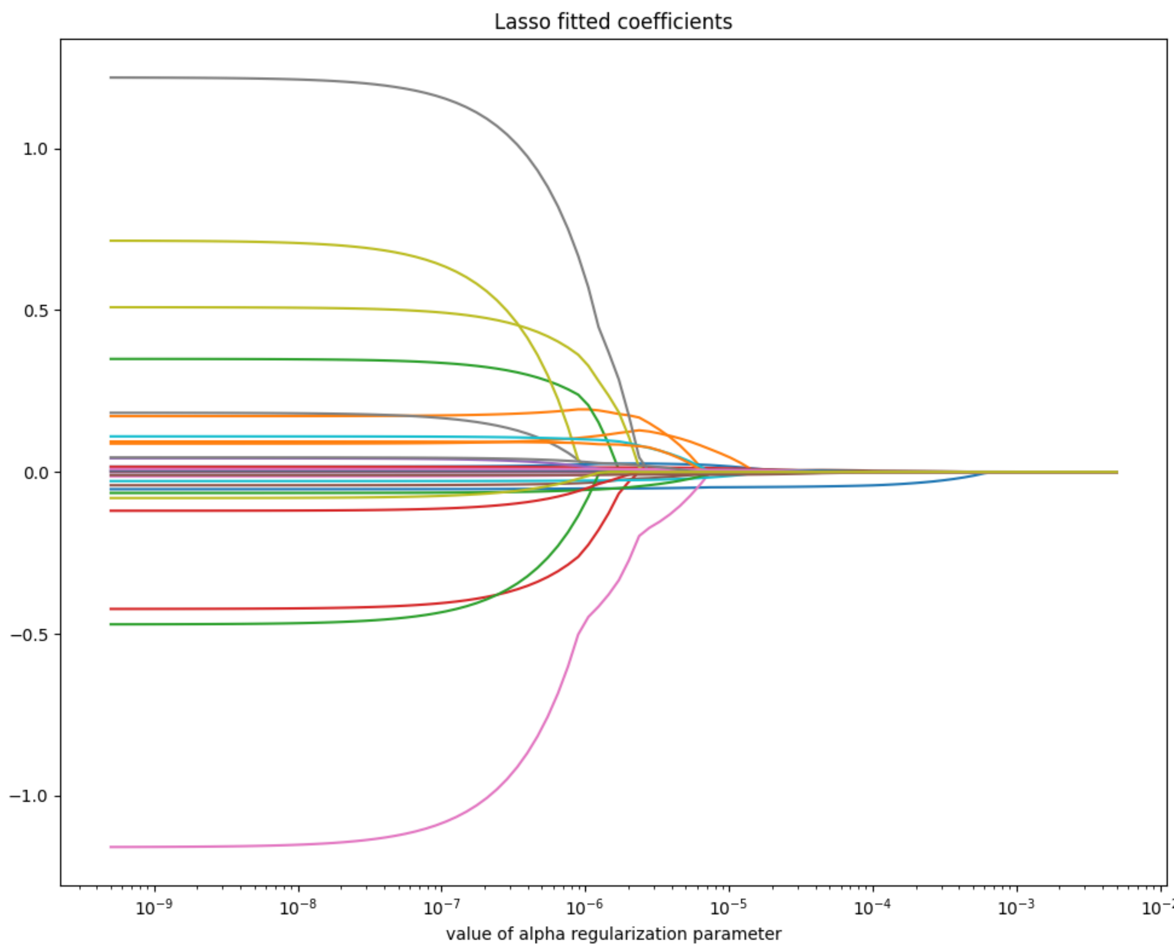
Lasso Regression (Least Absolute Shrinkage and Selection Operator) is a linear regression model that includes **L1 regularization**, which adds the sum of the absolute values of the coefficients as a penalty term to the loss function. This encourages sparsity, meaning it shrinks some coefficients to exactly zero, effectively performing feature selection. The objective function is:

$$\min_{w} ||y - Xw||_2^2 + \alpha ||w||_1$$

where α is a hyperparameter controlling the strength of regularization: a higher α increases shrinkage, leading to more coefficients being set to zero. Lasso regression is particularly useful when dealing with high-dimensional data where many features may be irrelevant, by reducing model complexity and improving interpretability. However, with correlated features, it tends to arbitrarily select one and ignore the others.

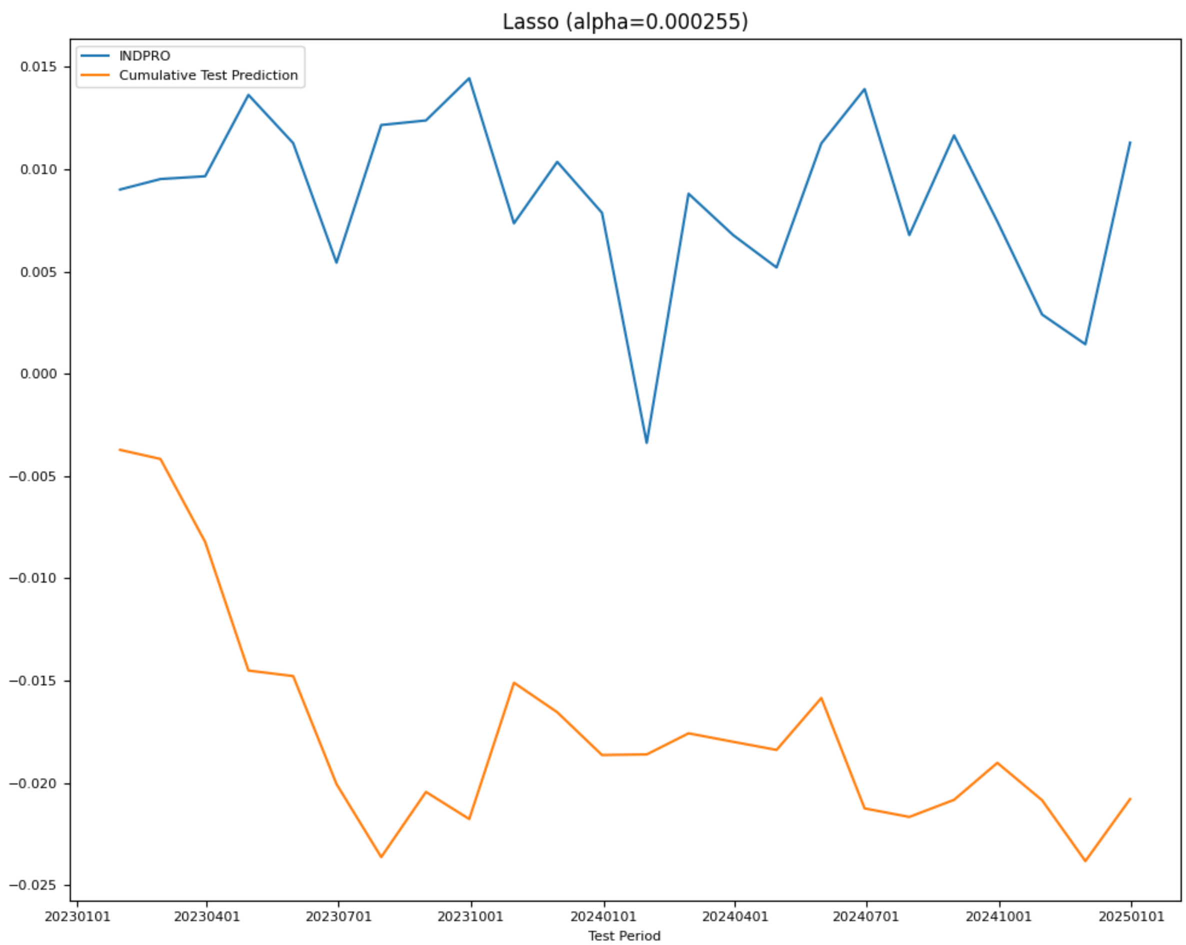
```
from sklearn.linear_model import Lasso, LassoCV
alphas = 10**np.linspace(-2, -9, 100)*0.5  # for parameter tuning
X_train, X_test, Y_train, Y_test = ts_split(X, Y)
scale = StandardScaler().fit(X_train)
X_train = scale.transform(X_train)
X_test = scale.transform(X_test)
```

```
# Plot fitted coefficients vs regularization
coefs = [Lasso(max_iter=10000, alpha=alpha)\
         .fit(X_subset, Y_train).coef_  for alpha in alphas]
fig, ax = plt.subplots(figsize=(10, 8))
ax.plot(alphas, coefs)
ax.set_xscale('log')
ax.set_xlabel('value of alpha regularization parameter')
ax.set_title('Lasso fitted coefficients')
plt.tight_layout()
```




```
# LassoCV 10-Fold CV
model = LassoCV(alphas=None,
                cv=5,
                n_jobs=5,
                verbose=VERBOSE,
                max_iter=20000).fit(X_train, Y_train)
name = f"Lasso (alpha={model.alpha_:.3g})"
Y_pred = Series(index=Y_test.index,
                data=model.predict(X_test).reshape((-1,)))
test[name] = mean_squared_error(Y_test, Y_pred)
train[name] = mean_squared_error(Y_train, model.predict(X_train))
final_models[name] = model
```

```
fig, ax = plt.subplots(figsize=(10, 8))
plot_date(pd.concat([Y_test.cumsum(), Y_pred.cumsum()], axis=1),
          legend1=[target_id, 'Cumulative Test Prediction'],
          title=name,
          xlabel='Test Period',
          fontsize=8,
          ax=ax)
plt.tight_layout()
```




```
DataFrame({'name': name,
           'train': np.sqrt(train[name]),
           'test': np.sqrt(test[name])}, index=['RMSE'])
```

```
                         name     train      test
RMSE  Lasso (alpha=0.000255)  0.006279  0.006808
```

```
# Display nonzero coefs
nonzero = np.sum(np.abs(model.coef_) > 0)
argsort = np.flip(np.argsort(np.abs(model.coef_)))[:nonzero]
df = DataFrame({'series_id': columns_unmap(X.columns[argsort])[0],
                'lags': columns_unmap(X.columns[argsort])[1],
                'desc': [alf.header(s)
                         for s in columns_unmap(X.columns[argsort])[0]],
                'coef': model.coef_[argsort]}).round(6).set_index('series_id')
print("Lasso: Nonzero Coefficients")
df
```

```
Lasso: Nonzero Coefficients
```

```
            lags                                               desc      coef
series_id
CLAIMS         1                                     Initial Claims -0.005356
SRVPRD         1                  All Employees, Service-Providing -0.001268
M2REAL         2                               Real M2 Money Stock  0.000893
IPNMAT         3  Industrial Production: Nondurable Goods Materials  0.000751
USGOVT         1                        All Employees, Government -0.000716
...          ...                                                ...       ...
M2SL           3                                                 M2  0.000020
COMPAPFF       3             3-Month Commercial Paper Minus FEDFUNDS -0.000020
UEMPLT5        3             Number Unemployed for Less Than 5 Weeks  0.000019
CUSR0000SAS    3  Consumer Price Index for All Urban Consumers: ...  0.000009
RPI            3                              Real Personal Income  0.000002

[82 rows x 3 columns]
```

29.2.5 Decision Tree

Decision trees partition the predictor space into regions, making predictions based on mean values (for regression) or mode (for classification).

- A **decision tree** consists of a sequence of splitting rules that divide observations into different regions. These trees are typically drawn upside down, with the leaves at the bottom.
- **Terminal nodes** or **leaves** represent the final partitions of the predictor space where observations are grouped.
- **Internal nodes** are points in the tree where the predictor space is split.
- **Branches** connect the nodes and represent different decision paths.
- A **stump** refers to a decision tree with only a single split (one internal node).

Decision trees are constructed using **recursive binary splitting**, a top-down, greedy algorithm. The process begins with all observations in a single region, and at each step, the predictor space is split into two new branches based on the best possible split at that moment (without considering future steps). The best split is determined by selecting the predictor and cutpoint that minimize a chosen cost function, such as the **Residual Sum of Squares (RSS)** in regression. The process continues recursively until a stopping criterion is met, such as a minimum number of observations per node.

Growing a tree until all leaves are pure (containing only one class) often leads to overfitting. Tree complexity is measured by the number of nodes, and controlling this complexity is crucial. If trees are allowed to grow too large, they may fit the training data well but perform poorly on unseen data. A balance must be found where accuracy on the test set is maximized before further tree growth begins to decrease performance.

Overfitting can be mitigated by pruning the tree using cost complexity pruning. A fully grown tree is simplified by removing less important splits to minimize the following function:

$$\sum_{m=1}^{|T|} \sum_{x_i \in R_m} (y_i - \hat{y}_{R_m})^2 + \alpha |T|$$

where $|T|$ is the number of terminal nodes, R_m represents a region in the predictor space, and $\hat{y}_{R_m}$ is the predicted response for that region. Pruning is controlled by a tuning parameter α, where higher values result in smaller trees that generalize better.

For classification tasks, different impurity measures determine the best binary splits:

- **Classification error rate**: Measures the proportion of incorrectly classified observations in a node. While intuitive, it is not sensitive enough for effective tree growth.

$$\hat{\rho}_{m,c} = \frac{n_{m,c}}{n_m}$$

where n_m is the number of observations in node m and $n_{m,c}$ is the number of observations in node m belonging to class c.

- **Gini index**: Measures total variance across classes. A lower Gini index indicates that most observations in a node belong to a single class.

$$G = \sum_{k=1}^{K} \hat{\rho}_{mk}(1 - \hat{\rho}_{mk})$$

where $\hat{\rho}_{mk}$ represents the proportion of training observations in node m that belong to class k.

- **Entropy**: Measures node impurity. Lower entropy values indicate purer nodes.

$$D = -\sum_{k=1}^{K} \hat{\rho}_{mk} \log \hat{\rho}_{mk}$$

Deviance is a statistical measure used to assess model performance in classification problems. It is derived from the likelihood function and is a measure of how well the predicted probabilities match the actual class labels. It is calculated as:

$$Deviance = -2 \sum_{m=1}^{g} \sum_{c=1}^{w} n_{m,c} \ln \hat{p}_{m,c}$$

where:

- g is the number of terminal nodes (leaf nodes) in the forest.
- w is the number of classes.
- $n_{m,c}$ is the number of observations in node m belonging to class c.
- $\hat{p}_{m,c}$ is the predicted probability of class c in node m.

with the **residual mean deviance** given by: $\frac{\text{deviance}}{n-1}$

A lower deviance indicates better model performance, meaning the predicted class probabilities align more closely with the true class labels.

Advantages:

- Easy to interpret and visualize, even for non-experts.
- Often reflect human decision-making processes.
- Can handle qualitative predictors without requiring dummy variables.

Disadvantages:

- Less accurate than some other regression and classification models.
- Highly sensitive to small changes in data, making them non-robust.
- Tend to overfit categorical variables.

Decision tree performance can be significantly improved using **ensemble methods** such as:

- **Bagging (Bootstrap Aggregating)**: Reduces variance by averaging predictions from multiple decision trees trained on different bootstrap samples.
- **Random Forests**: A variant of bagging that introduces additional randomness by selecting a random subset of features at each split.
- **Boosting**: Sequentially builds trees where each new tree corrects the errors of the previous ones, reducing bias and improving predictive performance.

By aggregating multiple decision trees, these ensemble methods enhance generalization and reduce overfitting, making decision trees more robust and effective for complex predictive modeling.

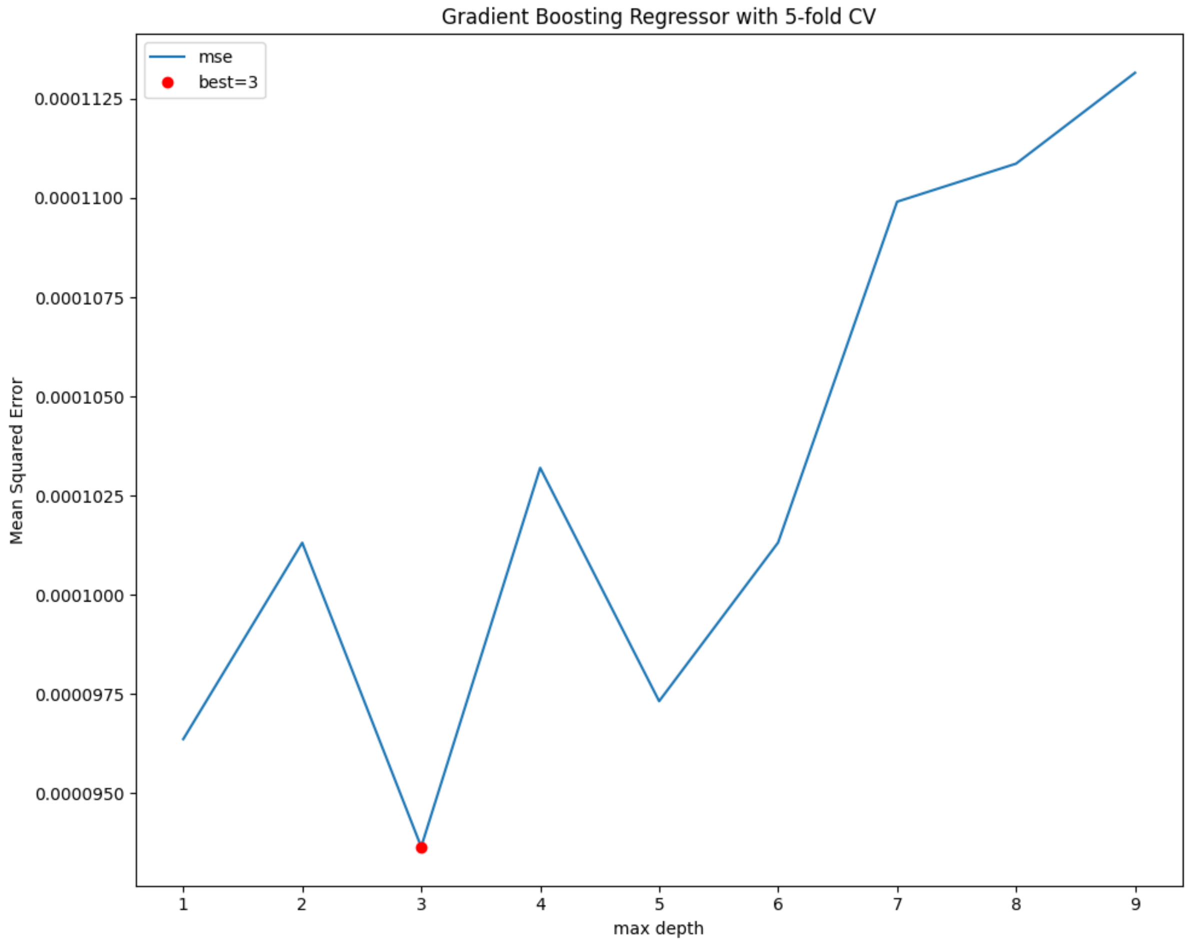
29.2.6 Gradient boosting

Gradient Boosting is an ensemble learning technique that builds models sequentially, where each new model corrects the errors of the previous ones. It uses decision trees as weak learners and minimizes the loss function by optimizing the model in the direction of the gradient of the loss. Key parameters include `n_estimators` (number of trees), `learning_rate` (step size for updates, controlling how much each tree contributes), and `max_depth` (tree depth, preventing overfitting). Gradient Boosting can be computationally expensive and prone to overfitting if not regularized using techniques like early stopping or shrinkage (learning_rate tuning).

```
from sklearn.ensemble import GradientBoostingRegressor
X_train, X_test, Y_train, Y_test = ts_split(X, Y)
scale = StandardScaler().fit(X_train)
X_train = scale.transform(X_train)
X_test = scale.transform(X_test)
```

```
# tune max_depth with 5-fold CV
n_splits=5
kf = KFold(n_splits=n_splits, shuffle=True, random_state=0)
mse = Series(dtype=float)
for i in range(1, 10): # tune max_depth for best performance
    boosted = GradientBoostingRegressor(max_depth=i, random_state=0)
    score = cross_val_score(boosted,
                            X_train,
                            Y_train,
                            cv=kf,
                            n_jobs=5,
                            verbose=VERBOSE,
                            scoring='neg_mean_squared_error').mean()
    mse.loc[i] = -score
```

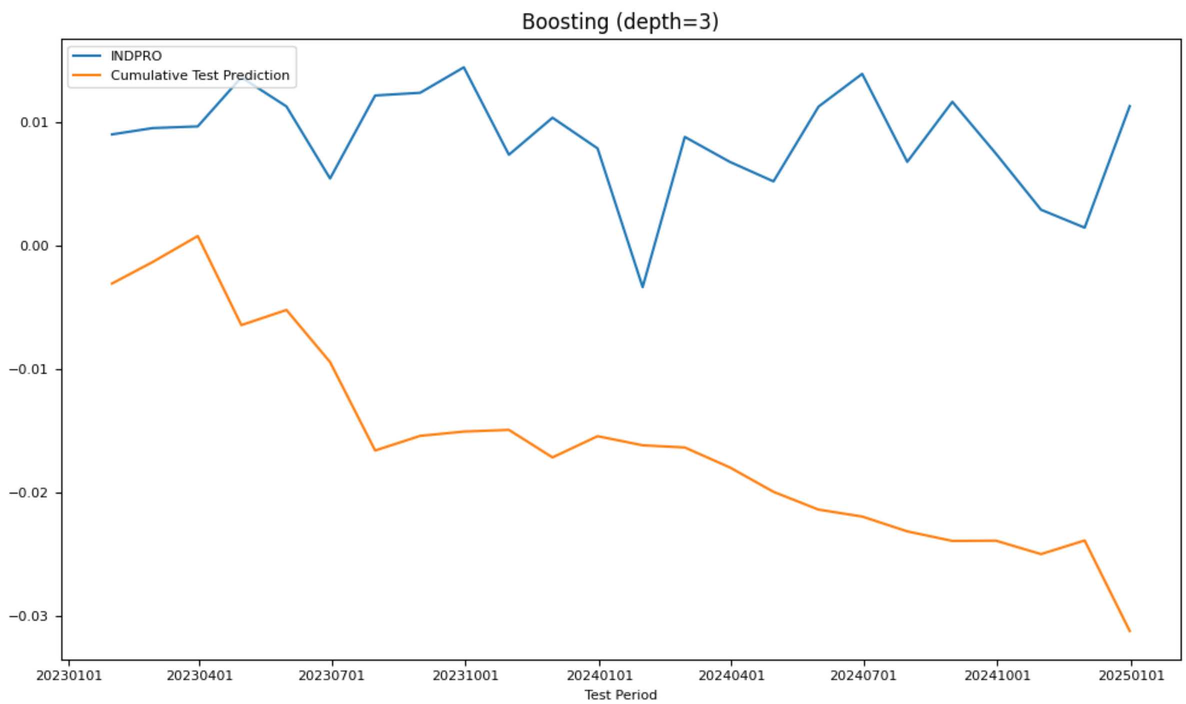
```
fig, ax = plt.subplots(figsize=(10, 8))
mse.plot(ax=ax, ylabel='Mean Squared Error', xlabel='max depth',
         title=f"Gradient Boosting Regressor with {n_splits}-fold CV")
best = mse.index[mse.argmin()]
ax.plot(best, mse.loc[best], "or")
ax.legend(['mse', f"best={best}"])
plt.tight_layout()
```




```
# evaluate train and test MSE
name = f"Boosting (depth={best})"
model = GradientBoostingRegressor(max_depth=best,
                                  random_state=0).fit(X_train, Y_train)
```

```
Y_pred = Series(index=Y_test.index,
                data=model.predict(X_test).reshape((-1,)))
test[name] = mean_squared_error(Y_test, Y_pred)
train[name] = mean_squared_error(Y_train, model.predict(X_train))
final_models[name] = model
```

```
fig, ax = plt.subplots(figsize=(10, 6))
plot_date(pd.concat([Y_test.cumsum(), Y_pred.cumsum()], axis=1),
          legend1=[target_id, 'Cumulative Test Prediction'],
          title=name,
          xlabel='Test Period',
          fontsize=8,
          ax=ax)
plt.tight_layout()
```




```
DataFrame({'name': name,
           'train': np.sqrt(train[name]),
           'test': np.sqrt(test[name])}, index=['RMSE'])
```

```
                    name     train      test
RMSE  Boosting (depth=3)  0.002704  0.007407
```

Feature importances:

```
# Show feature importance
top_n = 10
imp = Series(model.feature_importances_, index=X.columns).sort_values()
print(f"Gradient Boosting: Top {top_n} Feature Importances")
DataFrame.from_dict({i+1: {'importance': imp[s],
                           'series_id': s.split('_')[0],
                           'lags': s.split('_')[1],
                           'description': alf.header(s.split('_')[0])}
                     for i, s in enumerate(np.flip(imp.index[-top_n:]))},
                    orient='index')
```

```
Gradient Boosting: Top 10 Feature Importances
```

```
    importance series_id lags  \
1     0.140886  BUSLOANS    1
2     0.136621    CLAIMS    1
3     0.031817   UEMPLT5    3
4     0.030436    M2REAL    3
5     0.029806    USGOOD    1
6     0.027816      M2SL    1
7     0.022606    IPNMAT    1
```

(continues on next page)

(continued from previous page)

```
8     0.020286   DMANEMP    1
9     0.019784    EXCAUS    1
10    0.018134    UNRATE    1

                                           description
1   Commercial and Industrial Loans, All Commercia...
2                                       Initial Claims
3             Number Unemployed for Less Than 5 Weeks
4                                  Real M2 Money Stock
5                         All Employees, Goods-Producing
6                                                   M2
7   Industrial Production: Nondurable Goods Materials
8                           All Employees, Durable Goods
9   Canadian Dollars to U.S. Dollar Spot Exchange ...
10                                   Unemployment Rate
```

29.2.7 Random Forest

Random Forest is an ensemble learning method that builds multiple decision trees and combines their predictions to improve accuracy and reduce overfitting. It operates by randomly selecting subsets of the training data and features (using bootstrap sampling and feature bagging) to create diverse trees. The final prediction is determined by majority voting (for classification) or averaging (for regression). Key parameters include `n_estimators` (number of trees), `max_depth`** (maximum depth of each tree), `max_features` (number of features considered for splitting), and `min_samples_split` (minimum samples needed to split a node). Feature importance scores can be extracted to interpret which variables influence predictions most. Random Forest handles missing data well and mitigates overfitting through averaging multiple models.

```
from sklearn.ensemble import RandomForestRegressor
X_train, X_test, Y_train, Y_test = ts_split(X, Y)
```

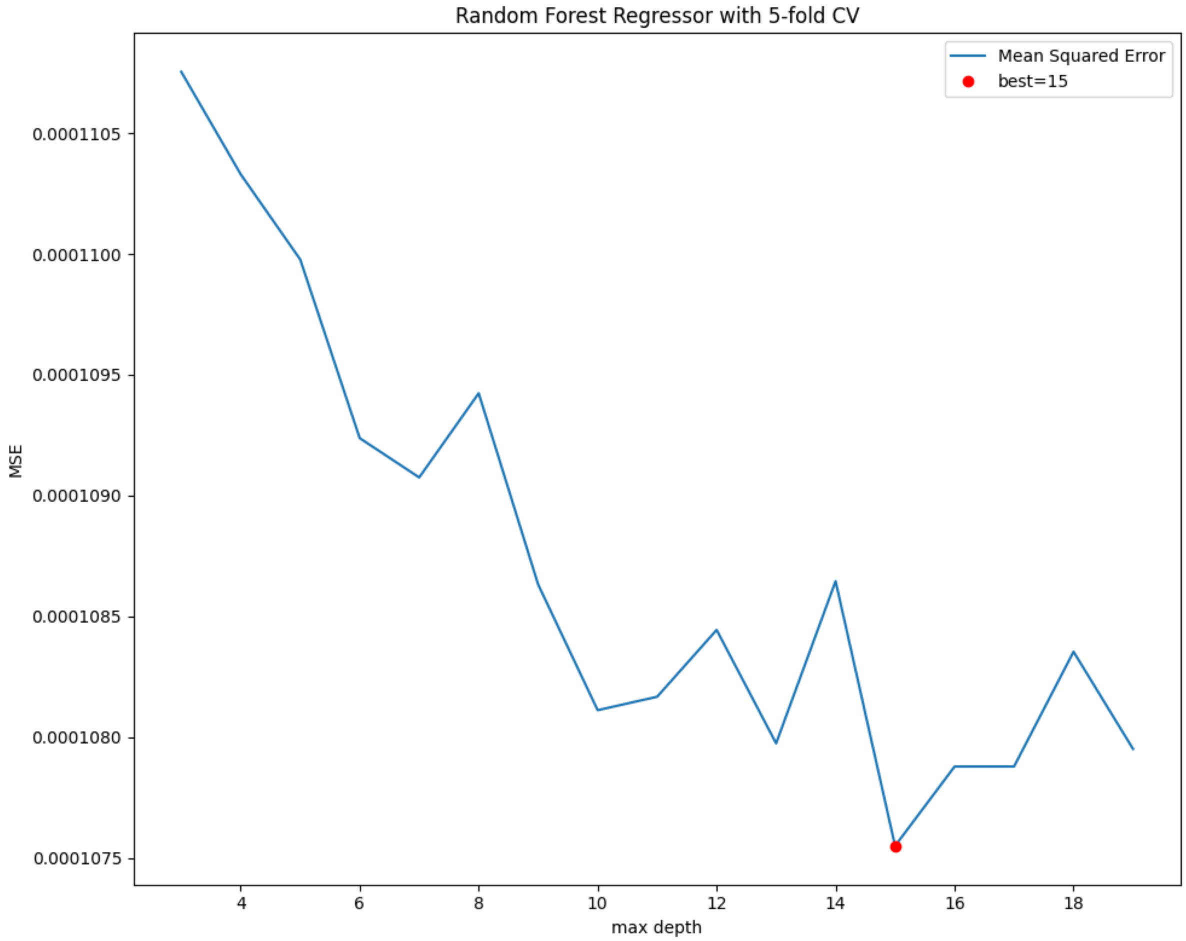
```
# tune max_depth with 5-fold CV
n_splits=5
kf = KFold(n_splits=n_splits,
           shuffle=True,
           random_state=0)
mse = Series(dtype=float)
for i in range(3, 20): #tune for best performance
    model = RandomForestRegressor(max_depth=i, random_state=0)
    score = cross_val_score(model,
                            X_train,
                            Y_train,
                            cv=kf,
                            n_jobs=5,
                            verbose=VERBOSE,
                            scoring='neg_mean_squared_error').mean()
    mse.loc[i] = -score
    #print(i, np.sqrt(abs(score)))
```

```
fig, ax = plt.subplots(figsize=(10, 8))
mse.plot(ax=ax, ylabel='MSE', xlabel='max depth',
         title=f"Random Forest Regressor with {n_splits}-fold CV")
best = mse.index[mse.argmin()]
```

(continues on next page)

(continued from previous page)

```
ax.plot(best, mse.loc[best], "or")
ax.legend(['Mean Squared Error', f"best={best}"])
plt.tight_layout()
```




```
name = f"RandomForest (depth={best})"
model = RandomForestRegressor(max_depth=best,
                              random_state=0).fit(X_train, Y_train)
```

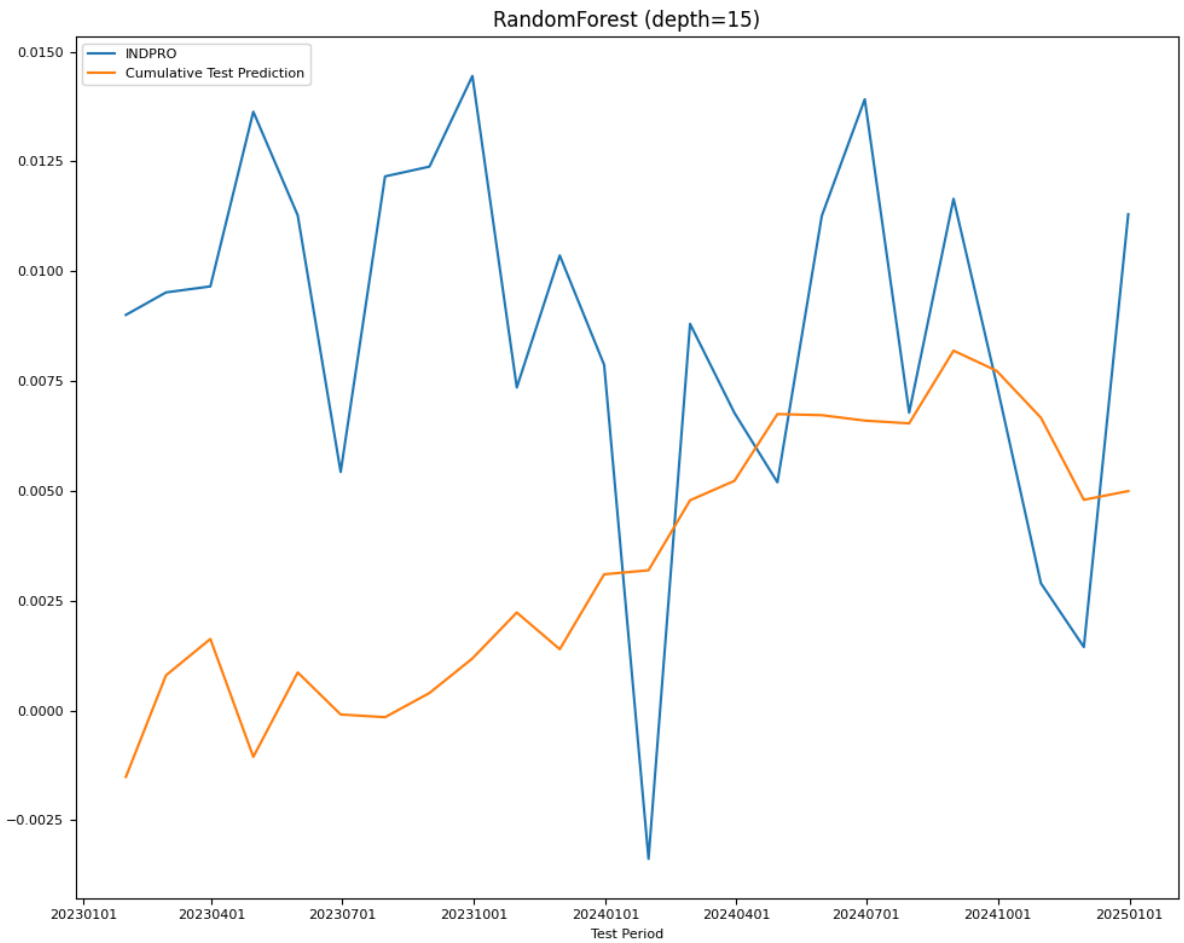
```
Y_pred = Series(index=Y_test.index,
                data=model.predict(X_test).reshape((-1,)))
test[name] = mean_squared_error(Y_test, Y_pred)
train[name] = mean_squared_error(Y_train, model.predict(X_train))
final_models[name] = model
```

```
fig, ax = plt.subplots(figsize=(10, 8))
plot_date(pd.concat([Y_test.cumsum(), Y_pred.cumsum()], axis=1),
          legend1=[target_id, 'Cumulative Test Prediction'],
          title=name,
          xlabel='Test Period',
          fontsize=8,
          ax=ax)
```

(continues on next page)

(continued from previous page)

```
plt.tight_layout()
```




```
DataFrame({'name': name,
           'train': np.sqrt(train[name]),
           'test': np.sqrt(test[name])}, index=['RMSE'])
```

```
                         name    train      test
RMSE  RandomForest (depth=15)  0.00376  0.005875
```

Feature importance scores:

```
# Feature importance
top_n = 10
imp = Series(model.feature_importances_, index=X.columns).sort_values()
print(f"Random Forest: Top {top_n} Feature Importances")
DataFrame.from_dict({i+1: {'importance': imp[s],
                           'series_id': s.split('_')[0],
                           'lags': s.split('_')[1],
                           'description': alf.header(s.split('_')[0])}
                     for i, s in enumerate(np.flip(imp.index[-top_n:]))},
                    orient='index')
```

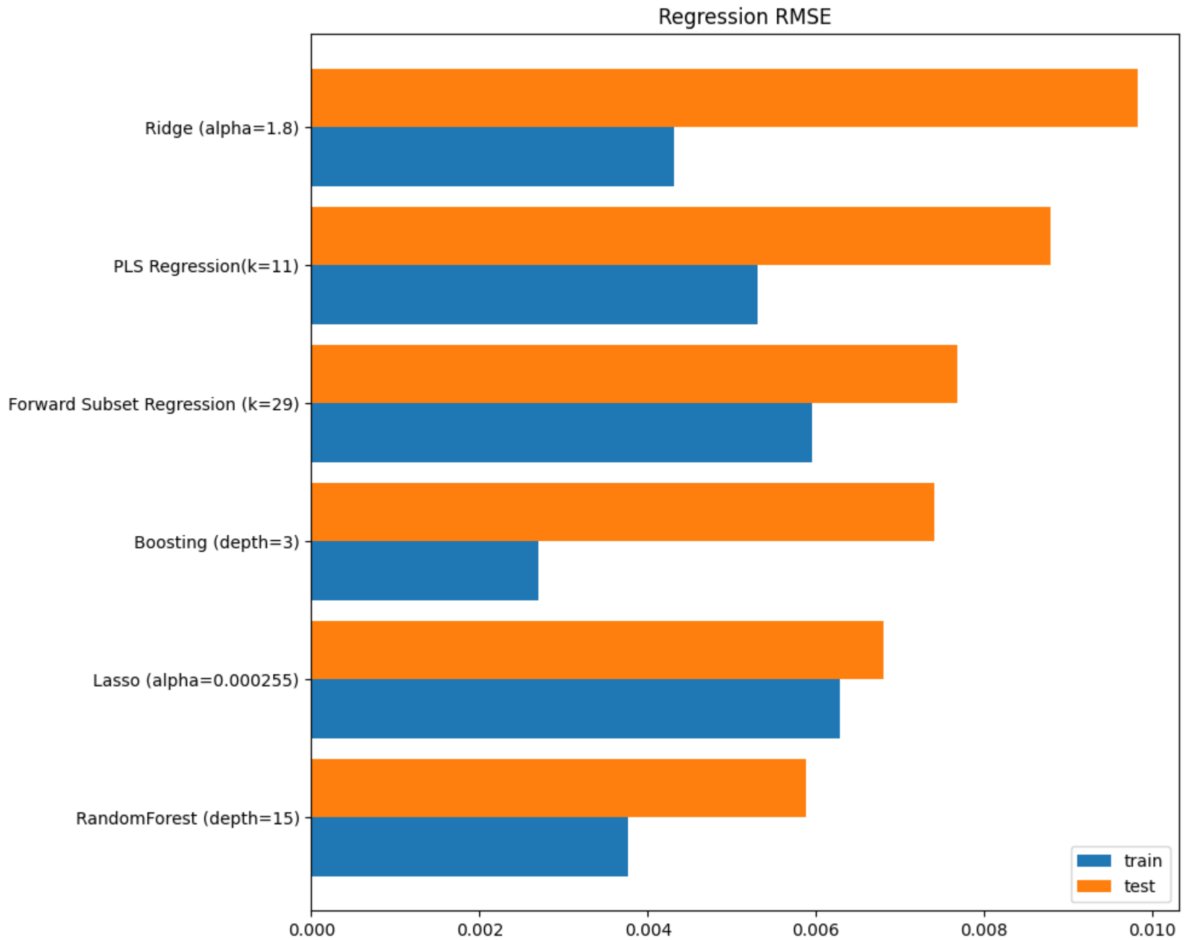
```
Random Forest: Top 10 Feature Importances
```

```
    importance series_id lags  \
1     0.051226    CLAIMS    1
2     0.046733  BUSLOANS    1
3     0.044055      M2SL    1
4     0.015825    USGOOD    1
5     0.015268    UNRATE    1
6     0.014617   UEMPLT5    3
7     0.014576   IPCONGD    1
8     0.014529   UEMPLT5    1
9     0.014496      M1SL    1
10    0.013855   DMANEMP    1

                                          description
1                                      Initial Claims
2   Commercial and Industrial Loans, All Commercia...
3                                                  M2
4                      All Employees, Goods-Producing
5                                   Unemployment Rate
6             Number Unemployed for Less Than 5 Weeks
7               Industrial Production: Consumer Goods
8             Number Unemployed for Less Than 5 Weeks
9                                                  M1
10                       All Employees, Durable Goods
```

Summary:

The **root mean squared error (RMSE)** performance of all the models, based on train and test samples, is summarized in the following chart:

```
fig, ax = plt.subplots(figsize=(10, 8))
pd.concat([np.sqrt(r.to_frame()) for r in [train, test]], axis=1)\
  .sort_values('test')\
  .plot.barh(ax=ax, width=0.85)
ax.yaxis.set_tick_params(labelsize=10)
ax.set_title('Regression RMSE')
ax.figure.subplots_adjust(left=0.35)
plt.tight_layout()
```



References:

Gareth James, Daniela Witten, Trevor Hastie, Robert Tibshirani. "An Introduction to Statistical Learning with Applications in R". New York, Springer, 2013.

CHAPTER

THIRTY

DEEP LEARNING

May your choices reflect your hopes, not your fears –Nelson Mandela

We explore the application of deep learning techniques for text classification, specifically focusing on categorizing US companies based on their industry sectors. Using business description texts extracted from SEC 10-K filings, we apply natural language processing (NLP) methods and deep averaging networks (DAN) to classify firms according to the Fama-French 10-sector scheme. The analysis includes preprocessing textual data, leveraging pre-trained word embeddings for semantic representation, and evaluating various training strategies to optimize predictive accuracy and generalization performance.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import random
import time
import pandas as pd
from pandas import DataFrame, Series
from collections import Counter
import matplotlib.pyplot as plt
from sklearn.metrics import confusion_matrix
from sklearn.preprocessing import LabelEncoder
from sklearn.model_selection import train_test_split
import seaborn as sns
from tqdm import tqdm
import torch
from torch import nn
import torchinfo
from textblob import TextBlob
from tqdm import tqdm
from finds.database import SQL, RedisDB
from finds.unstructured import Edgar, Vocab
from finds.structured import BusDay, CRSP, PSTAT
from finds.readers import Sectoring
from finds.utils import Store
from secret import credentials, paths, CRSP_DATE
VERBOSE = 0
outdir = paths['scratch']
store = Store(outdir, ext='pkl')
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
print(f"{device=}")
print(f"{torch.cuda.is_available()=}")      # Should return True
print(f"{torch.cuda.device_count()=}")      # Number of available GPUs
print(f"{torch.cuda.current_device()=}")    # Current GPU index
print(f"{torch.cuda.get_device_name(0)=}")  # Name of the GPU
```

```
device=device(type='cuda')
torch.cuda.is_available()=True
torch.cuda.device_count()=1
torch.cuda.current_device()=0
torch.cuda.get_device_name(0)='NVIDIA GeForce RTX 3080 Laptop GPU'
```

```
sql = SQL(**credentials['sql'], verbose=VERBOSE)
user = SQL(**credentials['user'], verbose=VERBOSE)
bd = BusDay(sql)
rdb = RedisDB(**credentials['redis'])
crsp = CRSP(sql, bd, rdb, verbose=VERBOSE)
pstat = PSTAT(sql, bd, verbose=VERBOSE)
ed = Edgar(paths['10X'], zipped=True, verbose=VERBOSE)
```

30.1 Industry text classification

We begin by extracting a universe of US-domiciled common stocks at the start of the most recent year, along with their corresponding 10-K business descriptions from SEC filings. The target categories for our text classification task are drawn from the Fama-French 10-sector classification scheme.

```
# Retrieve universe of stocks as of start of latest year
univ = crsp.get_universe(bd.endmo(CRSP_DATE-10000))
CRSP_DATE
```

```
20241231
```

```
# lookup company names
comnam = crsp.build_lookup(source='permno', target='comnam', fillna="")
univ['comnam'] = comnam(univ.index)
```

```
# lookup ticker symbols
ticker = crsp.build_lookup(source='permno', target='ticker', fillna="")
univ['ticker'] = ticker(univ.index)
```

```
# lookup sic codes from Compustat, and map to FF 10-sector code
sic = pstat.build_lookup(source='lpermno', target='sic', fillna=0)
industry = Series(sic[univ.index], index=univ.index)
industry = industry.where(industry > 0, univ['siccd'])
sectors = Sectoring(sql, scheme='codes10', fillna='')   # supplement from crosswalk
univ['sector'] = sectors[industry]
```

```
# retrieve 2023 10K business descriptions text
item, form = 'bus10K', '10-K'
rows = DataFrame(ed.open(form=form, item=item))
found = rows[rows['date'].between(bd.begyr(CRSP_DATE), bd.endyr(CRSP_DATE))]\
             .drop_duplicates(subset=['permno'], keep='last')\
             .set_index('permno')
```

30.1.1 Textblob

The `TextBlob` library simplifies common NLP tasks such as part-of-speech tagging, lemmatization, noun phrase extraction, sentiment analysis, and spelling correction. It provides friendly access to functionalities derived from NLTK and integrates with WordNet.

- https://textblob.readthedocs.io/en/dev/quickstart.html

For our task, TextBlob is employed to tokenize business descriptions and extract nouns. We filter the documents to retain only those containing at least 100 valid nouns to ensure robust semantic representation.

```
bus = {}
for permno in tqdm(found.index):
    if permno not in univ.index:
        continue
    doc = TextBlob(ed[found.loc[permno, 'pathname']].lower())  # tokenize and tag
    nouns = [word for word, tag in doc.tags
             if tag in ['NN', 'NNS'] and word.isalpha() and len(word) > 2]
    if len(nouns) > 100:
        bus[permno] = nouns
permnos = list(bus.keys())
```

```
100%|██████████| 4488/4488 [17:23<00:00,  4.30it/s]
```

30.1.2 Word Embeddings

Word embeddings are dense, numerical vector representations capturing semantic and syntactic meanings of words. These embeddings place words into a continuous vector space, positioning semantically similar or related words closely together. Word embeddings can be generated through neural network-based approaches or matrix factorization methods.

1. **Word2Vec** (Mikolov et al., 2013): Word2Vec utilizes shallow neural networks, usually comprising a single hidden layer, to learn embeddings from textual data. It has two primary training approaches:
    - **Skip-gram**: Predicts context words given a center word, effectively capturing representations of rare words.
    - **Continuous Bag of Words (CBOW)**: Predicts a center word from surrounding context words, typically faster and better for frequent words.
2. **GloVe (Global Vectors for Word Representation)** (Pennington et al., 2014): GloVe generates embeddings based on matrix factorization of global word-word co-occurrence statistics. Unlike Word2Vec, which relies on local context predictions, GloVe considers overall word pair co-occurrences, resulting in globally consistent embeddings.

Pre-trained GloVe vectors (300-dimensional) are utilized to represent the extracted words as embeddings.

```
# Load GloVe embeddings, source: "https://nlp.stanford.edu/data/glove.6B.zip"
embeddings_dim = 300  # dimension of GloVe embeddings vector

filename = paths['scratch'] / f"glove.6B.{embeddings_dim}d.txt.zip"
embeddings = pd.read_csv(filename, sep=" ", quoting=3,
                         header=None, index_col=0, low_memory=True)
embeddings.index = embeddings.index.astype(str).str.lower()
print(embeddings.shape)
```

```
(400000, 300)
```

30.1.3 Word vector arithmetic

Word embeddings reflect linguistic relationships through geometric relationships in vector space. Embeddings can be arithmetically combined and manipulated to uncover analogies and semantic similarities. However, these mathematical relationships are generally approximate and can highlight potential biases inherent in training data, such as implicit gender biases.

```
from sklearn.neighbors import NearestNeighbors
analogies = ["man king woman", "paris france tokyo", "big bigger cold"]
for analogy in analogies:
    words = analogy.lower().split()
    vectors = {word: embeddings.loc[word].values for word in words}
    vec = vectors[words[1]] - vectors[words[0]] + vectors[words[2]]

    sim = NearestNeighbors(n_neighbors=1).fit(embeddings)
    neighbors = sim.kneighbors(vec.reshape((1, -1)), n_neighbors=2,
                               return_distance=False).flatten().tolist()
    neighbors = [k for k in neighbors if embeddings.index[k] not in words]
    print(f"{words[1]} - {words[0]} + {words[2]} =",
          [embeddings.index[k] for k in neighbors])
```

```
king - man + woman = ['queen']
france - paris + tokyo = ['japan']
bigger - big + cold = ['colder']
```

30.1.4 Data preparation

We construct a custom vocabulary (`Vocab`) mapping each word to an index, encoding each document as a list of these indices. The pre-trained GloVe embedding matrix is adapted to include only words present in our corpus-specific vocabulary. Sector labels are converted into numerical values using LabelEncoder. The dataset is then stratified and split into training and testing subsets to maintain balanced class distributions.

```
words = Counter()
for nouns in bus.values():
    words.update(list(nouns))
vocab = Vocab(words.keys())
print('vocab len:', len(vocab))
```

```
vocab len: 85891
```

```
labels = []
x_all = []
for permno, nouns in bus.items():
    x = vocab.get_index([noun for noun in nouns])
    if sum(x):
        labels.append(univ.loc[permno, 'sector'])
        x_all.append(x)
class_encoder = LabelEncoder().fit(labels)     # .inverse_transform()
y_all = class_encoder.transform(labels)
```

```
store['dan'] = dict(y_all=y_all, x_all=x_all)
```

```
# retrieve from previously stored
y_all, x_all = store['dan'].values()
```

```
# relativize embeddings to words in vocab
vocab.set_embeddings(embeddings)
print(vocab.embeddings.shape)
```

```
(85891, 300)
```

```
vocab.dump(outdir / f"dan{embeddings_dim}.pkl")
```

```
# load vocab
vocab.load(outdir / f"dan{embeddings_dim}.pkl")
```

30.2 Feedforward neural networks

Neural networks are computational models inspired by the human brain. They are built from layers of simple computational units that transform input data to output predictions. Deep neural networks alternate between linear layers and non-linear activations, and can approximate any continuous function (Universal Approximation Theorem).

- **Neurons** are the basic computational units or nodes of a neural network. Each neuron receives input, processes it using a weighted sum and a bias term, and then applies an activation function to produce an output, which is then passed to the neurons in the next layer.
- **Activation functions** are the nonlinear mathematical functions applied to neurons in a neural network. They introduce non-linearity into the model, enabling it to learn and represent complex patterns in the data. Common activation functions include ReLU (Rectified Linear Unit), sigmoid, and tanh.
- **Input Layer** is the first layer of a neural network which directly receives the input data. Each neuron in the input layer represents one feature of the input.
- **Hidden Layers**, between the input layer and the output layer, take input from the previous layer of neurons, apply weights, biases, and activation functions, and pass the output to the next layer.
- **Output Layer** is the final layer of the neural network and it produces the network's output. Its neurons represent the predictions or classifications made by the network. The number of neurons in the output layer corresponds to the number of output classes or the dimensionality of the output. For classification tasks, softmax or sigmoid functions are often used in the output layer to provide probability distributions of the class predictions.

Feedforward neural networks (FFNNs) are the simplest form of neural networks, where the data flows in one direction (a forward pass) and the connections do not form a cycle. A **Multilayer Perceptron (MLP)** is a type of FFNN which must has at least one hidden layer: MLPs are composed of an input layer, one or more hidden layers, and an output layer, with non-linear activation functions applied between layers.

Optimization is the process of adjusting model parameters to align its predictions with true targets.

- **Loss function** measures how well a neural network's output matches the true label or target. During training, the goal is to minimize this loss. Common loss functions L1 (Mean Absolute Error) and L2 (Mean Squared Error) for regression tasks, and Cross-Entropy for classification tasks.
- **Stochastic Gradient Descent (SGD)** is an optimization method used to train neural networks by updating parameters using gradients from a single (or small batch of) data point(s) at each step. It allows efficient updates even on massive datasets, with the ability to escape local minima due to its noise.

- **Backpropagation** is used for training neural networks by updating the weights of neurons based on the error (loss) of the network's predictions: it involves calculating the gradient of the loss function with respect to each weight by using the chain rule of calculus, and propagating these gradients backward from the output layer to the input layer.
- **Computation Graph** is a graphical representation of the sequence of operations used to compute the forward pass and the backward pass for backpropagation. PyTorch's modules automatically constructs the computation graph and computes gradients, hence simplifying the implementation of neural networks.
- **Initialization** refers to the process of setting the initial values of the weights in a neural network before training begins. Poor initialization can lead to slow convergence or getting stuck in local minima. Common initialization methods include Xavier (Glorot) and He initialization

Training deep neural networks involves carefully tuning several key components to ensure effective learning and generalization.

- **Learning Rate**: If too low, training is slow; too high and loss spikes. A learning rate schedules (e.g. cosine annealing) is more efficient than a fixed learning rate.
- **Adam** (Adaptive Moment Estimation) is an optimization algorithm for training neural networks which improves on stochastic gradient descent and achieves good performance on problems with large, high-dimensional data sets. It adapts the learning rate for each parameter by computing adaptive learning rates from estimates of first and second moments of the gradients. **AdamW** improves the performance of Adam in deep networks by applying weight decay directly to model parameters separately from gradient-based updates.
- **Hyperparameters** are parameters not learned by the neural network during training. They are set before training and control how the network learns. Examples include: Learning rate (size of each update step in gradient descent); Number of epochs (how many times the model sees the entire dataset); and Model size (number of layers, units per layer).
- **Batching** divides the training data into smaller subsets called batches, rather than training the model on the entire dataset at once, which can be computationally intensive and inefficient. It also gives speedup compared to training the network one sample at a time due to more efficient matrix operations.
- **Dropout** is a regularization technique during training, where a random subset of neurons is "dropped out" or set to zero at each iteration. This reduces overfitting by ensuring that the model does not rely too heavily on any particular subset of neurons. Geoffrey Hinton, et al. in their 2012 paper that first introduced dropout. They found that using a simple method of 50% dropout for all hidden units and 20% dropout for input units achieve improved results with a range of neural networks on different problem types. It is not used on the output layer.

30.2.1 Deep Averaging Networks

Deep Averaging Network (DAN) is a straightforward feedforward neural network architecture used for text classification. It averages embeddings of document words and feeds this representation through multiple hidden layers to predict class labels. Key properties include:

- **Embedding Layer**: Uses pre-trained GloVe vectors (frozen or fine-tuned).
- **Fully Connected Layers**: Transform embeddings into classification scores.
- **Nonlinear Activations**: Employ ReLU for non-linearity.
- **Output Layer**: Applies `LogSoftmax` for multi-class predictions.
- **Dropout Layers**: Prevent overfitting.
- **Xavier Initialization**: Stabilizes training.

We investigate training strategies, such as frozen embeddings (fast, prevents overfitting on small data), fine-tuned embeddings (task-specific optimization but resource-intensive), and dropout regularization (enhances generalization).

```python
class DAN(nn.Module):
    """Deep Averaging Network for classification"""
    def __init__(self,
                 vocab_dim,
                 num_classes,
                 hidden,
                 embedding,
                 freeze=True):
        super().__init__()
        self.embedding = nn.EmbeddingBag.from_pretrained(embedding)
        self.embedding.weight.requires_grad = not freeze
        D = nn.Dropout(0.0)
        V = nn.Linear(vocab_dim, hidden[0])
        nn.init.xavier_uniform_(V.weight)
        L = [D, V]
        self.drops = [D]
        for in_dim, out_dim in zip(hidden, hidden[1:] + [num_classes]):
            L.append(nn.ReLU())    # nonlinearity layer
            D = nn.Dropout(0.0)
            self.drops.append(D)
            L.append(D)            # dropout layer
            W = nn.Linear(in_dim, out_dim)    # dense linear layer
            nn.init.xavier_uniform_(W.weight)
            L.append(W)
        self.network = nn.Sequential(*L)
        self.classifier = nn.LogSoftmax(dim=-1)  # output is (N, C) logits

    def set_dropout(self, dropout):
        if dropout:
            self.drops[0].p = 0.2    # input layer
            for i in range(1, len(self.drops)):    # hidden layers
                self.drops[i].p = 0.5
        else:
            for i in range(len(self.drops)):
                self.drops[i].p = 0.0

    def set_freeze(self, freeze):
        """To freeze part of the model (embedding layer)"""
        self.embedding.weight.requires_grad = not freeze

    def forward(self, x):
        """Return tensor of log probabilities"""
        return self.classifier(self.network(self.embedding(x)))

    def predict(self, x):
        """Return predicted int class of input tensor vector"""
        return torch.argmax(self(x), dim=1).int().tolist()

    def save(self, filename):
        """save model state to filename"""
        return torch.save(self.state_dict(), filename)

    def load(self, filename):
        """load model name from filename"""
        self.load_state_dict(torch.load(filename, map_location='cpu'))
        return self
```

Split the data into stratified (i.e. equal class proportions) train and test set

```python
# Stratified train_test split
num_classes = len(np.unique(labels))
train_index, test_index = train_test_split(
    np.arange(len(y_all)), stratify=y_all, random_state=42, test_size=0.2)
print(len(x_all), len(y_all), len(train_index), len(test_index), num_classes)
#Series(labels).value_counts().rename('count').to_frame()
pd.concat([Series(np.array(labels)[train_index]).value_counts().rename('Train'),
           Series(np.array(labels)[test_index]).value_counts().rename('Test')],
           axis=1)
```

```
3474 3474 2779 695 10
```

```
       Train  Test
Hlth     657   164
Other    612   153
HiTec    554   139
Manuf    275    69
Shops    246    62
Durbl    131    33
NoDur    114    28
Enrgy     81    20
Utils     72    18
Telcm     37     9
```

```python
# Specify model and training parameters
layers = 2
hidden_size = 32
model = DAN(embeddings_dim,
            num_classes,
            hidden=[hidden_size] * layers,
            embedding=torch.FloatTensor(vocab.embeddings)).to(device)
torchinfo.summary(model)
```

```
=================================================================
Layer (type:depth-idx)                   Param #
=================================================================
DAN                                      --
├─EmbeddingBag: 1-1                      (25,767,300)
├─Sequential: 1-2                        --
│    └─Dropout: 2-1                      --
│    └─Linear: 2-2                       9,632
│    └─ReLU: 2-3                         --
│    └─Dropout: 2-4                      --
│    └─Linear: 2-5                       1,056
│    └─ReLU: 2-6                         --
│    └─Dropout: 2-7                      --
│    └─Linear: 2-8                       330
├─LogSoftmax: 1-3                        --
=================================================================
Total params: 25,778,318
Trainable params: 11,018
Non-trainable params: 25,767,300
=================================================================
```

30.2.2 Training

Training employs

- **Adam optimizer** for adaptive learning rates.
- **Negative Log Likelihood (NLLLoss)** for multi-class classification.
- Batch training with shuffled data to improve generalization.
- Padding of variable-length word index lists to form uniform-length input tensors.
- Evaluation of both training and test performance per epoch.

```
batch_sz = 16
lr = 0.001
num_epochs = 50
optimizer = torch.optim.Adam(model.parameters(), lr=lr)
loss_function = nn.NLLLoss()
```

Helper function to batch and form an input for neural network. Pads each sample to have lengths equal to the max, and convert to Long tensor type.

```
def form_input(docs):
    """Pad lists of index lists to form batch of equal lengths"""
    lengths = [len(doc) for doc in docs]   # length of each doc
    max_length = max(1, max(lengths))      # to pad so all lengths equal max
    out = [doc + ([0] * (max_length-n)) for doc, n in zip(docs, lengths)]
    return torch.LongTensor(out)
```

```
accuracy = []
for imodel, (freeze, dropout) in enumerate([(True, False), (True, True), (False,␣
↪True)]):
    model.set_freeze(freeze=freeze)
    model.set_dropout(dropout=dropout)
    accuracy.append(dict())

    # Loop over epochs
    for epoch in tqdm(range(num_epochs)):
        tic = time.time()

        # Form batches
        random.shuffle(train_index)
        batches = [train_index[i:(i+batch_sz)]
                   for i in range(0, len(train_index), batch_sz)]

        # Train in batches
        total_loss = 0.0
        model.train()
        for batch in batches:  # train by batch
            x = form_input([x_all[idx] for idx in batch]).to(device)
            y = torch.LongTensor([y_all[idx] for idx in batch]).to(device)
            model.zero_grad()                     # reset model gradient
            log_probs = model(x)                  # run model
            loss = loss_function(log_probs, y)    # compute loss
            total_loss += float(loss)
            loss.backward()                       # loss step
            optimizer.step()                      # optimizer step
```

(continues on next page)

(continued from previous page)

```
        model.eval()
        model.save(outdir / f"dan{embeddings_dim}.pt")

        if VERBOSE:
            print(f"Loss {epoch}/{num_epochs} {(freeze, dropout)}:" +
                  f"{total_loss:.1f}")

        with torch.no_grad():   # evaluate test error
            test_pred = [model.predict(form_input([x_all[i]]).to(device))[0]
                         for i in test_index]
            test_gold = [y_all[idx] for idx in test_index]
            test_correct = (np.array(test_pred) == np.array(test_gold)).sum()
            train_pred = [model.predict(form_input([x_all[i]]).to(device))[0]
                          for i in train_index]
            train_gold = [y_all[idx] for idx in train_index]
            train_correct = (np.array(train_pred) == np.array(train_gold)).sum()
            accuracy[imodel][epoch] = {
                'loss': total_loss,
                'train': train_correct/len(train_gold),
                'test': test_correct/len(test_gold)}

            if VERBOSE:
                print(freeze,
                      dropout,
                      epoch,
                      int(time.time() - tic),
                      optimizer.param_groups[0]['lr'],
                      train_correct/len(train_gold),
                      test_correct/len(test_gold))
```

```
  0%|          | 0/50 [00:00<?, ?it/s]
```

```
100%|██████████| 50/50 [03:51<00:00,  4.62s/it]
100%|██████████| 50/50 [03:51<00:00,  4.64s/it]
100%|██████████| 50/50 [04:02<00:00,  4.85s/it]
```

30.2.3 Evaluation

Evaluation includes computing confusion matrices of prediction errors for both training and testing data.

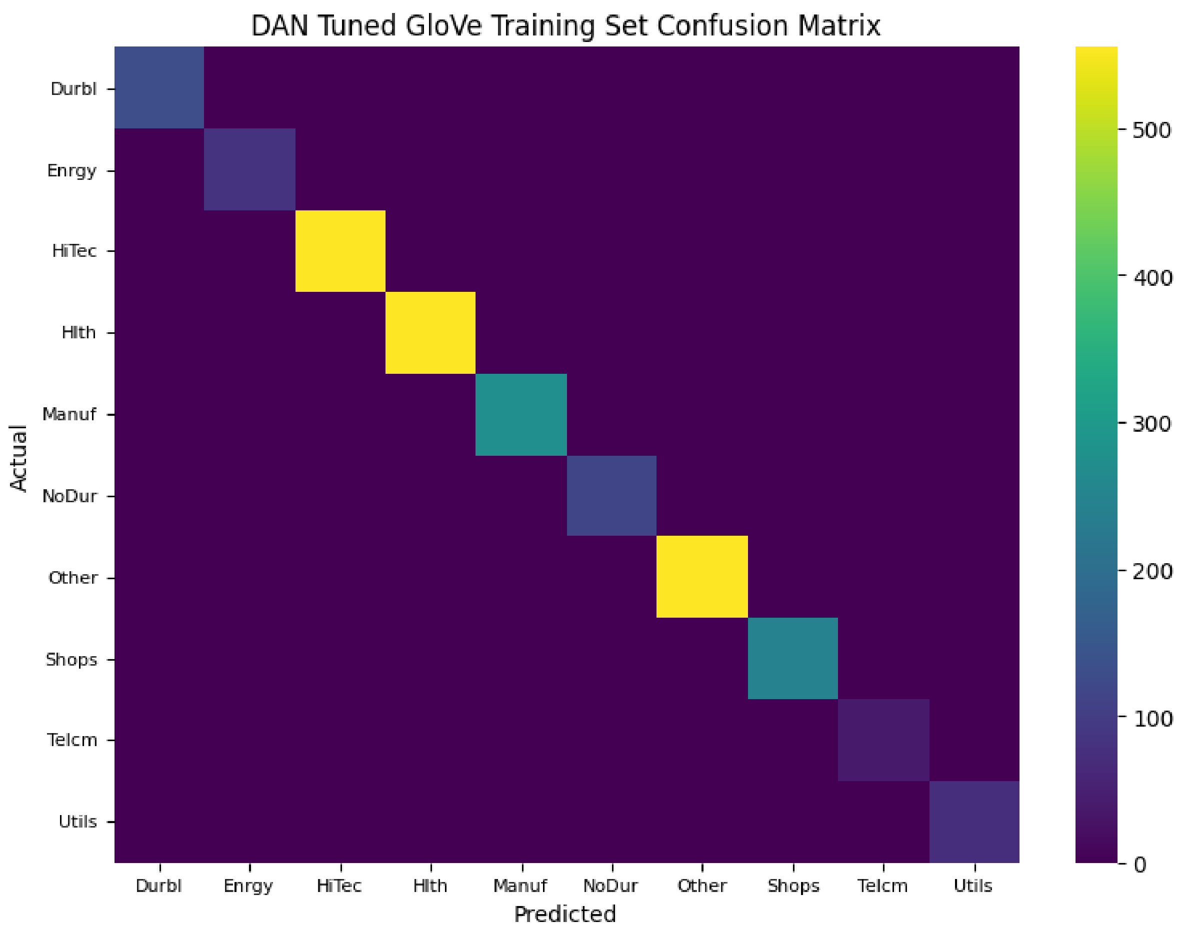
```
classes = class_encoder.classes_
cf_train = DataFrame(confusion_matrix(train_gold, train_pred),
                     index=pd.MultiIndex.from_product([['Actual'], classes]),
                     columns=pd.MultiIndex.from_product([['Predicted'], classes]))
cf_test = DataFrame(confusion_matrix(test_gold, test_pred),
                    index=pd.MultiIndex.from_product([['Actual'], classes]),
                    columns=pd.MultiIndex.from_product([['Predicted'], classes]))
```

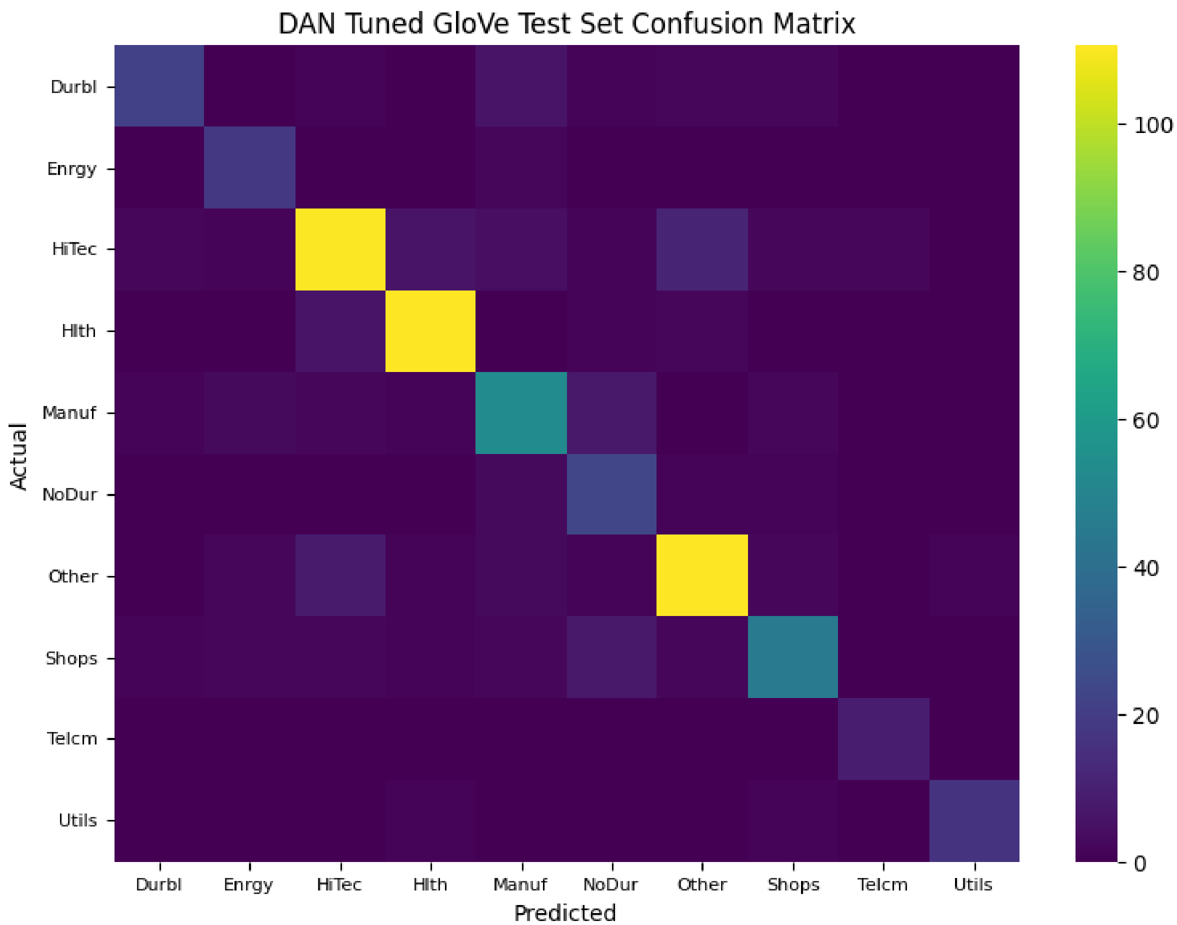
```
for num, (title, cf) in enumerate({'Training': cf_train,
                                   'Test': cf_test}.items()):
    fig, ax = plt.subplots(num=1+num, clear=True, figsize=(8, 6))
    sns.heatmap(cf, ax=ax, annot= False, fmt='d', cmap='viridis', robust=True,
```

(continues on next page)

(continued from previous page)

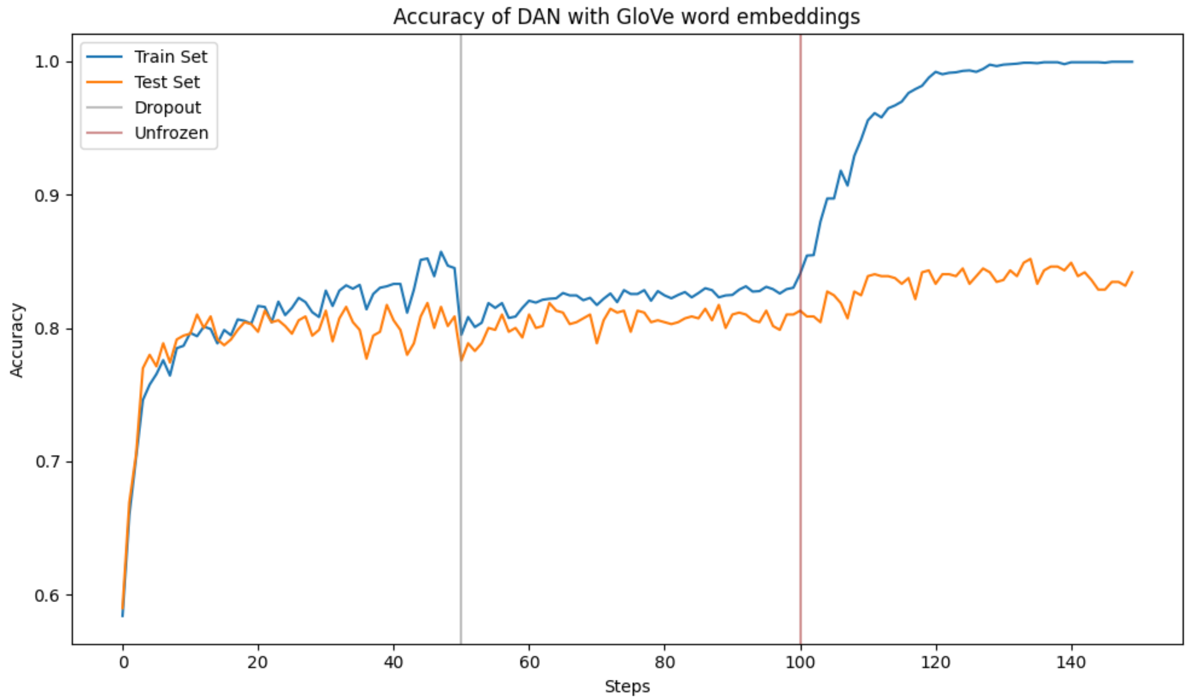
```
              yticklabels=class_encoder.classes_,
              xticklabels=class_encoder.classes_)
    ax.set_title(f'DAN Tuned GloVe {title} Set Confusion Matrix')
    ax.set_xlabel('Predicted')
    ax.set_ylabel('Actual')
    ax.yaxis.set_tick_params(labelsize=8, rotation=0)
    ax.xaxis.set_tick_params(labelsize=8, rotation=0)
    plt.subplots_adjust(left=0.35, bottom=0.25)
    plt.tight_layout()
```





Initially, embeddings are frozen, then fine-tuned, with dropout introduced last, to highlight generalization improvements and the challenge of overfitting.

```
train_accuracy = pd.concat([Series([epoch['train'] for epoch in acc.values()])
                            for acc in accuracy],
                           ignore_index=True)
test_accuracy = pd.concat([Series([epoch['test'] for epoch in acc.values()])
                           for acc in accuracy],
                          ignore_index=True)

fig, ax = plt.subplots(num=1, clear=True, figsize=(10, 6))
train_accuracy.plot(ax=ax)
test_accuracy.plot(ax=ax)
ax.axvline(len(accuracy[0]), c='grey', alpha=0.5)
ax.axvline(len(accuracy[0]) + len(accuracy[1]), c='brown', alpha=0.5)
ax.set_title(f'Accuracy of DAN with GloVe word embeddings')
ax.set_xlabel('Steps')
ax.set_ylabel('Accuracy')
ax.legend(['Train Set', 'Test Set','Dropout', 'Unfrozen'], loc='upper left')
plt.tight_layout()
```



When embeddings are frozen, the model overfits the training data, achieving 100% training accuracy. When dropout regularization is enabled, the test set accuracy slightly improves.

```
# Accuracy when frozen embeddings, unfrozen and with dropouts
p = (len(accuracy[0]) - 1, len(accuracy[0]) + len(accuracy[1]) - 1, -1)
print("Accuracy")
DataFrame({'frozen': [train_accuracy.iloc[p[0]], test_accuracy.iloc[p[0]]],
           'dropout': [train_accuracy.iloc[p[1]], test_accuracy.iloc[p[1]]],
           'unfrozen': [train_accuracy.iloc[p[2]], test_accuracy.iloc[p[2]]]},
          index=['train', 'test'])
```

```
Accuracy
```

	frozen	dropout	unfrozen
train	0.844908	0.830155	0.999640
test	0.808633	0.810072	0.841727

References:

Geoffrey E. Hinton, Nitish Srivastava, Alex Krizhevsky, Ilya Sutskever, Ruslan R. Salakhutdinov, July 2012, "Improving neural networks by preventing co-adaptation of feature detectors"

Jeffrey Pennington, Richard Socher, and Christopher D. Manning. 2014. GloVe: Global Vectors for Word Representation.

Tomas Mikolov, Kai Chen, Greg Corrado, Jeffrey Dean, 2013, "Efficient Estimation of Word Representations in Vector Space"

Greg Durrett, 2021-2024, "CS388 Natural Language Processing course materials", retrieved from https://www.cs.utexas.edu/~gdurrett/courses/online-course/materials.html

Philipp Krähenbühl, 2020-2024, "AI394T Deep Learning course materials", retrieved from https://www.philkr.net/dl_class/material and https://ut.philkr.net/deeplearning/

Philipp Krähenbühl, 2025, "AI395T Advances in Deep Learning course materials", retrieved from https://ut.philkr.net/advances_in_deeplearning/

CHAPTER

THIRTYONE

CONVOLUTIONAL NEURAL NETWORKS

Life can only be understood backwards; but it must be lived forwards. - Søren Kierkegaard

Convolutional Neural Networks (CNNs) are particularly effective for analyzing structured data like images and sequences. By leveraging convolutional layers, CNNs extract hierarchical patterns from raw inputs for complex tasks such as image classification and time series prediction. We explore the application of **Temporal Convolutional Networks (TCNs)** for capturing dependencies in economic time series. The results from modeling multiple time series data such as CPI components are compared with with classical models like Vector Autoregression (VAR).

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import random
import torch
import torch.nn as nn
import torchinfo
from statsmodels.tsa.api import VAR
from sklearn.preprocessing import StandardScaler
from sklearn.metrics import mean_squared_error
from finds.structured import BusDay
from finds.readers import Alfred
from secret import credentials
# %matplotlib qt
VERBOSE = 0
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")

# train-test split date
split_date = '2021-12-01'    # training period up to this date
```

31.1 Convolutions

Convolutional layers are memory-efficient neural network components designed to process spatially structured data, such as images. Unlike fully connected (linear) layers, which require vast numbers of parameters, convolutional layers use localized, shared filters called **kernels** to capture patterns in input data. Mathematically, a convolution operates as follows:

$$y_{i,j,k} = \sum_{l=1}^{C_1} \sum_{m=0}^{h-1} \sum_{n=0}^{w-1} x_{l,j+m,k+n} \cdot \omega_{i,l,m,n}$$

Here, ω is a small kernel (e.g., 3×3) that slides over the image, performing element-wise multiplications and summing the results.

31.1.1 Image Filters

Convolutions can be interpreted as applying image processing filters, such as:

- Box filter (averaging): $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
- Edge detection filter: $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Thus, convolutional layers serve as feature extractors, enabling models to detect edges, textures, and patterns crucial for tasks like image classification and segmentation.

31.1.2 Receptive field and output size

Convolutions operate on local patches of an image, but by stacking multiple layers, they can expand their **receptive field**, allowing the network to capture broader context. Several parameters affect the output size and receptive field:

- Padding: Adds extra pixels around the input to control output size and preserve borders.
- Stride: Controls how far the filter moves across the input, affecting downsampling.
- Dilation: Inserts zeros between kernel elements, expanding the receptive field without increasing parameters.
- Transposed convolution (up-convolution): A learnable upsampling method used to increase output size, often employed in image segmentation models like **U-Net**.

31.1.3 Computer vision

CNNs process images based on :

- Recurring Patterns: They detect similar structures that appear across different images.
- Multi-Scale Patterns: They recognize features ranging from small edges to large object shapes.
- Local Invariance: They take advantage of the fact that neighboring pixels often have similar values.
- Semantic Grouping: They group together pixels that belong to the same object based on shared patterns.

They excel at computer vision tasks that involve recognizing structured patterns at multiple levels of abstraction. These tasks include:

- **Image Classification**: CNNs are highly effective at identifying what object is present in an image by detecting low-level patterns (like edges and textures) and gradually building up to high-level semantic features (like object categories).
- **Object Detection**: By capturing patterns at various scales and identifying object parts, CNNs can localize and label multiple objects within an image, even if they vary in size or position.
- **Semantic Segmentation**: CNNs perform well at assigning a class label to each pixel in an image by grouping together pixels that form the same object or region.

AlexNet (2012) was the first deep network to outperform non-deep vision systems, winning the ImageNet challenge competition and kicking off the Deep Learning revolution. Winning the 2015 competition, ResNet introduced shortcut "residual connections" for gradients in convolutional architectures. The U-Net (2016) model designed a symmetric hourglass-shaped architecture dsemantic segmentation, combining down-sampling to capture context with up-sampling to produce higher output resolution. More recently, Vision Transformer (ViT) models have incorporated transformer encoders by dividing images into patches, treating each patch as a token.

31.2 Temporal convolutional networks (TCN)

Temporal Convolutional Networks (TCNs) are deep convolutional architectures designed for sequence data. They utilize causal and dilated convolutions along with residual connections to model long-range dependencies efficiently. Unlike CNNs for images, TCNs handle sequential data such as time series, text, and audio.

- Causal Convolutions: Ensure that each output at time t only depends on current and past inputs —preserving the temporal order:

$$y_t = \sum_{i=0}^{k-1} w_i x_{t-i}$$

- Dilated Convolutions: Introduce gaps between filter (kernel) elements to capture long-range dependencies without expanding filter size:

$$y_t = \sum_{i=0}^{k-1} w_i x_{t-d \cdot i}$$

- Residual Connections: Allow gradients to flow efficiently through deep networks, mitigating vanishing gradient issues:

$$\text{Output} = x + F(x)$$

Key hyperparameters of TCNs include:

- `kernel_size`: Size of convolutional filter.
- `dropout`: Regularization to prevent overfitting.
- `blocks`: Number of stacked convolutional layers.
- `dilation`: Grows exponentially (e.g., 1, 2, 4, $\cdots$).
- `activation`: Non-linear activation, typically ReLU.

```python
class TCN(torch.nn.Module):
    class CausalConv1dBlock(torch.nn.Module):
        """Conv1d block with ReLU, skip, dropout, dilation and padding"""

        def __init__(self, in_channels, out_channels, kernel_size, dilation,
                     dropout):
            super().__init__()

            # print('kernel', kernel_size, 'dilation', dilation)
            self.network = torch.nn.Sequential(
                torch.nn.ConstantPad1d(((kernel_size-1)*dilation, 0), 0),
                torch.nn.Conv1d(in_channels, out_channels, kernel_size,
                                dilation=dilation),
                torch.nn.ReLU(),
                torch.nn.ConstantPad1d(((kernel_size-1)*dilation, 0), 0),
                torch.nn.Conv1d(out_channels, out_channels, kernel_size,
```

(continues on next page)

(continued from previous page)

```
                                dilation=dilation),
                torch.nn.ReLU(),
                torch.nn.Dropout(dropout))
            self.skip = lambda x: x
            if in_channels != out_channels:   # downsample for skip if necessary
                self.skip = torch.nn.Conv1d(in_channels, out_channels, 1)

        def forward(self, x):
            return self.network(x) + self.skip(x)  # with skip connection


    def __init__(self, n_features, blocks, kernel_size, dropout):
        """TCN model by connecting multiple convolution layers"""
        super().__init__()
        in_channels = n_features
        L = []
        for dilation, hidden in enumerate(blocks):
            L.append(self.CausalConv1dBlock(in_channels=in_channels,
                                            out_channels=hidden,
                                            kernel_size=kernel_size,
                                            dilation=2**dilation,
                                            dropout=dropout))
            in_channels = hidden
        self.network = torch.nn.Sequential(*L) if L else lambda x: x
        if L:
            self.classifier = torch.nn.Conv1d(in_channels, n_features, 1)
        else:
            self.classifier = torch.nn.Sequential(
                torch.nn.ConstantPad1d((kernel_size-1, 0), 0),
                torch.nn.Conv1d(in_channels, n_features, kernel_size))

    def forward(self, x):
        """input is (B, n_features, L)), linear expects (B, * n_features)"""
        return self.classifier(self.network(x))

    def save(self, filename):
        """save model state to filename"""
        return torch.save(self.state_dict(), filename)

    def load(self, filename):
        """load model name from filename"""
        self.load_state_dict(torch.load(filename, map_location='cpu'))
        return self
```

31.2.1 Data preparation

The dataset comprises economic time series of CPI components obtained from FRED (Federal Reserve Economic Data). The data, covering various CPI categories (e.g., food, housing, transportation), is log-transformed and differenced for stationarity and standardized using `StandardScaler` to mean 0 and variance 1.

```
alf = Alfred(api_key=credentials['fred']['api_key'], verbose=-1)
vspans = alf.date_spans('USREC')    # recession periods for plots
```

```python
# CPI for U.S. City Average: Monthly, Seasonally Adjusted
# https://fred.stlouisfed.org/release/tables?rid=10&eid=34483
# 'CUSR0000SEEA'
series_ids = ['CPIFABSL', 'CPIHOSSL', 'CPIAPPSL', 'CPITRNSL', 'CPIMEDSL', 'CPIOGSSL']
df = pd.concat([alf(s, log=1, diff=1) for s in series_ids], axis=1)\
       .dropna()\
       .sort_index()
df.index = BusDay.to_datetime(df.index)
df.index.freq = 'M'     # set index to datetime type and freq = 'M'
```

```
/tmp/ipykernel_1025513/4272565358.py:9: FutureWarning: 'M' is deprecated and will
↪be removed in a future version, please use 'ME' instead.
  df.index.freq = 'M'     # set index to datetime type and freq = 'M'
```

```python
names = [s[s.find(':')+2:s.find(' in ')] for s in alf.header(series_ids)]
names
```

```
['Food and Beverages',
 'Housing',
 'Apparel',
 'Transportation',
 'Medical Care',
 'Other Goods and Services']
```

```python
# Standardize the data data
scaler = StandardScaler().fit(df)
scaled_data = DataFrame(scaler.transform(df), columns=names, index=df.index)
scaled_data
```

```
            Food and Beverages   Housing   Apparel  Transportation  \
date
1967-02-28           -1.520716 -1.129800  0.533990        0.266586
1967-03-31           -0.805655 -1.129800  0.124553       -0.267488
1967-04-30           -1.522780 -0.063784  0.529140        0.263339
1967-05-31           -0.805655 -0.067262  0.122142       -0.003280
1967-06-30            1.339539 -1.129800  0.524347       -0.267488
...                        ...       ...       ...             ...
2024-10-31           -0.252031  0.063171 -2.110286       -0.203592
2024-11-30           -0.053379 -0.037266 -0.114479        0.027062
2024-12-31           -0.180743 -0.209529 -0.015224        0.767819
2025-01-31            0.090331 -0.037301 -3.137367        0.794814
2025-02-28           -0.350673  0.133190  0.934107       -0.605692

            Medical Care  Other Goods and Services
date
1967-02-28     -0.248585                 -1.017195
1967-03-31     -0.253160                 -0.284868
1967-04-30      0.988995                 -1.017195
1967-05-31     -0.266687                 -0.286982
1967-06-30      0.962232                 -0.289083
...                  ...                       ...
2024-10-31     -0.692758                 -0.172972
2024-11-30     -0.702581                  0.085335
```

(continues on next page)

(continued from previous page)

```
2024-12-31     -1.058184                   -1.113129
2025-01-31     -0.701262                   -1.822241
2025-02-28     -0.563596                    0.499572

[697 rows x 6 columns]
```

```
ntrain = sum(scaled_data.index < split_date)
M = scaled_data.shape[1]      # M is number of time series
```

31.2.2 Training

The TCN is trained to predict the next time step of the CPI components using past observations. Training involves splitting data into train and test sets, and using the Adam optimizer to minimize mean squared error (MSE) between predictions and actual values.

```
# Model training parameters
seq_len = 8          # length of each input sequence for TCN
batch_size = 16
step_size = 30       # learning rate scheduler step size
lr = 0.01            # initial learning rate
num_lr = 3
num_epochs = step_size * num_lr
results = {}         # to collect evaluate results
train_loss = {}
test_loss = {}
```

```
# Form input data from training set
n_features = scaled_data.shape[1]     # number of input planes
train_exs = [scaled_data.iloc[(i - seq_len):(i + 1)].values
             for i in range(seq_len, ntrain)]
```

```
# train_ex should have dimension (batch size, channels, sequence length+1)
train_ex = torch.tensor(scaled_data.values[:ntrain].T)[None,:,:].float().to(device)
```

First, a baseline model comprising just a single **1D Conv layer** is trained to predict next time step

```
model = torch.nn.Conv1d(n_features, n_features, kernel_size=1).to(device)
print(model)
print(torchinfo.summary(model))
modelname = "1D-Convolution"
train_loss[modelname] = []
test_loss[modelname] = []
optimizer = torch.optim.Adam(model.parameters())
loss_function = nn.MSELoss()
```

```
Conv1d(6, 6, kernel_size=(1,), stride=(1,))
=================================================================
Layer (type:depth-idx)                   Param #
=================================================================
Conv1d                                   42
```

(continues on next page)

(continued from previous page)

```
=================================================================
Total params: 42
Trainable params: 42
Non-trainable params: 0
=================================================================
```

```
for epoch in range(num_epochs):
    for _ in range(batch_size):
        total_loss = 0.0
        model.train()
        model.zero_grad()
        X = train_ex[:,:,:-1]
        Y = train_ex[:,:,1:]
        output = model(X)
        loss = loss_function(output, Y)  # calculated over all outputs
        total_loss += float(loss)
        loss.backward()
        optimizer.step()

    model.eval()
    train_loss[modelname].append(total_loss)

    X = torch.tensor(scaled_data.values.T)[None,:,:].float().to(device)
    pred = model(X).cpu().detach().numpy()[0,:,:].T
    test_loss[modelname] = mean_squared_error(scaled_data.values[ntrain:],
                                              pred[ntrain-1:-1])
results[modelname] = {
    'Train Error': mean_squared_error(scaled_data.values[1:ntrain],
                                        pred[:ntrain-1]),
    'Test Error': mean_squared_error(scaled_data.values[ntrain:],
                                       pred[ntrain-1:-1])
}
```

Save the fitted weights to compare with classical Vector Autoregression (VAR) models.

```
conv1d_weights = np.vstack([model.bias.cpu().detach().numpy(),
                            model.weight.cpu().detach().numpy()[:,:,0].T])
```

Next, we train various TCN configurations, utilizing StepLR learning rate scheduler and shuffled batches, while varying:

- number of layers (blocks): 1, 2
- different kernel sizes: 1, 2
- dropout rates: 0, 0.5.

```
# [1,1,0], [1,2,0], [2,1,0], [2,2,0], [1,1,0.5], [1,2,0.5], [2,1,0.5], [2,2,0.5]
for block, kernel_size, dropout in [[1,1,0], [1,2,0], [2,1,0], [2,2,0],  [2,2,0.3]]:
    modelname = f"TCN(b={block},k={kernel_size},d={dropout:.1f})"
    train_loss[modelname] = []
    test_loss[modelname] = []

    # Set model, optimizer, loss function and learning rate scheduler
    model = TCN(n_features=n_features,
                blocks=[n_features]*block,
                kernel_size=kernel_size,
```

(continues on next page)

(continued from previous page)

```
                dropout=dropout).to(device)
    print()
    print('******', modelname, '******')
    print(model)
    print(torchinfo.summary(model))

    optimizer = torch.optim.Adam(model.parameters(), lr=lr)
    scheduler = torch.optim.lr_scheduler.StepLR(
        optimizer, gamma=0.1, step_size=step_size
    )
    loss_function = nn.MSELoss()

    # Run training loop over num_epochs with batch_size
    num_epochs = step_size * num_lr
    for epoch in range(num_epochs):

        # shuffle indxs into batches
        idxs = np.arange(len(train_exs))
        random.shuffle(idxs)
        batches = [idxs[i:min(len(idxs), i + batch_size)]
                   for i in range(0, len(idxs), batch_size)]

        # train by batch
        total_loss = 0.0
        model.train()
        for batch in batches:
            # input has shape (batch_size, n_features, seq_len)
            # Creating a tensor from a list of numpy.ndarrays is extremely slow.
            nparray = np.array([[train_exs[idx][seq] for idx in batch]
                                 for seq in range(seq_len+1)])
            train_ex = torch.tensor(nparray).permute(1, 2, 0).float().to(device)
            model.zero_grad()
            X = train_ex[:,:,:-1]
            Y = train_ex[:,:,1:]
            output = model(X)
            loss = loss_function(output, Y)  # calculated over all outputs
            total_loss += float(loss) / len(batches)
            loss.backward()
            optimizer.step()
            scheduler.step()

        model.eval()
        train_loss[modelname].append(total_loss)
        if VERBOSE and (epoch % (step_size//2)) == 0:
            print(epoch, num_epochs, optimizer.param_groups[0]['lr'], total_loss)

        # Compute MSE of one-period ahead forecast error in train and test sets
        X = torch.tensor(scaled_data.values.T)[None,:,:].float().to(device)
        pred = model(X).cpu().detach().numpy()[0,:,:].T
        test_loss[modelname].append(mean_squared_error(scaled_data.values[ntrain:],
                                                       pred[ntrain-1:-1]))

    results[modelname] = {
        'Train Error': mean_squared_error(scaled_data.values[1:ntrain],
                                          pred[:ntrain-1]),
        'Test Error': mean_squared_error(scaled_data.values[ntrain:],
```

(continues on next page)

(continued from previous page)

```
                                          pred[ntrain-1:-1])
    }
    #print('Blocks:', block, 'Kernel size:', kernel_size, results[modelname])
    #print(pd.concat(res, axis=1).T)
```

```
****** TCN(b=1,k=1,d=0.0) ******
TCN(
  (network): Sequential(
    (0): CausalConv1dBlock(
      (network): Sequential(
        (0): ConstantPad1d(padding=(0, 0), value=0)
        (1): Conv1d(6, 6, kernel_size=(1,), stride=(1,))
        (2): ReLU()
        (3): ConstantPad1d(padding=(0, 0), value=0)
        (4): Conv1d(6, 6, kernel_size=(1,), stride=(1,))
        (5): ReLU()
        (6): Dropout(p=0, inplace=False)
      )
    )
  )
  (classifier): Conv1d(6, 6, kernel_size=(1,), stride=(1,))
)
=================================================================
Layer (type:depth-idx)                   Param #
=================================================================
TCN                                      --
├─Sequential: 1-1                        --
│    └─CausalConv1dBlock: 2-1            --
│    │    └─Sequential: 3-1              84
├─Conv1d: 1-2                            42
=================================================================
Total params: 126
Trainable params: 126
Non-trainable params: 0
=================================================================

****** TCN(b=1,k=2,d=0.0) ******
TCN(
  (network): Sequential(
    (0): CausalConv1dBlock(
      (network): Sequential(
        (0): ConstantPad1d(padding=(1, 0), value=0)
        (1): Conv1d(6, 6, kernel_size=(2,), stride=(1,))
        (2): ReLU()
        (3): ConstantPad1d(padding=(1, 0), value=0)
        (4): Conv1d(6, 6, kernel_size=(2,), stride=(1,))
        (5): ReLU()
        (6): Dropout(p=0, inplace=False)
      )
    )
  )
  (classifier): Conv1d(6, 6, kernel_size=(1,), stride=(1,))
)
=================================================================
Layer (type:depth-idx)                   Param #
=================================================================
```

(continues on next page)

(continued from previous page)

```
TCN                                        --
├─Sequential: 1-1                          --
│    └─CausalConv1dBlock: 2-1              --
│    │    └─Sequential: 3-1                156
├─Conv1d: 1-2                              42
=================================================================
Total params: 198
Trainable params: 198
Non-trainable params: 0
=================================================================

****** TCN(b=2,k=1,d=0.0) ******
TCN(
  (network): Sequential(
    (0): CausalConv1dBlock(
      (network): Sequential(
        (0): ConstantPad1d(padding=(0, 0), value=0)
        (1): Conv1d(6, 6, kernel_size=(1,), stride=(1,))
        (2): ReLU()
        (3): ConstantPad1d(padding=(0, 0), value=0)
        (4): Conv1d(6, 6, kernel_size=(1,), stride=(1,))
        (5): ReLU()
        (6): Dropout(p=0, inplace=False)
      )
    )
    (1): CausalConv1dBlock(
      (network): Sequential(
        (0): ConstantPad1d(padding=(0, 0), value=0)
        (1): Conv1d(6, 6, kernel_size=(1,), stride=(1,), dilation=(2,))
        (2): ReLU()
        (3): ConstantPad1d(padding=(0, 0), value=0)
        (4): Conv1d(6, 6, kernel_size=(1,), stride=(1,), dilation=(2,))
        (5): ReLU()
        (6): Dropout(p=0, inplace=False)
      )
    )
  )
  (classifier): Conv1d(6, 6, kernel_size=(1,), stride=(1,))
)
=================================================================
Layer (type:depth-idx)                     Param #
=================================================================
TCN                                        --
├─Sequential: 1-1                          --
│    └─CausalConv1dBlock: 2-1              --
│    │    └─Sequential: 3-1                84
│    └─CausalConv1dBlock: 2-2              --
│    │    └─Sequential: 3-2                84
├─Conv1d: 1-2                              42
=================================================================
Total params: 210
Trainable params: 210
Non-trainable params: 0
=================================================================

****** TCN(b=2,k=2,d=0.0) ******
```

(continues on next page)

(continued from previous page)

```
TCN(
  (network): Sequential(
    (0): CausalConv1dBlock(
      (network): Sequential(
        (0): ConstantPad1d(padding=(1, 0), value=0)
        (1): Conv1d(6, 6, kernel_size=(2,), stride=(1,))
        (2): ReLU()
        (3): ConstantPad1d(padding=(1, 0), value=0)
        (4): Conv1d(6, 6, kernel_size=(2,), stride=(1,))
        (5): ReLU()
        (6): Dropout(p=0, inplace=False)
      )
    )
    (1): CausalConv1dBlock(
      (network): Sequential(
        (0): ConstantPad1d(padding=(2, 0), value=0)
        (1): Conv1d(6, 6, kernel_size=(2,), stride=(1,), dilation=(2,))
        (2): ReLU()
        (3): ConstantPad1d(padding=(2, 0), value=0)
        (4): Conv1d(6, 6, kernel_size=(2,), stride=(1,), dilation=(2,))
        (5): ReLU()
        (6): Dropout(p=0, inplace=False)
      )
    )
  )
  (classifier): Conv1d(6, 6, kernel_size=(1,), stride=(1,))
)
=================================================================
Layer (type:depth-idx)                   Param #
=================================================================
TCN                                      --
├─Sequential: 1-1                        --
│    └─CausalConv1dBlock: 2-1            --
│    │    └─Sequential: 3-1              156
│    └─CausalConv1dBlock: 2-2            --
│    │    └─Sequential: 3-2              156
├─Conv1d: 1-2                            42
=================================================================
Total params: 354
Trainable params: 354
Non-trainable params: 0
=================================================================

****** TCN(b=2,k=2,d=0.3) ******
TCN(
  (network): Sequential(
    (0): CausalConv1dBlock(
      (network): Sequential(
        (0): ConstantPad1d(padding=(1, 0), value=0)
        (1): Conv1d(6, 6, kernel_size=(2,), stride=(1,))
        (2): ReLU()
        (3): ConstantPad1d(padding=(1, 0), value=0)
        (4): Conv1d(6, 6, kernel_size=(2,), stride=(1,))
        (5): ReLU()
        (6): Dropout(p=0.3, inplace=False)
      )
```

(continues on next page)

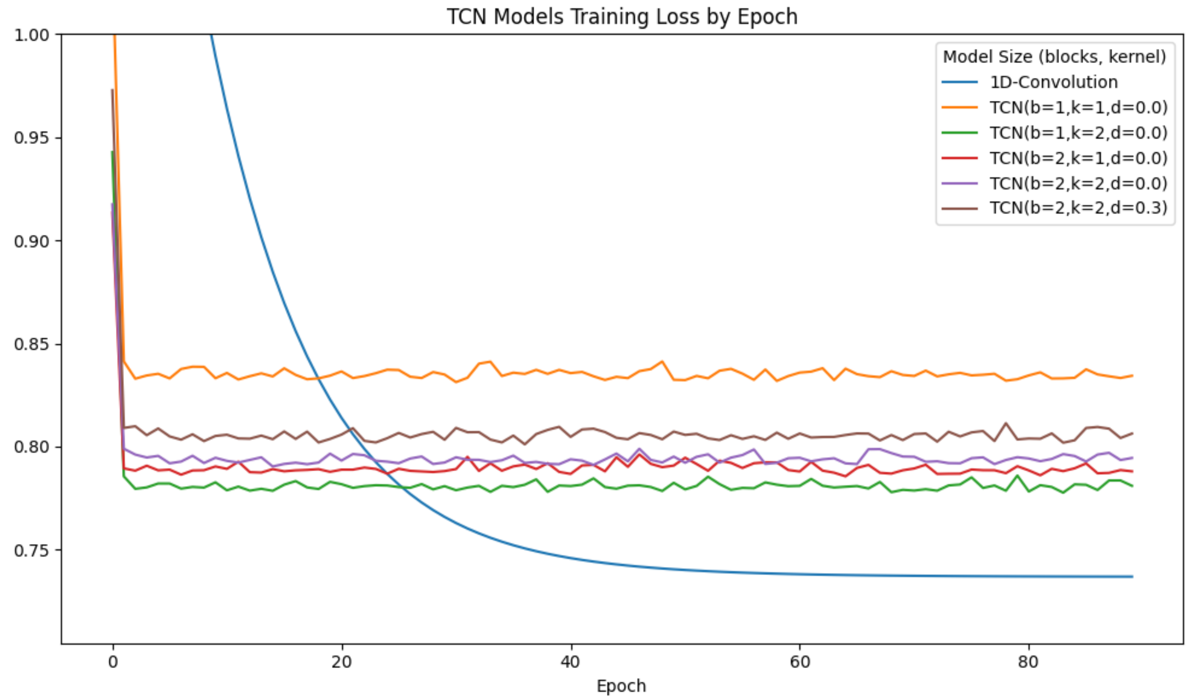
(continued from previous page)

```
    )
    (1): CausalConv1dBlock(
      (network): Sequential(
        (0): ConstantPad1d(padding=(2, 0), value=0)
        (1): Conv1d(6, 6, kernel_size=(2,), stride=(1,), dilation=(2,))
        (2): ReLU()
        (3): ConstantPad1d(padding=(2, 0), value=0)
        (4): Conv1d(6, 6, kernel_size=(2,), stride=(1,), dilation=(2,))
        (5): ReLU()
        (6): Dropout(p=0.3, inplace=False)
      )
    )
  )
  (classifier): Conv1d(6, 6, kernel_size=(1,), stride=(1,))
)
=================================================================
Layer (type:depth-idx)                   Param #
=================================================================
TCN                                      --
├─Sequential: 1-1                        --
│    └─CausalConv1dBlock: 2-1            --
│    │    └─Sequential: 3-1              156
│    └─CausalConv1dBlock: 2-2            --
│    │    └─Sequential: 3-2              156
├─Conv1d: 1-2                            42
=================================================================
Total params: 354
Trainable params: 354
Non-trainable params: 0
=================================================================
```

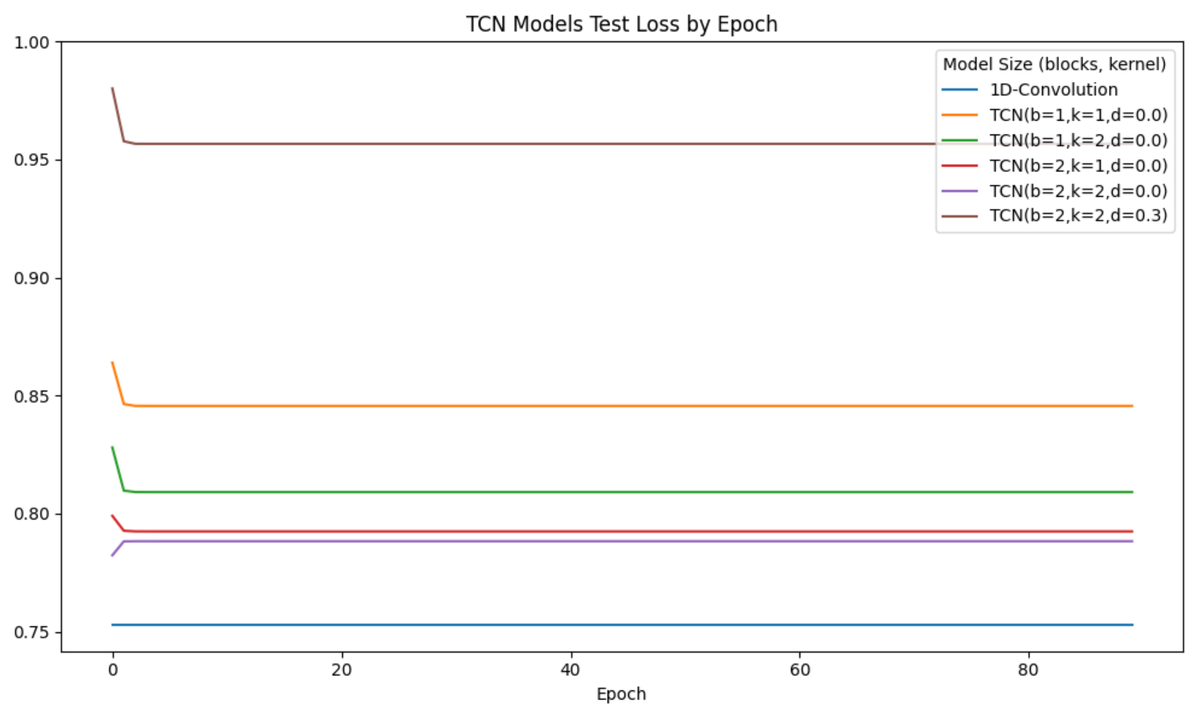
31.2.3 Evaluation

Training and test errors (MSE) from one-step-ahead forecasting are collected across models. Additionally, training and testing loss curves are plotted to analyze convergence and overfitting tendencies of different configurations.

```
fig, ax = plt.subplots(figsize=(10, 6))
DataFrame(train_loss).plot(ax=ax)
ax.set_ylim(top=1.0)
ax.set_title(f"TCN Models Training Loss by Epoch")
ax.set_xlabel('Epoch')
ax.legend(title='Model Size (blocks, kernel)')
plt.tight_layout()
```




```
fig, ax = plt.subplots(figsize=(10, 6))
DataFrame(test_loss).plot(ax=ax)
ax.set_ylim(top=1.0)
ax.set_title(f"TCN Models Test Loss by Epoch")
ax.set_xlabel('Epoch')
ax.legend(title='Model Size (blocks, kernel)')
plt.tight_layout()
```



```
print('Sorted by Test Error')
DataFrame(results).T.sort_values('Test Error')
```

```
Sorted by Test Error
```

```
                      Train Error  Test Error
1D-Convolution           0.736907    0.753075
TCN(b=2,k=2,d=0.0)       0.786185    0.788295
TCN(b=2,k=1,d=0.0)       0.788457    0.792443
TCN(b=1,k=2,d=0.0)       0.775419    0.809113
TCN(b=1,k=1,d=0.0)       0.832984    0.845615
TCN(b=2,k=2,d=0.3)       0.790285    0.956657
```

31.3 Vector Autoregression

Vector Autoregression (VAR) is a statistical time series model that captures linear interdependencies across multiple time series.

```
var_model = VAR(scaled_data.iloc[:ntrain], freq='ME')
```

31.3.1 Lag order

The lagged coefficients estimated from the Vector Autoregression help predict multi-step future outcomes. The optimal lag order (p) can be selected using infomation criteria such as AIC, BIC, HQIC, or FPE.

```
# up to max number p of VAR(p) lags
maxlags = 6

print("Optimal number of VAR(p) lags selected by various IC")
DataFrame({ic: var_model.fit(maxlags=maxlags, ic=ic).k_ar
           for ic in ['aic', 'fpe', 'hqic', 'bic']},
          index=['optimal p:'])\
          .rename_axis(columns='IC:')
```

```
Optimal number of VAR(p) lags selected by various IC
```

```
IC:         aic  fpe  hqic  bic
optimal p:    3    3     2    2
```

```
# Fit VAR(p) models
var_models = {p: var_model.fit(p) for p in range(1, maxlags+1)} # fit models
```

```
# Show model summary for VAR(1)
print(var_models[1].summary())
```

```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Sat, 15, Mar, 2025
Time:                     04:58:13
--------------------------------------------------------------------
No. of Equations:         6.00000    BIC:                   -1.67401
Nobs:                     657.000    HQIC:                  -1.84967
Log likelihood:          -4907.30    FPE:                   0.140733
AIC:                     -1.96089    Det(Omega_mle):        0.132063
--------------------------------------------------------------------
Results for equation Food and Beverages
==============================================================================================
                                 coefficient       std. error           t-stat       ␣
↪        prob
----------------------------------------------------------------------------------
↪-----------
const                              -0.000685         0.035822           -0.019       ␣
↪       0.985
L1.Food and Beverages               0.309512         0.037889            8.169       ␣
↪       0.000
L1.Housing                          0.212849         0.043404            4.904       ␣
↪       0.000
L1.Apparel                         -0.002978         0.038364           -0.078       ␣
↪       0.938
L1.Transportation                  -0.006074         0.038082           -0.159       ␣
↪       0.873
L1.Medical Care                    -0.007239         0.042771           -0.169       ␣
↪       0.866
L1.Other Goods and Services         0.010009         0.037379            0.268       ␣
↪       0.789
==============================================================================================

Results for equation Housing
==============================================================================================
                                 coefficient       std. error           t-stat       ␣
↪        prob
----------------------------------------------------------------------------------
↪-----------
const                              -0.016126         0.027926           -0.577       ␣
↪       0.564
L1.Food and Beverages               0.163358         0.029537            5.531       ␣
↪       0.000
L1.Housing                          0.494378         0.033836           14.611       ␣
↪       0.000
L1.Apparel                          0.038625         0.029907            1.291       ␣
↪       0.197
L1.Transportation                   0.074603         0.029687            2.513       ␣
↪       0.012
L1.Medical Care                     0.211794         0.033342            6.352       ␣
↪       0.000
L1.Other Goods and Services        -0.022649         0.029139           -0.777       ␣
↪       0.437
==============================================================================================

Results for equation Apparel
```

(continues on next page)

(continued from previous page)

```
===============================================================================================
                                    coefficient       std. error           t-stat      ␣
↪       prob
-----------------------------------------------------------------------------------
↪-----------
const                                 -0.004137         0.036418           -0.114      ␣
↪      0.910
L1.Food and Beverages                  0.032202         0.038519            0.836      ␣
↪      0.403
L1.Housing                             0.108030         0.044125            2.448      ␣
↪      0.014
L1.Apparel                             0.137027         0.039002            3.513      ␣
↪      0.000
L1.Transportation                      0.187727         0.038715            4.849      ␣
↪      0.000
L1.Medical Care                        0.116103         0.043482            2.670      ␣
↪      0.008
L1.Other Goods and Services            0.007090         0.038000            0.187      ␣
↪      0.852
===============================================================================================

Results for equation Transportation
===============================================================================================
                                    coefficient       std. error           t-stat      ␣
↪       prob
-----------------------------------------------------------------------------------
↪-----------
const                                  0.001347         0.034599            0.039      ␣
↪      0.969
L1.Food and Beverages                  0.021051         0.036595            0.575      ␣
↪      0.565
L1.Housing                             0.070974         0.041921            1.693      ␣
↪      0.090
L1.Apparel                             0.032549         0.037054            0.878      ␣
↪      0.380
L1.Transportation                      0.425719         0.036782           11.574      ␣
↪      0.000
L1.Medical Care                        0.050082         0.041310            1.212      ␣
↪      0.225
L1.Other Goods and Services           -0.035207         0.036103           -0.975      ␣
↪      0.329
===============================================================================================

Results for equation Medical Care
===============================================================================================
                                    coefficient       std. error           t-stat      ␣
↪       prob
-----------------------------------------------------------------------------------
↪-----------
const                                  0.033077         0.029060            1.138      ␣
↪      0.255
L1.Food and Beverages                  0.024634         0.030737            0.801      ␣
↪      0.423
L1.Housing                             0.232885         0.035211            6.614      ␣
↪      0.000
L1.Apparel                             0.044440         0.031122            1.428      ␣
```

(continues on next page)

(continued from previous page)

```
↪      0.153
L1.Transportation                   0.005782         0.030893            0.187     ␣
↪      0.852
L1.Medical Care                     0.431033         0.034697           12.423     ␣
↪      0.000
L1.Other Goods and Services         0.114476         0.030323            3.775     ␣
↪      0.000
===============================================================================================

Results for equation Other Goods and Services
===============================================================================================
                                 coefficient       std. error           t-stat     ␣
↪       prob
-----------------------------------------------------------------------------------
↪-----------
const                              -0.009490         0.037496           -0.253     ␣
↪      0.800
L1.Food and Beverages               0.026291         0.039660            0.663     ␣
↪      0.507
L1.Housing                          0.076689         0.045432            1.688     ␣
↪      0.091
L1.Apparel                          0.094758         0.040157            2.360     ␣
↪      0.018
L1.Transportation                   0.018115         0.039861            0.454     ␣
↪      0.649
L1.Medical Care                     0.260564         0.044769            5.820     ␣
↪      0.000
L1.Other Goods and Services         0.009675         0.039126            0.247     ␣
↪      0.805
===============================================================================================

Correlation matrix of residuals
                            Food and Beverages   Housing   Apparel  Transportation␣
↪ Medical Care  Other Goods and Services
Food and Beverages                    1.000000  0.147358  0.085820       -0.000563␣
↪     0.014162                -0.010059
Housing                               0.147358  1.000000  0.093314        0.132546␣
↪     0.094535                 0.019075
Apparel                               0.085820  0.093314  1.000000        0.126653␣
↪     0.037713                 0.025531
Transportation                       -0.000563  0.132546  0.126653        1.000000␣
↪    -0.054420                -0.012936
Medical Care                          0.014162  0.094535  0.037713       -0.054420␣
↪     1.000000                 0.153701
Other Goods and Services             -0.010059  0.019075  0.025531       -0.012936␣
↪     0.153701                 1.000000
```

31.3.2 Var(1) and Conv1d

The coefficients learned by VAR(1) models are compared to the learned weights of Conv1D layers, illustrating how classical linear models relate to convolution-based neural approaches in time series forecasting.

```
print('Coefficients of VAR(1) model')
DataFrame(np.vstack([var_models[1].intercept, var_models[1].coefs[0].T]),
          columns=var_models[1].names, index=var_models[1].exog_names).round(4)
```

```
Coefficients of VAR(1) model
```

```
                             Food and Beverages  Housing  Apparel  \
const                                   -0.0007  -0.0161  -0.0041
L1.Food and Beverages                    0.3095   0.1634   0.0322
L1.Housing                               0.2128   0.4944   0.1080
L1.Apparel                              -0.0030   0.0386   0.1370
L1.Transportation                       -0.0061   0.0746   0.1877
L1.Medical Care                         -0.0072   0.2118   0.1161
L1.Other Goods and Services              0.0100  -0.0226   0.0071

                             Transportation  Medical Care  \
const                                0.0013        0.0331
L1.Food and Beverages                0.0211        0.0246
L1.Housing                           0.0710        0.2329
L1.Apparel                           0.0325        0.0444
L1.Transportation                    0.4257        0.0058
L1.Medical Care                      0.0501        0.4310
L1.Other Goods and Services         -0.0352        0.1145

                             Other Goods and Services
const                                         -0.0095
L1.Food and Beverages                          0.0263
L1.Housing                                     0.0767
L1.Apparel                                     0.0948
L1.Transportation                              0.0181
L1.Medical Care                                0.2606
L1.Other Goods and Services                    0.0097
```

```
print('Tensor weights of Conv1D')
DataFrame(conv1d_weights, columns=names, index=['bias'] + names).round(4)
```

```
Tensor weights of Conv1D
```

```
                          Food and Beverages  Housing  Apparel  \
bias                                 -0.0011  -0.0167  -0.0041
Food and Beverages                    0.3154   0.1677   0.0322
Housing                               0.1997   0.4808   0.1080
Apparel                              -0.0014   0.0396   0.1370
Transportation                       -0.0037   0.0773   0.1877
Medical Care                         -0.0018   0.2197   0.1160
Other Goods and Services              0.0105  -0.0227   0.0072

                          Transportation  Medical Care  \
bias                              0.0014        0.0330
```

(continues on next page)

(continued from previous page)

```
Food and Beverages                    0.0210        0.0253
Housing                               0.0713        0.2302
Apparel                               0.0326        0.0447
Transportation                        0.4256        0.0063
Medical Care                          0.0493        0.4323
Other Goods and Services             -0.0349        0.1146

                           Other Goods and Services
bias                                        -0.0086
Food and Beverages                           0.0233
Housing                                      0.0861
Apparel                                      0.0936
Transportation                               0.0166
Medical Care                                 0.2514
Other Goods and Services                     0.0115
```

31.3.3 Evaluation

The forecasting accuracy of VAR models, measured by train and test MSE, is compared across different lag orders.

```python
# Calculate forecast errors for each observation and model
test_errors = {p: list() for p in range(maxlags+1)}
train_errors = {p: list() for p in range(maxlags+1)}
```

```python
for i in range(maxlags, len(scaled_data)-1):
    data = scaled_data.iloc[i].values

    # test or train sample
    var_errors = train_errors if i < ntrain else test_errors

    # error of unconditional mean forecast
    var_errors[0].append(mean_squared_error(data, scaled_data.iloc[:ntrain].mean()))

    # accumulate to error of VAR(p) model forecasts
    for p in range(1, maxlags+1):
        pred = var_models[p].forecast(scaled_data.iloc[:i].values, 1)
        var_errors[p].append(mean_squared_error(data.reshape(1, -1), pred))
```

```python
# Collect mean test and train set errors of all VAR(p) models
print('VAR models train and test set errors')
out = DataFrame({'Train Error': {f"VAR({p})" : np.mean(errors)
                                 for p, errors in train_errors.items()},
                 'Test Error':  {f"VAR({p})" : np.mean(errors)
                                 for p, errors in test_errors.items()}})
out
```

```
VAR models train and test set errors
```

```
        Train Error  Test Error
VAR(0)     1.009966    0.936672
VAR(1)     0.739496    0.757588
```

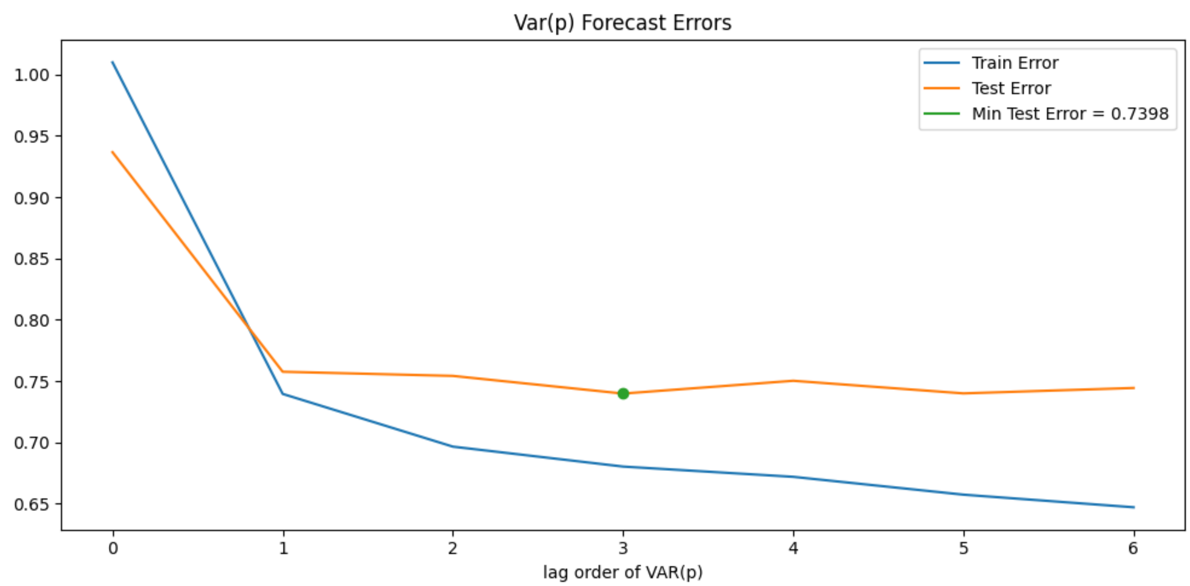
(continues on next page)

(continued from previous page)

```
VAR(2)      0.696483    0.754189
VAR(3)      0.680224    0.739767
VAR(4)      0.671862    0.750212
VAR(5)      0.657304    0.739927
VAR(6)      0.647010    0.744329
```

Error plots are generated to determine the optimal lag order for best predictive performance.

```python
# Plot Errors
fig, ax = plt.subplots(1, 1, figsize=(10, 5))
ax.plot(np.arange(len(out)), out['Train Error'], color="C0")
ax.plot(np.arange(len(out)), out['Test Error'], color="C1")
ax.plot([], [], color="C2")   # dummy for legend labels
argmin = out['Test Error'].argmin()
ax.plot(argmin, out.iloc[argmin]['Test Error'], 'o', color="C2")
ax.set_title(f'Var(p) Forecast Errors')
ax.set_xlabel('lag order of VAR(p)')
ax.legend(['Train Error', 'Test Error',
           f'Min Test Error = {out.iloc[argmin]["Test Error"]:.4f}'],
          loc='upper right')
plt.tight_layout()
```



References:

Philipp Krähenbühl, 2020-2024, "AI394T Deep Learning course materials", retrieved from https://www.philkr.net/dl_class/material and https://ut.philkr.net/deeplearning/

Philipp Krähenbühl, 2025, "AI395T Advances in Deep Learning course materials", retrieved from https://ut.philkr.net/advances_in_deeplearning/

CHAPTER

THIRTYTWO

RECURRENT NEURAL NETWORKS

History doesn't repeat itself, but it often rhymes - Mark Twain

We analyze the application of **Recurrent Neural Networks (RNNs)** for forecasting multivariate time series data, focusing on U.S. Consumer Price Index (CPI) components. RNN models are trained under different configurations to compare their predictive performance and investigate the behavior of their hidden states. Additionally, we examine how the temporal patterns learned by RNNs relate to the latent factors uncovered by Dynamic Factor Models (DFMs), a classical econometric approach for modeling co-movements in time series data.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import matplotlib.pyplot as plt
import gc
import statsmodels.api as sm
import torch
import torch.nn as nn
import torchinfo
from sklearn.preprocessing import StandardScaler
from sklearn.metrics import mean_squared_error
from tqdm import tqdm
from finds.structured import BusDay
from finds.readers import Alfred
from secret import credentials
import warnings
```

```
# %matplotlib qt
VERBOSE = 0
if not VERBOSE:  # Suppress FutureWarning messages
    warnings.simplefilter(action='ignore', category=FutureWarning)
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")

# train-test split date
split_date = '2021-12-01'
```

32.1 Sequence modeling

Recurrent Neural Networks (RNNs) are powerful models for learning from sequential data, enabling tasks such as language modeling, translation, and time series forecasting by maintaining a memory of past inpformation. Unlike feed-forward networks, RNNs process sequences one step at a time, using a hidden state that carries information across time steps to capture temporal dependencies. Variants like Elman and Jordan networks provide basic feedback mechanisms, while more advanced models such as LSTMs and GRUs address training challenges like vanishing gradients. However, RNNs remain inherently sequential, making them slower to train and less parallelizable compared to modern architectures like convolutional or transformer models.

32.1.1 Recurrent units

RNNs retain temporal information through recurrent connections, applying the same computation repeatedly at each time step while maintaining a **hidden state (memory)**. The hidden state h_t is updated based on the current input x_t and the previous hidden state h_{t-1}:

$$h_t = f_h(x_t, h_{t-1}, \theta_h)$$

$$h_t = f_h(x_t, h_{t-1}, \theta_h)$$

where the initial hidden state h_0 is typically initialized to zero, θ_h are learnable parameters, and f_h is the recurrent function (e.g., tanh, ReLU, GRU, LSTM).

Training RNNs is more complex than training feedforward neural networks due to dependencies across time steps.

RNNs are unfolded through time, applying the same parameters (shared weights) at each step, and trained using standard backpropogation, known as **Backpropagation Through Time (BPTT)** when applied over multiple time steps. This makes training computationally expensive, and prone to **vanishing gradients** (gradients shrink) and **exploding gradients** (gradients blow up), which respectively limit the model's ability to learn long-range dependencies and cause unstable updates.

32.1.2 LSTM networks

Long Short-Term Memory (LSTM) networks mitigate the vanishing gradient problem by introducing a **cell state** and multiple **gating mechanisms** that regulate information flow. Key components include:

- Cell state c_t: A "memory" that runs through the network, modified by gates.
- Hidden state h_t: Output of the LSTM at each time step.
- Input x_t: Current input at time t.
- Previous state h_{t-1}, c_{t-1}: From the last time step.
- Gates:
  - Forget gate f_t: Decides what information to discard from the cell state.
  - Input gate i_t: Controls how much new information flows into the cell state.
  - Output gate o_t: Determines what part of the cell state should be output as h_t.

A cell update combines the past cell state and new input to update c_t. The h_t output is updated from c_t and previous its previous state h_{t-1}.

32.1.3 GRU networks

Gated Recurrent Unit (GRU) networks are a simplified version of LSTMs that combine cell state and hidden state into a single vector, using fewer gates:

- Update gate (similar to combined forget/input gate): Controls what part of the past state to keep.
- Reset gate: Controls how to combine new input with the previous memory.
- Single hidden state h_t: Serves as both memory and output.

GRUs are computationally more efficient than LSTM, and perform comparably on many tasks.

32.2 Elman network

An **Elman network** is a basic RNN where recurrence occurs within the hidden layer:

$$h_t = f(x_t, h_{t-1})$$

Multiple Elman layers can be stacked to capture longer-term dependencies and more complex temporal patterns. For every element in the input sequence, each layer computes the following function:

$$h_t = \tanh(x_t\, W_{ih}^T \; + b_{ih} \; + h_{t-1}\, W_{hh}^T \; + b_{hh} \;)$$

where

- h_t is the hidden state at time t,
- x_t is the input at time t,
- h_{t-1} is the hidden state of the previous layer at time $t-1$ or the initial hidden state at time 0, and
- b' s and W' s are the learnable bias and weights

We implement a single-layer Elman RNN using PyTorch' s standard `RNN` module to process sequential data, with `Dropout` regularization to reduce overfitting.

```
class Elman(nn.Module):
    def __init__(self, n_features, hidden_size, dropout, num_layers=1):
        super().__init__()
        self.hidden_size = hidden_size
        self.output_size = n_features
        self.num_layers = num_layers
        self.dropout = nn.Dropout(dropout)
        self.rnn = nn.RNN(input_size=n_features,
                          hidden_size=hidden_size,
                          num_layers=num_layers)
        self.o2o = nn.Linear(hidden_size, n_features)

    def forward(self, x, hidden):
        x = self.dropout(x)      # drop out input layer
        output, hidden = self.rnn(x, hidden)
        output = self.o2o(output[-1:, :])
        return output, hidden

    def init_hidden(self):
        return torch.zeros(self.num_layers, self.hidden_size)
```

32.2.1 Data preparation

CPI time series data for multiple components (e.g., food, housing) are collected from FRED. The data are log-transformed and differenced to ensure stationarity, then standardized (using `StandardScaler`) to have mean 0 and variance 1. The time series data are split into training followed by testing sets using a defined cutoff date.

```
# Max number of hidden states (RNN) or factors (Dynamic Model)
K = 2
```

```
# number of out-of-sample forecasts to predict
nforecast = 3
```

```
# Load time series from FRED
alf = Alfred(api_key=credentials['fred']['api_key'], verbose=VERBOSE)
vspans = alf.date_spans('USREC')  # recession periods
```

```
# CPI for U.S. City Average: Monthly, Seasonally Adjusted
# https://fred.stlouisfed.org/release/tables?rid=10&eid=34483
# 'CUSR0000SEEA'
series_ids = ['CPIFABSL', 'CPIHOSSL', 'CPIAPPSL', 'CPITRNSL', 'CPIMEDSL', 'CPIOGSSL']
df = pd.concat([alf(s, log=1, diff=1) for s in series_ids], axis=1)\
       .dropna()\
       .sort_index()
df.index = BusDay.to_datetime(df.index)
df.index.freq = 'ME'     # set index to datetime type and freq = 'M'
```

```
names = [s[s.find(':')+2:s.find(' in ')] for s in alf.header(series_ids)]
names
```

```
['Food and Beverages',
 'Housing',
 'Apparel',
 'Transportation',
 'Medical Care',
 'Other Goods and Services']
```

```
# Standardize the data data
scaler = StandardScaler().fit(df)
scaled_data = DataFrame(scaler.transform(df), columns=names, index=df.index)
scaled_data
```

```
             Food and Beverages   Housing   Apparel  Transportation  \
date
1967-02-28            -1.520716 -1.129800  0.533990        0.266586
1967-03-31            -0.805655 -1.129800  0.124553       -0.267488
1967-04-30            -1.522780 -0.063784  0.529140        0.263339
1967-05-31            -0.805655 -0.067262  0.122142       -0.003280
1967-06-30             1.339539 -1.129800  0.524347       -0.267488
...                         ...       ...       ...             ...
2024-10-31            -0.252031  0.063171 -2.110286       -0.203592
2024-11-30            -0.053379 -0.037266 -0.114479        0.027062
2024-12-31            -0.180743 -0.209529 -0.015224        0.767819
```

(continues on next page)

(continued from previous page)

```
2025-01-31           0.090331 -0.037301 -3.137367       0.794814
2025-02-28          -0.350673  0.133190  0.934107      -0.605692

            Medical Care  Other Goods and Services
date
1967-02-28     -0.248585                 -1.017195
1967-03-31     -0.253160                 -0.284868
1967-04-30      0.988995                 -1.017195
1967-05-31     -0.266687                 -0.286982
1967-06-30      0.962232                 -0.289083
...                  ...                       ...
2024-10-31     -0.692758                 -0.172972
2024-11-30     -0.702581                  0.085335
2024-12-31     -1.058184                 -1.113129
2025-01-31     -0.701262                 -1.822241
2025-02-28     -0.563596                  0.499572

[697 rows x 6 columns]
```

```python
# Create input data for RNN
ntrain = sum(scaled_data.index < split_date)
ntest = len(scaled_data.index) - ntrain - 1
n_features = scaled_data.shape[1]
data = scaled_data.values
```

32.2.2 Training

RNN models with different hidden sizes ($K = 1, 2$) are trained using an Adam optimizer and a `StepLR` learning rate scheduler, which reduces the learning rate during training for better convergence.

```python
# Train model
num_layers = 1
dropout = 0.0
lr = 0.01         # starting learning rate
step_size = 100   # number of steps per learning rate
num_lr = 3        # number of learning rate periods
num_epochs = step_size * num_lr

train_loss = {}
hidden_states = {}
for hidden_size in range(1, K+1):
    torch.manual_seed(0)
    model = Elman(n_features=n_features,
                  hidden_size=hidden_size,
                  dropout=dropout,
                  num_layers=num_layers).to(device)
    print(model)
    torchinfo.summary(model)

    # Set optimizer and learning rate scheduler
    optimizer = torch.optim.Adam(model.parameters(), lr=lr)
    scheduler = torch.optim.lr_scheduler.StepLR(optimizer,
                                                step_size=step_size,
```

(continues on next page)

(continued from previous page)

```
                                                        gamma=0.1)
    loss_function = nn.MSELoss()
    train_loss[hidden_size] = []
    for epoch in tqdm(range(num_epochs)):   # Run training loop per epoch
        model.train()
        model.zero_grad()
        hidden = model.init_hidden().to(device)
        loss = torch.FloatTensor([0]).to(device)
        for i in range(ntrain):
            x = torch.FloatTensor(data[[i], :]).to(device)
            y = torch.FloatTensor(data[[i+1], :]).to(device)
            output, hidden = model(x, hidden)
            l = loss_function(output, y)
            loss += l
        loss.backward()
        optimizer.step()
        scheduler.step()

        model.eval()
        train_loss[hidden_size].append(float(loss)/ntrain)
        #if VERBOSE:
        #    print(epoch, train_loss[hidden_size][-1], scheduler.get_last_lr())

    # collect predictions and hidden states, and compute mse
    with torch.no_grad():    # reduce memory consumption for eval
        loss_function = nn.MSELoss()
        hidden = model.init_hidden().to(device)
        hidden_states[hidden_size] = [hidden.cpu().numpy().flatten()]
        y_pred = [np.zeros(n_features)]
        for i in range(ntrain + ntest):
            x = torch.FloatTensor(data[[i], :]).to(device)
            y = torch.FloatTensor(data[[i+1], :]).to(device)
            output, hidden = model(x, hidden)
            hidden_states[hidden_size].append(hidden.cpu().numpy().flatten())
            y_pred.append(output.cpu().numpy().flatten())

        # k-step ahead forecast at end of period
        for i in range(nforecast):
            x = y
            y, hidden = model(x, hidden)
            y_pred.append(y.cpu().numpy().flatten())
        print(f"train MSE (hidden={hidden_size}):",
              mean_squared_error(data[1:ntrain+1, :], y_pred[1:ntrain+1]))
        print(f"test MSE (hidden={hidden_size}):",
              mean_squared_error(data[ntrain+1:ntrain+ntest+1, :],
                                 y_pred[ntrain+1:ntrain+ntest+1]))
```

```
Elman(
  (dropout): Dropout(p=0.0, inplace=False)
  (rnn): RNN(6, 1)
  (o2o): Linear(in_features=1, out_features=6, bias=True)
)
```

```
100%|██████████| 300/300 [01:42<00:00,  2.93it/s]
```

```
train MSE (hidden=1): 0.8150888820868842
test MSE (hidden=1): 0.7845472493024133
Elman(
  (dropout): Dropout(p=0.0, inplace=False)
  (rnn): RNN(6, 2)
  (o2o): Linear(in_features=2, out_features=6, bias=True)
)
```

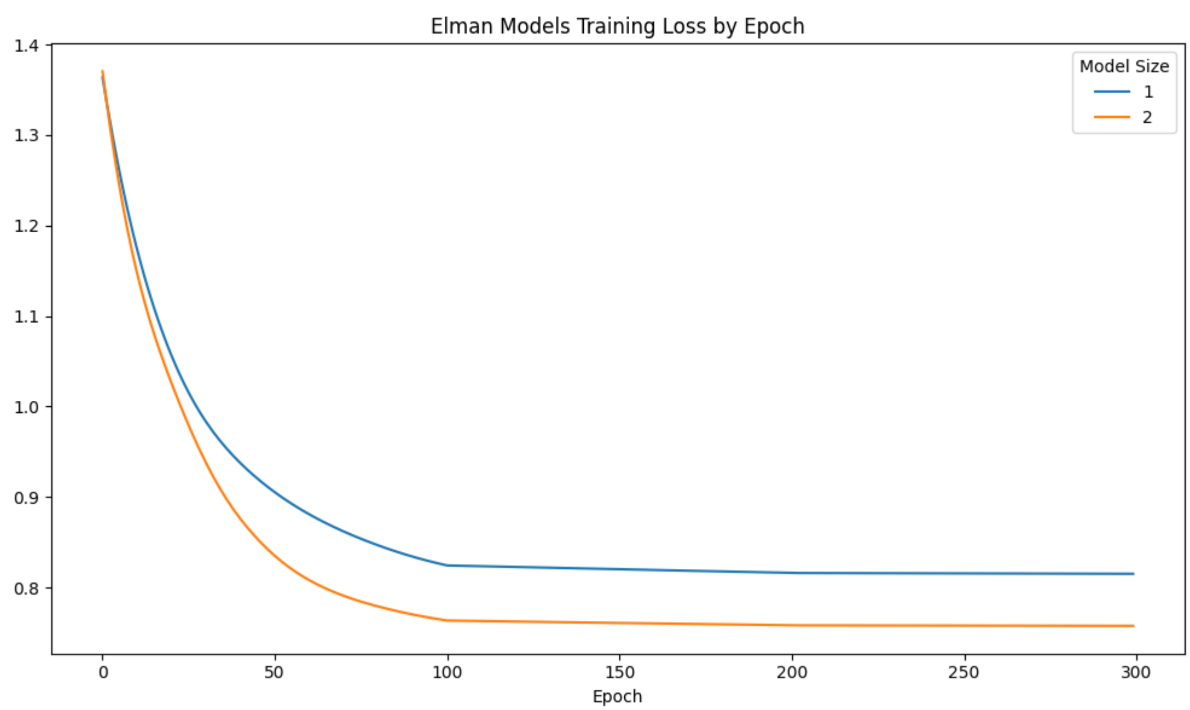
```
100%|██████████| 300/300 [01:38<00:00,  3.04it/s]
```

```
train MSE (hidden=2): 0.757587880335504
test MSE (hidden=2): 0.7211778265327311
```

32.2.3 Evaluation

The training and testing MSE for each RNN model are calculated for performance comparison. Training loss curves across epochs are plotted to assess convergence, and out-of-sample forecasts for 3 time steps ahead, after the end of the sample period, are visualized.

```
fig, ax = plt.subplots(figsize=(10, 6))
DataFrame(train_loss).plot(ax=ax)
ax.set_title(f"Elman Models Training Loss by Epoch")
ax.set_xlabel('Epoch')
ax.legend(title='Model Size')
plt.tight_layout()
```

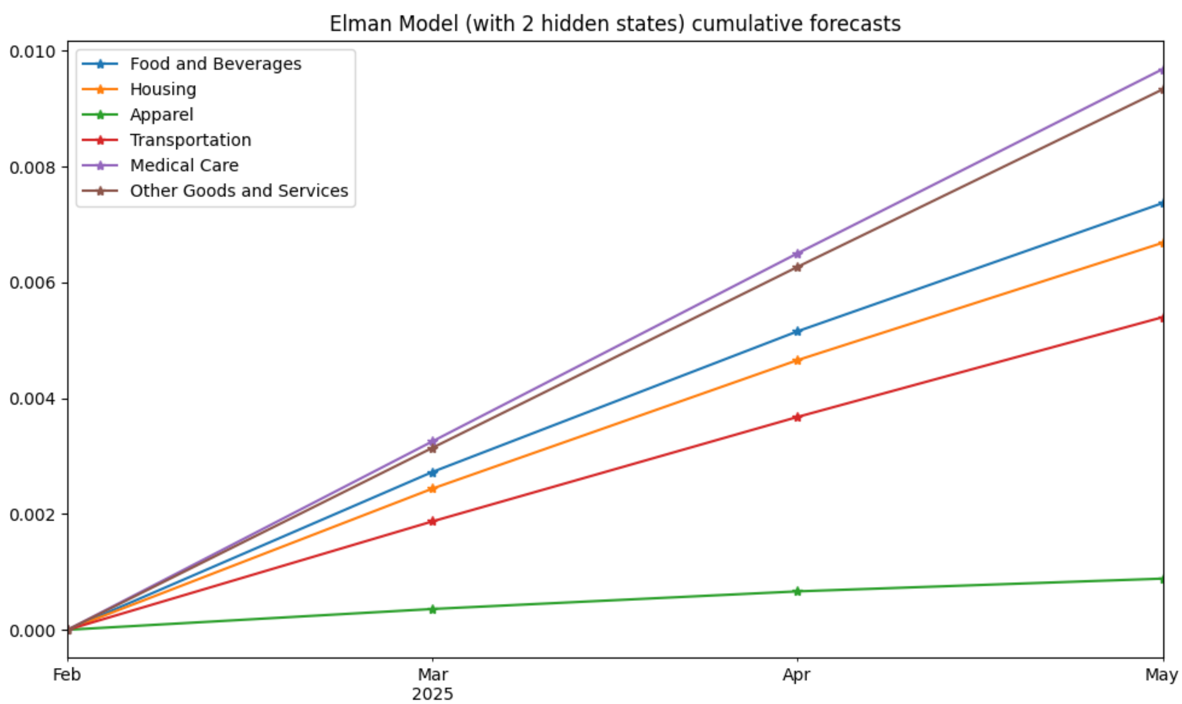


```python
# Show forecasts of last model
pred = scaler.inverse_transform(y_pred)     # undo standardization
t = [scaled_data.index[-1] + pd.DateOffset(months=i) for i in range(nforecast + 1)]
forecasts = DataFrame(np.vstack((np.zeros(n_features), pred[-nforecast:])),
                      index=pd.PeriodIndex(t, freq='M'), columns=scaled_data.columns)
print("Monthly forecasts from RNN Model")
forecasts
```

```
Monthly forecasts from RNN Model
```

```
         Food and Beverages   Housing   Apparel  Transportation  Medical Care  \
2025-02            0.000000  0.000000  0.000000        0.000000      0.000000
2025-03            0.002727  0.002439  0.000359        0.001873      0.003255
2025-04            0.002429  0.002219  0.000304        0.001803      0.003247
2025-05            0.002214  0.002026  0.000220        0.001723      0.003180

         Other Goods and Services
2025-02                  0.000000
2025-03                  0.003144
2025-04                  0.003125
2025-05                  0.003065
```

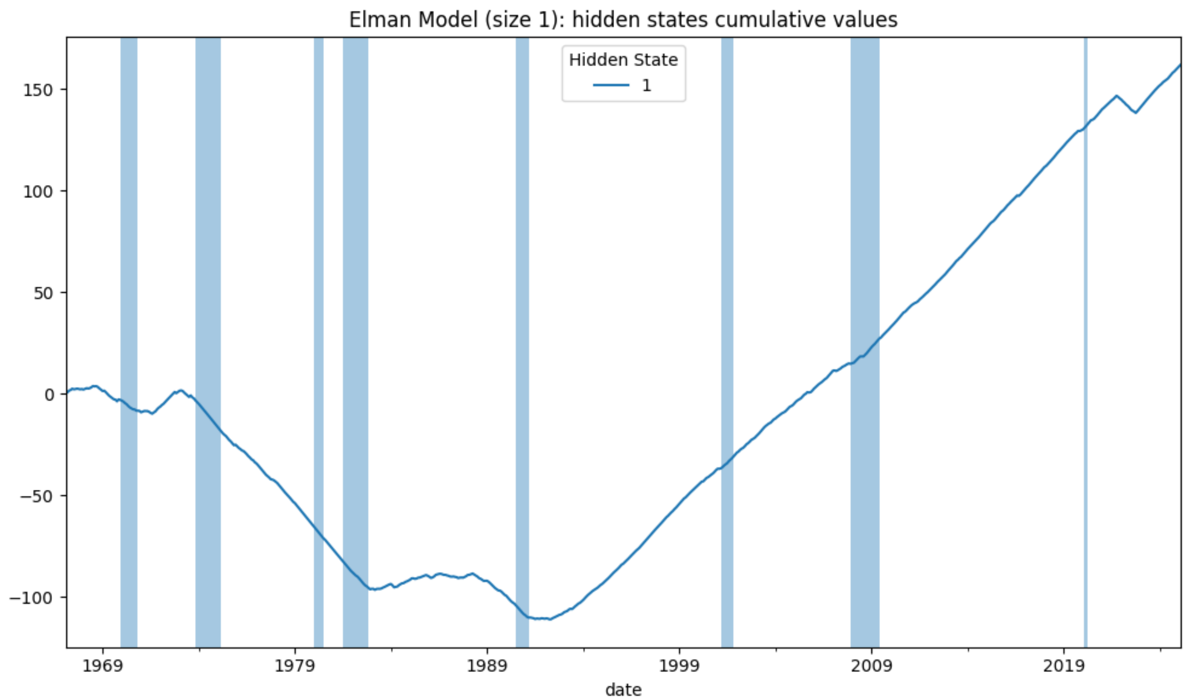
```python
# Plot forecasts
fig, ax = plt.subplots(figsize=(10, 6))
forecasts.cumsum().plot(ax=ax, marker='*')
ax.set_title(f"Elman Model (with {K} hidden states) cumulative forecasts")
plt.tight_layout()
```

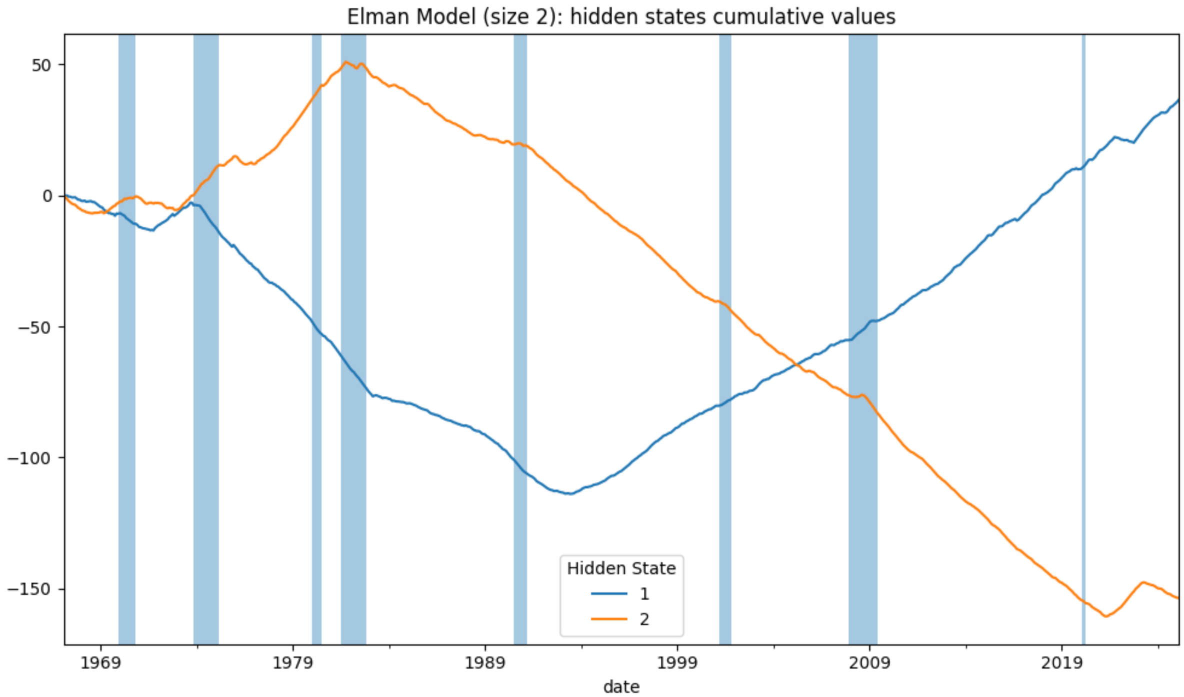


32.2.4 Hidden states

Hidden states from the last training epoch are collected, and their cumulative sums are plotted to illustrate how the internal memory of the Elman RNN evolves over time.

```
# Plot RNN hidden states values
for k, hidden in hidden_states.items():
    fig, ax = plt.subplots(figsize=(10, 6))
    hidden = DataFrame(np.array(hidden), index=scaled_data.index,
                       columns=[f"{i+1}" for i in range(k)])
    hidden.cumsum().plot(ax=ax, style='-')
    for a,b in vspans:
        if a >= min(hidden.index):
            ax.axvspan(a, min(b, max(hidden.index)), alpha=0.4)
    ax.legend(title='Hidden State')
    ax.set_title(f"Elman Model (size {k}): hidden states cumulative values")
    plt.tight_layout()
```





32.3 Dynamic Factor Models

This statistical model captures co-movements in time series using latent factors:

The basic model is:

$$y_t = \Lambda f_t + \epsilon_t$$

$$f_t = A_1 f_{t-1} + \cdots + A_2 f_{t-2} + u_t$$

where:

- y_t is observed data at time t
- ϵ_t is idiosyncratic disturbance at time t
- f_t is the unobserved factor at time t
- $u_t \sim N(0, Q)$ is the factor disturbance at time t
- Λ is referred to as the matrix of factor loadings
- A_i are matrices of autoregression coefficients

We use `DynamicFactorMQ` from `statsmodels`, which employs an **Expectation-Maximization (EM)** algorithm for fitting, and so can accommodate a large number of observed variables. This can handle any collection of blocks of factors, including different factor autoregression orders, and AR(1) processes for idiosyncratic disturbances. The model allows incorporate monthly/quarterly mixed frequency data, making it suitable for **nowcasting**.

- https://www.statsmodels.org/dev/examples/notebooks/generated/statespace_dfm_coincident.html

We fit models with varying lag orders (p) to find an optimal autoregressive structure.

```
# Fit ar lags with best BIC
dynamic_factors = dict()
models = {}
K = 2
for ar in range(1, 5):
    mod = sm.tsa.DynamicFactorMQ(endog=scaled_data,
                                 factors=1,                # num factor blocks
                                 factor_multiplicities=K,  # num factors in block
                                 factor_orders=ar,         # order of factor VAR
                                 idiosyncratic_ar1=True)
    fitted = mod.fit(disp=20 * bool(VERBOSE),
                     maxiter=1000,
                     full_output=True)
    models[ar] = dict(bic=fitted.bic,
                      mse=fitted.mse,
                      summary=fitted.summary().tables[0],
                      predict=fitted.predict(),
                      forecast=fitted.forecast(nforecast),
                      params=len(fitted.param_names))
    dynamic_factors[ar] = DataFrame(fitted.factors.filtered)
    dynamic_factors[ar].columns = np.arange(1, K+1)
    print(DataFrame(dict(bic=fitted.bic,
                         mse=fitted.mse,
                         parameters=len(fitted.param_names)),
                    index=[ar]))
    del fitted
    del mod
    gc.collect()
```

```
            bic       mse  parameters
1  10478.491794  4.275815          31
            bic       mse  parameters
2  10499.374971  4.272672          35
            bic       mse  parameters
3  10506.462669  4.243688          39
            bic       mse  parameters
4  10517.472768  4.231597          43
```

32.3.1 Lag order

Bayesian Information Criterion (BIC) is used to select the optimal lag order

```
# dynamic model with best bic
best, model = min(models.items(), key=lambda item: item[1]['bic'])
mse = mean_squared_error(scaled_data, model['predict'])
print('Best lag:', best, '  bic:', model['bic'])
print(model['summary'])
```

```
Best lag: 1   bic: 10478.49179401509
                                   Dynamic Factor Results
===========================================================================================
Dep. Variable:     "Food and Beverages", and 5 more   No. Observations:                   ␣
↪      697
```

(continues on next page)

(continued from previous page)

```
Model:                      Dynamic Factor Model   Log Likelihood                
↪-5137.771
                          + 2 factors in 1 blocks   AIC                          
↪10337.541
                            + AR(1) idiosyncratic   BIC                          
↪10478.492
Date:                            Sat, 15 Mar 2025   HQIC                         
↪10392.038
Time:                                    05:18:21   EM Iterations                
↪      182
Sample:                                02-28-1967
                                     - 02-28-2025
Covariance Type:                     Not computed
===========================================================================================
```

32.3.2 Evaluation

The models are evaluated based on **train/test MSE**. The 3-step-ahead forecasts, as of the end of the sample period, are also plotted.

```python
# Show prediction errors
print('Dynamic Factor Model Train MSE:',
      mean_squared_error(scaled_data.iloc[1:ntrain+1],
                         model['predict'].iloc[1:ntrain+1]))
print('Dynamic Factor Model Test MSE:',
      mean_squared_error(scaled_data.iloc[ntrain+1:],
                         model['predict'].iloc[ntrain+1:]))
print('Number of parameters:', model['params'])
```

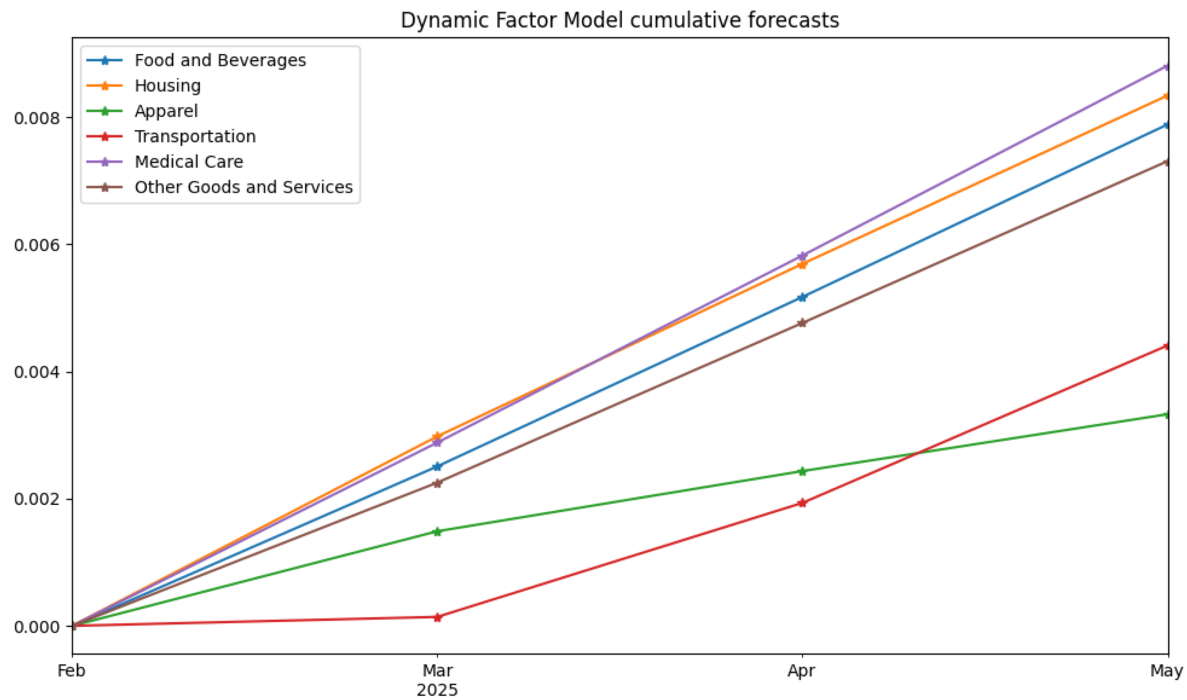
```
Dynamic Factor Model Train MSE: 0.7131933696697343
Dynamic Factor Model Test MSE: 0.7405157478690289
Number of parameters: 31
```

```python
model_out = scaler.inverse_transform(model['forecast'].iloc[:nforecast])
model_out = DataFrame(np.vstack((np.zeros(n_features), model_out)),
                      index=pd.PeriodIndex(t, freq='M'), columns=scaled_data.columns)
print("Monthly forecasts from Dynamic Factor Model")
model_out
```

```
Monthly forecasts from Dynamic Factor Model
```

```
         Food and Beverages   Housing   Apparel  Transportation  Medical Care  \
2025-02            0.000000  0.000000  0.000000        0.000000      0.000000
2025-03            0.002508  0.002980  0.001488        0.000141      0.002883
2025-04            0.002666  0.002714  0.000947        0.001794      0.002942
2025-05            0.002714  0.002649  0.000894        0.002477      0.002987

         Other Goods and Services
2025-02                  0.000000
2025-03                  0.002253
2025-04                  0.002512
2025-05                  0.002546
```

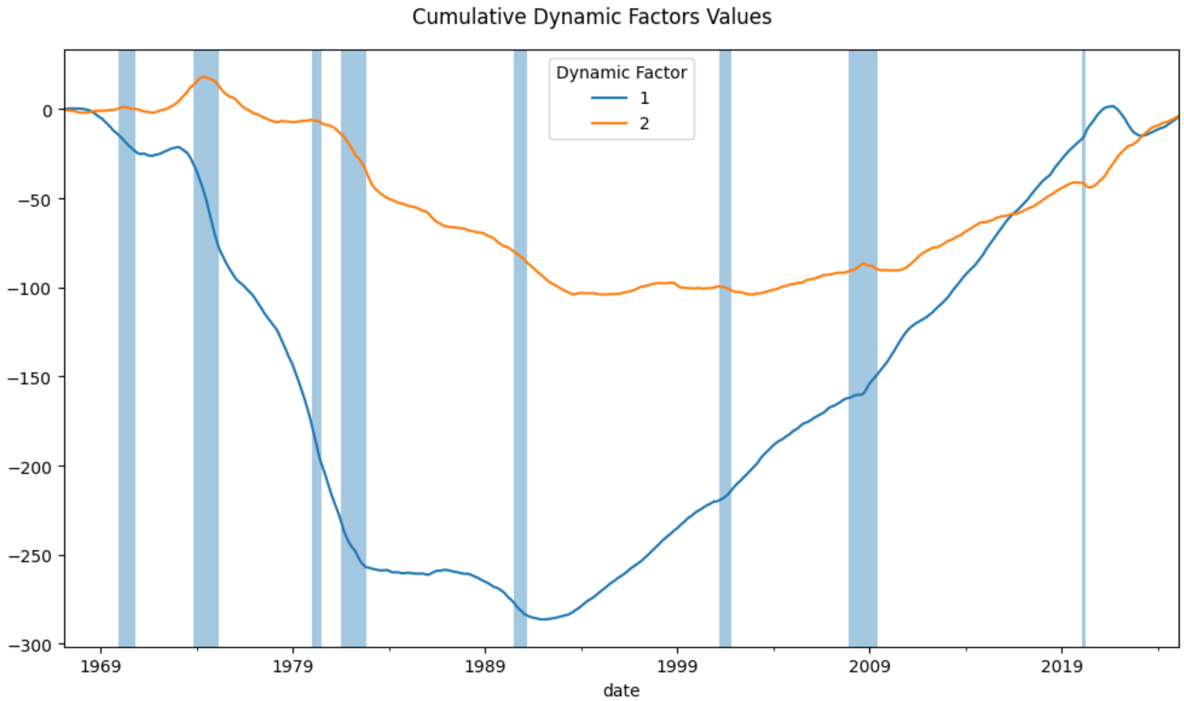
```
# Plot forecasts
fig, ax = plt.subplots(figsize=(10, 6))
model_out.cumsum().plot(ax=ax, marker='*')
ax.set_title(f"Dynamic Factor Model cumulative forecasts")
plt.tight_layout()
```



32.3.3 Dynamic factors

Dynamic factors represent unobserved common drivers of the time series, and their cumulative sums highlight underlying trends or cycles. In macroeconomic datasets, these latent factors often correspond to broad economic forces or business cycles. These cumulative factors are plotted to visualize their temporal patterns.

```
# Plot dynamic factors
dynamic_factor = dynamic_factors[best]
fig, ax = plt.subplots(figsize=(10, 6))
dynamic_factor.cumsum().plot(ax=ax, style='-')
ax.legend(title='Dynamic Factor')
for a,b in vspans:
    if a >= min(dynamic_factor.index):
        ax.axvspan(a, min(b, max(dynamic_factor.index)), alpha=0.4)
plt.suptitle(f"Cumulative Dynamic Factors Values")
plt.tight_layout()
```



The hidden states from the simple Elman RNN are compared to DFM latent factors, and with R-squared (R^2) statistics reported to measure the degree of overlap.

```
# RNN hidden state values explained by dynamic factors
rsq = dict()
for k, hidden_state in enumerate(np.array(hidden_states[2]).T):
    rsq[k+1] = sm.OLS(hidden_state, sm.add_constant(dynamic_factor).fillna(0))\
                 .fit()
print('Proportion of variance of RNN hidden state values '
      'explained by dynamic factors:',
      np.mean([r.rsquared for r in rsq.values()]).round(4))
DataFrame({k: [r.rsquared, r.f_pvalue] for k, r in rsq.items()},
          index=['R-square', 'pvalue'])\
          .rename_axis(columns='Hidden State')\
          .round(4)
```

```
Proportion of variance of RNN hidden state values explained by dynamic factors: 0.
↪7471
```

```
Hidden State        1        2
R-square       0.7494   0.7447
pvalue         0.0000   0.0000
```

References:

Philipp Krähenbühl, 2020-2024, “AI394T Deep Learning course materials” , retrieved from https://www.philkr.net/dl_class/material

CHAPTER

THIRTYTHREE

REINFORCEMENT LEARNING

I have not failed. I' ve just found 10,000 ways that won' t work - Thomas A. Edison

We combine financial modeling with reinforcement learning (RL) to evaluate static and adaptive retirement spending strategies. Traditional approaches, such as the **4% rule**, assume fixed withdrawal rates and asset allocations, that may not adapt well to changing market conditions. Leveraging historical economic data, simulations, and deep learning techniques, we apply RL to learn dynamic strategies which adjust asset allocations based on financial conditions, minimizing the risk of retirees outliving their savings.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
from pandas import DataFrame, Series
import pandas as pd
import numpy as np
import math
import random
import matplotlib.pyplot as plt
import seaborn as sns
from tqdm import tqdm
import numpy as np
import gymnasium as gym
from gymnasium import spaces
from stable_baselines3 import DQN
from typing import List, Tuple
from finds.database import SQL
from finds.structured import BusDay
from finds.utils import Store, subplots, set_xticks
import torch
from secret import credentials, paths
store = Store(paths['scratch'])
#pd.set_option('display.max_rows', None)
VERBOSE = 0
gym.logger.min_level = gym.logger.ERROR  # Suppress warnings
```

```
# open connections
sql = SQL(**credentials['sql'], verbose=VERBOSE)
bd = BusDay(sql, verbose=VERBOSE)
outdir = paths['scratch'] / 'RL'
```

33.1 Retirement spending policy

A **retirement spending policy** guides how retirees withdraw funds from their savings to sustain their lifestyle while minimizing the risk of depleting assets. The key elements include the **withdrawal strategy**, which defines the initial withdrawal rate (e.g., the 4% rule) and its adjustments over time (e.g., inflation-linked or dynamic withdrawals); **asset allocation**, which balances stocks, bonds, and other investments to optimize growth and risk; **time horizon**, representing the expected duration of withdrawals, typically spanning 20-30 years or more; and **market conditions**, including interest rates, inflation, and asset returns, which influence portfolio sustainability. Effective policies seek to balance spending needs, longevity risk, and market fluctuations to minimize the risk of outliving their assets.

Benz, Ptak and Rekenthaler (2022) found that "For retirees who seek a fixed real withdrawal from their portfolio in retirement, a starting withdrawal rate of 3.8% is safe in Morningstar's model over a 30-year time horizon, assuming a 90% success rate (defined here as a 90% likelihood of not running out of funds) and a balanced portfolio."

33.1.1 SBBI data

The **Stocks, Bonds, Bills, and Inflation (SBBI)** dataset provides historical monthly, quarterly, and yearly total returns and yields for major U.S. asset classes, including large-cap stocks, small-cap stocks, corporate bonds, government bonds, and inflation. This data, which dates back to 1926, is commonly used for retirement portfolio simulations.

```python
# Read sbbi data
sbbi_file = paths['data'] / 'SBBI/stocks-bonds-bills-and-inflation-data.xlsx'
df = pd.read_excel(sbbi_file,
                   sheet_name=0,
                   skiprows=list(range(9)) + [10],
                   header=0,
                   usecols='A,B,G,P',
                   index_col=0)
columns_official = df.columns.tolist()
df.columns = ['stocks', 'bonds', 'inflation']
df.index = bd.to_date(df.index)
df
```

```
            stocks     bonds  inflation
19260131  0.000000  0.013756   0.000000
19260228 -0.038462  0.006313   0.000000
19260331 -0.057471  0.004129  -0.005587
19260430  0.025305  0.007589   0.005618
19260531  0.017918  0.001412  -0.005587
...            ...       ...        ...
20240831  0.024257       NaN   0.000814
20240930  0.021357  0.100107   0.001604
20241031 -0.009069 -0.046509   0.001151
20241130  0.058701  0.068325  -0.000542
20241231 -0.023838 -0.044901   0.000355

[1188 rows x 3 columns]
```

33.1.2 Scenario generator

To create realistic economic simulations, we extract different 30-year retirement periods from the 100-year span of historical data. These scenarios help model how varying economic conditions impact retirement outcomes.

```
# Scenario generator: episode, backtest a sample path
class Episodes:
    def __init__(self, data: DataFrame, T: int, num_loops: int = 1):
        self.data = np.log(1 + data)
        self.high = list(self.data.max() + self.data.std()*.1)
        self.low = list(self.data.min() - self.data.std()*.1)
        self.T = T              # number of years per episode
        self.M = (T + 1) * 12   # number of monthly observations per episode
        self.num_loops = num_loops

    def __len__(self):
        return (len(self.data) - self.M + 1) * self.num_loops

    def __iter__(self):
        rows = []
        for i in range(self.num_loops):
            rows += np.random.permutation(len(self.data) - self.M + 1).tolist()
        for t in rows:
            df = self.data.iloc[t:(t + self.M), :].reset_index(drop=True)
            yield df.groupby(df.index // 12)\
                    .sum()\
                    .set_index(self.data.index[(t + 11):(t + self.M):12])
```

Summary statistics of all 30-year episodes:

```
T = 30
means = {s: [] for s in df.columns}
episodes = Episodes(df, T=T)
for episode in iter(episodes):
    for s in df.columns:
        means[s].append(episode[s].mean())
print('30-year sample periods:')
DataFrame({s: np.mean(x) for s, x in means.items()} | {'N': len(episodes)},
          index=['annualized mean'])
```

```
30-year sample periods:
```

	stocks	bonds	inflation	N
annualized mean	0.105185	0.05502	0.03604	817

We assess the effectiveness of a fixed annual withdrawal strategy by analyzing its performance under different conditions. Key inputs for such strategies include: ortfolio asset allocation (e.g., stock/bond mix); market environment (e.g., past returns and inflation rates); and expected duration of withdrawals.

For example, a typical rule of a fixed 4% withdrawal rate (adjusted for inflation) for a 50% stock / 50% bond portfolio can be evaluated across rolling 30-year periods to estimate the likelihood that a retiree’s savings will last the full duration.

The `Base` model is a simple **buy-and-hold** strategy where funds are invested in a fixed allocation at the start of retirement and are not rebalanced, even if allocation weights drift over time. Withdrawals remain fixed as a percentage of initial wealth, adjusted for inflation.

```
class BaseModel:
    """Buy-and-hold allocation model where assets weights drift from initial"""

    name = 'Buy-and-Hold'
    def __init__(self, T: int, W: List[float]):
        assert W[-1] > 0          # spend must be positive
        self.initial = dict(T=int(T), W=np.array(W))

    def reset(self, market_changes: List[float]):
        self.T = int(self.initial["T"])
        self.W = np.array(self.initial["W"])
        return list(market_changes)

    def step(self, action: List, market_changes: List) -> Tuple:
        assert self.T > 0
        self.W[:-1] = np.array(action).flatten() * self.W[:-1].sum() # rebalance
        self.W = self.W * np.exp(np.array(market_changes))           # price changes
        self.T = self.T - 1
        wealth = sum(self.W[:-1])  # remaining wealth
        if wealth < self.W[-1]:
            truncated = True          # is not enough for spend
            self.W[:-1] = 0.0
        else:
            truncated = False         # is sufficient for spend
            spend = self.W[-1] * self.W[:-1] / wealth  # allocate and deduct spending
            self.W[:-1] = self.W[:-1] - spend
        terminated = not self.T or truncated
        return terminated, truncated

    def predict(self, obs: List) -> List:
        """Allow initial asset allocation to drift"""
        return self.W[:-1] / sum(self.W[:-1])
```

The Fixed model adds annual rebalancing, ensuring that the portfolio maintains a constant stock/bond allocation throughout retirement.

```
class FixedModel(BaseModel):
    """Allocation model where assets are rebalanced to fixed weights"""

    name = 'Annual-Rebalance'
    def predict(self, obs: List) -> List:
        """Action to rebalance asset allocation to fixed initial weight"""
        return self.initial['W'][:-1] / sum(self.initial['W'][:-1])
```

To assess the risk of retirees running out of money, we simulate rolling 30-year periods and track shortfalls. Our starting scenarios assume a 50/50 stock-bond allocation and a 4% inflation-adjusted withdrawal rule. The probability of shortfall measures the fraction of simulations where assets were depleted before reaching 30 years.

```
alloc = 50    # 50-50 stocks/bonds initial allocation
rule = 4.0    # 4 percent spending policy
model = BaseModel(T=T, W=[alloc, 100-alloc, rule])
```

```
result = {}
for n, episode in enumerate(iter(episodes)):
    obs = model.reset(episode.iloc[0])
```

(continues on next page)

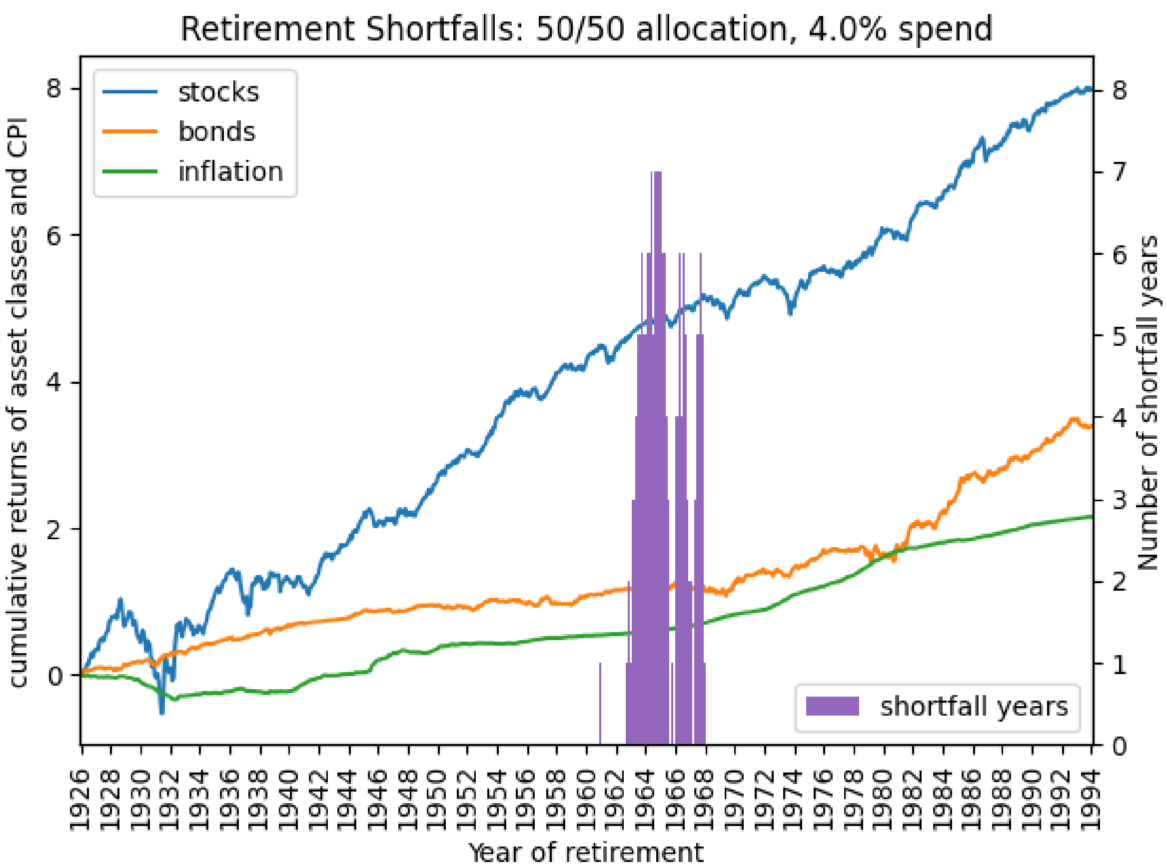
(continued from previous page)

```
    for year in episode.index[1:]:
        obs = episode.loc[year].to_list()
        action = model.predict(obs)
        terminated, truncated = model.step(action, obs)
        if truncated:
            break
    result[episode.index[0]] = model.T
prob = np.mean(np.array(list(result.values())) != 0)
print('Number of 30-year scenerios:', len(episodes))
print('Probability of shortfall:    ', round(prob, 4))
```

```
Number of 30-year scenerios: 817
Probability of shortfall:     0.0747
```

The graph illustrates the years in which retirees ran out of money, with bar heights representing the number of years their assets fell short of the goal. Additionally, plots of compounded asset returns and inflation highlight how periods of lower investment returns and higher inflation increased the risk of shortfall.

```
fails = Series(result).sort_index().rename('shortfall_years')
market = df.reindex(fails.index).cumsum()
fails.index = fails.index.astype(str).str.slice(0,4)
market.index = fails.index
ax = market.plot()
ax.set_title(f"Retirement Shortfalls: {alloc}/{100-alloc} allocation, {rule}% spend")
ax.set_ylabel('cumulative returns of asset classes and CPI')
ax.set_xlabel('Year of retirement')
ax.legend(loc='upper left')
##
bx = ax.twinx()
fails.plot(kind='bar', width=1.0, color='C4', ax=bx)
bx.set_ylabel('Number of shortfall years')
bx.legend(['shortfall years'], loc='lower right')
set_xticks(ax=ax, nskip=23, rotation=90)

plt.tight_layout()
```



33.1.3 Historical simulations

Next, we expanded the analysis to include a broader range of asset allocations (0% to 100% in stocks) and withdrawal rates (3% to 5%) to evaluate their impact on portfolio longevity.

```
# range of spending policy rules
rules = np.arange(3, 5.1, 0.1)

# simulate fixed and initial equity allocations from 0 to 100%
allocs = np.arange(0, 105, 5)
```

```
TAIL = 0.95
tail = int(100 * (1 - TAIL))
def compute_shortfall(x, q):
    """Compute average number of years of shortfall given tail probability level, q"""
    x = sorted(x)    # each simulation's results (years of shortfall)
    q = int(q * len(x))
    return x[q], np.mean(x[q:])
```

```
for num, Model in enumerate([BaseModel, FixedModel]):
    fail = DataFrame(columns=rules, index=allocs, dtype=float)
    shortfall = DataFrame(columns=rules, index=allocs, dtype=float)
    quantile = DataFrame(columns=rules, index=allocs, dtype=float)
```

(continues on next page)

(continued from previous page)

```
    for alloc in tqdm(allocs):
        for rule in rules:

            # Evaluate for this allocation strategy and spending policy
            model = Model(T=T, W=[alloc, 100-alloc, rule])
            result = []
            for n, episode in enumerate(iter(episodes)): # for every 30-year sample
                obs = model.reset(episode.iloc[0])
                for year in episode.index[1:]:
                    obs = episode.loc[year].to_list()
                    action = model.predict(obs)
                    terminated, truncated = model.step(action, obs)
                    if truncated:
                        break
                result.append(model.T)
            fail.loc[alloc, rule] = np.mean(np.array(result) != 0)
            quantile.loc[alloc, rule], shortfall.loc[alloc, rule] = compute_
↪shortfall(result, TAIL)
    store[model.name] = dict(fail=fail, shortfall=shortfall)
```

```
100%|██████████| 21/21 [07:33<00:00, 21.57s/it]
100%|██████████| 21/21 [07:32<00:00, 21.57s/it]
```

We compare the proportion of scenarios where the `Base` buy-and-hold strategy performed better or worse than the `Fixed` strategy with annual rebalancing.

```
#for model in [BaseModel, FixedModel]:
#    fail = store[model.name]['fail']
#    print(f"Probability of Shortfall: with {model.name} allocation")
#    print(fail.iloc[::-1, :].round(2).to_string())
print('Buy-and-hold outperformed Annual-Rebalance:',
      round(np.mean(store[FixedModel.name]['fail'] > store[BaseModel.name]['fail']),␣
↪3))
print('Annual-Rebalance outperformed Buy-and-Hold:',
      round(np.mean(store[FixedModel.name]['fail'] < store[BaseModel.name]['fail']),␣
↪3))
```

```
Buy-and-hold outperformed Annual-Rebalance: 0.374
Annual-Rebalance outperformed Buy-and-Hold: 0.297
```

33.1.4 Risk of spending shortfall

To measure the likelihood and severity of depleting funds before the end of retirement, we use two key metrics:

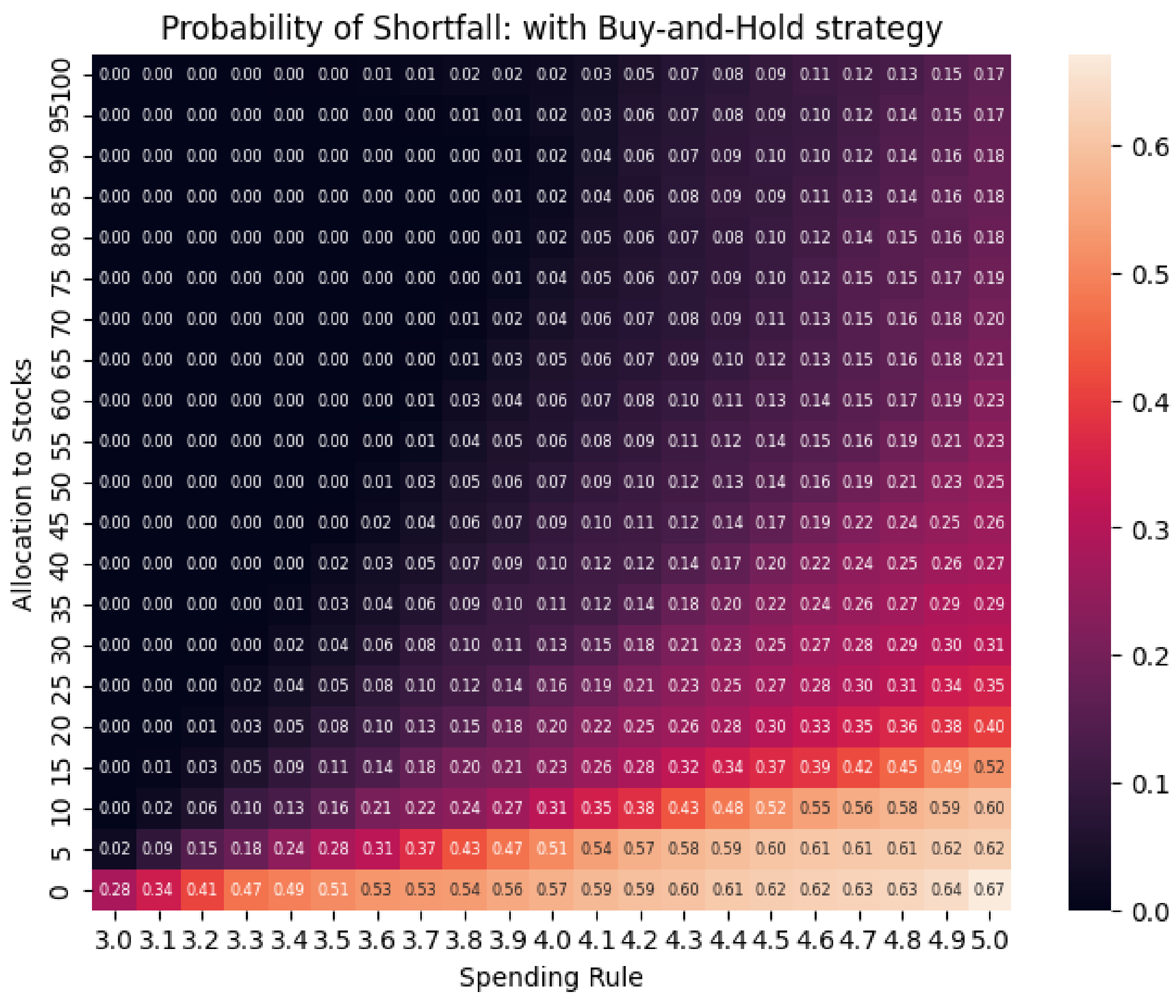
- **Probability of shortfall:** The percentage of simulations where retirees outlived their assets.
- **Expected shortfall period:** This metric quantifies the severity of shortfalls by estimating how many years retirees would be without funds in the worst-case scenarios (e.g., the worst 5% of simulations).

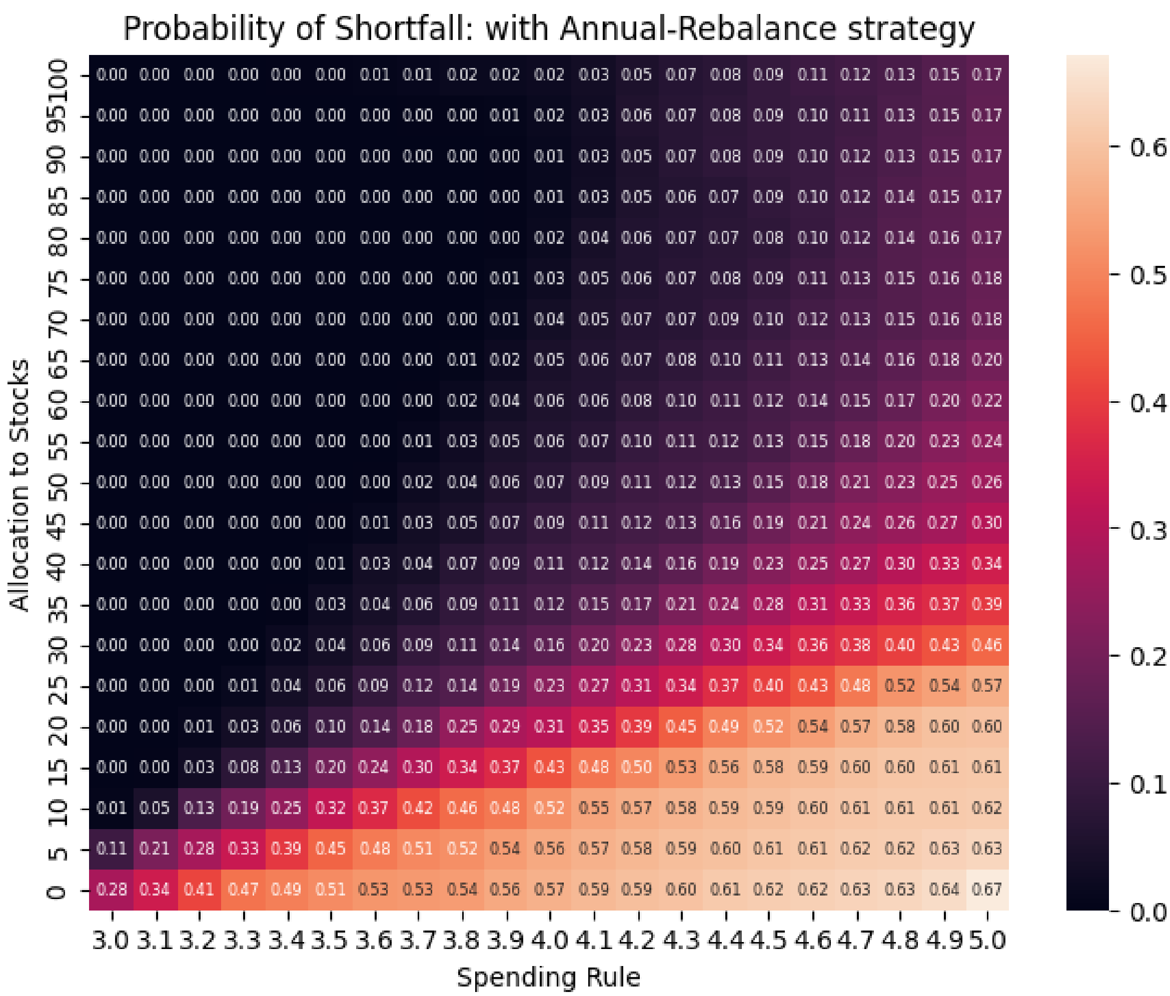
```
for model in [BaseModel, FixedModel]:
    fail = store[model.name]['fail'].iloc[::-1, :]
    plt.figure(figsize=(8, 6))
```

(continues on next page)

(continued from previous page)

```
    sns.heatmap(fail, annot=True, cbar=True, fmt='.2f', annot_kws=dict(fontsize='xx-
↪small'),
                xticklabels=np.round(fail.columns, 1), yticklabels=fail.index)

    # Labels and title
    plt.xlabel("Spending Rule")
    plt.ylabel("Allocation to Stocks")
    plt.title(f"Probability of Shortfall: with {model.name} strategy")
    plt.show()
```





Probability of shortfall:

Contour lines in the probability of shortfall graph show the combinations of asset allocation and withdrawal rates that result in the same likelihood of running out of money.

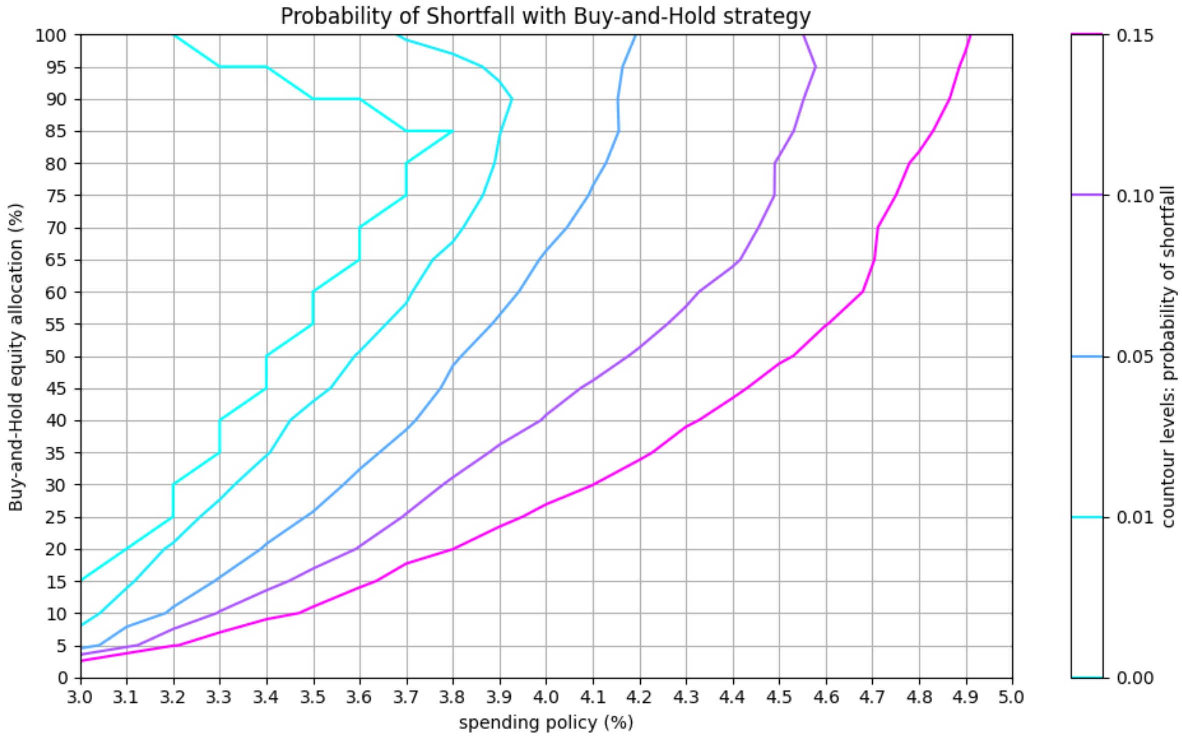
```python
def plot_contour(Z, levels, title, label):
    """Helper to plot contour lines at given levels"""
    X, Y = np.meshgrid(rules, allocs)
    fig, ax = plt.subplots(figsize=(10, 6))
    cp = ax.contour(X, Y, Z, levels=levels, cmap='cool')
    ax.set_title(f"{title} with {model.name} strategy")
    ax.set_xticks(rules)
    ax.set_xlabel('spending policy (%)')
    ax.set_yticks(allocs)
    ax.set_ylabel(f'{model.name} equity allocation (%)')
    ax.grid(which='both')
    fig.colorbar(cp, label=label)
    plt.tight_layout()
```

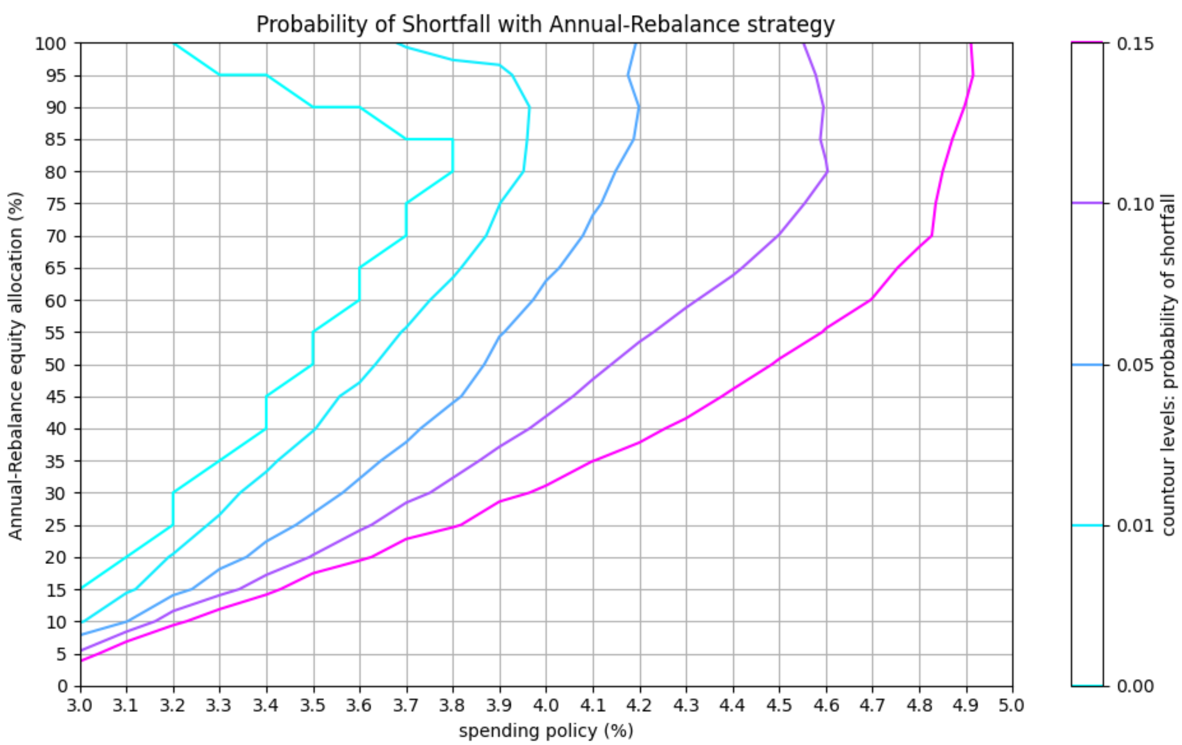
```python
for model in [BaseModel, FixedModel]:
    fail = store[model.name]['fail']
    plot_contour(fail, levels=[0.0, .01,.05,.1,.15],
```

(continues on next page)

(continued from previous page)

```
                title="Probability of Shortfall",
                label="countour levels: probability of shortfall")
```





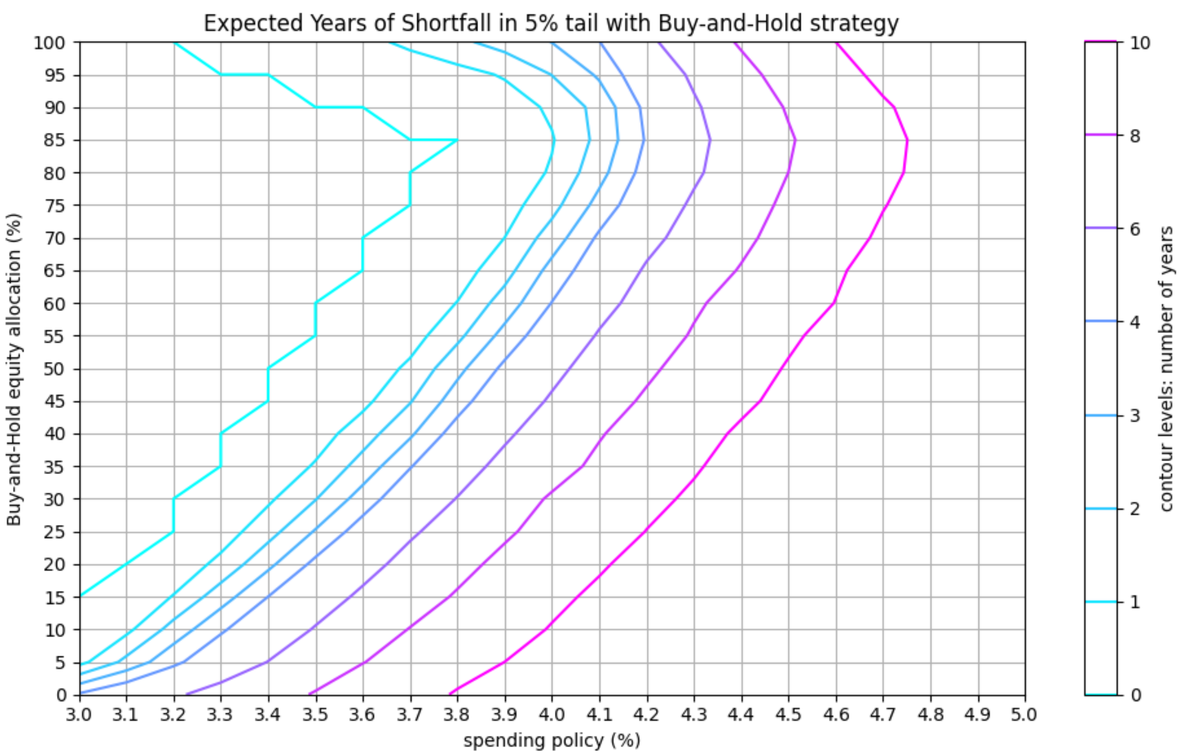
Expected shortfall period:

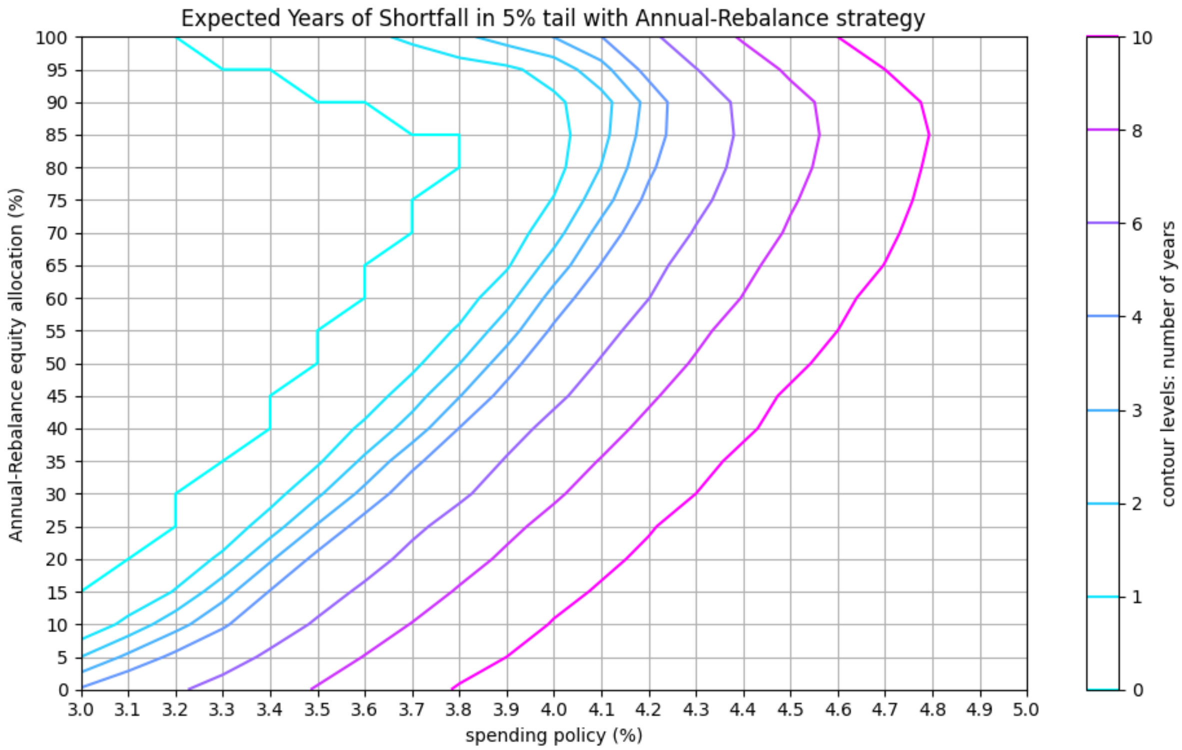
This metric estimates the potential duration of financial shortfalls, which helps retirees plan for worst-case scenarios.

```
#for model in [BaseModel, FixedModel]:
#    shortfall = store[model.name]['shortfall']
#    print(f"Expected Years of Shortfall in {tail}% tail: (model.name) allocation")
#    print(shortfall.iloc[::-1, :].round(1).to_string())
print('Buy-and-hold outperformed Annual-Rebalance:',
      round(np.mean(store[FixedModel.name]['shortfall'] > store[BaseModel.name][
 ↪'shortfall']), 3))
print('Annual-Rebalance outperformed Buy-and-Hold:',
      round(np.mean(store[FixedModel.name]['shortfall'] < store[BaseModel.name][
 ↪'shortfall']), 3))
```

```
Buy-and-hold outperformed Annual-Rebalance: 0.059
Annual-Rebalance outperformed Buy-and-Hold: 0.61
```

```
for model in [BaseModel, FixedModel]:
    shortfall = store[model.name]['shortfall']
    plot_contour(shortfall, levels=[0, 1, 2, 3, 4, 6, 8, 10],
                 title=f"Expected Years of Shortfall in {tail}% tail",
                 label="contour levels: number of years")
```





33.2 Deep reinforcement learning

Unlike static asset allocation models, **reinforcement learning (RL)** can learn optimal strategies that adapt to market conditions and remaining wealth.

33.2.1 Gymnasium environment

The `Gymnasium` (formerly OpenAI Gym) library provides a Pythonic interface for reinforcement learning problems. `Stable Baselines3` (SB3) implements a set of RL algorithms in PyTorchf OpenAI's Gym library.

https://gymnasium.farama.org/index.html

https://github.com/DLR-RM/stable-baselines3

33.2.2 State space

During training, the RL model learns to predict optimal spending actions based on the current financial state. The **state space** includes:

- current wealth and allocation,
- recent market (equity and bonds) and inflation changes
- spending amount
- years since retirement

33.2.3 Actions

Exploitation: Selects the action with the highest expected value, based on past training data (e.g., Q-learning, SARSA).

Exploration: Tries alternative strategies to discover better long-term policies.

33.2.4 Reward function

- If assets are depleted, the model applies a severe penalty of -100 times the square of remaining years: $-(100T^2)$
- If wealth is positive, the reward is proportional to the wealth-to-spending coverage ratio: $\sqrt{\frac{W}{S(T+1)}}$

```
FAIL = 100    # failure reward factor
class CustomEnv(gym.Env):
    """Custom gymnasium environment, using Episodes scenario generator"""
    def __init__(self, model: BaseModel, episodes: Episodes):
        super().__init__()
        self.model = model            # for stepping through a 30-year episode
        self.episodes = episodes      # for generating a sample 30-year episode
        self.iterator = iter(self.episodes)  # iterator to reset a 30-year sample
        T = self.model.initial['T']
        W = self.model.initial['W']
        low = np.array([0] + episodes.low + [0]*len(W))
        high = np.array([T] + episodes.high + [T]*len(W))
        self.observation_space = spaces.Box(low=low, high=high)
        self.action_space = spaces.Discrete(21)
        #self.action_space = spaces.Box(low=0.0, high=1.0)

    def reset(self, seed=0):
        super().reset(seed=seed)

        # generate a fresh 30-year episode
        self.episode = next(self.iterator, None)
        if self.episode is None:
            self.iterator = iter(self.episodes)
            self.episode = next(self.iterator)
        self.n = 0

        # return initial observations
        deltas = self.model.reset(self.episode.iloc[self.n])
        obs = [self.model.T] + list(deltas) + self.model.W.tolist()
        return obs, {}


    def step(self, action):
        S = self.model.W[-1]              # amount to spend at t-1
        W = np.sum(self.model.W[:-1])     # wealth at t-1
        T = self.model.T                  # year remaining till termination

        # Convert action to asset allocation weights
        action = action * 0.05
        action = np.array([action, 1-action])

        # Grab next market move at time t
        self.n = self.n + 1
        deltas = self.episode.iloc[self.n].tolist()
```

(continues on next page)

(continued from previous page)

```
        # Apply rebalance wealth and market move (t=1)
        terminated, truncated = self.model.step(action, deltas)

        # Calculate reward (t=1)
        reward = -(T*T*FAIL) if truncated else math.sqrt(W / (S*T))

        # Return as next observation
        obs = [T] + list(deltas) + self.model.W.tolist()
        return obs, reward, terminated, truncated, {}
```

Helpers to evaluate trained model

```
def evaluate(env, model, episodes):
    """Return success likelihood, shortfalls and asset allocation actions"""
    result = []
    actions = {t: [] for t in range(episodes.T + 1)}  # to store predicted actions
    for n, episode in enumerate(iter(episodes)):
        obs, info = env.reset()
        terminated = False
        while not terminated:
            action, _states = model.predict(np.array(obs))
            actions[env.model.T].append(float(action))
            obs, rewards, terminated, truncated, info = env.step(action)
        result.append(env.model.T if truncated else 0)
    #print(n, truncated, action, rewards, env.model.W)
    return np.mean(np.array(result) != 0), *compute_shortfall(result, TAIL), actions
```

33.2.5 Deep Q-Network (DQN)

We use **Deep Q-Networks (DQN)** from `Stable Baselines3`, which is designed for discrete action spaces. This approach employs **deep reinforcement learning** to maximize wealth longevity while adapting asset allocation strategies dynamically.

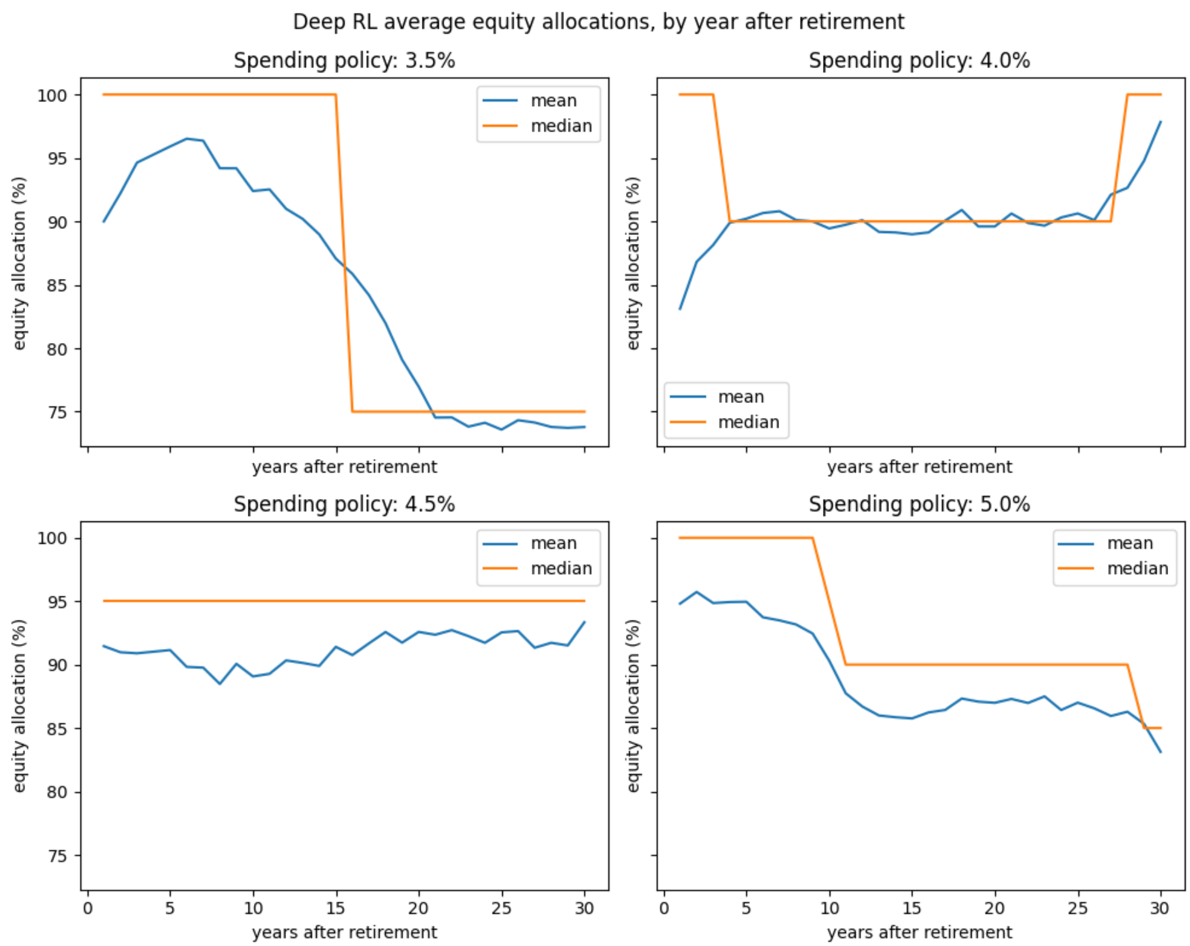
```
TIMESTEPS = int(5e5)
initial_alloc = 50
fail, actions, shortfall, quantile = {}, {}, {}, {}
for rule in tqdm(rules):  # train a model for each spending rule

    # define and train model for this spending rule
    name = str(round(rule, 1))
    W = [initial_alloc, 100-initial_alloc, rule]
    env = CustomEnv(model=BaseModel(T=T, W=W), episodes=episodes)
    clf = DQN('MlpPolicy', env, verbose=VERBOSE)
    clf.learn(total_timesteps=TIMESTEPS)
    clf.save(f"{outdir}/{name}")

    # evaluate model
    test_clf = clf.load(f"{outdir}/{name}", env=None)
    fail[name], quantile[name], shortfall[name], actions[name] =\
        evaluate(env, test_clf, episodes)

    store['dqn'] = dict(fail=fail, shortfall=shortfall,
                        quantile=quantile, actions=actions)
```

```
100%|███████████| 21/21 [2:00:19<00:00, 343.79s/it]
```

```
print(DataFrame(fail, index=["Deep RL"]).round(2).to_string())
```

```
          3.0   3.1   3.2   3.3   3.4  3.5  3.6   3.7   3.8   3.9   4.0   4.1   4.2  ␣
↪4.3  4.4   4.5   4.6   4.7   4.8   4.9   5.0
Deep RL  0.0   0.0   0.0   0.0   0.01  0.0  0.0  0.01  0.02  0.02  0.03  0.06  0.06  0.
↪08  0.1  0.11  0.14  0.12  0.16  0.21  0.19
```

```
# Plot average allocations over time
labels = list(np.arange(T) + 1)
fig, axs = subplots(nrows=2, ncols=2, figsize=(10, 8), sharex=True, sharey=True)
plt.suptitle('Deep RL average equity allocations, by year after retirement')
for ax, rule in zip(axs, ["3.5", "4.0", "4.5", "5.0"]):
    y = [[a*5 for a in actions[rule][i]] for i in labels]
    mean = [np.mean(a) for a in y]
    median = [np.median(a) for a in y]
    ax.plot(labels, mean, label='mean')
    ax.plot(labels, median, label='median')
    ax.set_title(f"Spending policy: {rule}%")
    ax.set_xlabel('years after retirement')
    ax.set_ylabel('equity allocation (%)')
    ax.legend()
plt.tight_layout()
```



References:

Richard S. Sutton and Andrew G. Barto, 2018, "Reinforcement Learning: An Introduction", MIT Press.

Christine Benz, Jeffrey Ptak John Rekenthaler, Dec. 12, 2022, "The State of Retirement Income: 2022. A look at how higher bond yields, lower equity valuations, and inflation affect starting safe withdrawal rates", Morningstar Portfolio and Planning Research.

CHAPTER THIRTYFOUR

LANGUAGE MODELING

Attention is all you need - Vaswani et al

Transformers, built on the **attention** mechanism, are neural network models designed to process variable-length sequences and capture complex dependencies in language without relying on recurrence or convolution. By leveraging self-attention, multi-head attention, and positional encodings, transformers can model long-range relationships between words for tasks like text generation, translation, and summarization. We apply transformer-based models to language modeling of Federal Reserve meeting minutes, introducing perplexity as a key evaluation metric and exploring decoding strategies such as nucleus sampling to generate coherent and diverse text.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
from typing import Callable, List
import math
import numpy as np
import pandas as pd
from pandas import Series, DataFrame
import bisect
import matplotlib.pyplot as plt
from nltk.tokenize import wordpunct_tokenize as tokenize
import torch
import torch.nn as nn
import torch.nn.functional as F
import torch.optim as optim
from torch.optim import Adam
from torch.optim.lr_scheduler import StepLR
from torch.utils.data import Dataset, DataLoader
import torchinfo
from tqdm import tqdm
from finds.database.mongodb import MongoDB
from finds.unstructured import Unstructured, Vocab
from secret import credentials, paths
# %matplotlib qt
VERBOSE = 0
```

```
mongodb = MongoDB(**credentials['mongodb'], verbose=VERBOSE)
fomc = Unstructured(mongodb, 'FOMC')
outdir = paths['scratch']
device = 'cuda' if  torch.cuda.is_available() else 'cpu'
print('Device:', device)
```

```
Device: cuda
```

34.1 Transformers

Transformers are a neural network architecture built entirely on attention mechanisms, designed to process variable-length sequential data without relying on recurrence (as in RNNs) or convolution (as in CNNs). They are especially effective for natural language processing (NLP) tasks such as language modeling, translation, and text generation.

Traditional models like RNNs and CNNs face limitations when processing language data. RNNs are sequential, making them slow to train and difficult to parallelize, and they suffer from vanishing gradients. CNNs, while powerful for structured patterns like images, are less suited for variable-length, syntactically complex language sequences.

Because language involves variable lengths, hierarchical syntax, and long-range dependencies, attention allows the model to focus on relevant parts of the input when generating outputs. To introduce positional information, since Transformers lack an inherent sense of sequential order, positional encodings to provide information about work positions are added to the token embeddings. In autoregressive tasks, **causal masks** are applied to ensure that each token only attends to previous tokens, not future ones.

34.1.1 Attention mechanism

Attention is a mechanism that enables models to reason about a **set of elements and their relationships**, dynamically weighting the importance of different parts of the input.

Attention is essentially a set operator designed to reason about a set of elements and their relationships, dynamically weighting the importance of different parts of the input when producing each element of the output sequence. Unlike RNNs, attention does not require sequential processing, making it highly parallelizable and efficient for sequence modeling.

The inputs to the attention operator are called:

- Queries (Q): What we' re looking for.
- Keys (K): Labels that help identify relevant content.
- Values (V): The actual content or information to aggregate.

In **self-attention**, these inputs are all derived from the same input, allowing each input element to attend to every other element. Each input token is represented by three vectors: *query*, *key*, and *value*. Attention scores are computed as a scaled dot product between queries and keys, and these scores weight the values to produce a contextualized representation:

$$\text{Attention}(Q, K, V) = \text{Softmax}\left(\frac{QK^\top}{\sqrt{C}}\right)V$$

Learnable weight matrices W_Q, W_K, W_V enable queries, keys, and values, respectively, to adapt to different input patterns:

$$\text{Attention}(X; W_Q, W_K, W_V) = \text{Attention}(XW_Q, XW_K, XW_V)$$

where X is the embedded input.

Cross-attention is a mechanism used in encoder-decoder transformer architectures for tasks such as machine translation, where queries (Q) come from one source (typically the decoder) and keys (K) and values (V) come from another source (typically the encoder output)..

Multi-head attention runs multiple independent attention layers (heads) in parallel. Each head learns different ways of attending to the input, capturing different aspects of the relationships. Heads are concatenated and linearly projected back to the output dimension.

34.1.2 Masked attention

Auto-regressive prediction is used for sequence generation tasks such as text completion and translation. To ensure predictions are causal (based only on past and current information), causal masks hide future tokens when computing attention, The mask is just a upper-triangular matrix applied to the attention scores to block future tokens, ensuring that each word can only attend to itself and earlier words. This prevents the model from "cheating" by looking at future tokens when generating or predicting a sequence, preserving the causal structure required for tasks like language modeling and text completion.

34.1.3 Positional encoding

Since Transformers treat inputs as sets of tokens without inherent order, **positional encodings** provide sequence information. These are added to token embeddings to enable the model to understand word positions.
Types of positional encodings include:

- Absolute Positional Embeddings: Add a fixed position index to each input token. Although simple and straightforward, this method is not generalizable to longer sequences than seen during training.
- Relative Positional Embeddings: encode pairwise distances between tokens. These relative distances are bounded and reusable, hence independent of the total length of the sequence.
- Sinusoidal Positional Embeddings: Predefined using sine and cosine functions of different frequencies. This can capture relative position information which generalizes to sequences longer than trained on. $\mathrm{PE}(n, 2i) = \sin\left(\frac{n}{10000^{2i/C}}\right), \quad \mathrm{PE}(n, 2i+1) = \cos\left(\frac{n}{10000^{2i/C}}\right)$
- Rotary Positional Embeddings (RoPE): Combines both absolute and relative positional information through a rotation operation, which extrapolates well to longer contexts and is widely adopted in large language models (LLMs).
- Learnable Positional Embeddings: Initialized randomly and learned during training like token embeddings. This fully flexible, but performance may degrade if sequence length varies significantly between training and testing.

```
class PositionalEncoding(nn.Module):
    """Positional encoder, learned with an embeddings layer"""
    def __init__(self, d_model: int, max_len: int, dropout: float= 0.0):
        super().__init__()
        self.dropout = nn.Dropout(dropout)
        self.emb = nn.Embedding(num_embeddings=max_len, embedding_dim=d_model)

    def forward(self, x):
        """
        Args:
            x: Tensor, shape [seq_len, batch_size, embedding_dim]
        """
        to_embed = torch.LongTensor(np.asarray(range(0, x.size(1))))\
                        .to(x.device)
        embedded = self.emb(to_embed)
        embedded = self.dropout(embedded)
        return x + embedded.unsqueeze(0)

    """ Alternate positional encodings with sine function
    def __init__(self, d_model: int, max_len: int, dropout: float = 0.1):
        super().__init__()
        self.dropout = nn.Dropout(p=dropout)
        position = torch.arange(0, max_len, dtype=torch.float).unsqueeze(1)
        div_term = torch.exp(torch.arange(0, d_model, 2)
                             * (-math.log(10000.0) / d_model))
        pe = torch.zeros(max_len, d_model)
```

(continues on next page)

(continued from previous page)

```
        pe[:, 0::2] = torch.sin(position * div_term)
        pe[:, 1::2] = torch.cos(position * div_term)
        pe = pe[:, None, :]
        self.register_buffer('pe', pe)

    def forward(self, x):
        x = x + self.pe[:x.size(1), 0, :]
        return self.dropout(x)
    """
```

34.1.4 Transformer layers

A transformer-based neural network is built from repeated blocks of Transformer **layers**, each consisting of:

- Multi-Head Attention: Combines several attention "heads" that learn different relationships between tokens. Each head performs: $Attention$(Q, K, V) = \text{softmax}\left(\frac{QK^\top}{\sqrt{C}}\right) V$ and the final output is concatenated and linearly projected back to match input dimensions.
- Feedforward Neural Network: Applies a two-layer fully connected network (MLP) with non-linearity (typically ReLU) in between: MLP(x) = \text{ReLU}(\text{Linear}_1(x)) \to \text{Linear}_2$$
- Residual Connections: Directly connect input and output of each sub-layer.
- Layer Normalization: Normalizes inputs within each layer.

The input sentence is split into parts (characters, words, or "tokens"). The model takes token embeddings with positional encodings, applies layers of attention and MLPs, and outputs contextualized representations of each token.

```
class Transformer(nn.Module):
    """Transformer neural network"""
    def __init__(self, seq_len: int, vocab_size: int, d_model: int, nhead: int,
                 num_layers: int, dim_feedforward: int, dropout: float):
        super().__init__()

        # model dimensions
        self.seq_len = seq_len
        self.vocab_size = vocab_size
        self.d_model = d_model

        # define layers
        self.embedding = nn.Embedding(num_embeddings=vocab_size,
                                      embedding_dim=d_model)
        self.positional = PositionalEncoding(max_len=seq_len,
                                             d_model=d_model,
                                             dropout=dropout)

        layer = nn.TransformerEncoderLayer(d_model=d_model,
                                           nhead=nhead,
                                           dim_feedforward=dim_feedforward,
                                           dropout=dropout,
                                           batch_first=True)
        self.encoder = nn.TransformerEncoder(encoder_layer=layer,
                                             num_layers=num_layers)
        self.decoder = nn.Linear(in_features=d_model,
                                 out_features=vocab_size)
```

(continues on next page)

(continued from previous page)

```
        # initialize weights
        self.embedding.weight.data.uniform_(-0.1, 0.1)
        self.decoder.weight.data.uniform_(-0.1, 0.1)
        self.decoder.bias.data.zero_()

    def causal_mask(self, sz: int, device: str = 'cpu'):
        """returns upper triu set to -inf"""
        return nn.Transformer.generate_square_subsequent_mask(sz=self.seq_len,
                                                              device=device)

    def forward(self, x):
        if len(x.shape) == 1:
            x = x[None, :]
        assert x.size(-1) == self.seq_len
        x = self.embedding(x) * math.sqrt(self.d_model)  # embedding
        x = self.positional(x)                           # position encoding
        x = self.encoder(x, mask=self.causal_mask(sz=len(x), device=x.device))
        x = self.decoder(x)                              # linear layer
        x = F.log_softmax(x, dim=-1)                     # classify
        return x

    def save(self, filename):
        """save model state to filename"""
        return torch.save(self.state_dict(), filename)

    def load(self, filename):
        """load model name from filename"""
        self.load_state_dict(torch.load(filename, map_location='cpu'))
        return self
```

34.2 Language modeling

Language modeling is the task of estimating the probability distribution over word sequences. By learning this distribution from large text corpora, models capture linguistic structure, enabling downstream tasks like translation and text generation.

34.2.1 Perplexity

Accuracy (measuring whether the predicted word is exactly correct) is not a meaningful metric for language models. Predicting the exact next word in a sequence is highly uncertain and difficult, so accuracy would be very low even for strong models. Instead, we care about how well the language model assigns probability distributions over possible next words. Perplexity quantifies how well the it predicts the test set, calculated as the exponential of the average negative log likelihood over the test set:

$$\text{Perplexity} = \exp\left(-\frac{1}{N}\sum_{i=1}^{N}\log P(w_i|w_{i-1},\ldots,w_{i-n+1})\right)$$

- N is the total number of words in the test set.
- $P(w_i|w_{i-1},\ldots,w_{i-n+1})$ is the probability assigned by the language model to the word w_i given its context $w_{i-1},\ldots,w_{i-n+1}$.

Intuitively, perplexity measures how surprised the model is by the text. It can be interpreted as the geometric mean of the inverse probabilities assigned by the model, hence lower perplexity indicates better model performance and generalization.

```
def get_next_log_probs(model, context: List[str], unk=UNK):
    """log P(word | context) where word ranges over the vocab"""
    # pad to length seq_len
    if len(context) > model.seq_len:
        context = context[-model.seq_len:]
    elif len(context) < model.seq_len:
        context = ([unk] * (model.seq_len - len(context))) + context
    context = torch.LongTensor(vocab.get_index(context))\
                   .to(device)\
                   .unsqueeze(0)
    output = model(context)
    logits = output[0, -1, :]
    return logits.cpu().detach().numpy()
```

```
def get_perplexity(model, sentence: List[str]) -> float:
    """Compute perpexlity score"""

    context = [UNK] + sentence
    log_probs = 0
    for i in range(len(context) - len(sentence), len(context)):
        log_probs += get_next_log_probs(model=model, context=context[:i])[vocab.get_
↪index(context[i])]
    return np.exp(-log_probs / len(sentence))
```

34.2.2 Fedspeak

The Fed has a jargon all of its own, with Alan Blinder coining the term *Fedspeak* to describe the "turgid dialect of English" used by Federal Reserve Board chairs. We explore language modeling of minutes text from all FOMC meetings since 1993.

The text data are tokenized and converted into indices in a `Vocab` object. PyTorch `Dataset` and `DataLoader` tools simplify the processing of chunks and batches of the data. The most recent document is held-out from training to serve as the test set for perplexity evaluation.

```
# Retrieve and preprocess FOMC minutes text
dates = fomc['minutes'].distinct('date')        # check dates stored in MongoDB
docs = Series({doc['date']: [w.lower() for w in tokenize(doc['text'])]
               for doc in fomc.select('minutes')},
              name='minutes').sort_index()
UNK = " "
vocab = Vocab(set().union(*docs.tolist()), unk=UNK)
print(f"{len(vocab)=}, {len(docs)=}: {min(dates)}-{max(dates)}")
```

```
len(vocab)=8675, len(docs)=256: 19930203-20250129
```

```
# Pytorch Dataset and DataLoader
class FOMCDataset(Dataset):
    """Subclass of torch Dataset

    Notes:
```

(continues on next page)

(continued from previous page)

```
      All subclasses should overwrite __getitem__(),
      supporting fetching a data sample for a given key. Subclasses
      could also optionally overwrite __len__(), which is expected to
      return the size of the dataset
    """
    def __init__(self, text: Series, seq_len: int, get_index: Callable[[str], int]):
        self.text = text
        self.seq_len = seq_len
        self.get_index = get_index
        self.counts = np.cumsum([len(s) // seq_len for s in text])

    def __len__(self):
        return self.counts[-1]

    def __getitem__(self, idx):
        assert 0 <= idx < len(self), "idx out of range"
        doc = bisect.bisect_right(self.counts, idx)

        start = (idx - (self.counts[doc-1] if doc > 0 else 0)) * self.seq_len
        end = start + self.seq_len
        chunk = self.text.iloc[doc][start:end]
        return (torch.LongTensor([0] + self.get_index(chunk[:-1])),
                torch.LongTensor(self.get_index(chunk)))

# length of input sequence
seq_len = 30

# split last document to be test set
test_len = 1
test_set = docs.iloc[-test_len:].tolist()
train_set = FOMCDataset(docs.iloc[:-test_len], seq_len, vocab.get_index)
dataloader = DataLoader(train_set, batch_size=32, shuffle=True)
DataFrame({'docs': len(docs)-test_len, 'chunks': len(train_set)}, index=['Train'])
```

```
       docs  chunks
Train   255   54877
```

Create the model:

```
# Create the model
lr = 0.0001
step_size = 30
num_epochs = 100 #step_size * 1
```

```
d_model = 512 #512
nhead = 4 # 4
num_layers = 3 # 2
dim_feedforward = 2048 # 512 #1024
dropout = 0.3 # 0.3 # 0.2
```

```
model = Transformer(seq_len=seq_len,
                    vocab_size=len(vocab),
                    d_model=d_model,
                    nhead=nhead,
```

(continues on next page)

(continued from previous page)

```
                    num_layers=num_layers,
                    dim_feedforward=dim_feedforward,
                    dropout=dropout).to(device)
torchinfo.summary(model)
```

```
==========================================================================================
Layer (type:depth-idx)                                                  Param #
==========================================================================================
Transformer                                                             --
├─Embedding: 1-1                                                        4,441,600
├─PositionalEncoding: 1-2                                               --
│    └─Dropout: 2-1                                                     --
│    └─Embedding: 2-2                                                   15,360
├─TransformerEncoder: 1-3                                               --
│    └─ModuleList: 2-3                                                  --
│    │    └─TransformerEncoderLayer: 3-1                                3,152,384
│    │    └─TransformerEncoderLayer: 3-2                                3,152,384
│    │    └─TransformerEncoderLayer: 3-3                                3,152,384
├─Linear: 1-4                                                           4,450,275
==========================================================================================
Total params: 18,364,387
Trainable params: 18,364,387
Non-trainable params: 0
==========================================================================================
```

Train the model:

```
# Specify training parameters
criterion = nn.NLLLoss().to(device)
optimizer = Adam(model.parameters(), lr=lr)
scheduler = StepLR(optimizer, step_size=step_size, gamma=0.1)
```

```
perplexity = []
losses = []
for epoch in tqdm(range(num_epochs)):
    model.train()
    for train_ex, target_ex in dataloader:
        optimizer.zero_grad()
        train_ex, target_ex = train_ex.to(device), target_ex.to(device)
        output = model(train_ex)
        loss = criterion(output.view(-1, len(vocab)), target_ex.view(-1))
        loss.backward()
        optimizer.step()
    scheduler.step()

    # Evaluate perplexity on test set
    model.eval()
    perplexity.append(np.mean([get_perplexity(model, s) for s in test_set]))
    losses.append(loss.item())
    if VERBOSE:
        print(f"Epoch: {epoch}, Loss: {loss.item()}, Perplexity: {perplexity[-1]}")
model.save(outdir / f"transformer{nhead}_{dim_feedforward}.pt")
```

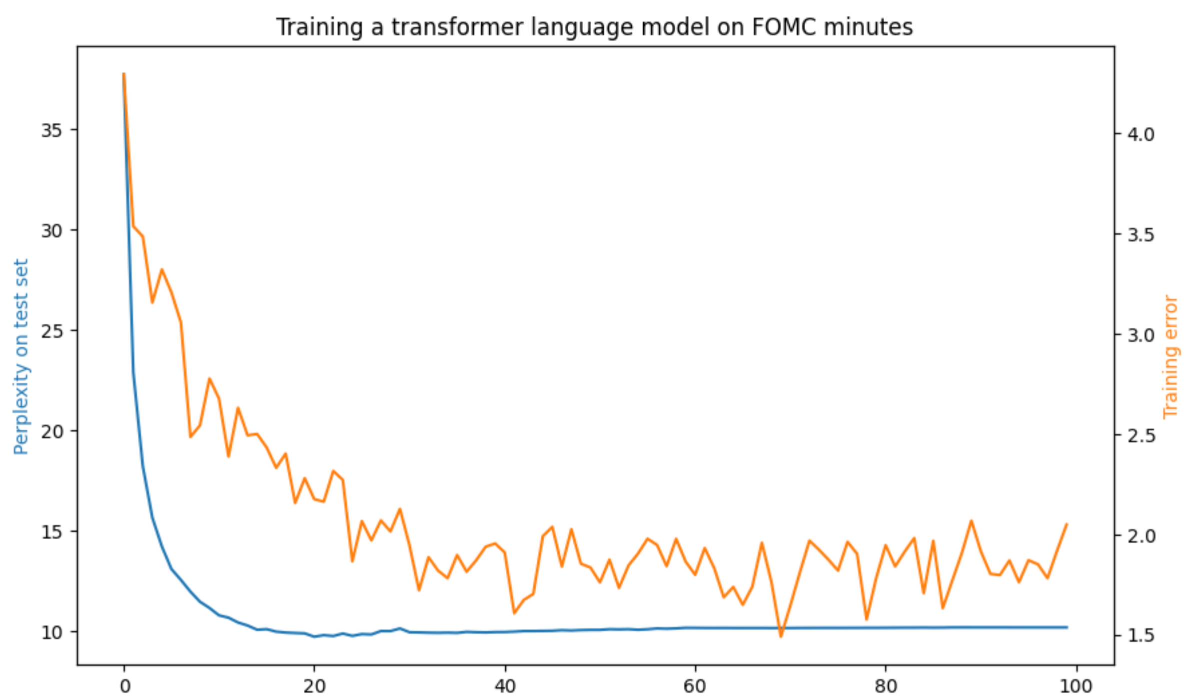
```
100%|██████████| 100/100 [1:03:53<00:00, 38.33s/it]
```

```python
# save model checkpoint
import warnings
with warnings.catch_warnings():
    warnings.filterwarnings('ignore')  ## ignore the weights_only=True future warning
    model.load(outdir / f"transformer{nhead}_{dim_feedforward}.pt")
```

Evaluate the model:

```python
# Plot perplexity
fig, ax = plt.subplots(figsize=(10,6))
ax.plot(perplexity, color="C0")
ax.set_ylabel('Perplexity on test set', color="C0")
bx = ax.twinx()
bx.plot(losses, color="C1")
bx.set_ylabel('Training error', color="C1")
plt.title('Training a transformer language model on FOMC minutes')
print('Perplexity:', perplexity[-1], '  Loss:', losses[-1])
```

```
Perplexity: 10.185370762878241   Loss: 2.0497496128082275
```



34.2.3 Decoding

Decoding refers to the process of generating a sequence of words based on learned probabilities. Language models generate text by sampling from a probability distribution over the next word $P(y_i|y_1, ..., y_{i-1})$, given previous words:

- **Greedy** approach: At each step of generation, the word with the highest probability according to the model is selected as the next word. While simple and computationally efficient, this results in repetitive or less diverse outputs.
- **Beam search** maintains a fixed number (beam width) of partial candidate sequences of words.At each step, it expands all possible next words for each candidate, keeping the top k based on their joint probabilities. This allows exploration of multiple promising paths, but can be computationally expensive, and may still produce suboptimal outputs due to early pruning.
- **Nucleus Sampling** samples from the smallest set of k words whose cumulative probability mass exceeds a predefined threshold p. This approach promotes diversity in generated text by allowing for the possibility of sampling from a larger set of words.

```
def get_nucleus_sequence(model, n: int, p: float, context: List[str] = []):
    """Sample sequence of words given context using nucleus sampling"""
    if not context:
        context = [UNK]

    for i in range(n):
        probs = np.exp(get_next_log_probs(model, context))
        probs_sorted = sorted(probs, reverse=True)
        probs_cum = np.cumsum(probs_sorted)
        num_drop = sum(probs_cum > p)
        threshold = probs_sorted[-num_drop]
        probs[probs < threshold] = 0.
        probs /= sum(probs)
        choice = vocab.get_word(np.random.choice(len(probs), p=probs))
        context.append(choice)
        #print(i, drop, len(probs), len(probs_sorted))
    return context
```

```
import textwrap
wrapper = textwrap.TextWrapper(width=80, fix_sentence_endings=True)
```

Finally, nucleus sampling with $p = 0.95$ is used to generate new text conditioned on starting contexts, balancing diversity and coherence.

```
n, p = seq_len * 4, 0.95
for context in ['the financial markets' , 'participants noted that']:

    # generate from context with nuclear sampling
    words = get_nucleus_sequence(model, n=n, p=p, context=context.split())

    # pretty-print the output
    out = ''
    is_end = True
    is_space = ''
    for w in words:
        if not w.isalnum():
            out += w
        else:
            if is_end:
```

(continues on next page)

(continued from previous page)

```
                w = w.capitalize()
            out += is_space + w
        is_end = w in ['!', '?', '.']
        is_space = ' '*bool(w not in ["'", '-', '-'])
    print(f"{context.upper()}...")
    print(wrapper.fill(out))
    print()
```

```
THE FINANCIAL MARKETS...
The financial markets.  In addition, the tga and the resulting decline in the
soma portfolio would result in a combination of shifts in the composition of
reserve liabilities, and a waning volume of credit allocation liquidity.  In
that regard, the appropriate course of monetary policy, a number of participants
noted that purchases of longer-term securities were faced by the likely onset of
the financial crisis in mid-december.  Labor market conditions improved further
in january but expanded modestly on balance over the intermeeting period.
Consumer price inflation— as measured by the 12-month percentage change in the
price index for personal consumption expenditures( pce)— was elevated in march

PARTICIPANTS NOTED THAT...
Participants noted that recent indicators and orders pointed to somewhat more
moderate expansion of spending for equipment and software.  The nominal deficit
on u.  S. Trade in goods and services was significantly larger in the third
quarter than in the previous quarter.  The value of exports of goods and
services also increased considerably in july, with increases widespread by
categories.  Imports of services rose more than exports.  The increase in
imports was concentrated in consumer goods, however, consumer goods, and
services, which decreased exports of capital goods.  Imports of services in july
and august were expanding briskly; the gains were concentrated in industrial
supplies, semiconductors, and services.
```

References:

https://pytorch.org/tutorials/beginner/transformer_tutorial.html

Jay Alammar, The Illustrated Transformer, retrieved from https://jalammar.github.io/illustrated-transformer/

Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N. Gomez, Lukasz Kaiser, and Illia Polosukhin. 2017. Attention Is All You Need. https://arxiv.org/abs/1706.03762

Greg Durrett, 2021-2024, "CS388 Natural Language Processing course materials", retrieved from https://www.cs.utexas.edu/~gdurrett/courses/online-course/materials.html

Philipp Krähenbühl, 2020-2024, "AI394T Deep Learning course materials", retrieved from https://www.philkr.net/dl_class/material and https://ut.philkr.net/deeplearning/

CHAPTER

THIRTYFIVE

LARGE LANGUAGE MODELS

I didn't have time to write a short letter, so I wrote a long one instead - Mark Twain

We introduce large language models (LLMs) through a financial natural language processing (NLP) task: summarizing the *Quantitative and Qualitative Disclosures About Market Risk* sections of 10-K reports. To assess performance, we compare the overlap and readability of summaries generated by GPT-4o-mini, a proprietary closed-source model, and DeepSeek-R1-14B, an open-source model that can be downloaded and run locally. Small language models, particularly those trained using techniques like **distillation**, can closely approximate the performance of larger models while offering lower latency and reduced memory requirements.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import textwrap
from pprint import pprint
from rouge_score import rouge_scorer
from tqdm import tqdm
from finds.database import SQL, RedisDB
from finds.unstructured import Edgar
from finds.structured import BusDay, CRSP, PSTAT
from finds.readers import Sectoring
from secret import paths, credentials
VERBOSE = 0
```

```
sql = SQL(**credentials['sql'], verbose=VERBOSE)
user = SQL(**credentials['user'], verbose=VERBOSE)
bd = BusDay(sql)
rdb = RedisDB(**credentials['redis'])
crsp = CRSP(sql, bd, rdb, verbose=VERBOSE)
pstat = PSTAT(sql, bd, verbose=VERBOSE)
ed = Edgar(paths['10X'], zipped=True, verbose=VERBOSE)
```

35.1 OpenAI GPT models

Large language models are built using transformer-based deep learning architectures and pre-trained on massive text corpora. GPT models, short for Generative Pre-trained Transformers, use an autoregressive approach to learn the structure of language by predicting the next token given the previous ones. The transformer architectures allows these models to capture long-range dependencies in text, making them particularly powerful for understanding context and generating fluent text. Modern LLMs extend this base with techniques like instruction tuning and reinforcement learning from human feedback (RLHF), which improve their usability and alignment with human intent.

- **Pre-training** teaches the model general language patterns from large amounts of raw text data. This process builds a foundational base that can be fine-tuned for specific tasks later.
- **Instruction tuning** guides the model to follow specific types of tasks or instructions.
- **RLHF** improves output quality by training the model to reflect human preferences.

BERT (Bidirectional Encoder Representations from Transformers), released by Google in October 2018 not long after the seminal "Attention is All You Need" paper, pioneered transformers-based models for NLP tasks.

OpenAI's GPT series, from GPT-2 to GPT-o3, demonstrated increasingly powerful capabilities due to the scale of their parameters and training data, containing billions and now trillions of adjustable weights in their deep neural networks. GPT-3 represented a fundamental shift in AI, demonstrating how scaling models alone could achieve generalization. It also introduced *In-Context Learning*, allowing models to learn from examples in the prompt without fine-tuning. GPT-4 expanded the context length to 128K **tokens** (which are how LLMs represent the fundamental units of text, which can be as small as single characters or as large as whole words), significantly improving its ability to understand and summarize long documents. These models, however, are only available through proprietary APIs.

LLM	Number of Parameters	Context Length
BERT-Base	110 million	512
BERT-Large	340 million	512
GPT-2	1.5 billion	1K
GPT-3.5	175 billion	4K
GPT-4	~1 trillion	128K

```
gpt_name = "gpt-4o-mini"
```

35.1.1 LangChain framework

A modular framework for building applications with language models, such as **LanChain** simplifies the process of integrating language models with external data sources and other AI tools. It abstracts over the underlying LLM API (OpenAI, Ollama, etc.) and allows users to create chains of prompts, tools, and logic for custom NLP workflows.

```
# Initializes an OpenAI model using LangChain. temperature=0 ensures deterministic␣
↪outputs
from langchain_openai import ChatOpenAI
gpt_model = ChatOpenAI(model_name=gpt_name, temperature=0, **credentials['openai'])
pprint(gpt_model.to_json())
```

```
{'id': ['langchain', 'chat_models', 'openai', 'ChatOpenAI'],
 'kwargs': {'model_name': 'gpt-4o-mini',
            'openai_api_key': {'id': ['OPENAI_API_KEY'],
                               'lc': 1,
```

(continues on next page)

(continued from previous page)

```
                                    'type': 'secret'},
            'temperature': 0.0},
'lc': 1,
'name': 'ChatOpenAI',
'type': 'constructor'}
```

Temperature controls randomness in generation: lower values yield more deterministic responses, while higher values lead to more creative or diverse outputs.

35.1.2 Open and closed models

A large language model consists of three key components:

- Architecture: The structure of the model (e.g., Transformer-based).
- Weights: The learned parameters that define the model's behavior.
- Training code & data: The scripts and datasets used to train the model.

LLMs are categorized as:

- **Closed models**: API-only, no access to weights or training data (e.g., GPT-4).
- **Open models**: Model weights are available, but full training details are not (e.g., LLaMA, Qwen).
- **Open-source models**: Full transparency including architecture, code, data, and weights (e.g., DeepSeek-R1).

35.2 DeepSeek-R1 model

DeepSeek-R1 is a powerful open-weight language model released by DeepSeek in January 2025, with size ranging from from 1.3B to 236B parameters across different variants. Supporting a context length up to 128K tokens with a GPT-style transformer decoder-only architecture, it was trained with 6-10T tokens from multilingual internet sources. Furthermore, DeepSeek-R1 was fine-tuned to implement chain-of-thought reasoning without explicit prompting. Its training process included:

- synthetic dataset of thousands of long-form CoT examples
- group relative policy optimization, a reinforcement learning that improved its ability to solve challenging problems
- fine-tuning using a final round of reinforcement learning to boost its reasoning accuracy, helpfulness and harmlessness.

The model exposes its reasoning during inference, a departure from the typical black-box approach of other models, allowing users to witness the model's "thinking process" as it works through problems.

35.2.1 Distilled models

Distillation compresses LLMs by transferring knowledge from a large *teacher* model to a smaller *student* model.

- Knowledge Distillation (KD): Student learns from the teacher's output probabilities (soft targets) in addition to true labels (hard targets).
- Intermediate Layer Distillation: Transfers information from internal layers.
- Data Augmentation: Uses teacher-generated samples to expand the training set.

LLM distillation is expected to become an even more important practice in the AI world. Examples include GPT-4o distilled into GPT-4o-mini, or DeepSeek-R1 variants trained on Llama and Qwen to preserve reasoning capabilities with fewer parameters.

Distilled versions of DeepSeek-R1 are available in various sizes, including 1.5B, 7B, 14B, 32B, and 70B parameters. These models used DPO (Direct Preference Optimization) or supervised fine-tuning on synthetic highly-curated datasets generated by the larger R1 models, retaining 90–95% of teacher model performance with lower latency.

https://ollama.com/library/deepseek-r1

```
# model name in Ollama
model_name = "deepseek-r1:14b"
```

35.2.2 Small language models

Small language models (SLMs) are smaller in scale and scope than large language models (LLMs), with number of parameters ranging from a few million to a few billion. Requiring less memory and computational power, they can be deployed in resource-constrained environments such as edge devices, mobile apps and off-line situations where AI inferencing (when a model generates a response to a user's query) must be done without a data network.

35.2.3 Ollama server

Ollama simplifies running open-source LLMs locally. After installing the Ollama runtime and pulling a model (e.g., `deepseek-r1:14b`), it can serve requests on localhost. It provides a simple API for creating, running, and managing models, as well as a library of pre-built models. This allows experimentation with high-performance LLMs, improving accessibility, privacy, and latency.

https://github.com/ollama/ollama

1. Install Ollama (https://ollama.com/)
   - `curl https://ollama.ai/install.sh | sh`
   - `ls -ltra` which ollama``
   - `ollama --version`
2. Pull a model (stored in /usr/share/ollama/.ollama/models/)
   - `ollama pull deepseek-r1:14b`
   - `ollama list`
3. Serve an LLM
   - `ollama run deepseek-r1:14b` # uses GPU
   - `ollama ps`

4. or Linux service
   - `sudo systemctl status ollama # service status`
   - `sudo systemctl disable ollama # disable so it does not start up again upon reboot`
   - `sudo systemctl stop ollama # stop service`
   - `sudo systemctl restart ollama # restart service`
   - `sudo rm /etc/systemd/system/ollama.service # delete service file`
   - `sudo rm $(which ollama) # remove ollama binary`
5. Endpoint
   - `curl http://localhost:11434/api/generate -d '{"model": "deepseek-r1:14b", "prompt":"Why is the sky blue?"}'`

```python
# Initializes a local LLM (DeepSeek-R1) using Ollama
from langchain_ollama.llms import OllamaLLM
model = OllamaLLM(model=model_name, temperature=0)
pprint(model.to_json())
```

```
{'id': ['langchain_ollama', 'llms', 'OllamaLLM'],
 'lc': 1,
 'name': 'OllamaLLM',
 'repr': "OllamaLLM(model='deepseek-r1:14b', temperature=0.0)",
 'type': 'not_implemented'}
```

35.3 Text summarization

Summarization condenses lengthy documents into concise outputs. LLMs can perform abstractive summarization, generating summaries in their own words rather than extracting sentences. Summarization is a core NLP benchmark, critical for a wide variety of applications.

35.3.1 Natural language processing (NLP) tasks

These tasks play a crucial role in the field of **natural language processing**, challenging research and applications that have enhanced how machines understand and interact with human language. The performance of LLM's on these tasks are commonly evaluated using large benchmark datasets, such as MMLU (undergraduate level knowledge), GSM-8K (grade-school math), HumanEval (coding), GPQA (graduate-level questions), and MATH (math word problems). However, the intepretation of these results should be tempered by the inadvertent risk that some benchmark examples found their way in the data set used for training models.

- Natural Language Inference (NLI), also known as textual entailment, is the task of determining the relationship between two sentences, i.e. predict whether one sentence (the hypothesis) logically follows from another sentence (the premise).
- Named Entity Recognition (NER) involves identifying and classifying named entities within a text into predefined categories such as person names, organizations, locations, dates, etc.
- Text Generation is the process of generating coherent and contextually relevant text given a certain input or prompt.
- Machine Translation (MT) is the task of automatically translating text from one language to another.

- Text Summarization involves creating a concise summary of a longer text while preserving its key information and meaning.
- Reading comprehension requires models to read a passage of text and answer questions about it, demonstrating understanding of the text. Some challenges when developing and evaluating reading comprehension models include:
  - Artifacts, which refer to incorrect or misleading information generated by models that do not reflect the true content of the text but rather exploit patterns in the training data
  - Adversarial attacks, which are instances where models fail due to intentional manipulation or perturbation of the input, aiming to mislead or deceive the model.
  - Multihop reasoning, which refers to the ability of a model to connect multiple pieces of information or "hops" across the text to arrive at an answer.
- Question-Answering (QA) systems that automatically answer questions posed by humans in natural language, either based on a given context or dataset (known as closed-QA) or diverse topics from any domen (open-QA).
- Sentiment Analysis is the task of determining the sentiment or emotional tone expressed in a piece of text, such as positive, negative, or neutral.

35.3.2 10-K Market risk disclosures

We focus on Item 7A of the 10-K reports: *Quantitative and Qualitative Disclosures About Market Risk*. After retrieving and filtering disclosures from the SEC's EDGAR database, only the largest firms with sufficiently long reports are retained. One representative document per sector is selected for summarization.

```
# Retrieve universe of stocks
beg, end = 20240101, 20240331
univ = crsp.get_universe(bd.endmo(beg, -1))
```

```
# lookup company names
comnam = crsp.build_lookup(source='permno', target='comnam', fillna="")
univ['comnam'] = comnam(univ.index)
```

```
# lookup sic codes from Compustat, and map to FF 10-sector code
sic = pstat.build_lookup(source='lpermno', target='sic', fillna=0)
industry = Series(sic[univ.index], index=univ.index)
industry = industry.where(industry > 0, univ['siccd'])
sectors = Sectoring(sql, scheme='codes10', fillna='')   # supplement from crosswalk
univ['sector'] = sectors[industry]
```

```
# Load Disclosure about Market Risk text from 10-K's
item, form = 'qqr10K', '10-K'
rows = DataFrame(ed.open(form=form, item=item))
found = rows[rows['date'].between(beg, end)]\
    .drop_duplicates(subset=['permno'], keep='last')\
    .set_index('permno')
```

```
# Keep largest decile of stocks
found = found.loc[found.index.intersection(univ.index[univ['decile'] == 1])]
```

```
# Require minimum length of text
docs = {permno: ed[found.loc[permno, 'pathname']].lower()
```

(continues on next page)

(continued from previous page)

```
        for permno in found.index}
permnos = [permno for permno, doc in docs.items() if len(doc)>2000]
found = found.join(Series(docs, name='item').reindex(permnos), how='inner')
docs = univ.loc[found.index].groupby('sector').sample(1)
```

35.3.3 Generation

A LangChain pipeline is used to apply two models (DeepSeek-R1 via Ollama and GPT-4o-mini via OpenAI) to generate summaries. Model endpoints are configured with deterministic settings (temperature = 0). A prompt template and output parser are defined to extract core content, looping through each 10-K document. Summaries are generated and collected for analysis.

```
summary = {}  # to collect generated summaries
```

Define Langchain input prompt template

```
from langchain_core.prompts import ChatPromptTemplate
prompt_template = """
{role}.
Please summarize this risk report in about 300 words in prose form:

{text}
"""
prompt = ChatPromptTemplate.from_template(prompt_template)
```

Select Langchain output parser

```
from langchain_core.output_parsers import StrOutputParser
parser = StrOutputParser()
```

```
def collect_summaries(model, role="You are a helpful AI assistant."):
    """Helper to iterate over companies and generate summaries of risk reports"""
    summ = {}
    for i, permno in enumerate(docs.index):
        print(f'===== {i+1}/{len(docs)}.', univ.loc[permno, 'comnam'], '=====')
        chain = prompt | model | parser
        response = chain.invoke({"role": role, "text": found.loc[permno, 'item']})
        print("\n".join([textwrap.fill(s, width=80) for s in response.split('\n')]))
        print()
        summ[permno] = response.split('</think>')[-1]    # remove model's "thinking"
    return summ
```

Generate summaries with DeepSeek-R1-14b model

```
summary[model_name] = collect_summaries(model)
```

```
===== 1/10. PACCAR INC =====
<think>
Okay, so I need to summarize this risk report into about 300 words. Let me read
through it carefully first.
```

(continues on next page)

(continued from previous page)

```
The report is about market risks and derivative instruments, focusing on
interest rates, currencies, and commodities. It mentions that the figures are in
millions. The company uses hedging programs to manage these risks, as described
in Note P.

Starting with interest-rate risk: They measure this by estimating how a 100
basis point increase would affect fair values. In 2023, assets like cash
equivalents and fixed rate loans show potential losses, while liabilities such
as fixed rate term debt and swaps show gains. The total for 2023 is a loss of
$17.7 million, which is better than the previous year's $1.1 million loss.

Next, currency risk: They hedge against several currencies like CAD, EUR, GBP,
etc. A 10% unfavorable change in exchange rates would cause losses of $259.7
million in 2023 and $216.6 million in 2022. But these are offset by changes in
the underlying exposures.

Then, commodity price risk: They use forward contracts to hedge against price
fluctuations for production inputs. A 10% unfavorable change would lead to
losses of $3.3 million in 2023 and $2.5 million in 2022, which are also offset
by hedged exposures.

I should structure the summary to cover each risk category with their respective
figures and explanations. Make sure it's concise, around 300 words, and flows
well.
</think>

The risk report outlines a company's strategies and exposures related to market
risks, including interest rates, currencies, and commodities, as of December 31,
2023, and 2022. The company employs hedging programs to mitigate these risks.

**Interest-Rate Risk:**
The company assesses the impact of a 100 basis point increase in interest rates
on the fair value of its assets and liabilities. In 2023, cash equivalents and
marketable debt securities showed potential losses of $29.2 million, while
fixed-rate loans indicated a $146.5 million loss. On the liability side, fixed
rate term debt and swaps resulted in gains of $156.8 million and $1.2 million,
respectively. The net interest-rate risk for 2023 was a manageable $17.7 million
loss, an improvement from the previous year's $1.1 million loss.

**Currency Risk:**
To manage exchange rate fluctuations, the company hedges against major
currencies like the Canadian dollar and Euro. A 10% unfavorable change in these
rates could lead to losses of $259.7 million in 2023 and $216.6 million in 2022.
These potential losses are largely offset by corresponding changes in the value
of hedged exposures, indicating effective risk management.

**Commodity Price Risk:**
The company uses forward contracts to stabilize earnings against adverse
commodity price movements, particularly for truck production inputs. A 10%
unfavorable change in commodity prices could result in $3.3 million and $2.5
million losses in 2023 and 2022, respectively. These figures reflect the
company's ability to mitigate risks through hedging.

In summary, the company demonstrates a structured approach to managing market
risks, with effective use of derivatives and hedging strategies to minimize
potential financial impacts across interest rates, currencies, and commodities.
```

(continues on next page)

(continued from previous page)

```
===== 2/10. PHILLIPS 66 =====
<think>
Okay, so I'm trying to understand all these risks mentioned in the document. Let
me start by reading through them carefully.

First, there are market conditions like fluctuations in prices and margins for
NGLs, crude oil, natural gas, and refined products. That makes sense because
energy prices can be really volatile due to things like supply and demand
changes or geopolitical events.

Then there's government policies affecting pricing, regulation, taxation,
especially exports. I know that export policies can have a big impact on supply
and demand, so this is an important factor.

Capacity constraints in pipelines, storage, and fractionation facilities are
another risk. If these infrastructure issues arise, it could limit how much
product they can transport, leading to bottlenecks or higher costs.

OPEC and non-OPEC actions influence supply and demand, which affects prices. I
remember that OPEC's decisions can cause significant shifts in the market.

The success of DCP LP integration is mentioned, including achieving synergies.
This probably refers to a business strategy where they're combining operations,
so if this doesn't go as planned, it could hurt their performance.

Unexpected technical difficulties or cost increases during construction or
operation are risks too. Construction delays can be costly and disrupt
production.

Drilling and production volumes around midstream assets are another point. If
the wells aren't producing as expected, it affects the company's revenue from
those assets.

Permits and regulations compliance are also risks. They need to get permits for
projects, and if they can't or if regulations change, it could delay things or
require more spending.

Savings and cost reductions from business transformation initiatives might not
happen as planned. If they don't achieve these savings, their financial goals
could be at risk.

Renewable fuels policies and climate change regulations are factors too. Changes
in these areas could affect demand for traditional fuels or require new
investments.

Economic and political developments like the Russia-Ukraine war can impact
markets. Also, things like inflation, interest rates, and expropriation of
assets pose risks.

Public health crises, like pandemics, can disrupt operations and reduce demand
for their products. The recovery after such events is also uncertain.

Capital projects might not be completed on time or within budget. Delays here
can lead to cost overruns and project failures.
```

(continues on next page)

(continued from previous page)

```
Asset dispositions or acquisitions could face challenges if they don't complete
them successfully. This includes both the sale of assets and buying new ones.

Litigation or government actions could disrupt operations, leading to legal fees
or operational changes.

Accidents, weather events, civil unrest, etc., can damage facilities and
interrupt operations, causing financial losses.

Meeting sustainability goals is another risk. If they don't reduce GHG emissions
as planned or develop new technologies, it could affect their reputation and
operations.

New products might not be accepted by the market, leading to wasted investments.

Monetary conditions and exchange controls can impact international trade and
profitability.

Environmental regulations requiring significant investments or reducing demand
for their products are risks. They might have to spend a lot on compliance or
face reduced sales.

Liability from environmental issues like cleanup costs is another concern.

Changes in laws and regulations, including alternative energy mandates, could
force them to adapt their business models.

Climate change concerns might lead to changes in how they operate or increase
expenses through litigation or other costs.

Impairment charges could occur if the value of assets decreases due to market
conditions or strategic decisions.

Access to capital is crucial; if it's limited or more expensive, they might not
be able to fund their operations effectively.

Counterparties going bankrupt could lead to financial losses from unpaid debts
or obligations.

Cybersecurity incidents can compromise sensitive data and lead to legal
liabilities or operational disruptions.

Joint ventures that the company doesn't control could make decisions that
negatively impact their own business.

Lastly, all these factors are generally described in Item 1A of the risk factors
in the report.

I think I've covered all the points. Now, how do I summarize this? Maybe by
categorizing them into market risks, regulatory risks, operational risks,
financial risks, and environmental/sustainability risks. That way, it's easier
to understand and address each category separately.
</think>

The document outlines a comprehensive list of risks that could impact the
company's performance and operations. These risks can be categorized as follows:
```

(continues on next page)

(continued from previous page)

```
1. **Market Risks**:
   - Volatility in energy prices (NGLs, crude oil, natural gas, refined
products).
   - Fluctuations in margins and demand for refining and petrochemical products.
   - Impact of OPEC and non-OPEC actions on supply and demand.

2. **Regulatory and Policy Risks**:
   - Changes in government policies affecting pricing, regulation, taxation, and
exports.
   - Compliance with environmental regulations (GHG emissions, renewable fuels).
   - Influence of international monetary conditions and exchange controls.

3. **Operational Risks**:
   - Capacity constraints in transportation infrastructure (pipelines, storage).
   - Delays or cost overruns in construction and capital projects.
   - Challenges in permit acquisition and regulatory compliance.
   - Risks from joint ventures not under the company's control.

4. **Financial Risks**:
   - Failure to achieve expected savings from business transformation
initiatives.
   - Impairment charges due to asset value declines.
   - Limitations in accessing capital or increased funding costs.
   - Counterparty risks (e.g., bankruptcy leading to financial losses).

5. **Environmental and Sustainability Risks**:
   - Meeting sustainability goals and adapting to climate change regulations.
   - Development of new technologies and products that may not gain market
acceptance.

6. **Geopolitical and Public Health Risks**:
   - Impact of geopolitical events (e.g., Russia-Ukraine war) on markets.
   - Disruptions from public health crises (e.g., pandemics).

7. **Legal and Reputational Risks**:
   - Litigation risks and potential liabilities from environmental issues.
   - Cybersecurity threats leading to data breaches or operational disruptions.

By categorizing these risks, the company can better manage and mitigate
potential challenges across different areas of its operations.

===== 3/10. MASTERCARD INC =====
<think>
Okay, so I need to summarize this risk report from MasterCard's 2023 Form 10-K
about market risks, specifically items related to quantitative and qualitative
disclosures. The user wants a summary in around 300 words, written in prose
form.

First, I'll read through the provided text carefully to understand the key
points. It seems like the report is divided into sections on foreign exchange
risk and interest rate risk.

Under foreign exchange risk, MasterCard uses derivative contracts to manage
currency exposures from transactions and asset/liability fluctuations. They
mention a hypothetical 10% adverse change leading to potential losses of around
```

(continues on next page)

(continued from previous page)

```
$414 million in 2023 and $94 million in 2022. There's also something about
short-term settlement activities limiting their exposure to a few days, so the
impact isn't material even with a 10% change. Additionally, they talk about net
investment hedges for foreign subsidiaries but didn't have any as of December
31, 2023.

For interest rate risk, MasterCard has available-for-sale debt investments and
fixed-rate debt. They mention that a 100 basis point adverse change wouldn't
cause material losses in either year. They use derivative contracts to hedge
some of their fixed-rate debt exposure.

I need to structure this summary clearly, probably starting with an overview of
market risk, then breaking down into foreign exchange and interest rate risks
separately. I should highlight the strategies they use, like derivatives for
hedging, and mention the hypothetical losses but also note that these are before
considering offsets from hedged activities. It's important to convey that their
exposure is managed effectively through these measures.

I'll make sure to keep it concise, around 300 words, using clear and
professional language without getting too technical. I should avoid jargon where
possible and ensure each section flows logically into the next.
</think>

The risk report outlines Mastercard's approach to managing market risks,
including foreign exchange and interest rate risks, which can lead to financial
losses due to adverse changes in currency values or interest rates. The company
actively monitors these risks and employs strategies such as derivative
financial instruments to mitigate potential impacts.

**Foreign Exchange Risk:**
Mastercard manages foreign exchange risk through derivative contracts aimed at
hedging currency exposures from transactions and asset/liability fluctuations.
These contracts help reduce transaction gains or losses resulting from currency
fluctuations against the U.S. dollar and euro. A hypothetical 10% adverse change
in currency values could result in fair value losses of approximately $414
million (2023) and $94 million (2022). However, Mastercard's short-term
settlement activities typically limit exposure to a few days between
transactions and settlements, minimizing material impacts from such changes.
Additionally, the company may use derivatives to hedge its net investment in
foreign subsidiaries, though no such hedges were designated as of December 31,
2023.

**Interest Rate Risk:**
The company's available-for-sale debt investments include fixed and variable
rate securities sensitive to interest rate fluctuations. A hypothetical 100
basis point adverse change in interest rates would not result in material losses
for its investments or derivative contracts used to hedge fixed-rate debt as of
December 31, 2023 and 2022.

Overall, Mastercard employs robust risk management strategies, including the use
of derivatives, to effectively mitigate market risks and safeguard against
potential financial losses.

===== 4/10. BRISTOL MYERS SQUIBB CO =====
<think>
Okay, so I need to summarize this risk report into about 300 words. Let me read
```

(continues on next page)

(continued from previous page)

```
through it carefully first.

The report is about market risks the company faces, specifically from currency
exchange rates and interest rates. They use derivatives like forwards and swaps
to hedge these risks but don't trade with them.

Looking at foreign exchange risk: a lot of their revenue and cash flows are
affected by changes in Euro and Japanese Yen. They use forward contracts and put
options to manage this. There's also transaction risk from non-functional
currencies, which they offset with forwards not designated as hedges. They hedge
net investments with affiliates using these contracts too.

They estimate that a 10% appreciation in the hedged currencies would decrease
the fair value of their FX contracts by certain amounts, reducing earnings.
Cross-currency swaps are used for long-term debt and foreign subsidiaries; here,
a 10% change affects the swap values differently.

For interest rate risk: they use fixed-to-floating swaps to balance their debt.
They also analyze how changes in rates affect their marketable securities and
long-term debt. A 1% increase in rates doesn't have a big impact on earnings but
does decrease the fair value of long-term debt.

Credit risk is managed by investing only with high-quality institutions,
diversifying derivatives across counterparties to reduce default risks.

I need to condense all this into a coherent summary without getting too
technical. Make sure to mention each type of risk (FX and interest rate), how
they hedge, the impact of currency appreciation/depreciation, and their credit
risk management strategies.
</think>

The company faces market risks primarily from fluctuations in foreign exchange
rates and interest rates. To mitigate these risks, it employs derivative
financial instruments such as forwards, swaps, and put options, which are used
for hedging rather than trading.

**Foreign Exchange Risk:**
A significant portion of the company's revenue, earnings, and cash flows are
exposed to changes in Euro and Japanese Yen exchange rates. The company uses
foreign currency forward contracts and purchased local currency put option
contracts to manage risks from intercompany transactions. Additionally, these
forwards are used to hedge net investment exposures in international affiliates.
Cross-currency swaps are employed to manage long-term debt denominated in Euros.

The impact of a 10% appreciation in hedged currencies is estimated to decrease
the fair value of foreign exchange contracts and increase/decrease cross-
currency swap values, affecting earnings over time.

**Interest Rate Risk:**
The company uses fixed-to-floating interest rate swaps to balance its debt
structure. Sensitivity analysis shows that a 1% increase in short-term or long-
term interest rates would not significantly impact earnings but could decrease
the fair value of long-term debt.

**Credit Risk Management:**
Investments are limited to high-credit-quality institutions, and derivatives are
```

(continues on next page)

(continued from previous page)

```
diversified across counterparties to mitigate default risks.

In summary, the company effectively manages market risks through strategic use
of financial instruments while adhering to conservative credit policies.

===== 5/10. CARRIER GLOBAL CORP =====
<think>
Okay, so I need to summarize this risk report about market risks for a company.
The user provided the entire section under item 7a, which includes foreign
currency exposures, commodity price exposures, and interest rate exposures. They
also mentioned that there's been no significant change in their market risk
exposure for the year ended December 31, 2023.

First, I should read through each part carefully to understand what each section
is talking about.

Starting with foreign currency exposures: The company operates internationally,
so they're exposed to exchange rate fluctuations. Their non-U.S. operations have
different functional currencies, which affects their reported results when the
U.S. dollar strengthens or weakens against those currencies. They manage
material currency exposures at the legal entity level but don't hedge
translation risk. However, for specific acquisitions like VCS and TCC, they used
financial instruments like window forward contracts and cross-currency swaps to
mitigate risks related to paying in foreign currencies.

Next, commodity price exposures: The company is affected by volatile commodity
prices and uses fixed-price contracts when appropriate. They also face fuel
costs for shipping but don't have any hedges in place as of December 31, 2023.

Interest rate exposures: Most of their long-term debt has fixed rates, so they
don't expect much impact from interest rate fluctuations.

I need to condense all this into about 300 words. I should structure it by first
introducing the overall market risks (currency, commodities, interest rates),
then go into each category with key points, and conclude without significant
changes in exposure.

I should make sure to mention their strategies for managing these risks, like
using swaps and contracts, but also note where they don't hedge, such as
translation risk. I'll need to keep the language clear and concise, avoiding too
much jargon so it's accessible.

Let me outline the summary:

1. Introduction: Briefly state that the company faces market risks from
currency, commodities, and interest rates, with no significant change in 2023.
2. Foreign Currency Exposures: Explain their international operations leading to
exchange rate risks. Mention active management at entity level but no hedging of
translation risk. Highlight specific actions taken for VCS and TCC acquisitions
using financial instruments.
3. Commodity Price Exposures: Discuss volatility in commodity prices, use of
fixed-price contracts, and absence of hedges for fuel costs.
4. Interest Rate Exposures: Note that most debt is fixed-rate, so minimal
expected impact from rate changes.
5. Conclusion: Reiterate no significant change in overall market risk exposure.
```

(continues on next page)

(continued from previous page)

```
I should ensure each section flows logically into the next, providing a
comprehensive yet concise overview without getting bogged down in details. Also,
I need to watch the word count to stay around 300 words, so I'll aim for about
6-7 sentences per main point.
</think>

The company faces market risks primarily from fluctuations in foreign currency
exchange rates, commodity prices, and interest rates, with no significant
changes in exposure during 2023.

**Foreign Currency Exposures:** The company's global operations expose it to
exchange rate risks, particularly affecting its U.S. dollar reporting. While
they manage material currency exposures at the entity level without hedging
translation risk, specific strategies were employed for acquisitions. For
instance, window forward contracts were used for the VCS acquisition to mitigate
euro-denominated risks, and cross-currency swaps were utilized for the TCC
acquisition to hedge yen-denominated investments.

**Commodity Price Exposures:** The company deals with volatile commodity prices
by using fixed-price contracts where appropriate. However, they lack hedges
against fuel cost fluctuations impacting shipping expenses.

**Interest Rate Exposures:** With most long-term debt having fixed rates, the
company anticipates minimal impact from interest rate changes.

In conclusion, the company's market risk management remains consistent, focusing
on strategic hedging for specific risks while acknowledging areas where exposure
is accepted without mitigation.

===== 6/10. LULULEMON ATHLETICA INC =====
<think>
Okay, so I need to summarize this risk report into about 300 words. Let me read
through it carefully first.

The report starts with market risks, specifically foreign currency exchange and
translation risks. It mentions that their international subsidiaries use local
currencies as functional currencies, which are then converted to USD for
consolidated financial statements. Exchange rate fluctuations affect reported
revenues, expenses, assets, and liabilities. In 2023, revenue was lower by $89.8
million compared to 2022 due to exchange rates.

They also talk about translation risk where foreign currency differences are
recorded in other comprehensive income (OCI). They have a Canadian subsidiary
with significant net assets, so they use forward contracts to hedge part of this
exposure. The impact on OCI was an increased loss of $9 million, including hedge
gains.

Then there's transaction risk from intercompany transactions and inventory
purchases in non-functional currencies. They hold cash and other monetary assets
in different currencies and have forward contracts to hedge these. The
sensitivity analysis shows that a 10% depreciation in the USD would affect
derivative fair values by $29.8 million, but this is offset by changes in hedged
items.

They plan to use more derivatives in the future for additional currency pairs
but won't trade them for profit. There's also mention of interest rate risk from
```

(continues on next page)

(continued from previous page)

```
their revolving credit facility, which has a variable rate. As of now, there are
no borrowings except for $6.3 million in letters of credit. They don't do any
hedging yet but might in the future.

Cash and cash equivalents are held in short-term deposits and money market
funds, totaling $2.2 billion. Interest rates affect the interest generated on
these balances. Credit risk is managed by dealing with reputable institutions,
though some deposits exceed insured limits. They monitor counterparties' credit
standing to minimize risks.

Inflation is another concern, particularly wage increases affecting operating
margins in 2022 and 2023. Higher transportation costs and raw material prices
could hurt margins if selling prices don't rise or cost efficiencies aren't
found.

I need to condense all this into a coherent summary without exceeding 300 words.
I'll structure it by main risk categories: foreign currency, interest rate,
credit, and inflation risks. Highlight the key points like revenue impact,
hedging strategies, sensitivity analysis, current financial status, and future
plans regarding derivatives. Also, mention the effects of inflation on margins.

I should ensure that each section is concise, using bullet points or clear
paragraphs to separate the different types of risk. Make sure to include the
main figures like $89.8 million revenue drop, $2.2 billion cash, and sensitivity
results. Avoid jargon where possible and keep it in prose form as requested.

Let me start drafting:

- Foreign currency exchange and translation risks: Explain how exchange rates
affect financial statements, mention the 2023 revenue drop, their use of forward
contracts to hedge, and the impact on OCI.

- Transaction risk: Discuss intercompany transactions and inventory purchases in
non-functional currencies, their hedging strategies with forward contracts, and
sensitivity analysis results.

- Interest rate risk: Describe the credit facility's variable rates, current
borrowing status, and future plans for hedging if needed.

- Credit risk: Talk about holding cash in reputable institutions, monitoring
counterparties, and minimal losses experienced.

- Inflation: Highlight past impacts on margins due to wages and freight costs,
potential future effects, and reliance on price increases or cost efficiencies.

I need to make sure each section flows logically and stays within the word
limit. Let me check for any redundancies or areas that can be more concise.
</think>

The risk report outlines several key market risks faced by the company,
primarily focusing on foreign currency exchange, translation, transaction,
interest rate, credit, and inflationary risks.

**Foreign Currency and Translation Risks:** The company's international
subsidiaries use local currencies, which are converted to USD for consolidated
financial statements. Exchange rate fluctuations significantly impact reported
```

(continues on next page)

(continued from previous page)

```
revenues, expenses, assets, and liabilities. In 2023, revenue decreased by $89.8
million due to unfavorable exchange rates. To mitigate translation risk, the
company records foreign currency differences in other comprehensive income
(OCI). They also use forward contracts to hedge risks associated with their
significant Canadian subsidiary, resulting in a $9 million increase in OCI
losses, including hedge gains.

**Transaction Risks:** The company faces transaction risks from intercompany
transactions and inventory purchases denominated in non-functional currencies.
To manage these risks, they hold cash and other monetary assets in various
currencies and employ forward contracts for hedging. Sensitivity analysis
indicates that a 10% USD depreciation would affect derivative fair values by
$29.8 million, offset by changes in hedged items.

**Interest Rate Risks:** The company's revolving credit facility exposes it to
variable interest rates. Currently, there are no borrowings except for $6.3
million in letters of credit. While they do not engage in hedging activities at
present, they plan to use derivatives for future currency pairs without trading
for profit.

**Credit Risks:** The company manages credit risks by holding cash in reputable
financial institutions and monitoring counterparty credit standing. Although
some deposits exceed insured limits, they have experienced minimal losses and
maintain investment-grade relationships to mitigate risks.

**Inflation Risks:** Inflationary pressures, such as rising wages and
transportation costs, have impacted operating margins in recent years. Sustained
increases could adversely affect margins if selling prices do not rise or cost
efficiencies are not achieved. The company remains vigilant about these factors
to ensure margin stability.

This structured approach ensures clarity and conciseness, effectively conveying
the key risks and strategies outlined in the report within 300 words.

===== 7/10. AIRBNB INC =====
<think>
Okay, so I need to summarize this risk report about market risks that Airbnb is
facing. The user provided the entire section from item 7a, which talks about
both quantitative and qualitative disclosures regarding market risks. They
specifically mentioned foreign currency risk and investment risk.

First, I should read through the entire text carefully to understand all the
points made. It starts by saying that their global operations expose them to
various market risks, mainly foreign currency and investment risks.

Under foreign currency exchange risk, they mention that they support over 40
currencies, with some key ones like euro, British pound, etc. Their revenue and
costs are in these currencies, which can fluctuate against the US dollar,
affecting their financial results. They have several exposures: revenue and cost
of revenue, funds held, unbilled amounts, and intercompany balances.

They use foreign currency derivatives to hedge these risks but don't eliminate
them entirely. There's a mention that if there was an adverse 10% change in
exchange rates as of Dec 31, 2023, they would lose about $20 million.

Then, investment and interest rate risk: they have a large cash and short-term
```

(continues on next page)

(continued from previous page)

```
investments portfolio, mainly in debt securities. They don't invest for trading
or speculation, so their exposure to interest rate changes is limited because
their investments are short-term. A 100 basis point increase would decrease
their portfolio by $20 million.

I need to condense all this into about 300 words. I should structure it with an
introduction, then sections on foreign currency risk and investment risk,
highlighting the key points like hedging strategies, potential losses from
exchange rate changes, and their approach to managing interest rates through
short-term investments.

I also need to make sure the summary is in prose form, clear and concise,
without using bullet points or technical jargon. I should mention the specific
currencies, the use of derivatives, the amounts at risk, and their investment
strategies.

Wait, let me check if there are any other important details. They talk about how
guests can pay in different currencies, which introduces more currency risks
because hosts might choose a different payment currency. Also, they have
separate funds for bookings, which adds to their foreign exchange exposures.

In the investment section, it's important to note that they focus on preserving
capital and liquidity, so they stick to safe, short-term investments. This
minimizes their exposure to interest rate fluctuations but still, a significant
change could impact them.

I should also include the specific figures: $20 million loss from a 10% exchange
rate change and the same figure for a 100 basis point increase in interest
rates. These numbers show the potential financial impact.

Putting it all together, I'll start with an overview of their market risks, then
break down each type, explain the exposures, their hedging strategies, and the
potential losses. Conclude by summarizing their approach to managing these
risks.
</think>

Airbnb faces significant market risks primarily due to its global operations,
which expose it to foreign currency exchange risk and investment risk.

**Foreign Currency Exchange Risk:**
Airbnb operates in over 40 currencies, with key exposures including the euro,
British pound, Canadian dollar, Australian dollar, Brazilian real, and Mexican
peso. Fluctuations in these currencies against the US dollar can impact its
financial results through various channels: revenue and costs denominated in
foreign currencies, funds held on behalf of customers, unbilled amounts from its
"Pay Less Upfront" program, and intercompany balances. To mitigate this risk,
Airbnb uses foreign currency derivative contracts to hedge forecasted exposures,
though it does not eliminate all risks. A 10% adverse exchange rate change as of
December 31, 2023, could result in a $20 million loss.

**Investment and Interest Rate Risk:**
Airbnb's investment portfolio, primarily consisting of short-term, highly-liquid
debt securities, is managed to preserve capital and meet liquidity needs. As of
December 31, 2023, it held $6.9 billion in cash and cash equivalents, $3.2
billion in short-term investments, and $5.9 billion in funds receivable. The
portfolio's short maturity minimizes sensitivity to interest rate changes. A
```

(continues on next page)

(continued from previous page)

```
hypothetical 100 basis point increase in rates could decrease the investment
portfolio's fair value by $20 million. Airbnb avoids speculative investments and
trading activities, focusing on stability rather than risk.

In summary, while Airbnb employs hedging strategies and conservative investment
practices to manage risks, significant currency fluctuations or interest rate
changes could still impact its financial performance.

===== 8/10. MERCADOLIBRE INC =====
<think>
Okay, so I'm trying to understand this document about MercadoLibre's risks and
sensitivities. It seems like it's part of their financial disclosures, maybe in
their annual report or something similar. Let me break down what each section is
saying.

First, the foreign exchange risk. They have a significant amount of revenue from
Brazil, Argentina, and other Latin American countries. Since they're dealing
with multiple currencies, especially the Brazilian real and Argentine peso,
which can be volatile, this could affect their financials. If the real or peso
weakens against the dollar, their reported revenue in dollars might decrease
because when converted, those currencies would buy fewer dollars. That makes
sense because if you have a lot of revenue in a currency that's dropping, your
overall revenue in USD terms goes down.

Next is interest rate risk. They have long-term debt, which means they're
exposed to changes in interest rates. If rates go up, the value of their
existing debt might decrease, and their borrowing costs could increase if they
need to refinance or take on more debt. I'm not entirely sure how sensitive they
are to rate changes, but it's something to watch, especially with global
interest rates potentially rising.

Then there's commodity price risk. They're exposed to fuel and electricity
prices because these are operational costs. If oil prices go up, their delivery
and logistics costs would increase, affecting their margins. Similarly, higher
electricity prices could impact their operations, especially in countries where
energy is a significant cost. This seems like a manageable risk but could have
noticeable effects if prices spike.

Equity price risk is tied to their LTRP plans. These are long-term retention
programs that pay out based on stock performance. The value of these obligations
is sensitive to the company's stock price. If the stock goes up, the payouts
increase, which could strain their finances. Conversely, a drop in stock price
reduces the payout liability. They've done a sensitivity analysis showing how a
40% change in stock price affects the obligation. That's pretty significant; a
40% drop would reduce the liability from $418 million to about $251 million,
which is a substantial difference.

Inflation risk is another factor, especially in countries like Argentina where
inflation has been high. High inflation can erode purchasing power and affect
consumer behavior. If people have less money due to inflation, they might spend
less online, impacting MercadoLibre's sales. Additionally, rising costs for
goods and services could squeeze margins if the company can't pass these costs
onto customers.

I'm a bit confused about how exactly the equity price affects their obligations.
It seems like it's based on the stock price relative to a fixed average from
```

(continues on next page)

(continued from previous page)

```
previous years. So each payment is tied to whether the current stock price is
higher or lower than that average. If the stock does well, they have to pay
more; if not, less.

I'm also thinking about how these risks interact. For example, inflation could
lead to higher interest rates, which would then affect their debt costs. Or a
weaker currency might make imports more expensive, contributing to inflation.
It's like a web of interconnected risks that could amplify each other in certain
economic conditions.

Another point is the sensitivity analysis for equity price risk. They show a
static present value and how it changes with different stock prices. This helps
investors understand how exposed they are to stock price fluctuations. A 40%
change seems extreme, but it's useful to see the range of possible outcomes.

I wonder if there are any mitigating factors or strategies MercadoLibre uses to
manage these risks. For example, do they hedge their currency exposure? How
flexible are their costs in the face of inflation? And how do they structure
their LTRP plans to balance retention with financial risk?

Overall, this section is highlighting several key areas where MercadoLibre faces
potential volatility and challenges. It's important for investors to consider
these risks when evaluating the company's long-term prospects.
</think>

**MercadoLibre Risk Analysis Summary**

1. **Foreign Exchange Risk:**
   - **Exposure:** Significant revenue from Brazil, Argentina, and other Latin
American countries exposed to currency fluctuations, particularly the Brazilian
real and Argentine peso.
   - **Impact:** Currency devaluation can reduce reported USD revenue when
converted.

2. **Interest Rate Risk:**
   - **Exposure:** Long-term debt subject to interest rate changes.
   - **Impact:** Rising rates may decrease debt value and increase borrowing
costs.

3. **Commodity Price Risk:**
   - **Exposure:** Operational costs tied to fuel and electricity prices.
   - **Impact:** Price spikes can increase operational expenses and reduce
margins.

4. **Equity Price Risk:**
   - **Exposure:** Long-term retention programs (LTRP) linked to stock
performance.
   - **Impact:** Stock price fluctuations affect payout obligations; a 40% drop
reduces liability from $418 million to $251 million.

5. **Inflation Risk:**
   - **Exposure:** High inflation in countries like Argentina affecting
purchasing power and operational costs.
   - **Impact:** Could reduce consumer spending and squeeze margins if cost
increases aren't offset.
```

(continues on next page)

(continued from previous page)

```
6. **Interconnected Risks:**
   - Inflation may lead to higher interest rates, affecting debt costs and
potentially exacerbating currency issues.

7. **Mitigation Strategies:**
   - Consider hedging strategies for currency exposure, flexible cost
structures, and balanced LTRP designs to manage financial risks.

**Conclusion:** MercadoLibre faces multiple interconnected risks that could
impact its financial health. Investors should assess these factors alongside the
company's risk management strategies when evaluating long-term prospects.

===== 9/10. A T & T INC =====
<think>
Okay, so I'm trying to understand this document about AT&T's financial report.
It looks like it's an audit report from their independent registered public
accounting firm. Let me go through it step by step.

First, the report is divided into two main sections: one on the internal control
over financial reporting and another on the consolidated financial statements.
Both have an opinion section, a basis for opinion, and some additional details.

Starting with the opinion on internal control over financial reporting. The
auditors state that they've audited AT&T's internal controls as of December 31,
2023, based on the COSO criteria. They conclude that the company maintained
effective internal control over financial reporting in all material respects.
That sounds positive; it means the auditors think AT&T has good systems in place
to manage their finances properly.

Next, looking at the basis for this opinion. It mentions that management is
responsible for maintaining these controls and assessing their effectiveness.
The auditors' role was to express an opinion based on their audit, which they
did by following PCAOB standards. They obtained an understanding of the internal
controls, assessed risks, tested the design and operation of the controls, and
performed other necessary procedures. Their conclusion is that AT&T's controls
are effective.

Moving on to the report on the consolidated financial statements. The auditors
also express an unqualified opinion here, meaning they didn't find any material
issues with the financial statements. They were prepared in accordance with
GAAP, which is standard for public companies in the U.S.

The basis for this opinion section explains that the auditors are responsible
for expressing opinions on whether the financial statements present fairly, in
all material respects, the company's financial position and results of
operations. They did this by auditing according to PCAOB standards, assessing
risks, testing accounting policies, significant estimates, transactions, and
disclosures.

There are also two specific sections discussing areas where the audit was
particularly complex: internal control over financial reporting and the
impairment evaluations for certain reporting units. The auditors explain that
these areas required a deeper dive because they involve subjective assumptions
and complex methodologies. They tested controls, assessed assumptions against
industry trends, performed sensitivity analyses, and involved specialists to
ensure everything was in order.
```

(continues on next page)

(continued from previous page)

```
The report ends with a note that the auditing firm has been serving AT&T since
1999, which shows a long-standing relationship, likely indicating consistency
and familiarity with the company's operations.

I'm trying to think if there's anything I might be missing. The document seems
comprehensive, covering both the internal controls and financial statements. It
mentions specific areas like impairment evaluations and valuation models, which
are known for being complex and subject to judgment. The auditors' involvement
of specialists in these areas makes sense because it adds depth to their
analysis.

I wonder if there were any material weaknesses or significant deficiencies
mentioned, but from what I can see, the opinions are unqualified, so no major
issues were found. It's also worth noting that the report is dated February 23,
2024, which would typically be around the time when earnings are released, so
it's timely.

Overall, this audit report seems to provide assurance to stakeholders that
AT&T's financial reporting is reliable and their internal controls are
effective. The detailed explanation of the auditing process gives confidence in
the independence and thoroughness of the audit.
</think>

The audit report on AT&T provides a comprehensive assessment of both its
internal control over financial reporting and its consolidated financial
statements, concluding with an unqualified opinion in both areas. Here's a
structured summary:

### Key Findings:
1. **Internal Control Over Financial Reporting:**
   - The auditors found that AT&T maintained effective internal controls as of
December 31, 2023, based on the COSO framework.
   - The company's systems provide reasonable assurance regarding the
reliability of financial reporting and compliance with GAAP.

2. **Consolidated Financial Statements:**
   - The financial statements were prepared in accordance with GAAP and present
AT&T's financial position accurately.
   - No material issues were identified, leading to an unqualified opinion.

### Methodology:
- Auditors followed PCAOB standards, assessing risks, testing controls, and
involving specialists for complex areas like impairment evaluations.
- Detailed procedures included testing assumptions against industry trends and
performing sensitivity analyses.

### Areas of Complexity:
- **Impairment Evaluations:** Involved subjective assumptions and complex
valuation methods. The auditors ensured these were sound through thorough
testing and specialist involvement.
- **Internal Controls:** The effectiveness was validated through a comprehensive
audit process, confirming the controls' operational efficiency.

### Conclusion:
The report assures stakeholders of AT&T's financial integrity and control
```

(continues on next page)

(continued from previous page)

```
systems. The unqualified opinions indicate no significant issues, reflecting
well on the company's financial governance. The long-standing relationship with
the auditing firm underscores consistency and familiarity in their work.

===== 10/10. REPUBLIC SERVICES INC =====
<think>
Okay, so I need to summarize this risk report into about 300 words. Let me read
through it carefully first.

The report starts with Item 7A, which is about quantitative and qualitative
disclosures on market risks. The main areas discussed are interest rate risk,
fuel price risk, and commodities price risk.

Starting with interest rate risk: The company's major exposure is to changes in
U.S. interest rates and fluctuations in SOFR (which I think stands for Secured
Overnight Financing Rate). They manage this by using a mix of fixed and floating
rate debt. Their variable rate debt is valued at fair value because the rates
are current, but this can change over time. They've used swap agreements as
hedges before, but it looks like they might not be doing that anymore or maybe
have changed their strategy.

Looking at the table provided, it shows fixed and variable rate debts across
different years up to 2028. The total fair value is around $9.8 billion. They
also mention having $2.23 billion in floating rate debt and swap contracts with
a notional value of $350 million. If interest rates go up or down by 100 basis
points, their annual interest expense would change by about $20 million. But
this doesn't account for new borrowings or economic impacts.

Next is fuel price risk: Fuel costs are a big part of their operations. They try
to hedge against fuel price changes when it's economically practical, but as of
December 31, 2023, they had no hedges in place. They charge fuel recovery fees
to most customers, but not all. A 20-cent per gallon change in diesel would
affect their fuel costs by about $27 million and the fees by $36 million. They
also mention other petrochemical-based products used in operations, which could
increase costs if prices go up.

Lastly, commodities price risk: They deal with recycled materials like old
corrugated containers and newsprint. Market volatility affects these prices. In
the past, they've used swaps and collars as hedges, but none are currently in
place. A $10 per ton change would impact both revenue and operating income by
about $10 million. Their recycling revenue dropped from $359.1 million in 2022
to $312.3 million in 2023.

Putting this all together, the summary should cover each risk area with key
points: how they're managed, current positions (like hedges), and potential
financial impacts. I need to make sure it's concise, around 300 words, and flows
well.
</think>

The risk report outlines the company's exposure to various market risks,
including interest rates, fuel prices, and commodities.

**Interest Rate Risk:** The company manages this through a mix of fixed and
floating rate debt, with variable rate debt valued at fair value. As of December
31, 2023, their total debt was approximately $9.8 billion, with $2.23 billion in
floating rate debt and swap contracts. A 100 basis point interest rate change
```

(continues on next page)

(continued from previous page)

```
could alter annual interest expenses by about $20 million.

**Fuel Price Risk:** Fuel costs are significant, and while the company charges
recovery fees to most customers, not all are covered. A 20-cent per gallon price
change in diesel would affect fuel costs by $27 million and fees by $36 million.
Additionally, petrochemical-based product costs may rise with fuel prices.

**Commodities Price Risk:** The company markets recycled materials, which face
market volatility. Despite past hedging strategies, no hedges were active as of
the report date. A $10 per ton price change could impact revenue and operating
income by $10 million each. Recycling revenue decreased from $359.1 million in
2022 to $312.3 million in 2023.

In summary, the company faces notable risks from fluctuating interest rates,
fuel prices, and commodities markets, with strategies in place to mitigate some
of these impacts.
```

Show ollama processes

```
!ollama ps
```

```
NAME               ID              SIZE     PROCESSOR    UNTIL
deepseek-r1:14b    ea35dfe18182    11 GB    100% GPU     4 minutes from now
```

Generate summaries with OpenAI GPT-4o-mini model

```
summary[gpt_name] = collect_summaries(gpt_model)
```

```
===== 1/10. PACCAR INC =====
The risk report outlines the company's exposure to market risks, specifically
focusing on interest rate, currency, and commodity price risks, with figures
presented in millions.

In terms of interest rate risks, the company employs hedging programs to
mitigate exposure to fluctuations. The report quantifies the potential impact of
a 100 basis point increase in interest rates on the fair value of interest-
sensitive assets and liabilities. For 2023, the fair value losses for cash
equivalents and marketable debt securities amounted to $29.2 million, while
fixed-rate loans reflected a loss of $146.5 million. Conversely, fixed-rate term
debt showed gains of $156.8 million, and interest-rate swaps contributed a gain
of $1.2 million, resulting in a total net loss of $17.7 million for the year,
compared to a loss of $1.1 million in 2022.

Regarding currency risks, the company utilizes foreign currency exchange
contracts to hedge against fluctuations in various currencies, including the
Canadian dollar and the euro. The sensitivity analysis indicates a potential
loss of $259.7 million from a 10% unfavorable change in exchange rates as of
December 31, 2023, an increase from a loss of $216.6 million in 2022. However,
these losses are expected to be offset by corresponding changes in the values of
the underlying hedged exposures.

Lastly, the report addresses commodity price risks, where the company engages in
commodity forward contracts to stabilize earnings and cash flows against adverse
price movements. The potential loss in fair value from a 10% unfavorable change
```

(continues on next page)

(continued from previous page)

```
in commodity prices is estimated at $3.3 million for 2023, up from $2.5 million
in 2022, with similar offsetting effects from the underlying hedged exposures.
Overall, the report highlights the company's proactive approach to managing
market risks through various hedging strategies.

===== 2/10. PHILLIPS 66 =====
The risk report outlines the market risks faced by the company and its
subsidiaries, primarily stemming from fluctuations in commodity prices, interest
rates, and foreign currency exchange rates. The company is particularly exposed
to the prices of crude oil, refined petroleum products, natural gas liquids
(NGL), natural gas, renewable feedstock, and electric power. To manage these
risks, the company employs derivative contracts, including futures, forwards,
swaps, and options, which help convert fixed-price contracts to floating market
prices and optimize supply chain value.

The report emphasizes the company's policy of remaining exposed to market prices
while using derivatives to balance physical systems, meet refinery requirements,
and manage cash flow risks associated with price fluctuations. A Value at Risk
(VaR) model is utilized to estimate potential losses from adverse market
changes, indicating that the VaR for derivative instruments as of December 31,
2023, was immaterial to cash flows and operations.

Interest rate risk is another significant concern, as the company holds both
fixed-rate and variable-rate debt. Fixed-rate debt can lead to changes in fair
value due to market interest rate fluctuations, while variable-rate debt exposes
the company to short-term interest expense changes. The report provides detailed
tables of the company's debt instruments, highlighting their sensitivity to
interest rate changes.

Additionally, the company faces foreign currency risk from its international
operations but generally does not hedge this exposure. Risk monitoring is
overseen by the CEO and CFO, ensuring that risks related to commodity prices,
interest rates, and foreign exchange rates are effectively managed. The report
concludes with a cautionary note regarding forward-looking statements,
emphasizing the uncertainties and risks that could impact future performance,
including market conditions, regulatory changes, and geopolitical events.

===== 3/10. MASTERCARD INC =====
The risk report outlines the company's exposure to market risk, specifically
focusing on interest rate and foreign currency exchange rate fluctuations.
Market risk refers to potential economic losses from adverse changes in these
factors. The company has limited exposure to such risks, and management actively
monitors and implements policies to govern funding, investments, and the use of
derivative financial instruments to mitigate these risks.

To manage foreign currency risk, the company utilizes foreign exchange
derivative contracts to hedge against anticipated receipts and disbursements in
currencies other than its functional currency. This strategy aims to minimize
transaction gains and losses due to currency fluctuations, particularly against
the U.S. dollar and euro. A hypothetical 10% adverse change in the value of
functional currencies could lead to significant fair value losses on outstanding
foreign exchange derivatives, estimated at approximately $414 million and $94
million for the years ending December 31, 2023, and 2022, respectively, before
considering any offsetting effects.

Additionally, the company faces foreign exchange risk from daily settlement
```

(continues on next page)

(continued from previous page)

```
activities, which it manages through short-duration derivative contracts.
However, a similar hypothetical 10% adverse change would not materially impact
the fair value of these contracts. The company also has exposure related to the
translation of net investments in foreign subsidiaries, although as of December
31, 2023, it had no designated net investment hedges.

Regarding interest rate risk, the company holds both fixed and variable-rate
securities. It maintains a policy of investing in high-quality securities while
ensuring liquidity and diversification. A hypothetical adverse change of 100
basis points in interest rates would not significantly affect the fair value of
these investments or the company's interest rate derivative contracts related to
fixed-rate debt. Overall, the company employs various strategies to manage its
market risk effectively.

===== 4/10. BRISTOL MYERS SQUIBB CO =====
The risk report outlines the company's exposure to market risks, particularly
from fluctuations in currency exchange rates and interest rates. To mitigate
these risks, the company employs various derivative financial instruments,
although these are not used for trading purposes. The report highlights
significant foreign exchange risks, particularly related to the euro and
Japanese yen, which affect the company's revenues, earnings, and cash flow. To
manage these risks, the company utilizes foreign currency forward contracts and
purchased local currency put options, primarily for intercompany transactions.
Additionally, these contracts help hedge against foreign currency exposures
related to net investments in international affiliates.

The report estimates that a 10% appreciation in the currencies being hedged
against the U.S. dollar would lead to a decrease in the fair value of foreign
exchange contracts by $409 million and $782 million as of December 31, 2023, and
2022, respectively. Conversely, cross-currency swap contracts, which are used to
manage risks from long-term debt in euros, would see an increase in fair value
by $46 million in 2023, while decreasing by $73 million in 2022 under similar
currency appreciation scenarios.

Regarding interest rate risk, the company employs fixed-to-floating interest
rate swap contracts to balance its debt portfolio. A sensitivity analysis
indicates that a 1% increase in interest rates would not materially impact
earnings. However, it is estimated that such an increase would decrease the fair
value of long-term debt by $3.0 billion in 2023 and $2.6 billion in 2022.

Lastly, the report addresses credit risk associated with counterparties in
derivative transactions. The company maintains a strict investment policy to
minimize credit risk, ensuring that investments are made only with high-quality
institutions and diversifying counterparties to mitigate potential defaults. For
further details, the report refers to additional financial statements and
supplementary data.

===== 5/10. CARRIER GLOBAL CORP =====
The risk report outlines the company's exposure to market risks, including
fluctuations in foreign currency exchange rates, interest rates, and commodity
prices, which could affect its financial performance. As of December 31, 2023,
there has been no significant change in the company's exposure to these market
risks.

In terms of foreign currency exposure, the company operates globally, which
subjects it to exchange rate fluctuations relative to its reporting currency,
```

(continues on next page)

(continued from previous page)

```
the U.S. dollar. Many of its international operations use currencies other than
the U.S. dollar, meaning that the company's reported results can vary based on
the strength or weakness of the dollar against these currencies. While the
company actively manages material currency exposures related to transactions at
the legal entity level, it does not hedge against currency translation risk.

The report highlights specific transactions, such as the acquisition of the VCS
business, where 80% of the euro-denominated purchase price was paid in cash,
exposing the company to exchange rate risks. To mitigate this risk, the company
utilized window forward contracts, with changes in their fair value reflected in
other income or expense. Similarly, for the TCC acquisition, the company
employed cross currency swaps and a Japanese term loan facility to hedge against
foreign currency translation risks associated with its investments in
subsidiaries operating in yen.

Regarding commodity price exposure, the company faces volatility in the prices
of certain commodities and shipping fuel costs. While it uses fixed-price
contracts to manage some of this exposure, it currently does not have any
commodity hedge contracts in place. Lastly, the report notes that most of the
company's long-term debt carries fixed interest rates, insulating it from
significant impacts due to fluctuations in market interest rates.

===== 6/10. LULULEMON ATHLETICA INC =====
The risk report outlines various market risks faced by the company, focusing on
foreign currency exchange risk, interest rate risk, credit risk, and inflation.

Foreign currency exchange risk is primarily associated with the translation of
financial statements from local currencies of international subsidiaries into
U.S. dollars. In 2023, fluctuations in exchange rates resulted in a revenue
decrease of $89.8 million compared to 2022. The company records foreign currency
translation differences as other comprehensive income (loss) within
stockholders' equity. A significant portion of net assets is held in Canadian
dollars, and the company uses forward currency contracts to hedge against
translation exposure. The translation of Canadian subsidiaries contributed to an
increase in other comprehensive loss of $9 million, despite net investment hedge
gains. Additionally, transaction risk arises from intercompany transactions and
inventory purchases in currencies other than the subsidiaries' functional
currencies. As of January 28, 2024, the company had forward currency contracts
to hedge against foreign currency revaluation gains and losses.

Interest rate risk is linked to the company's revolving credit facility, which
has a variable interest rate. As of January 28, 2024, there were no borrowings
under this facility, but the company may consider using derivative financial
instruments in the future to mitigate potential losses if a significant balance
arises.

Credit risk is minimal, as the company holds cash with reputable financial
institutions and invests in AAA-rated money market funds. The company actively
monitors the creditworthiness of its counterparties to limit exposure.

Lastly, inflation poses a risk to operating results, particularly due to rising
costs in wages, transportation, and raw materials. Increased costs may adversely
affect operating margins if selling prices do not adjust accordingly. The report
emphasizes the importance of managing these risks to maintain financial
stability.
```

(continues on next page)

(continued from previous page)

```
===== 7/10. AIRBNB INC =====
The risk report outlines the market risks faced by the company, primarily
focusing on foreign currency risk and investment risk due to its extensive
global operations. In 2023, the company conducted transactions in over 40
currencies, with significant exposure to the euro, British pound, Canadian
dollar, Australian dollar, Brazilian real, and Mexican peso. This exposure
arises from international revenue and expenses, which are subject to
fluctuations in foreign currency exchange rates against the U.S. dollar.
Consequently, a strengthening U.S. dollar can negatively impact financial
results, while a weakening dollar can be beneficial.

The company faces foreign currency risks related to various aspects, including
revenue from bookings in foreign currencies, funds receivable and payable, and
intercompany balances. To mitigate these risks, the company employs foreign
currency derivative contracts aimed at managing forecasted foreign-denominated
revenue and other related balances. However, these hedges do not completely
eliminate the impact of currency fluctuations, and the company may opt not to
hedge certain exposures due to economic or accounting considerations. A
hypothetical adverse change of 10% in foreign currency exchange rates could lead
to a loss of approximately $20 million.

Additionally, the report addresses investment and interest rate risk,
particularly concerning the company's investment portfolio. As of December 31,
2023, the company held $6.9 billion in cash and cash equivalents and $3.2
billion in short-term investments, primarily in high-quality debt securities.
The company aims to preserve capital and maintain liquidity without
significantly increasing risk, avoiding speculative investments. Due to the
short maturities of its investments, the portfolio is relatively insensitive to
interest rate changes, with a potential $20 million decrease in value
anticipated from a hypothetical 100 basis point increase in interest rates.

===== 8/10. MERCADOLIBRE INC =====
The risk report outlines the market risks faced by the company, primarily
stemming from macroeconomic instability and fluctuations in interest rates and
foreign currency exchange rates, particularly with the Brazilian real, Argentine
peso, and Mexican peso. These factors can significantly impact the value of the
company's financial assets and liabilities. The company also faces risks related
to its long-term retention programs (LTRPs), which involve cash payments to
employees that vary based on the market price of its stock.

With substantial international operations, the company is exposed to foreign
currency risks that can adversely affect its financial results. It engages in
transactions in various foreign currencies and charges its international
subsidiaries for the use of intellectual property and corporate services. To
mitigate these risks, the company employs foreign currency exchange forward
contracts and currency swaps, although these hedges do not completely eliminate
the impact of currency fluctuations.

As of December 31, 2023, the company reported significant cash and cash
equivalents, receivables, and investments in foreign currencies, totaling over
$12 billion. The report highlights a consolidated loss of $615 million due to
foreign currency fluctuations, particularly in Argentina, where government
restrictions on accessing U.S. dollars have exacerbated losses.

Interest rate changes also pose risks to the company's earnings and cash flows,
affecting the cost of financing and the returns on investments. The report notes
```

(continues on next page)

(continued from previous page)

```
that a hypothetical increase in interest rates could lead to increased financial
liabilities.

Additionally, the company’ s LTRPs expose it to equity price risk, with a total
contractual obligation fair value of $418 million as of December 31, 2023. The
report includes sensitivity analyses showing how changes in equity prices could
impact the company's financial obligations related to these programs. Overall,
the report emphasizes the complexities and potential financial impacts of market
risks on the company's operations.

===== 9/10. A T & T INC =====
The risk report outlines AT&T Inc.'s exposure to market risks, primarily from
fluctuations in interest rates and foreign currency exchange rates, which affect
its cost of capital. The company employs a strategic approach to manage these
risks, utilizing derivatives such as interest rate swaps, locks, and cross-
currency swaps, strictly for hedging purposes rather than speculative trading.
The report indicates that there are no anticipated changes to these risk
management strategies in the near future.

A significant factor in estimating postretirement benefit obligations is the
weighted-average discount rate, which has seen increased volatility and is
currently lower than historical averages. This results in higher obligations for
the company, although future increases in discount rates could lead to lower
obligations and improved funded status.

Interest rate risk is managed through a mix of fixed- and floating-rate debt,
with the majority of financial instruments being medium- to long-term fixed-rate
notes. The company has established limits on interest rate risk and closely
monitors its debt and derivatives portfolios. As of December 31, 2023, AT&T had
no interest rate locks but utilized cross-currency swaps to mitigate risks
associated with foreign-denominated debt.

Foreign exchange risk is addressed through contracts that hedge costs and debt
in foreign currencies. The report notes that AT&T's foreign-denominated debt has
been converted to fixed-rate U.S. dollars, effectively eliminating associated
risks. A sensitivity analysis is employed to assess the impact of market risk
exposures on the fair value of financial instruments.

Overall, the report emphasizes AT&T's commitment to maintaining financial
flexibility and managing risks effectively through established policies and
procedures, ensuring the integrity of its financial reporting and internal
controls.

===== 10/10. REPUBLIC SERVICES INC =====
The risk report outlines the company's exposure to market risks, particularly
focusing on interest rate risk, fuel price risk, and commodities price risk.

In terms of interest rate risk, the company is primarily affected by
fluctuations in U.S. interest rates and the Secured Overnight Financing Rate
(SOFR). To manage this risk, the company employs a mix of fixed and floating
rate debt. As of December 31, 2023, the carrying value of its variable rate debt
is close to its fair value, reflecting current market conditions. The company
has also utilized interest rate swap agreements as cash flow hedges to mitigate
the impact of interest rate fluctuations on its variable rate debt. The report
indicates that a 100 basis point change in interest rates could alter annual
interest expenses by approximately $20 million.
```

(continues on next page)

(continued from previous page)

```
Regarding fuel price risk, fuel costs are a significant operational expense for
the company. Although it charges fuel recovery fees to most customers, it cannot
do so universally. As of the end of 2023, the company had no fuel hedges in
place. A 20-cent per gallon change in diesel fuel prices could affect fuel costs
by about $27 million annually, while the corresponding change in fuel recovery
fees could be around $36 million.

Lastly, the report addresses commodities price risk, particularly concerning the
marketing of recycled materials. The company has experienced volatility in
commodity prices due to market supply and demand fluctuations. As of December
31, 2023, it had no hedges in place for recycling commodities. A $10 per ton
change in recycled commodity prices could impact annual revenue and operating
income by approximately $10 million. Revenue from recycling activities decreased
from $359.1 million in 2022 to $312.3 million in 2023. Overall, the report
highlights the company's proactive approach to managing these market risks while
acknowledging the inherent uncertainties.
```

35.3.4 Evaluation

ROUGE

Recall-Oriented Understudy for Gisting Evaluation (ROUGE) is a set of metrics used to evaluate the quality of summaries by comparing them to reference summaries or human-generated summaries.

- ROUGE-N measures the overlap of n-grams (contiguous sequences of n words) between the system-generated and the reference summaries
- ROUGE-L measures the longest common subsequence (LCS).

BLEU Bilingual Evaluation Understudy (BLEU) evaluates n-gram precision with a brevity penalty to discourage overly short outputs. Originally for machine translation, it is also used for summarization.

- N-gram Precision measures the overlap of n-grams (typically up to 4-grams) between the system-generated summary and the reference summary.
- Brevity Penalty penalizes overly short summaries that do not capture enough information from the reference summaries.
- Cumulative BLEU calculates the geometric mean of BLEU scores for 1-gram to n-gram, rewarding systems that produce more accurate translations across longer phrases.

```python
# These metrics compare overlapping n-grams to measure content similarity.
from rouge_score import rouge_scorer
```

```python
# computes ROUGE-1 and ROUGE-2 scores between model-generated summaries
def collect_rouge(target, prediction):
    """Helper to loop over companies to compute rouge scores of two risk summaries"""
    scores = {'rouge1': [], 'rouge2': []}
    scorer = rouge_scorer.RougeScorer(scores.keys(), use_stemmer=True)
    for permno in docs.index:
        score = scorer.score(target=target[permno], prediction=prediction[permno])
        for rouge_type in scores.keys():
            scores[rouge_type].append(Series(score[rouge_type]._asdict(),
                                             name=univ.loc[permno, 'comnam']))
    return scores
```

```python
# Display and compare rouge metric
def display_rouge(rouge_type, scores):
    """Helper to display rouge scores over the companies"""
    df = pd.concat(scores[rouge_type], axis=1)
    print(f"{rouge_type.upper()} metric:")
    return pd.concat([df, df.T.mean().rename('  average')], axis=1).T  # display
```

```python
# Compute rouge-1 and rouge-2 scores between gpt- and llama-generated summaries
scores = collect_rouge(target=summary[gpt_name], prediction=summary[model_name])
```

```python
display_rouge("rouge1", scores)
```

```
ROUGE1 metric:
```

	precision	recall	fmeasure
PACCAR INC	0.787234	0.742475	0.764200
PHILLIPS 66	0.366142	0.324042	0.343808
MASTERCARD INC	0.759184	0.628378	0.687616
BRISTOL MYERS SQUIBB CO	0.773913	0.585526	0.666667
CARRIER GLOBAL CORP	0.761628	0.451724	0.567100
LULULEMON ATHLETICA INC	0.560241	0.636986	0.596154
AIRBNB INC	0.720472	0.639860	0.677778
MERCADOLIBRE INC	0.467593	0.331148	0.387716
A T & T INC	0.393805	0.312281	0.348337
REPUBLIC SERVICES INC	0.783410	0.553746	0.648855
average	0.637362	0.520617	0.568823

```python
display_rouge("rouge2", scores)
```

```
ROUGE2 metric:
```

	precision	recall	fmeasure
PACCAR INC	0.483986	0.456376	0.469775
PHILLIPS 66	0.079051	0.069930	0.074212
MASTERCARD INC	0.381148	0.315254	0.345083
BRISTOL MYERS SQUIBB CO	0.441048	0.333333	0.379699
CARRIER GLOBAL CORP	0.350877	0.207612	0.260870
LULULEMON ATHLETICA INC	0.244713	0.278351	0.260450
AIRBNB INC	0.367589	0.326316	0.345725
MERCADOLIBRE INC	0.116279	0.082237	0.096339
A T & T INC	0.097778	0.077465	0.086444
REPUBLIC SERVICES INC	0.462963	0.326797	0.383142
average	0.302543	0.247367	0.270174

35.3.5 Role prompting

By adjusting the system prompt (e.g., "You are a patient teacher"), LLMs can be guided to produce more accessible summaries. This technique, known as **role prompting**, is helpful for tailoring the tone and persona of responses for specific audiences.

```
# generates simplified summaries for readability
summary['simple_deepseek'] = collect_summaries(
    model,
    role="You are a patient lower-school teacher, using simple words to explain to␣
↪your students in the fifth grade.")
```

```
===== 1/10. PACCAR INC =====
<think>
Okay, so I need to summarize this risk report for fifth graders. Let me read
through it carefully first.

The report talks about market risks and derivative instruments. It mentions
interest-rate risks, currency risks, and commodity price risks. Each section has
some numbers and explanations.

Starting with interest-rate risks: The company uses hedging programs to manage
how changes in interest rates affect them. They estimate the impact if there's a
100 basis point increase across all yield curves. There are tables showing
potential losses or gains for assets and liabilities in 2023 and 2022.

Next, currency risks: The company hedges against exchange rate fluctuations for
several currencies like Canadian dollar, euro, etc. They mention potential
losses from unfavorable changes in foreign exchange rates, with numbers for 2023
and 2022.

Then, commodity price risks: They use forward contracts to hedge prices of
commodities used in truck production. The report states the potential loss from
a 10% unfavorable change in commodity prices, again giving figures for both
years.

I need to simplify this into prose that fifth graders can understand. I should
avoid jargon and explain terms like "hedging" as strategies to reduce risk.
Maybe use analogies they can relate to, like protecting something with
insurance.

I'll structure it by introducing each type of risk, explain the method used
(hedging), mention the potential losses in simple terms, and note that these are
offset by other changes. I should keep sentences short and clear, avoiding
complex structures.

Let me make sure I cover all three risks: interest rates, currencies,
commodities. For each, a brief explanation of what it is, how they manage it,
and the associated numbers without getting too bogged down in specifics.

I think that's a good plan. Now, let me put it all together in a summary.
</think>

**Understanding Market Risks for Kids**

Imagine you have a big lemonade stand, and you want to protect yourself from
things that might make your profits go down. There are three main risks we talk
```

(continues on next page)

(continued from previous page)

```
about here: interest rates, currencies, and commodity prices.

1. **Interest Rates**: Just like how the bank gives you different rates if you
save or borrow money, companies have to deal with changing interest rates too.
If rates go up suddenly, it can affect how much money they make or owe. The
company uses something called hedging to protect against these changes. They
look at how much their assets and debts might change if interest rates went up
by a full percentage point. In 2023, they saw a small loss, but in 2022, it was
almost no loss.

2. **Currencies**: When you sell lemonade to people from other countries who use
different money, the exchange rate can affect how much you make. The company
uses contracts to protect against changes in these rates for currencies like the
Canadian dollar and euro. If these rates go 10% against them, they might lose
some money, but this was $259.7 million in 2023 and $216.6 million in 2022.
However, other factors balance this out.

3. **Commodity Prices**: Making lemonade needs lemons, right? If the price of
lemons goes up or down a lot, it affects your costs. The company uses forward
contracts to lock in prices for materials they need, like metals and plastics
for making trucks. A 10% bad change in these prices could cost them $3.3 million
in 2023 and $2.5 million in 2022, but again, other changes help reduce this
impact.

So, just like you might save money or buy insurance to protect your lemonade
stand, companies use strategies like hedging to manage these risks and keep
their profits safe.

===== 2/10. PHILLIPS 66 =====
<think>
Okay, so I'm trying to understand all these risks mentioned in the document. Let
me start by reading through them carefully.

First, there are market conditions like fluctuations in prices and margins for
NGLs, crude oil, natural gas, and refined products. That makes sense because
energy prices can be really volatile due to things like supply and demand
changes or geopolitical events.

Then there's government policies affecting pricing, regulation, taxation,
especially exports. I know that export policies can have a big impact on supply
and demand, so this is an important factor.

Capacity constraints in pipelines, storage, and fractionation facilities are
another risk. If these infrastructure issues arise, it could limit how much
product they can transport, leading to bottlenecks or higher costs.

OPEC and non-OPEC actions influence supply and demand, which affects prices. I
remember that OPEC's decisions can cause significant shifts in the market.

The success of DCP LP integration is mentioned, including achieving synergies.
This probably refers to a business strategy where they're combining operations,
so if this doesn't go as planned, it could hurt their performance.

Unexpected technical difficulties or cost increases during construction or
operation are risks too. Construction delays can be costly and disrupt
production.
```

(continues on next page)

(continued from previous page)

```
Drilling and production volumes around midstream assets are another point. If
the wells aren't producing as expected, it affects the company's revenue from
those assets.

Permits and regulations compliance are also risks. They need to get permits for
projects, and if they can't or if regulations change, it could delay things or
require more spending.

Savings and cost reductions from business transformation initiatives might not
happen as planned. If they don't achieve these savings, their financial goals
could be at risk.

Renewable fuels policies and climate change regulations are factors too. Changes
in these areas could affect demand for traditional fuels or require new
investments.

Economic and political developments like the Russia-Ukraine war can impact
markets. Also, things like inflation, interest rates, and expropriation of
assets pose risks.

Public health crises, like pandemics, can disrupt operations and reduce demand
for their products. The recovery after such events is also uncertain.

Capital projects might not be completed on time or within budget. Delays here
can lead to cost overruns and project failures.

Asset dispositions or acquisitions could face challenges if they don't complete
them successfully. This includes both the sale of assets and buying new ones.

Litigation or government actions could disrupt operations, leading to legal fees
or operational changes.

Accidents, weather events, civil unrest, etc., can damage facilities and
interrupt operations, causing financial losses.

Meeting sustainability goals is another risk. If they don't reduce GHG emissions
as planned or develop new technologies, it could affect their reputation and
operations.

New products might not be accepted by the market, leading to wasted investments.

Monetary conditions and exchange controls can impact international trade and
profitability.

Environmental regulations requiring significant investments or reducing demand
for their products are risks. They might have to spend a lot on compliance or
face reduced sales.

Liability from environmental issues like cleanup costs is another concern.

Changes in laws and regulations, including alternative energy mandates, could
force them to adapt their business models.

Climate change concerns might lead to changes in how they operate or increase
expenses through litigation or other costs.
```

(continues on next page)

(continued from previous page)

```
Impairment charges could occur if the value of assets decreases due to market
conditions or strategic decisions.

Access to capital is crucial; if it's limited or more expensive, they might not
be able to fund their operations effectively.

Counterparties going bankrupt could lead to financial losses from unpaid debts
or obligations.

Cybersecurity incidents can compromise sensitive data and lead to legal
liabilities or operational disruptions.

Joint ventures that the company doesn't control could make decisions that
negatively impact their own business.

Lastly, all these factors are generally described in Item 1A of the risk factors
in the report.

I think I've covered all the points. Now, how do I summarize this? Maybe by
categorizing them into market risks, regulatory risks, operational risks,
financial risks, and environmental/sustainability risks. That way, it's easier
to understand and address each category separately.
</think>

The document outlines a comprehensive list of risks that could impact the
company's performance and operations. These risks can be categorized as follows:

1. **Market Risks**:
   - Volatility in energy prices (NGLs, crude oil, natural gas, refined
products).
   - Fluctuations in margins and demand for refining and petrochemical products.
   - Impact of OPEC and non-OPEC actions on supply and demand.

2. **Regulatory and Policy Risks**:
   - Changes in government policies affecting pricing, regulation, taxation, and
exports.
   - Compliance with environmental regulations (GHG emissions, renewable fuels).
   - Influence of international monetary conditions and exchange controls.

3. **Operational Risks**:
   - Capacity constraints in transportation infrastructure (pipelines, storage).
   - Delays or cost overruns in construction and capital projects.
   - Challenges in permit acquisition and regulatory compliance.
   - Risks from joint ventures not under the company's control.

4. **Financial Risks**:
   - Failure to achieve expected savings from business transformation
initiatives.
   - Impairment charges due to asset value declines.
   - Limitations in accessing capital or increased funding costs.
   - Counterparty risks (e.g., bankruptcy leading to financial losses).

5. **Environmental and Sustainability Risks**:
   - Meeting sustainability goals and adapting to climate change regulations.
   - Development of new technologies and products that may not gain market
```

(continues on next page)

(continued from previous page)

```
acceptance.

6. **Geopolitical and Public Health Risks**:
   - Impact of geopolitical events (e.g., Russia-Ukraine war) on markets.
   - Disruptions from public health crises (e.g., pandemics).

7. **Legal and Reputational Risks**:
   - Litigation risks and potential liabilities from environmental issues.
   - Cybersecurity threats leading to data breaches or operational disruptions.

By categorizing these risks, the company can better manage and mitigate
potential challenges across different areas of its operations.

===== 3/10. MASTERCARD INC =====
<think>
Okay, so I need to explain this risk report to fifth graders. Let me read
through it again and try to understand what it's saying.

The report is about market risks that a company faces, specifically MasterCard,
I think. Market risk here refers to the possibility of losing money because of
changes in things like interest rates or foreign exchange rates. The company
talks about how they manage these risks by using derivatives and other financial
instruments.

First, there's something about foreign currency risk. They use derivative
contracts to handle the exposure from transactions that happen in different
currencies. These contracts help them reduce losses if the exchange rate
changes. For example, a 10% change could lead to a loss of around $414 million
in 2023 and $94 million in 2022. But they also mention that these numbers don't
account for the actual transactions they're hedging against.

Then there's interest rate risk. They have investments that are sensitive to
changes in interest rates, but a 1% change wouldn't cause significant losses
because their portfolio is diversified and they invest in high-quality
securities. They also use derivative contracts to hedge some of their fixed-rate
debt.

I need to simplify all this into about 300 words using simple language. I should
avoid jargon and explain concepts like derivatives, exchange rates, and interest
rates in a way that fifth graders can understand. Maybe compare it to something
they know, like saving money or exchanging currencies when traveling.

I'll start by introducing what market risk is, then talk about how the company
manages foreign currency risks using contracts, and then move on to interest
rate risks. I should also mention that these risks are managed through policies
and tools, but even with all that, there's still some potential for loss if
things change unexpectedly.

Let me structure it step by step: introduce market risk, explain foreign
exchange risk with an example, talk about how they manage it, then move to
interest rates, and finally wrap up by saying that while they take steps to
minimize these risks, there are still possible losses.
</think>

**Understanding Market Risks in Simple Terms**
```

(continues on next page)

(continued from previous page)

```
Imagine you have a piggy bank where you save your money. Sometimes, the value of
your savings can change based on things like how much interest you earn or
changes in exchange rates if you're saving different currencies.

Market risk is like the possibility that your piggy bank's value might go down
because of these changes. Companies, just like you, want to protect their money
from losing value.

One way they do this is by using something called derivatives, which are like
special contracts. For example, if a company does business in different
countries and gets money in other currencies, they can use these contracts to
protect against the risk that those currencies might lose value compared to
their home currency, like the US dollar or euro.

They also watch interest rates because if the bank where they keep their money
changes the interest rate, it affects how much they earn. By using more of these
special contracts, they can reduce the chance that their savings will be
affected by these changes.

Even though companies use these tools to protect themselves, there's still a
chance that unexpected changes could cause losses. So, while they take steps to
keep their money safe, they know there are no guarantees in the ever-changing
world of money and exchange rates.

===== 4/10. BRISTOL MYERS SQUIBB CO =====
<think>
Okay, so I need to explain this risk report to fifth graders. Let me read
through it again to make sure I understand the main points.

The report talks about market risks related to currency exchange rates and
interest rates. The company uses derivatives like forwards and swaps to hedge
these risks when it's cost-effective. They mention that all financial
instruments, including derivatives, are subject to counterparty credit risk,
which is part of their fair value measurement. Importantly, they don't use
derivatives for trading purposes.

Looking at foreign exchange risk, a big part of their revenue, earnings, and
cash flow are affected by changes in currency rates, mainly the euro and
Japanese yen. They use forward contracts and put options to manage risks from
intercompany transactions. There's also transaction risk from non-functional
currencies, which they handle with forwards but aren't designated as hedges.
Additionally, cross-currency swaps help manage long-term debt in euros and hedge
net investments.

They estimate that a 10% appreciation in the hedged currencies would decrease
the fair value of their foreign exchange contracts by certain amounts, reducing
earnings. Similarly, cross-currency swaps would have different impacts on their
fair values.

For interest rate risk, they use fixed-to-floating swaps to balance their debt
and analyze sensitivity to rate changes. A 1% increase in rates wouldn't harm
earnings much, but a larger increase in long-term rates would decrease the fair
value of their long-term debt.

Credit risk is managed by investing only with high-quality institutions and
diversifying derivatives across counterparties to reduce default risks.
```

(continues on next page)

(continued from previous page)

```
Now, I need to simplify this into about 300 words. I'll start by introducing
market risks, then break it down into foreign exchange and interest rate risks,
explaining how they use derivatives to hedge. Then touch on credit risk
management. I should avoid jargon and make each part clear for kids.
</think>

Imagine you have a big company that sells things around the world. Sometimes,
when they sell or buy stuff in other countries, the money they get can change
value because of something called exchange rates. It's like if you have dollars
and someone else has euros, and one day those euros might be worth more or less
compared to your dollars.

The company uses special tools called derivatives to protect themselves from
these changes. These tools are kind of like insurance policies that help them
guess what the money will look like in the future so they don't lose too much if
things change.

They also have to worry about interest rates, which are like the cost of
borrowing money. If the interest rates go up, it might make their debts more
expensive. But again, they use these derivative tools to balance things out and
not get hurt too badly.

Lastly, they make sure that the people they deal with (their counterparties) are
trustworthy so that if something goes wrong, they don't lose money because
someone else didn't pay them back.

So, in short, the company is careful about how it handles money changes and
interest rates using special tools to stay safe financially.

===== 5/10. CARRIER GLOBAL CORP =====
<think>
Okay, so I need to explain this risk report to fifth graders. Let me read
through it again and try to understand what it's saying.

The report talks about market risks like foreign currency exchange rates,
interest rates, and commodity prices affecting the company's operations and
financial condition. It says there haven't been significant changes in their
exposure to these risks in 2023.

First part is about foreign currency exposures. They have operations all over
the world, so when currencies change against the US dollar, it affects their
reported results. They manage some currency risks with things like window
forward contracts and cross-currency swaps, especially for big acquisitions. But
they don't hedge translation risk entirely.

Then there's commodity price exposure. They use fixed-price contracts sometimes
but didn't have any hedges at the end of 2023. Fuel costs for shipping are
another concern.

Interest rate exposure is low because most debt has fixed rates, so changes in
market rates won't affect them much.

I need to simplify all this into a summary that's easy for fifth graders to
understand. Maybe use examples they can relate to, like how exchanging money
when traveling can change how much you have, or how prices of things like oil
```

(continues on next page)

(continued from previous page)

```
can go up and down affecting costs.

I should avoid jargon and keep sentences short. Make sure each point is clear
and relatable.
</think>

The company faces risks from changes in foreign exchange rates, interest rates,
and commodity prices, which can affect their profits and financial health. These
risks haven't changed much over the past year.

**Foreign Currencies:**
They operate worldwide, so when other countries' currencies go up or down
against the US dollar, it affects how much money they make. They use strategies
like window forward contracts to manage some of these risks, especially for big
purchases. For example, if they buy something in euros and the euro goes up, it
might cost them more in dollars.

**Commodity Prices:**
They rely on materials whose prices can swing, like oil used for shipping.
Sometimes they lock in fixed prices to avoid sudden jumps, but at the end of
2023, they didn't have any such contracts.

**Interest Rates:**
Most of their loans have fixed rates, so even if market rates change, it won't
affect them much. It's like having a set monthly payment on a loan, no
surprises.

In summary, they manage some risks with smart financial tools but still face
uncertainties from global markets and material costs.

===== 6/10. LULULEMON ATHLETICA INC =====
<think>
Okay, so I need to explain this risk report to fifth graders. Let me read
through it again and try to understand what each part is saying.

First, the report talks about market risks, specifically foreign currency
exchange risk and translation risk. It mentions that their international
subsidiaries use local currencies as functional currencies, but the financial
statements are in US dollars. So when they translate from other currencies to
USD, changes in exchange rates affect the reported amounts. In 2023, their
revenue was lower by $89.8 million compared to 2022 because of this.

They also mention translation risk where differences from translating balance
sheets into USD are recorded as part of comprehensive income. They have a
Canadian subsidiary with significant net assets and use forward contracts to
hedge some of this exposure. The impact on their comprehensive loss was an
increase due to these hedges.

Then there's transaction risk, which comes from transactions in currencies other
than the functional currency—like intercompany transactions or inventory
purchases. They hold cash in different currencies too. As of a certain date,
they had forward contracts to hedge revaluation gains and losses on monetary
assets and liabilities of their subsidiaries.

They did a sensitivity analysis showing that a 10% depreciation in USD would
affect the fair value of derivatives by $29.8 million. But this is offset by
```

(continues on next page)

(continued from previous page)

```
changes in the underlying hedged items. They plan to use more derivatives in the
future for other currency pairs but don't trade them for profit.

Next, interest rate risk: their credit facility has variable rates, so if they
borrow money, changing rates could affect them. As of the date, they didn't have
any borrowings except for some letters of credit. They don't do any hedging now
but might in the future if needed.

They hold cash and equivalents in safe places like money market funds, which are
AAA-rated. Even though some deposits exceed insured limits, they haven't had any
losses. They monitor the financial institutions they deal with to minimize
credit risk.

Lastly, inflation could affect their costs—like wages, freight, and raw
materials—which might hurt their margins if they can't pass these costs onto
customers or find efficiencies.

So, putting this all together, I need to explain it in simple terms for fifth
graders. Maybe start by talking about exchanging money between countries and how
the value of money changes over time. Then explain that when companies do
business internationally, they have to convert currencies, which can lead to
gains or losses based on exchange rates. Use an example with a pretend company
to make it relatable.

I should also mention hedges as a way to protect against these risks, maybe
using the analogy of insurance. Keep each section short and use everyday
language so it's easy to understand.
</think>

**Explaining Market Risks in Simple Terms for Fifth Graders**

Imagine you're a kid who loves to travel and exchange money between countries.
When you go on vacation, you might take some dollars and exchange them into
another country's currency, like Canadian dollars or Euros. Now, companies that
do business all over the world also have to deal with exchanging money, but it
can be tricky because the value of these currencies changes over time.

**1. Foreign Currency Exchange Risk:**
Just like how you might get more or fewer toys when you exchange your money,
companies can end up with more or less "toys" (money) based on how the exchange
rate changes. For example, if a company earns money in Canada but reports it in
US dollars, a change in the value of the Canadian dollar against the US dollar
can make their earnings go up or down. In 2023, this change made their revenue
$89.8 million less than in 2022.

**2. Translation Risk:**
When companies translate their money from one country's currency to another,
like converting Canadian dollars back to US dollars for reports, the value can
change. This is called translation risk. Sometimes, these changes can affect how
much profit or loss a company shows on its financial statements.

**3. Transaction Risk:**
Companies also face risks when they do transactions in different currencies. For
example, if they buy something from another country and pay in that country's
currency, the value of that payment can change by the time it's converted back
to their home currency. To protect against this, companies might use "hedges,"
```

(continues on next page)

(continued from previous page)

```
which are like insurance policies to lock in exchange rates.

**4. Interest Rate Risk:**
Borrowing money is another area where companies face risks. If they take a loan
with an interest rate that changes over time, and if those rates go up, it could
cost them more money to pay back the loan. Companies keep an eye on this so they
can plan for any increases.

**5. Inflation:**
Inflation is when things like food, clothes, and other goods get more expensive
over time. For companies, this means their costs might go up too—like paying
more for materials or worker wages. If a company can't increase its prices
enough to cover these higher costs, it could affect their profits.

**6. Credit Risk:**
When companies deal with banks or other financial institutions, there's always a
chance that the other side might not be able to pay back what they owe. To
minimize this risk, companies work with reliable institutions and keep track of
how much money they're dealing with from each partner.

In summary, just like you have to be careful when exchanging money while
traveling, companies must manage these risks carefully to ensure their profits
aren't affected by changes in currency values, interest rates, or inflation.
Using tools like hedges and working with trustworthy institutions helps them
navigate these challenges smoothly.

===== 7/10. AIRBNB INC =====
<think>
Okay, so I need to summarize this risk report for fifth graders. Let me read
through it carefully first.

The report talks about market risks that Airbnb faces because they operate all
over the world. The main risks are foreign currency risk and investment risk.

Foreign currency risk is when changes in exchange rates affect their financial
results. They accept payments in over 40 currencies, so if the US dollar
strengthens or weakens, it can impact how much money they make or spend. For
example, if the US dollar gets stronger, their revenue from other countries
might decrease because those dollars are worth less.

They also mention that they use foreign currency derivatives to hedge against
some of these risks, but they don't eliminate all the effects. There's a 20
million loss mentioned if exchange rates were to change unfavorably.

Investment risk is related to their investment portfolio. They have a lot of
cash and short-term investments in things like debt securities. If interest
rates go up or down, it can affect how much they earn on these investments. A
100 basis point increase would decrease their portfolio by 20 million. But since
most of their investments are short-term, the risk from interest rate changes
isn't too bad.

I need to explain all this in simple terms for kids. Maybe use analogies they
can understand, like comparing currencies to different types of money and how
exchanging them can be tricky. Also, simplify investment risks by talking about
saving money in a piggy bank or something safe where small changes in interest
rates don't matter much.
```

(continues on next page)

(continued from previous page)

```
I should structure it into two main parts: foreign currency risk and investment
risk. Use everyday language and avoid jargon. Maybe include examples they can
relate to, like traveling abroad and exchanging money.
</think>

**Understanding Airbnb's Market Risks in Simple Terms**

Airbnb faces some financial risks because they operate all around the world,
dealing with many different currencies and investments. Let's break down these
risks into two main parts: foreign currency risk and investment risk.

1. **Foreign Currency Risk:**
   - Imagine you have money in one country's currency, like euros from France or
pounds from England. If the value of your home currency (like the US dollar)
changes, it can affect how much money you actually have when you exchange it.
   - Airbnb deals with over 40 currencies, which means they are exposed to
changes in exchange rates. For example, if the US dollar becomes stronger, the
money they make from other countries might decrease because those dollars are
worth less.
   - To protect against these changes, Airbnb uses something called foreign
currency derivatives, which are like insurance policies to reduce the impact of
bad exchange rate changes. However, even with this protection, there's still
some risk. If exchange rates were to change unfavorably, they could lose about
$20 million.

2. **Investment Risk:**
   - Airbnb keeps a lot of money in safe investments, like short-term debt
securities and government bonds. These are similar to putting money in a savings
account where it's secure but doesn't earn a lot.
   - If interest rates change, the value of these investments can go up or down.
For instance, if interest rates were to increase by 1%, their investment
portfolio could decrease by about $20 million.
   - However, since most of their investments are short-term, they aren't too
affected by small changes in interest rates. It's like having money in a piggy
bank where you don't need it right away, so minor fluctuations won't hurt much.

In summary, Airbnb manages risks from exchanging currencies and investing by
using protective measures, but there are still some uncertainties due to the
global nature of their business.

===== 8/10. MERCADOLIBRE INC =====
<think>
Okay, so I'm trying to understand this document about MercadoLibre's risks and
sensitivities. It seems like it's part of their financial disclosures, maybe in
their annual report or something similar. Let me break down what each section is
saying.

First, the foreign exchange risk. They have a significant amount of revenue from
Brazil, Argentina, and other Latin American countries. Since they're dealing
with multiple currencies, especially the Brazilian real and Argentine peso,
which can be volatile, this could affect their financials. If the real or peso
weakens against the dollar, their reported revenue in dollars might decrease
because when converted, those currencies would buy fewer dollars. That makes
sense because if you have a lot of revenue in a currency that's dropping, your
overall revenue in USD terms goes down.
```

(continues on next page)

(continued from previous page)

```
Next is interest rate risk. They have long-term debt, which means they're
exposed to changes in interest rates. If rates go up, the value of their
existing debt might decrease, and their borrowing costs could increase if they
need to refinance or take on more debt. I'm not entirely sure how sensitive they
are to rate changes, but it's something to watch, especially with global
interest rates potentially rising.

Then there's commodity price risk. They're exposed to fuel and electricity
prices because these are operational costs. If oil prices go up, their delivery
and logistics costs would increase, affecting their margins. Similarly, higher
electricity prices could impact their operations, especially in countries where
energy is a significant cost. This seems like a manageable risk but could have
noticeable effects if prices spike.

Equity price risk is tied to their LTRP plans. These are long-term retention
programs that pay out based on stock performance. The value of these obligations
is sensitive to the company's stock price. If the stock goes up, the payouts
increase, which could strain their finances. Conversely, a drop in stock price
reduces the payout liability. They've done a sensitivity analysis showing how a
40% change in stock price affects the obligation. That's pretty significant; a
40% drop would reduce the liability from $418 million to about $251 million,
which is a substantial difference.

Inflation risk is another factor, especially in countries like Argentina where
inflation has been high. High inflation can erode purchasing power and affect
consumer behavior. If people have less money due to inflation, they might spend
less online, impacting MercadoLibre's sales. Additionally, rising costs for
goods and services could squeeze margins if the company can't pass these costs
onto customers.

I'm a bit confused about how exactly the equity price affects their obligations.
It seems like it's based on the stock price relative to a fixed average from
previous years. So each payment is tied to whether the current stock price is
higher or lower than that average. If the stock does well, they have to pay
more; if not, less.

I'm also thinking about how these risks interact. For example, inflation could
lead to higher interest rates, which would then affect their debt costs. Or a
weaker currency might make imports more expensive, contributing to inflation.
It's like a web of interconnected risks that could amplify each other in certain
economic conditions.

Another point is the sensitivity analysis for equity price risk. They show a
static present value and how it changes with different stock prices. This helps
investors understand how exposed they are to stock price fluctuations. A 40%
change seems extreme, but it's useful to see the range of possible outcomes.

I wonder if there are any mitigating factors or strategies MercadoLibre uses to
manage these risks. For example, do they hedge their currency exposure? How
flexible are their costs in the face of inflation? And how do they structure
their LTRP plans to balance retention with financial risk?

Overall, this section is highlighting several key areas where MercadoLibre faces
potential volatility and challenges. It's important for investors to consider
these risks when evaluating the company's long-term prospects.
```

(continues on next page)

(continued from previous page)

```
</think>

**MercadoLibre Risk Analysis Summary**

1. **Foreign Exchange Risk:**
   - **Exposure:** Significant revenue from Brazil, Argentina, and other Latin
American countries exposed to currency fluctuations, particularly the Brazilian
real and Argentine peso.
   - **Impact:** Currency devaluation can reduce reported USD revenue when
converted.

2. **Interest Rate Risk:**
   - **Exposure:** Long-term debt subject to interest rate changes.
   - **Impact:** Rising rates may decrease debt value and increase borrowing
costs.

3. **Commodity Price Risk:**
   - **Exposure:** Operational costs tied to fuel and electricity prices.
   - **Impact:** Price spikes can increase operational expenses and reduce
margins.

4. **Equity Price Risk:**
   - **Exposure:** Long-term retention programs (LTRP) linked to stock
performance.
   - **Impact:** Stock price fluctuations affect payout obligations; a 40% drop
reduces liability from $418 million to $251 million.

5. **Inflation Risk:**
   - **Exposure:** High inflation in countries like Argentina affecting
purchasing power and operational costs.
   - **Impact:** Could reduce consumer spending and squeeze margins if cost
increases aren't offset.

6. **Interconnected Risks:**
   - Inflation may lead to higher interest rates, affecting debt costs and
potentially exacerbating currency issues.

7. **Mitigation Strategies:**
   - Consider hedging strategies for currency exposure, flexible cost
structures, and balanced LTRP designs to manage financial risks.

**Conclusion:** MercadoLibre faces multiple interconnected risks that could
impact its financial health. Investors should assess these factors alongside the
company's risk management strategies when evaluating long-term prospects.

===== 9/10. A T & T INC =====
<think>
Okay, so I'm trying to understand this document about AT&T's financial report.
It looks like it's an audit report from their independent registered public
accounting firm. Let me go through it step by step.

First, the report is divided into two main sections: one on the internal control
over financial reporting and another on the consolidated financial statements.
Both have an opinion section, a basis for opinion, and some additional details.

Starting with the opinion on internal control over financial reporting. The
```

(continues on next page)

(continued from previous page)

```
auditors state that they've audited AT&T's internal controls as of December 31,
2023, based on the COSO criteria. They conclude that the company maintained
effective internal control over financial reporting in all material respects.
That sounds positive; it means the auditors think AT&T has good systems in place
to manage their finances properly.

Next, looking at the basis for this opinion. It mentions that management is
responsible for maintaining these controls and assessing their effectiveness.
The auditors' role was to express an opinion based on their audit, which they
did by following PCAOB standards. They obtained an understanding of the internal
controls, assessed risks, tested the design and operation of the controls, and
performed other necessary procedures. Their conclusion is that AT&T's controls
are effective.

Moving on to the report on the consolidated financial statements. The auditors
also express an unqualified opinion here, meaning they didn't find any material
issues with the financial statements. They were prepared in accordance with
GAAP, which is standard for public companies in the U.S.

The basis for this opinion section explains that the auditors are responsible
for expressing opinions on whether the financial statements present fairly, in
all material respects, the company's financial position and results of
operations. They did this by auditing according to PCAOB standards, assessing
risks, testing accounting policies, significant estimates, transactions, and
disclosures.

There are also two specific sections discussing areas where the audit was
particularly complex: internal control over financial reporting and the
impairment evaluations for certain reporting units. The auditors explain that
these areas required a deeper dive because they involve subjective assumptions
and complex methodologies. They tested controls, assessed assumptions against
industry trends, performed sensitivity analyses, and involved specialists to
ensure everything was in order.

The report ends with a note that the auditing firm has been serving AT&T since
1999, which shows a long-standing relationship, likely indicating consistency
and familiarity with the company's operations.

I'm trying to think if there's anything I might be missing. The document seems
comprehensive, covering both the internal controls and financial statements. It
mentions specific areas like impairment evaluations and valuation models, which
are known for being complex and subject to judgment. The auditors' involvement
of specialists in these areas makes sense because it adds depth to their
analysis.

I wonder if there were any material weaknesses or significant deficiencies
mentioned, but from what I can see, the opinions are unqualified, so no major
issues were found. It's also worth noting that the report is dated February 23,
2024, which would typically be around the time when earnings are released, so
it's timely.

Overall, this audit report seems to provide assurance to stakeholders that
AT&T's financial reporting is reliable and their internal controls are
effective. The detailed explanation of the auditing process gives confidence in
the independence and thoroughness of the audit.
</think>
```

(continues on next page)

(continued from previous page)

```
The audit report on AT&T provides a comprehensive assessment of both its
internal control over financial reporting and its consolidated financial
statements, concluding with an unqualified opinion in both areas. Here's a
structured summary:

### Key Findings:
1. **Internal Control Over Financial Reporting:**
   - The auditors found that AT&T maintained effective internal controls as of
December 31, 2023, based on the COSO framework.
   - The company's systems provide reasonable assurance regarding the
reliability of financial reporting and compliance with GAAP.

2. **Consolidated Financial Statements:**
   - The financial statements were prepared in accordance with GAAP and present
AT&T's financial position accurately.
   - No material issues were identified, leading to an unqualified opinion.

### Methodology:
- Auditors followed PCAOB standards, assessing risks, testing controls, and
involving specialists for complex areas like impairment evaluations.
- Detailed procedures included testing assumptions against industry trends and
performing sensitivity analyses.

### Areas of Complexity:
- **Impairment Evaluations:** Involved subjective assumptions and complex
valuation methods. The auditors ensured these were sound through thorough
testing and specialist involvement.
- **Internal Controls:** The effectiveness was validated through a comprehensive
audit process, confirming the controls' operational efficiency.

### Conclusion:
The report assures stakeholders of AT&T's financial integrity and control
systems. The unqualified opinions indicate no significant issues, reflecting
well on the company's financial governance. The long-standing relationship with
the auditing firm underscores consistency and familiarity in their work.

===== 10/10. REPUBLIC SERVICES INC =====
<think>
Okay, so I need to summarize this risk report for fifth graders. Let me read
through it carefully first.

The report talks about market risks the company faces, specifically interest
rate risk, fuel price risk, and commodities price risk. Hmm, I should break each
section down into simpler terms.

Starting with interest rate risk: The company has both fixed and variable debt.
Fixed rates don't change, while variable ones do based on market rates. They use
swaps to manage this risk. If rates go up or down by 100 basis points, their
interest expense changes by about $20 million. I should explain what a swap is
in simple terms, maybe like an agreement to balance out the risks.

Next, fuel price risk: Fuel costs are a big part of their expenses. They try to
hedge against price changes but currently have no hedges in place. A 20-cent
change per gallon affects both their costs and what they charge customers.
Petrochemical products used in operations also cost money, so higher prices
```

(continues on next page)

(continued from previous page)

```
there would increase their costs too.

Lastly, commodities price risk: They sell recycled materials, and market
fluctuations affect their revenue. Previously, they used derivatives like swaps
to hedge this, but none are active now. A $10 per ton change affects both
revenue and operating income by about $10 million.

I need to make sure I explain terms like "hedging" in a way fifth graders can
understand, maybe as strategies to protect against price changes. Also, I should
avoid jargon and keep sentences short and clear.

Let me structure the summary with each risk as a separate point, using simple
language and examples they can relate to, like how changing fuel prices affect
their daily lives or school budget.
</think>

**Summary of Market Risks in Simple Terms**

The company faces several risks related to market changes that could impact its
financial health. Here's a breakdown of these risks explained in a way fifth
graders can understand:

1. **Interest Rate Risk:**
   - The company borrows money through loans with both fixed and variable
interest rates.
   - Fixed rates stay the same, while variable rates change based on market
conditions.
   - To manage these changes, they use something called "swaps," which are
agreements to balance out potential losses or gains from rate changes.
   - If interest rates go up or down by a certain amount (100 basis points),
their yearly interest expense would change by about $20 million. This is like
how a school might adjust its budget if the cost of supplies goes up or down.

2. **Fuel Price Risk:**
   - Fuel costs are a major expense for the company, similar to how a school
might spend money on transportation.
   - They try to protect against fuel price changes by using strategies called
"hedging," but currently, they don't have any such strategies in place.
   - A 20-cent change in diesel price per gallon would affect both their costs
and what they charge customers. For example, if diesel goes up by 20 cents,
their expenses increase, and they might need to charge more to cover this cost.

3. **Commodities Price Risk:**
   - The company sells recycled materials like cardboard and newspaper.
   - Market changes can cause prices of these materials to go up or down,
affecting both revenue and how much money they make.
   - They used to use tools like "swaps" and "collars" to manage these price
changes, but currently, they aren't using any such tools.
   - A $10 per ton change in recycled material prices would impact their yearly
revenue and profits by about $10 million. This is similar to how a lemonade
stand might adjust its prices if the cost of lemons fluctuates.

In summary, the company deals with risks from changing interest rates, fuel
prices, and recycled materials markets. They use strategies like swaps to manage
some of these risks but are currently exposed to others. Understanding these
risks helps them make better financial decisions, much like how planning for
price changes helps a school manage its budget effectively.
```

```
scores = collect_rouge(target=summary[gpt_name], prediction=summary['simple_deepseek
↪'])
```

```
display_rouge("rouge1", scores)
```

```
ROUGE1 metric:
```

```
                          precision    recall  fmeasure
PACCAR INC                 0.397394  0.408027  0.402640
PHILLIPS 66                0.366142  0.324042  0.343808
MASTERCARD INC             0.371681  0.283784  0.321839
BRISTOL MYERS SQUIBB CO    0.331731  0.226974  0.269531
CARRIER GLOBAL CORP        0.502674  0.324138  0.394130
LULULEMON ATHLETICA INC    0.317172  0.537671  0.398983
AIRBNB INC                 0.391304  0.440559  0.414474
MERCADOLIBRE INC           0.467593  0.331148  0.387716
A T & T INC                0.393805  0.312281  0.348337
REPUBLIC SERVICES INC      0.414439  0.504886  0.455213
  average                  0.395393  0.369351  0.373667
```

```
display_rouge("rouge2", scores)
```

```
ROUGE2 metric:
```

```
                          precision    recall  fmeasure
PACCAR INC                 0.133987  0.137584  0.135762
PHILLIPS 66                0.079051  0.069930  0.074212
MASTERCARD INC             0.066667  0.050847  0.057692
BRISTOL MYERS SQUIBB CO    0.053140  0.036304  0.043137
CARRIER GLOBAL CORP        0.134409  0.086505  0.105263
LULULEMON ATHLETICA INC    0.052632  0.089347  0.066242
AIRBNB INC                 0.093458  0.105263  0.099010
MERCADOLIBRE INC           0.116279  0.082237  0.096339
A T & T INC                0.097778  0.077465  0.086444
REPUBLIC SERVICES INC      0.150134  0.183007  0.164948
  average                  0.097753  0.091849  0.092905
```

Generate simple summaries with GPT-4o-mini

```
summary['simple_gpt-4o'] = collect_summaries(
    gpt_model,
    role="You are a patient lower-school teacher, using simple words to explain to␣
↪your students in the fifth grade.")
```

```
===== 1/10. PACCAR INC =====
The risk report talks about how a company manages different types of financial
risks, especially related to market changes. It focuses on three main areas:
interest rates, currency exchange rates, and commodity prices.

First, for interest-rate risks, the company looks at how changes in interest
rates can affect the value of its assets and debts. They estimate what would
```

(continues on next page)

(continued from previous page)

```
happen if interest rates suddenly went up by 1%. In 2023, the company faced
losses of $29.2 million from cash and marketable securities and $146.5 million
from fixed-rate loans. However, they also had gains from fixed-rate debts and
interest-rate swaps, leading to a total loss of $17.7 million, which was worse
than the previous year’ s loss of $1.1 million.

Next, the report discusses currency risks. The company uses contracts to protect
itself from changes in foreign currency values, especially with currencies like
the Canadian dollar and the euro. If the value of these currencies drops by 10%,
the company could lose $259.7 million in 2023, compared to $216.6 million in
2022. However, these losses would be balanced out by changes in the value of the
things they are protecting.

Lastly, the company also deals with commodity price risks. They use contracts to
lock in prices for materials needed to make trucks. If commodity prices drop by
10%, they could lose $3.3 million in 2023, up from $2.5 million in 2022. Again,
these losses would be offset by changes in the value of the materials they are
buying.

Overall, the report shows how the company tries to manage risks to protect its
finances from sudden changes in the market.

===== 2/10. PHILLIPS 66 =====
The risk report discusses the various market risks that our company and some of
its subsidiaries face. These risks come from changes in prices of crude oil,
natural gas, and other energy products, as well as fluctuations in interest
rates and foreign currency exchange rates. To manage these risks, we use special
financial contracts called derivatives.

Our main strategy is to stay connected to the market prices of these
commodities. We use derivatives to switch from fixed prices in our contracts
back to floating market prices. This helps us balance our supply and demand
needs and take advantage of market opportunities, like moving products to where
they can be sold for more money or storing them for better prices later. We also
use derivatives to protect our cash flow from price changes.

The use of these financial tools is carefully controlled by rules set by our
board of directors. We monitor our risks daily to ensure we stay within safe
limits. We use a method called Value at Risk (VaR) to estimate potential losses
from market changes, and as of the end of 2023, these risks were not significant
to our cash flow.

Additionally, we face interest rate risks from our debts. Fixed-rate debts can
lose value if interest rates rise, while variable-rate debts can lead to higher
interest costs. We provide detailed information about our debts and their
interest rates.

Lastly, we are also exposed to foreign currency risks due to our international
operations, but we generally do not take steps to protect against these risks.
Our CEO and CFO keep a close eye on all these risks to ensure the company
remains stable and successful.

===== 3/10. MASTERCARD INC =====
The risk report discusses market risk, which is the chance of losing money due
to changes in things like interest rates and foreign currency exchange rates.
The company, Mastercard, has a limited exposure to these risks and keeps a close
```

(continues on next page)

(continued from previous page)

```
watch on them. They have rules in place to manage their funding, investments,
and the use of special financial tools called derivatives to help reduce these
risks.

One type of risk they face is foreign exchange risk, which happens when they
deal with money in different currencies. To manage this risk, Mastercard uses
foreign exchange derivative contracts. These contracts help them handle expected
payments and receipts in currencies that are not their main currency, which is
usually the U.S. dollar or the euro. If the value of these currencies changes by
10%, it could lead to a loss of about $414 million on their contracts as of
December 31, 2023. However, they also have activities that can offset these
losses.

Mastercard also deals with foreign exchange risk when they have investments in
other countries. They may use contracts to protect these investments, but as of
December 31, 2023, they did not have any contracts specifically for this
purpose. In 2022, a similar 10% change in the U.S. dollar could have led to a
loss of about $203 million.

Another risk is interest rate risk, which affects their investments in bonds.
Mastercard aims to invest in high-quality bonds and manage their risks
carefully. A change in interest rates by 1% would not significantly impact the
value of their investments. They also use interest rate derivatives to protect
against changes in the value of their fixed-rate debt, and similar to foreign
exchange, a 1% change would not have a major effect on their contracts. Overall,
Mastercard actively manages these risks to protect their financial health.

===== 4/10. BRISTOL MYERS SQUIBB CO =====
The risk report discusses how a company faces market risks related to changes in
currency exchange rates and interest rates. To protect itself from these risks,
the company uses certain financial tools called derivatives, but only when they
are cost-effective. These derivatives are not used for trading but to manage
risks.

One major risk is foreign exchange risk, which happens when the value of money
in different countries changes. The company earns a lot of its money in euros
and Japanese yen, so it uses contracts called foreign currency forwards to help
manage this risk. These contracts help protect the company from losing money
when it buys or sells things in different currencies. However, these contracts
are not always perfect hedges, meaning they don’t cover all risks completely.

The report estimates that if the euro and yen become stronger against the U.S.
dollar by 10%, the company could lose a significant amount of money on its
foreign exchange contracts. Additionally, the company uses cross-currency swaps
to manage risks from long-term debts in euros and to protect its investments in
foreign businesses.

The report also talks about interest rate risk, which is the risk of losing
money when interest rates change. The company uses interest rate swaps to
balance its debts. If interest rates go up by 1%, the impact on the company’s
earnings would not be very large, but it could still affect the value of its
long-term debt.

Lastly, the company is careful about credit risk, which is the risk of losing
money if someone it does business with doesn’t pay. It only invests in high-
quality institutions and spreads its investments to reduce risk. Overall, the
```

(continues on next page)

(continued from previous page)

```
company takes steps to manage these financial risks to protect its earnings and
investments.

===== 5/10. CARRIER GLOBAL CORP =====
The risk report discusses the market risks that our company faces, which include
changes in foreign currency exchange rates, interest rates, and commodity
prices. These risks can affect how well we perform financially. For the year
ending December 31, 2023, there have not been any major changes in our exposure
to these market risks.

One important area is foreign currency exposure. Since we operate in many
countries, we deal with different currencies. Our main reporting currency is the
U.S. dollar, and when other currencies change in value compared to the dollar,
it can affect our financial results. We manage some of these currency risks
related to our sales and purchases, but we do not protect against all risks from
currency changes.

Recently, we made an acquisition and paid a large part of the purchase price in
euros. To protect ourselves from changes in the euro's value compared to the
U.S. dollar, we used special contracts called window forward contracts. These
contracts help us manage the risk of losing money due to currency fluctuations.

For another acquisition, we used cross currency swaps and a loan in Japanese yen
to help manage the risks related to the yen. These financial tools help us keep
track of changes in currency values and their impact on our investments.

We also face risks from the prices of commodities, which are materials we use to
make our products. We sometimes use fixed price contracts to help manage these
costs, but as of December 31, 2023, we do not have any contracts in place to
protect against changes in commodity prices.

Lastly, most of our long-term debt has fixed interest rates, so changes in
interest rates are not expected to significantly affect our financial results.
Overall, we are actively managing these risks to protect our company.

===== 6/10. LULULEMON ATHLETICA INC =====
The risk report discusses various financial risks that the company faces,
particularly related to foreign currency exchange, interest rates, credit, and
inflation.

**Foreign Currency Exchange Risk**: The company has international subsidiaries
that operate in local currencies, but its financial statements are in U.S.
dollars. This means that when the value of the U.S. dollar changes, it affects
how much revenue and expenses are reported. In 2023, the company reported $89.8
million less in revenue compared to 2022 due to these currency fluctuations. The
company uses forward currency contracts to protect itself from these risks,
especially with its Canadian subsidiary, which saw a loss of $9 million from
currency translation.

**Transaction Risk**: The company also faces risks when its subsidiaries conduct
transactions in currencies different from their main currency. They hold cash
and other assets in various currencies and use forward contracts to manage these
risks. As of January 28, 2024, the company had a liability of $2.2 million from
these contracts, and a 10% drop in the U.S. dollar could lead to a $29.8 million
decrease in value.
```

(continues on next page)

(continued from previous page)

```
**Interest Rate Risk**: The company has a credit facility that allows it to
borrow up to $400 million at variable interest rates. If it borrows a
significant amount, it could be affected by changes in interest rates.
Currently, they have no borrowings but may consider hedging against interest
rate risks in the future.

**Credit Risk**: The company keeps cash with reputable banks and invests in
high-rated money market funds. They are cautious about credit risk, which is the
risk of losing money if a bank fails to meet its obligations. They have not
faced any losses and believe this risk is low.

**Inflation**: Rising costs for products, wages, and transportation can hurt the
company's profits. In recent years, increased wages and shipping costs have
affected their operating margins. If these costs continue to rise without a
corresponding increase in product prices, it could impact their financial
health.

===== 7/10. AIRBNB INC =====
The risk report discusses the market risks faced by a company with global
operations, focusing on foreign currency risk and investment risk.

**Foreign Currency Risk:** The company operates in over 40 currencies, with key
currencies including the euro, British pound, Canadian dollar, Australian
dollar, Brazilian real, and Mexican peso. This means that when the value of
these currencies changes compared to the U.S. dollar, it can affect the
company's financial results. For example, if the U.S. dollar strengthens, the
company could lose money on its international sales. The company has various
sources of foreign currency risk, such as revenue from bookings made in other
currencies, funds held for customers, and payments to hosts. To manage this
risk, the company uses foreign currency derivative contracts, which help protect
against changes in exchange rates. However, these contracts do not completely
eliminate the risk, and the company may choose not to hedge certain exposures
due to costs or accounting reasons. If there were a significant change in
exchange rates, it could lead to substantial losses.

**Investment and Interest Rate Risk:** The company also faces risks related to
interest rates, which can affect the earnings from its investments. As of
December 31, 2023, the company had significant cash and short-term investments,
primarily in safe, liquid assets like government bonds and corporate debt. The
goal of these investments is to keep the money safe while ensuring it is
available when needed. Because the investments are mostly short-term, they are
not very sensitive to changes in interest rates. A hypothetical increase in
interest rates could lead to a small decrease in the value of the investment
portfolio, but the company does not expect to face major risks from interest
rate changes.

===== 8/10. MERCADOLIBRE INC =====
The risk report discusses the various market risks that our company faces due to
its business operations. These risks mainly come from changes in the economy,
interest rates, and currency exchange rates, especially with currencies like the
Brazilian real, Argentine peso, and Mexican peso. These changes can affect the
value of our financial assets and liabilities.

We also have long-term retention programs for employees, which involve cash
payments that depend on our stock price. This means that if our stock price
changes, the amount we pay to employees can also change.
```

(continues on next page)

(continued from previous page)

```
Since we operate in many countries, we deal with different currencies, which
exposes us to foreign currency risk. This can impact our financial results
because we earn money and spend money in various currencies. To manage this
risk, we use contracts that help protect us from unfavorable changes in currency
exchange rates. However, these contracts do not completely eliminate the risk.

As of December 31, 2023, we had significant amounts of cash and investments in
local currencies. Our subsidiaries mostly earn and spend money in their local
currencies, except for our Argentine subsidiaries, which use the U.S. dollar due
to high inflation. We also experienced a loss of $615 million in foreign
currency due to changes in the Argentine market.

Interest rates also affect our earnings and cash flow. Changes in interest rates
can impact the costs of loans and the income we earn from our investments. As of
December 31, 2023, we had various loans and investments that are sensitive to
interest rate changes.

Lastly, our long-term retention programs for employees are linked to our stock
price, meaning that if our stock price goes up or down, it affects how much we
owe to employees. Overall, these market risks can significantly impact our
financial performance.

===== 9/10. A T & T INC =====
The risk report discusses how AT&T Inc. manages market risks, particularly those
related to interest rates and foreign currency exchange rates. These risks can
affect the company's costs and financial stability. To handle these risks, AT&T
uses various financial tools called derivatives, such as interest rate swaps and
foreign currency contracts. These tools help the company control its financial
risks and maintain flexibility without engaging in risky trading practices.

One important aspect of the report is the company's approach to estimating its
future obligations for employee benefits, which relies on a discount rate. This
rate is influenced by the returns on high-quality corporate bonds. Recently,
these rates have been lower and more unstable than in the past, which means that
the company's obligations could be higher. If rates increase in the future, it
could lower these obligations and improve the company's financial health.

The report also highlights interest rate risk, as most of AT&T's financial
instruments are fixed-rate notes. Changes in interest rates can significantly
affect their value. To manage this risk, AT&T monitors its debt structure and
uses interest rate swaps to balance fixed and floating rates.

Additionally, AT&T addresses foreign exchange risk by converting foreign debt
into U.S. dollars through cross-currency swaps. This helps eliminate risks
related to currency fluctuations. The report includes details about the
company's financial instruments and their fair values as of December 31, 2023.

Overall, AT&T is committed to managing its market risks carefully to ensure
long-term financial stability and flexibility. The company regularly assesses
its financial strategies and maintains effective internal controls to support
accurate financial reporting.

===== 10/10. REPUBLIC SERVICES INC =====
The risk report discusses the financial risks related to interest rates, fuel
prices, and commodity prices that the company faces.
```

(continues on next page)

(continued from previous page)

```
**Interest Rate Risk:** The company is mainly affected by changes in interest
rates in the United States. To manage this risk, they use a mix of fixed and
variable interest rate debts. As of December 31, 2023, the company had a total
of about $10.4 billion in fixed-rate debt and around $2.6 billion in variable-
rate debt. They also use financial agreements called swap contracts to help
protect against changes in interest rates. If interest rates go up or down by
1%, the company’ s interest expenses could change by about $20 million each year.

**Fuel Price Risk:** Fuel costs are a significant expense for the company. They
may use fuel hedges, which are contracts to lock in fuel prices, but as of the
end of 2023, they had no such contracts in place. A change of 20 cents per
gallon in diesel fuel prices could affect their fuel costs by about $27 million
annually. They charge some customers for fuel recovery fees, but not all, which
can impact their revenue. In 2023, their fuel costs were $541.6 million, which
was lower than in 2022.

**Commodity Price Risk:** The company also deals with the prices of recycled
materials, like old cardboard and newspapers. Changes in supply and demand can
cause these prices to fluctuate. They have previously used financial tools to
manage this risk but had no hedges in place as of December 31, 2023. A $10
change in the price of recycled materials could affect their revenue and
operating income by about $10 million. In 2023, they earned $312.3 million from
recycling, down from $359.1 million in 2022.

Overall, the report highlights how the company is working to manage these
financial risks to protect its operations and profitability.
```

```python
scores = collect_rouge(target=summary[gpt_name], prediction=summary['simple_gpt-4o'])
```

Display rouge-1 scores for simple GPT-4o-mini summary

```python
display_rouge("rouge1", scores)
```

```
ROUGE1 metric:
```

```
                          precision    recall  fmeasure
PACCAR INC                 0.653571  0.612040  0.632124
PHILLIPS 66                0.551971  0.536585  0.544170
MASTERCARD INC             0.560261  0.581081  0.570481
BRISTOL MYERS SQUIBB CO    0.550000  0.542763  0.546358
CARRIER GLOBAL CORP        0.551155  0.575862  0.563238
LULULEMON ATHLETICA INC    0.576577  0.657534  0.614400
AIRBNB INC                 0.658621  0.667832  0.663194
MERCADOLIBRE INC           0.553333  0.544262  0.548760
A T & T INC                0.665480  0.656140  0.660777
REPUBLIC SERVICES INC      0.650794  0.667752  0.659164
  average                  0.597176  0.604185  0.600267
```

Display rouge-2 scores for simple GPT-4o-mini summary

```python
display_rouge("rouge2", scores)
```

```
ROUGE2 metric:
```

```
                         precision    recall  fmeasure
PACCAR INC                0.336918  0.315436  0.325823
PHILLIPS 66               0.244604  0.237762  0.241135
MASTERCARD INC            0.241830  0.250847  0.246256
BRISTOL MYERS SQUIBB CO   0.257525  0.254125  0.255814
CARRIER GLOBAL CORP       0.231788  0.242215  0.236887
LULULEMON ATHLETICA INC   0.243976  0.278351  0.260032
AIRBNB INC                0.359862  0.364912  0.362369
MERCADOLIBRE INC          0.254181  0.250000  0.252073
A T & T INC               0.257143  0.253521  0.255319
REPUBLIC SERVICES INC     0.382166  0.392157  0.387097
  average                 0.280999  0.283933  0.282281
```

35.3.6 Readability

Readability scores such as **Flesch-Kincaid** and **Gunning-Fog** assess how easy a summary is to read. These metrics are calculated based on sentence length, word complexity, and syllable count, and correspond to U.S. grade levels. Simpler summaries which score lower are suitable for broader audiences.

```
# applies Flesch-Kincaid and Gunning-Fog readability indexes to measure how complex␣
↪each summary is
from readability import Readability
fog = {permno: {name: Readability(summary[name][permno]).flesch_kincaid().grade_level
                for name in ['simple_deepseek', 'simple_gpt-4o', model_name, gpt_
↪name]}
       for permno in summary[gpt_name].keys()}
DataFrame(fog).T  # display grade-level
```

```
      simple_deepseek simple_gpt-4o deepseek-r1:14b gpt-4o-mini
60506               7             9              13          15
13356              12            10              12          17
91233              10            10              16          16
19393               9            10              15          15
19285               8            10              17          16
92203               9            11              15          15
20190               9            12              15          17
92221              12            10              12          16
66093              16            12              16          16
86228               9             8              10          12
```

```
fog = {permno: {name: Readability(summary[name][permno]).gunning_fog().grade_level
                for name in ['simple_deepseek', 'simple_gpt-4o', model_name, gpt_
↪name]}
       for permno in summary[gpt_name].keys()}
DataFrame(fog).T  # display grade-level
```

```
        simple_deepseek simple_gpt-4o   deepseek-r1:14b       gpt-4o-mini
60506                 9            12  college_graduate  college_graduate
13356           college       college           college  college_graduate
91233                12            12  college_graduate  college_graduate
```

(continues on next page)

(continued from previous page)

```
19393               12       college  college_graduate  college_graduate
19285               11       college  college_graduate  college_graduate
92203               12       college  college_graduate  college_graduate
20190               12       college  college_graduate  college_graduate
92221               12       college                12  college_graduate
66093  college_graduate       college  college_graduate  college_graduate
86228               12            12           college           college
```

References:

Greg Durrett, 2021-2024, "CS388 Natural Language Processing course materials", retrieved from https://www.cs.utexas.edu/~gdurrett/courses/online-course/materials.html

Philipp Krähenbühl, 2020-2024, "AI394T Deep Learning course materials", retrieved from https://www.philkr.net/dl_class/material and https://ut.philkr.net/deeplearning/

Philipp Krähenbühl, 2025, "AI395T Advances in Deep Learning course materials", retrieved from https://ut.philkr.net/advances_in_deeplearning/

CHAPTER

THIRTYSIX

LLM FINE-TUNING

To improve is to change; to be perfect is to change often - Winston Churchill

Large language models (LLMs) have demonstrated remarkable general capabilities, but tailoring them to specific tasks or domains may require fine-tuning –adjusting model weights by further training on task-specific data. We examine the fine-tuning of Meta’ s **Llama-3.1** model using tools from the Hugging Face ecosystem, applying efficient techniques such as quantization and low-rank adaptation (LoRA) to an industry text classification task using firm-level 10-K filings.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import numpy as np
import pandas as pd
from pandas import DataFrame, Series
import os
from tqdm import tqdm
from pathlib import Path
from pprint import pprint
import textwrap
import warnings
import bitsandbytes as bnb
import torch
from datasets import Dataset
from peft import LoraConfig, PeftConfig
from trl import SFTTrainer
from transformers import (AutoModelForCausalLM,
                          AutoTokenizer,
                          BitsAndBytesConfig,
                          pipeline,
                          logging)
import matplotlib.pyplot as plt
from sklearn.metrics import (accuracy_score,
                             classification_report,
                             confusion_matrix)
from sklearn.model_selection import train_test_split
from finds.database import SQL, RedisDB
from finds.unstructured import Edgar
from finds.structured import BusDay, CRSP, PSTAT
from finds.readers import Sectoring
from finds.utils import Store
from secret import paths, CRSP_DATE, credentials
logging.set_verbosity_error()
```

```
NUM_TRAIN_EPOCHS = 2    # 0 # 1
RESUME_FROM_CHECKPOINT = False    # False # True
```

(continues on next page)

(continued from previous page)

```
MAX_SEQ_LENGTH = 1024  #512 #2048
LOGGING_STEPS = 200
```

```
VERBOSE = 0
sql = SQL(**credentials['sql'], verbose=VERBOSE)
bd = BusDay(sql)
rdb = RedisDB(**credentials['redis'])
crsp = CRSP(sql, bd, rdb, verbose=VERBOSE)
pstat = PSTAT(sql, bd, verbose=VERBOSE)
ed = Edgar(paths['10X'], zipped=True, verbose=0)
store = Store('assets', ext='pkl')
permnos = list(store.load('nouns').keys())
print(f"{len(permnos)=}")    # comparable sample
```

```
len(permnos)=3474
```

36.1 Meta Llama-3.1 model

Meta' s **Llama 3.1** is an open-source large language model released in July 2024 under the Llama 3.1 Community License, permitting broad use, including commercial applications. Key highlights include:

- Model variants:
  - 8B: 8 billion parameters.
  - 70B: 70 billion parameters.
  - 405B: 405 billion parameters.
- Context length of up to 128,000 tokens.
- Pre-trained on over 15 trillion tokens sourced from publicly available datasets.
- Fine-tuned using supervised fine-tuning (SFT) and reinforcement learning with human feedback (RLHF).
- Multilingual support, including English, French, German, Hindi, Italian, Portuguese, Spanish, and Thai.

https://huggingface.co/meta-llama/Llama-3.1-8B-Instruct

```
base_model = 'meta-llama/Llama-3.1-8B-Instruct'
```

```
# Show current memory stats
gpu_stats = torch.cuda.get_device_properties(0)
max_memory = round(gpu_stats.total_memory / (1024**3), 3)
print(f"GPU = {gpu_stats.name}. Max memory = {max_memory} GB.")

def cuda_memory(title, trainer_stats=None):
    """Show final memory and optional trainer stats"""
    if torch.cuda.is_available():
        device = torch.device('cuda')
        total_memory = torch.cuda.get_device_properties(device).total_memory
        reserved_memory = torch.cuda.memory_reserved(device)
        allocated_memory = torch.cuda.memory_allocated(device)
        free_memory = total_memory - reserved_memory
```

(continues on next page)

(continued from previous page)

```
        print(f'------ {title.upper()} ------')
        if trainer_stats:
            print(f"{trainer_stats.metrics['train_runtime']} seconds used for␣
↪training.")
        print(f"Total memory: {total_memory / (1024**3):.2f} GB")
        print(f"Reserved memory: {reserved_memory / (1024**3):.2f} GB")
        print(f"Allocated memory: {allocated_memory / (1024**3):.2f} GB")
        print(f"Free memory: {free_memory / (1024**3):.2f} GB")
```

```
GPU = NVIDIA GeForce RTX 3080 Laptop GPU. Max memory = 15.739 GB.
```

36.2 Supervised fine-tuning (SFT)

Supervised Fine-Tuning is the process of enhancing a pre-trained language model by fine-tuning it on labeled input–output pairs using standard supervised learning. Common use cases include:

- Instruction tuning: The model learns to follow new instructions
- Chatbot fine-tuning (e.g., with help-desk data)
- Domain adaptation (e.g., legal, medical)

36.2.1 Huggingface framework

Several ecosystems support fine-tuning and training of LLMs. The Hugging Face Ecosystem includes:

- `transformers`: Model architectures and training components.
- Transformers Reinforcement Learning (`trl`): Training large language models (LLMs) with reinforcement learning techniques, especially for alignment tasks like RLHF (Reinforcement Learning with Human Feedback) and DPO (Direct Preference Optimization).
- `bitsandbytes`: Enables efficient low-bit model quantization, allowing large language models to run on limited GPU memory without much loss in performance.
- Parameter-Efficient Fine-Tuning (`peft`): Tools to fine-tune large language models by training only a small number of additional parameters.
- Accelerate: Distributed training optimization.
- `datasets`: For loading, processing, and managing datasets

It provides access to 100k+ pre-trained transformer models, and tools for efficient-tuning of these models using low memory and quantized weights.

If you encounter a gated model repository on Hugging Face, it means the model requires manual access approval from the authors before you can use or download it. You should log in to your huggingface.ro account, go to the Model Page, and click on the "Request Access" button –approval may take up to a few days. When authorized, make sure you have set your Hugging Face token in your environment (e.g. `huggingface-cli login`), see https://huggingface.co/settings/tokens

```
# Locations to save fine-tuned model weights
output_dir = str(Path(paths['scratch'], "fine-tuned-model"))   # training checkpoints
model_dir = str(Path(paths['scratch'], "Llama-3.1-8B-Instruct-FF-Sector"))  # final␣
↪model
```

```
from trl import SFTConfig
args = SFTConfig(
    output_dir=output_dir,                     # directory to save and repository id
    num_train_epochs=NUM_TRAIN_EPOCHS, ####1   # number of training epochs
    per_device_train_batch_size=2,    ####1    # batch size per device during training
    gradient_accumulation_steps=4,   ####8     # before performing a backward/update␣
↪pass
    gradient_checkpointing=True,               # use gradient checkpointing to save␣
↪memory
    optim="paged_adamw_32bit",
    logging_strategy="steps",                  # or "steps" or "no" or "epoch"
    logging_steps=LOGGING_STEPS, #### 1,
    learning_rate=2e-4,                        # learning rate, based on QLoRA paper
    weight_decay=0.001,
    fp16=True,
    bf16=False,
    max_grad_norm=0.3,                         # max gradient norm based on QLoRA paper
    max_steps=-1,
    warmup_ratio=0.03,                         # warmup ratio based on QLoRA paper
    group_by_length=False,
    lr_scheduler_type="cosine",                # use cosine learning rate scheduler
    report_to="tensorboard",
    max_seq_length=MAX_SEQ_LENGTH,   #512,  ### should be 1024? or MAX_CHARS // 4
    packing=False,
    dataset_kwargs={
    "add_special_tokens": False,
    "append_concat_token": False,
    }
)
```

36.2.2 Tokenizer

The `AutoTokenizer` in Hugging Face is a smart utility that automatically loads the correct tokenizer for a given pretrained model.

```
# Load the tokenizer and set the pad token id.
tokenizer = AutoTokenizer.from_pretrained(base_model)
tokenizer.pad_token_id = tokenizer.eos_token_id
```

36.2.3 Quantization

Quantization converts high-precision data to lower-precision data, for instance, by representing model weights and activation values as 4-bit or 8-bit integers instead of 32-bit floating point numbers. The `bitsandbytes` library for efficient low-bit model quantization is integrated with Hugging Face and works seamlessly with parameter-efficient fine-tuning like QLora.

```
# Load the Llama-3.1-8b-instruct model in 4-bit quantization to save GPU memory
bnb_config = BitsAndBytesConfig(
    load_in_4bit=True,
    bnb_4bit_use_double_quant=False,
    bnb_4bit_quant_type="nf4",
    bnb_4bit_compute_dtype="float16",
)
```

36.2.4 AutoModel

The `AutoModel` class in Hugging Face is a convenient interface that automatically loads the correct model architecture based on the model name or path. Its variants automatically load the correct model head (e.g., classification layer, decoder head) based on your specific task, e.g.

Class	Task	Output
`AutoModel`	Base model (no head)	Hidden states
`AutoModelForSequenceClassification`	Text classification (e.g. sentiment)	Class logits
`AutoModelForTokenClassification`	Token labeling (e.g. NER, POS)	Token-level logits
`AutoModelForQuestionAnswering`	Extractive QA	Start/end logits for answer spans
`AutoModelForCausalLM`	Text generation (GPT-style)	Next-token logits
`AutoModelForMaskedLM`	Mask filling (BERT-style)	Predictions for masked tokens
`AutoModelForSeq2SeqLM`	Translation, summarization (T5, BART)	Generated sequences
`AutoModelForMultipleChoice`	Multiple-choice QA (e.g. SWAG)	Choice logits
`AutoModelForVision2Seq`	Image captioning	Generated text
`AutoModelForImageClassification`	Vision tasks	Class logits
`AutoModelForSpeechSeq2Seq`	Speech translation	Generated text from audio

```
model = AutoModelForCausalLM.from_pretrained(
    base_model,
    device_map="auto",
    torch_dtype="float16",
    quantization_config=bnb_config,
)
model.config.use_cache = False
model.config.pretraining_tp = 1
```

36.2.5 Parameter-efficient fine-tuning

Parameter-Efficient Fine-Tuning (PEFT) is both a technique and a Hugging Face library for adapting large language models (LLMs) to new tasks by training only a small subset of parameters. Instead of updating the entire model, the base (pretrained) model is kept frozen, and lightweight, trainable components called **adapters** are added. These adapters typically involve only a few million parameters, making fine-tuning faster and more memory-efficient.

- **Low-rank factorization**: This is a compression technique which decomposes a large matrix of weights into a smaller, lower-rank matrix, resulting in a more compact approximation that requires fewer parameters and computations.
- **LoRA**: A small number of trainable low-rank matrices are added to the model's attention layers. The original weights are frozen and just these adapters are fine-tuned.
- **QLora**: Combines LoRA with Quantization: The base model is converted to 4-bit precision, reducing memory usage dramatically without losing much performance.

```
# Extract the linear module names from the model using the bits and bytes library.
def find_all_linear_names(model):
    cls = bnb.nn.Linear4bit
    lora_module_names = set()
    for name, module in model.named_modules():
        if isinstance(module, cls):
            names = name.split('.')
            lora_module_names.add(names[0] if len(names) == 1 else names[-1])
    if 'lm_head' in lora_module_names:  # needed for 16 bit
        lora_module_names.remove('lm_head')
    return list(lora_module_names)
modules = find_all_linear_names(model)
modules
```

```
['q_proj', 'down_proj', 'v_proj', 'gate_proj', 'o_proj', 'up_proj', 'k_proj']
```

```
# Configure LoRA for the target modules, task type, and other training arguments
peft_config = LoraConfig(
    lora_alpha=16,
    lora_dropout=0,
    r=64,
    bias="none",
    task_type="CAUSAL_LM",
    target_modules=modules,
)
```

36.3 Industry text classification

We fine-tune the model for classifying firms into ten Fama-French sector categories based on their business descriptions in 10-K filings. The text data for each U.S.-domiciled common stock is drawn from the most recent year's Business Description section of their 10-K filings.

Load 10-K business description text for industry classification task

```
# Retrieve universe of stocks
beg, end = bd.begyr(CRSP_DATE), bd.endyr(CRSP_DATE)
print(f"{beg=}, {end=}")
univ = crsp.get_universe(bd.endyr(CRSP_DATE, -1))

# lookup company names
comnam = crsp.build_lookup(source='permno', target='comnam', fillna="")
univ['comnam'] = comnam(univ.index)

# lookup company names
comnam = crsp.build_lookup(source='permno', target='comnam', fillna="")
univ['comnam'] = comnam(univ.index)

# lookup ticker symbols
ticker = crsp.build_lookup(source='permno', target='ticker', fillna="")
univ['ticker'] = ticker(univ.index)

# lookup sic codes from Compustat, and map to FF 10-sector code
sic = pstat.build_lookup(source='lpermno', target='sic', fillna=0)
```

(continues on next page)

(continued from previous page)

```
industry = Series(sic[univ.index], index=univ.index)
industry = industry.where(industry > 0, univ['siccd'])
sectors = Sectoring(sql, scheme='codes10', fillna='')   # supplement from crosswalk
univ['sector'] = sectors[industry]

# retrieve latest year's bus10K's
item, form = 'bus10K', '10-K'
rows = DataFrame(ed.open(form=form, item=item))
rows = rows[rows['date'].between(beg, end)]\
    .drop_duplicates(subset=['permno'], keep='last')\
    .set_index('permno')\
    .reindex(permnos)

# split documents into train/test sets
labels = univ.loc[permnos, 'sector']
class_labels = np.unique(labels)
print(f"{class_labels=}")

train_index, test_index = train_test_split(permnos,
                                           stratify=labels,
                                           random_state=42,
                                           test_size=0.2)
```

```
beg=20240102, end=20241231
class_labels=array(['Durbl', 'Enrgy', 'HiTec', 'Hlth', 'Manuf', 'NoDur', 'Other',
       'Shops', 'Telcm', 'Utils'], dtype=object)
```

36.3.1 HuggingFace `dataset` module

The training data are converted to LLM instruction statements, and implemented as a HuggingFace Dataset class. This class can be conveniently created from many different sources, including data files of various formats or from a generator function.

```
# Create LLM instruction statement
MAX_CHARS = MAX_SEQ_LENGTH * 2
class_text = "'" + "' or '".join(class_labels) + "'"
def generate_prompt(permno, test=False):
    text = ed[rows.loc[permno, 'pathname']].replace('\n','')[:MAX_CHARS]
    return f"""
Classify the text into one of these {len(class_labels)} classification labels:
{class_text}
and return the answer as the label.
text: {text}
label: {'' if test else univ.loc[permno, 'sector']}""".strip()
```

```
cuda_memory('before dataset')
```

```
------ BEFORE DATASET ------
Total memory: 15.74 GB
Reserved memory: 6.83 GB
Allocated memory: 5.63 GB
Free memory: 8.91 GB
```

```
X_train = DataFrame(columns=['text'], index=train_index,
                    data=[generate_prompt(permno, test=False) for permno in train_
↪index])
X_test = DataFrame(columns=['text'], index=test_index,
                   data=[generate_prompt(permno, test=True) for permno in test_index])
y_test = [univ.loc[permno, 'sector'] for permno in test_index]

train_data = Dataset.from_pandas(X_train[["text"]])
test_data = Dataset.from_pandas(X_test[["text"]])
print(textwrap.fill(train_data['text'][3]))
```

```
Classify the text into one of these 10 classification labels: 'Durbl'
or 'Enrgy' or 'HiTec' or 'Hlth' or 'Manuf' or 'NoDur' or 'Other' or
'Shops' or 'Telcm' or 'Utils'  and return the answer as the label.
text: ITEM 1. BUSINESS  OVERVIEW  B. RILEY FINANCIAL, INC. (NASDAQ:
RILY) (THE COMPANY IS A DIVERSIFIED FINANCIAL SERVICES PLATFORM THAT
DELIVERS TAILORED SOLUTIONS TO MEET THE STRATEGIC, OPERATIONAL, AND
CAPITAL NEEDS OF ITS CLIENTS AND PARTNERS. WE OPERATE THROUGH SEVERAL
CONSOLIDATED SUBSIDIARIES (COLLECTIVELY, B. RILEY THAT PROVIDE
INVESTMENT BANKING, BROKERAGE, WEALTH MANAGEMENT, ASSET MANAGEMENT,
DIRECT LENDING, BUSINESS ADVISORY, VALUATION, AND ASSET DISPOSITION
SERVICES TO A BROAD CLIENT BASE SPANNING PUBLIC AND PRIVATE COMPANIES,
FINANCIAL SPONSORS, INVESTORS, FINANCIAL INSTITUTIONS, LEGAL AND
PROFESSIONAL SERVICES FIRMS, AND INDIVIDUALS.   THE COMPANY
OPPORTUNISTICALLY INVESTS IN AND ACQUIRES COMPANIES OR ASSETS WITH
ATTRACTIVE RISK-ADJUSTED RETURN PROFILES TO BENEFIT OUR SHAREHOLDERS.
WE OWN AND OPERATE SEVERAL UNCORRELATED CONSUMER BUSINESSES AND INVEST
IN BRANDS ON A PRINCIPAL BASIS. OUR APPROACH IS FOCUSED ON HIGH
QUALITY COMPANIES AND ASSETS IN INDUSTRIES IN WHICH WE HAVE EXTENSIVE
KNOWLEDGE AND CAN BENEFIT FROM OUR EXPERIENCE TO MAKE OPERATIONAL
IMPROVEMENTS AND MAXIMIZE FREE CASH FLOW. OUR PRINCIPAL INVESTMENTS
OFTEN LEVERAGE THE FINANCIAL, RESTRUCTURING, AND OPERATIONAL EXPERTISE
OF OUR PROFESSIONALS WHO WORK COLLABORATIVELY ACROSS DISCIPLINES.   WE
REFER TO B. RILEY AS A PLATFORM BECAUSE OF THE UNIQUE COMPOSITION OF
OUR BUSINESS. OUR PLATFORM HAS GROWN CONSIDERABLY AND BECOME MORE
DIVERSIFIED OVER THE PAST SEVERAL YEARS. WE HAVE INCREASED OUR MARKET
SHARE AND EXPANDED THE DEPTH AND BREADTH OF OUR BUSINESSES BOTH
ORGANICALLY AND THROUGH OPPORTUNISTIC ACQUISITIONS. OUR INCREASINGLY
DIVERSIFIED PLATFORM ENABLES US TO INVEST OPPORTUNISTICALLY AND TO
DELIVER STRONG LONG-TERM INVESTMENT PERFORMANCE THROUGHOUT A RANGE OF
ECONOMIC CYCLES.   OUR PLATFORM IS COMPRISED OF MORE THAN 2,700
AFFILIATED PROFESSIONALS, INCLUDING EMPLOYEES AND INDEPENDENT
CONTRACTORS. WE ARE HEADQUARTERED IN LOS ANGELES, CALIFORNIA AND
MAINTAIN OFFICES THROUGHOUT THE U.S., INCLUDING IN NEW YORK, CHICAGO,
METRO DISTRICT OF COLUMBIA, AT label: Other
```

```
# verify max_seq_length sufficient
curr_max = 0
for row, data in enumerate(train_data):
    tokenized = tokenizer.tokenize(data['text'])
    curr_max = max(curr_max, len(tokenized))
#    print(f"{row=}, {len(tokenized)=}")
assert curr_max < args.max_seq_length
print(curr_max, f"{MAX_SEQ_LENGTH=}")
```

```
820 MAX_SEQ_LENGTH=1024
```

```
cuda_memory('after dataset')
```

```
------ AFTER DATASET ------
Total memory: 15.74 GB
Reserved memory: 6.83 GB
Allocated memory: 5.63 GB
Free memory: 8.91 GB
```

36.3.2 Pipeline

Hugging Face's pipeline function enables one-line use for easy inference, by simply specifying the model, tokenizer, generation parameters (e.g. sampling methdology, maximum new tokens), and task, e.g.:

- "text-classification" : Sentiment analysis, topic labeling
- "token-classification" : Named Entity Recognition (NER), POS tagging
- "question-answering" : Extractive QA from context
- "text-generation" : Generate text (GPT-style)
- "summarization" : Generate summaries from long text

```
# Use the text generation pipeline to predict labels from the "text"
def generate(prompt, model=model, tokenizer=tokenizer, verbose=False):
    """Generate a response"""
    pipe = pipeline(task="text-generation",
                    model=model,
                    tokenizer=tokenizer,
                    do_sample=False,
                    top_p=None,
                    top_k=None,
                    return_full_text=False,
                    max_new_tokens=4,   # 2
                    temperature=None)    # 0.1
    result = pipe(prompt)
    answer = result[0]['generated_text'].split("label:")[-1].strip()
    if verbose:
        print(f"{len(prompt)=}, {result=}, {answer=}")
    return answer

def predict(test, model, tokenizer, verbose=False):
    """Predict test set"""
    y_pred = []
    for i in tqdm(range(len(test))):
        prompt = test.iloc[i]["text"]
        answer = generate(prompt, model, tokenizer, verbose=verbose)
        # Determine the predicted category
        for category in class_labels:
            if category.lower() in answer.lower():
                y_pred.append(category)
                break
        else:
```

(continues on next page)

(continued from previous page)

```
            y_pred.append("none")
    return y_pred
```

Create function that will use the predicted labels and true labels to compute the overall accuracy, classification report, and confusion matrix.

```
def evaluate(y_true, y_pred):
    mapping = {label: idx for idx, label in enumerate(class_labels)}

    def map_func(x):
        return mapping.get(x, -1)  # Map to -1 if not found, should not occur with␣
↪correct data

    y_true_mapped = np.vectorize(map_func)(y_true)
    y_pred_mapped = np.vectorize(map_func)(y_pred)
    labels = list(mapping.values())
    target_names = list(mapping.keys())
    if -1 in y_pred_mapped:
        labels += [-1]
        target_names += ['none']

    # Calculate accuracy
    accuracy = accuracy_score(y_true=y_true_mapped, y_pred=y_pred_mapped)
    print(f'Accuracy: {accuracy:.3f}')

    # Generate classification report
    class_report = classification_report(y_true=y_true_mapped, y_pred=y_pred_mapped,
                                         target_names=target_names,
                                         labels=labels, zero_division=0.0)
    print('\nClassification Report:')
    print(class_report)

    # Generate confusion matrix
    conf_matrix = confusion_matrix(y_true=y_true_mapped, y_pred=y_pred_mapped,
                                   labels=labels)
    print('\nConfusion Matrix:')
    print(conf_matrix)
```

Evaluate accuracy before fine-tuning the model

```
y_pred = predict(X_test, model, tokenizer)
Series(y_pred).value_counts()
```

```
100%|██████████| 695/695 [05:45<00:00,  2.01it/s]
```

```
Manuf     217
NoDur     184
HiTec     109
Other      65
none       54
Hlth       24
Utils      15
Telcm      14
Shops       8
```

(continues on next page)

(continued from previous page)

```
Enrgy      4
Durbl      1
Name: count, dtype: int64
```

```
evaluate(y_test, y_pred)
```

```
Accuracy: 0.203

Classification Report:
              precision    recall  f1-score   support

       Durbl       0.00      0.00      0.00        33
       Enrgy       0.50      0.10      0.17        20
       HiTec       0.25      0.19      0.22       139
        Hlth       0.88      0.13      0.22       164
       Manuf       0.22      0.70      0.34        69
       NoDur       0.03      0.21      0.06        28
       Other       0.25      0.10      0.15       153
       Shops       0.75      0.10      0.17        62
       Telcm       0.36      0.56      0.43         9
       Utils       0.67      0.56      0.61        18
        none       0.00      0.00      0.00         0

    accuracy                           0.20       695
   macro avg       0.35      0.24      0.21       695
weighted avg       0.44      0.20      0.21       695


Confusion Matrix:
[[ 0  0  2  0 18 10  2  1  0  0  0]
 [ 0  2  0  0  8  8  2  0  0  0  0]
 [ 0  0 27  0 43 38 18  0  8  4  1]
 [ 0  0 73 21 15 22 11  0  1  1 20]
 [ 0  0  3  0 48 12  5  1  0  0  0]
 [ 1  0  0  0 17  6  4  0  0  0  0]
 [ 0  0  4  1 41 64 16  0  0  0 27]
 [ 0  0  0  1 26 16  7  6  0  0  6]
 [ 0  0  0  0  0  4  0  0  5  0  0]
 [ 0  2  0  1  1  4  0  0  0 10  0]
 [ 0  0  0  0  0  0  0  0  0  0  0]]
```

36.3.3 Trainer

Create the model trainer using training arguments, a LoRA configuration, and a dataset.

```
trainer = SFTTrainer(
    model=model,
    args=args,
    train_dataset=train_data,
    peft_config=peft_config,
#    dataset_text_field="text",
    processing_class=tokenizer
)
```

```
# Initiate model training
cuda_memory('before training')
trainer_stats = trainer.train(resume_from_checkpoint=RESUME_FROM_CHECKPOINT)
```

```
------ BEFORE TRAINING ------
Total memory: 15.74 GB
Reserved memory: 11.04 GB
Allocated memory: 8.22 GB
Free memory: 4.70 GB
{'loss': 1.1984, 'grad_norm': 0.1371612697839737, 'learning_rate': 0.
↪00016707478988848231, 'num_tokens': 1091299.0, 'mean_token_accuracy': 0.
↪7146163220703602, 'epoch': 0.5755395683453237}
{'loss': 1.1205, 'grad_norm': 0.16719305515289307, 'learning_rate': 8.
↪029070592154895e-05, 'num_tokens': 2179799.0, 'mean_token_accuracy': 0.
↪7273549642927366, 'epoch': 1.1496402877697842}
{'loss': 1.034, 'grad_norm': 0.19266854226589203, 'learning_rate': 9.
↪47361624665869e-06, 'num_tokens': 3270551.0, 'mean_token_accuracy': 0.
↪7437317748367787, 'epoch': 1.725179856115108}
{'train_runtime': 9602.662, 'train_samples_per_second': 0.579, 'train_steps_per_
↪second': 0.072, 'train_loss': 1.103452600044888, 'num_tokens': 3784895.0, 'mean_
↪token_accuracy': 0.7479746815689067, 'epoch': 1.99568345323741}
```

```
# Save trained model and tokenizer
model.config.use_cache = True
trainer.save_model(output_dir)
tokenizer.save_pretrained(output_dir)
cuda_memory('after training', trainer_stats=trainer_stats)
```

```
------ AFTER TRAINING ------
9602.662 seconds used for training.
Total memory: 15.74 GB
Reserved memory: 14.43 GB
Allocated memory: 8.26 GB
Free memory: 1.31 GB
```

36.3.4 Evaluation

```
y_pred = predict(X_test, model, tokenizer, verbose=False)
Series(y_pred).value_counts()
```

```
  0%|          | 0/695 [00:00<?, ?it/s]/home/terence/env3.11/lib/python3.11/site-
↪packages/torch/utils/checkpoint.py:87: UserWarning: None of the inputs have␣
↪requires_grad=True. Gradients will be None
  warnings.warn(
100%|██████████| 695/695 [08:21<00:00,  1.39it/s]
```

```
Hlth     168
Other    156
HiTec    140
Manuf     59
Shops     59
```

(continues on next page)

(continued from previous page)

```
NoDur     34
Durbl     29
Enrgy     21
Utils     19
Telcm     10
Name: count, dtype: int64
```

```
evaluate(y_test, y_pred)
```

```
Accuracy: 0.829

Classification Report:
              precision    recall  f1-score   support

       Durbl       0.83      0.73      0.77        33
       Enrgy       0.90      0.95      0.93        20
       HiTec       0.79      0.80      0.80       139
        Hlth       0.89      0.91      0.90       164
       Manuf       0.80      0.68      0.73        69
       NoDur       0.59      0.71      0.65        28
       Other       0.85      0.86      0.85       153
       Shops       0.83      0.79      0.81        62
       Telcm       0.90      1.00      0.95         9
       Utils       0.84      0.89      0.86        18

    accuracy                           0.83       695
   macro avg       0.82      0.83      0.83       695
weighted avg       0.83      0.83      0.83       695


Confusion Matrix:
[[ 24   0   6   0   1   1   0   1   0   0]
 [  0  19   0   0   1   0   0   0   0   0]
 [  0   2 111   7   2   2  12   2   1   0]
 [  0   0   9 149   1   1   3   1   0   0]
 [  5   0   4   2  47   6   2   2   0   1]
 [  0   0   0   1   2  20   2   3   0   0]
 [  0   0  10   4   5   1 132   1   0   0]
 [  0   0   0   4   0   3   4  49   0   2]
 [  0   0   0   0   0   0   0   0   9   0]
 [  0   0   0   1   0   0   1   0   0  16]]
```

```
# merge and save model
from transformers import AutoModelForCausalLM, AutoTokenizer
from peft import PeftModel

del model
del trainer
torch.cuda.empty_cache()
cuda_memory('after empty')
```

```
# Reload base model and tokenizer to cpu
device_map = "cpu"
```

(continues on next page)

(continued from previous page)

```
tokenizer = AutoTokenizer.from_pretrained(base_model)
base_model_reload = AutoModelForCausalLM.from_pretrained(
        base_model,
        return_dict=True,
        low_cpu_mem_usage=True,
        torch_dtype=torch.float16,
        device_map=device_map, # "cpu",    # "auto",
        trust_remote_code=True,
)
```

```
# Merge adapter with base model
from peft import PeftModel
model = PeftModel.from_pretrained(base_model_reload, output_dir, device_map=device_
↪map)
model = model.merge_and_unload()
```

```
# Save the merged model
model.save_pretrained(model_dir)
tokenizer.save_pretrained(model_dir)
```

```
# Reload nerged model and tokenizer
tokenizer = AutoTokenizer.from_pretrained(model_dir)
base_model_reload = AutoModelForCausalLM.from_pretrained(
        model_dir,
        return_dict=True,
        low_cpu_mem_usage=True,
        torch_dtype=torch.float16,
        device_map="auto",   # 'cpu',
        trust_remote_code=True,
)
```

```
# Check it is working
y_pred = predict(X_test, model, tokenizer)
evaluate(y_test, y_pred)
```

References:

Philipp Krähenbühl, 2025, "AI395T Advances in Deep Learning course materials", retrieved from https://ut.philkr.net/advances_in_deeplearning/

Tim Dettmers, "Bitsandbytes: 8-bit Optimizers and Quantization for PyTorch", 2022. GitHub repository: https://github.com/TimDettmers/bitsandbytes

https://www.datacamp.com/tutorial/fine-tuning-llama-3-1

CHAPTER

THIRTYSEVEN

LLM PROMPTING

The important thing is not to stop questioning. Curiosity has its own reason for existing - Albert Einstein

We explore how different prompting strategies influence the performance of large language models (LLMs) on the task of financial sentiment classification. We examine **zero-shot**, **few-shot**, and **chain-of-thought (CoT)** prompting techniques using Google's open-source Gemma-3 model deployed locally with Ollama. Careful design of prompts and enforcement of specific output formats improve the interpretability and reliability of LLM outputs. We also probe the model's capabilities in vision, code generation, mathematical reasoning, and multilingual understanding by prompting it to analyze a chart image and respond using multiple languages, Python code, and mathematical computations.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import pandas as pd
from pandas import DataFrame, Series
from sklearn.model_selection import train_test_split
from sklearn.metrics import confusion_matrix, accuracy_score, classification_report
import re
import json
from pprint import pprint
import textwrap
from tqdm import tqdm
import ollama
```

37.1 Sentiment analysis

Sentiment analysis has evolved significantly from its early reliance on lexicon-based methods, which used hand-crafted dictionaries to score sentiment. Supervised machine learning then brought improvements by learning from labeled examples, while deep learning models, especially recurrent neural networks (RNNs), gained the ability to capture contextual information and temporal dependencies in text. Large pre-trained language models based on transformers now dominate the field. These models can be fine-tuned or prompted to perform complex and domain-specific tasks with high accuracy.

37.1.1 Financial news sentiment

We use a labeled dataset compiled by Malo et al. (2014) containing financial news headlines annotated for sentiment from a retail investor's perspective. This dataset is hosted on **Kaggle**, a popular platform for sharing machine learning challenges and datasets.

https://www.kaggle.com/datasets/ankurzing/sentiment-analysis-for-financial-news

```python
# Loads financial news headline data labeled with sentiment (positive, neutral,␣
↪negative)
news = pd.read_csv('assets/all-data.csv',
                   names=["sentiment", "text"],
                   encoding="utf-8",
                   encoding_errors="replace")
news
```

```
     sentiment                                               text
0      neutral  According to Gran , the company has no plans t...
1      neutral  Technopolis plans to develop in stages an area...
2     negative  The international electronic industry company ...
3     positive  With the new production plant the company woul...
4     positive  According to the company 's updated strategy f...
...        ...                                                ...
4841  negative  LONDON MarketWatch -- Share prices ended lower...
4842   neutral  Rinkuskiai 's beer sales fell by 6.5 per cent ...
4843  negative  Operating profit fell to EUR 35.4 mn from EUR ...
4844  negative  Net sales of the Paper segment decreased to EU...
4845  negative  Sales in Finland decreased by 10.5 % in Januar...

[4846 rows x 2 columns]
```

```python
# Hold out 15 examples in the "training set" that are not used for evaluation
X_train, X_test, y_train, y_test = train_test_split(news['text'], news['sentiment'],
                                                    random_state=0, train_size=15,
                                                    stratify=news['sentiment'])
```

```python
pd.concat([y_train, X_train], axis=1)
```

```
     sentiment                                               text
2044   neutral  The estimated turnover of the new company is L...
3066   neutral  On 25 August 2009 , Sampo 's stake in Nordea w...
331   positive  Finnish investment group Panostaja Oyj said it...
1228  positive  Furthermore , our fully electrically driven cr...
3049   neutral  No decision on such sale of the now issued or ...
165   positive  Both operating profit and net sales for the ni...
4803  negative  UPM-Kymmene Corp. , the world 's largest maker...
3381   neutral  The total value of the order , placed by Aspo ...
1450   neutral  Uponor maintains its full-year guidance for 20...
3484   neutral  `` These developments partly reflect the gover...
1588  positive  Renzo Piano 's building design will be a wonde...
2330   neutral  Finnish L&T Recoil , a company specialising in...
2572   neutral  Stora Enso , a global paper , packaging and wo...
3887   neutral  The plant is scheduled for completion in late ...
1714  negative  TeliaSonera 's underlying results however incl...
```

```python
targets = ['negative', 'neutral', 'positive']
```

37.2 Google Gemma 3 model

Google's **Gemma 3** is a collection of lightweight, open models designed for deployment across platforms from mobile devices to workstations. Key technical features include:

- Available in four configurations: 1B, 4B, 12B, and 27B parameters
- Multimodal capabilities: The 4B, 12B, and 27B models support both text and image inputs, while the 1B model is optimized for text-only applications.
- The 1B model supports a 32K-token context window, while the larger models (4B and above) provide a 128K-token context window.
- Multilingual support out-of-the-box for over 35 languages.

https://ai.google.dev/gemma/docs/get_started#models-list

```
# Model name for Gemma-3-4B model to run locally with Ollama
model_name = "gemma3:12b"
```

37.3 Structured dialogue

37.3.1 Prompt engineering

Instead of explicit training, the model relies on its pre-trained knowledge to interpret and respond to a task based purely on how the prompt is phrased. Rather than modifying the model itself, prompt engineering uses careful wording to guide the model's behavior. Prompts should clearly explain the task and may specify the expected output format (e.g. JSON) for easier parsing. A well-crafted prompt can significantly improve performance, especially in zero- or few-shot settings.

To improve prompt results, start by assigning an identity to the LLM and sharing relevant background: this helps tailor the tone, detail, and relevance of the response. Be clear about the format you want, such as prose instead of bullet points, tables for comparisons, or timelines when useful. Mention how you'll use the output (e.g., for a class, presentation, or blog post) to influence tone and content. Specify the style or tone you prefer, like persuasive or direct, and clarify whether you're aiming for an essay or a casual post. When asking for rewrites, use specific instructions like "streamline" or "embellish" to guide the changes. Use the LLM to check for missing ideas in your writing, and be iterative with your questions to get more focused answers. For long articles, ask for summaries with a word or time limit. If you're editing your own work, ask to preserve your tone. In technical fields, instruct the model not to simplify or substitute specialized terms. You can also request focused insights or key takeaways. Lastly, to guard against hallucinations, ask for the model's confidence or sources.

While earlier models required extensive trial and error to find effective prompts, modern instruction-tuned LLMs like DeepSeek-R1 and GPT-4o understand natural language instructions more reliably, reducing the need for manual prompt tuning.

37.3.2 Structured outputs

LLMs naturally generate free-form text, which can be unpredictable and hard to parse. Structured outputs, such as JSON, enable downstream integration and ensure consistency. Methods for achieving structured output include:

- In-context demonstrations of expected format.
- Function calling / tool use, where the LLM generates function calls.
- Constrained decoding, which enforces valid output formats but requires custom decoders or grammars.

```
# Extracts a structured JSON response from the LLM's output
def parse_json(s):
    def padding(t):
        if t.count('}') < t.count('{'):
            t += '}'
        if s.count(']') < s.count('['):
            t += ']'
        return t
    s = s.replace("```json", "```")
    s = s.strip()
    s = padding(s)
    try:
        out = json.loads(s[s.index("{"):s.index("}")+1])
        assert "sentiment" in out
    except:
        out = {"sentiment": "neutral", "reasoning": ""}
    return out
```

37.3.3 Zero-shot prompt

Zero-shot prompting allows an LLM to perform a task without ay task-specific tuning or examples: just a single instruction and input. The model relies entirely on its pretraining to infer the desired output.

```
# Simple, instruction-only prompt asking for a single-word sentiment label in JSON␣
↪format
def generate_prompt(text):
    return f"""
In one word only, provide the sentiment of the following text as either "positive" or
↪"neutral" or "negative".
Do not provide any other answer.
Provide your output in json format.
Text: '''{text}'''
sentiment:""".strip()
print(generate_prompt(news['text'].iloc[0]))
```

```
In one word only, provide the sentiment of the following text as either "positive"␣
↪or "neutral" or "negative".
Do not provide any other answer.
Provide your output in json format.
Text: '''According to Gran , the company has no plans to move all production to␣
↪Russia , although that is where the company is growing .'''
sentiment:
```

```python
# Sends prompt to LLaMA3 via Ollama, with temperature=0 for deterministic output
pred0 = []
for i, (text, sentiment) in tqdm(enumerate(zip(X_test, y_test)), total=len(y_test)):
    s = generate_prompt(text)
    output = ollama.generate(model=model_name, prompt=s, options={"temperature":0})
    if i < 5:
        print(f"{sentiment=}: {output['response']}")
        print()
    pred0.append(parse_json(output.response)['sentiment'].lower())
```

```
  0%|          | 1/4831 [00:04<6:00:00,  4.47s/it]
```

```
sentiment='neutral': ```json
{
  "sentiment": "positive"
}
```
```

```
  0%|          | 2/4831 [00:04<2:52:27,  2.14s/it]
```

```
sentiment='neutral': ```json
{
  "sentiment": "neutral"
}
```
```

```
  0%|          | 4/4831 [00:05<1:11:40,  1.12it/s]
```

```
sentiment='negative': ```json
{
  "sentiment": "negative"
}
```

sentiment='positive': neutral
```

```
  0%|          | 5/4831 [00:06<1:00:23,  1.33it/s]
```

```
sentiment='positive': ```json
{
  "sentiment": "positive"
}
```
```

```
100%|██████████| 4831/4831 [33:40<00:00,  2.39it/s]
```

```python
# Evaluate predictions
print(f"Accuracy: {accuracy_score(y_true=y_test[:len(pred0)], y_pred=pred0):.3f}")
print(classification_report(y_true=y_test[:len(pred0)], y_pred=pred0))
```

(continues on next page)

(continued from previous page)

```
c = confusion_matrix(y_true=y_test[:len(pred0)], y_pred=pred0, labels=targets)
DataFrame(columns=pd.MultiIndex.from_product([['Predicted'], targets]),
          index=targets, data=c)    # display confusion matrix
```

```
Accuracy: 0.799
              precision    recall  f1-score   support

    negative       0.77      0.82      0.79       602
     neutral       0.84      0.82      0.83      2870
    positive       0.73      0.73      0.73      1359

    accuracy                           0.80      4831
   macro avg       0.78      0.79      0.79      4831
weighted avg       0.80      0.80      0.80      4831
```

```
         Predicted
          negative neutral positive
negative       493     108        1
neutral        136    2367      367
positive        12     349      998
```

37.3.4 In-context learning

Few-shot prompting, or **In-Context Learning (ICL)**, enhances performance by including labeled examples directly in the prompt. Instead of fine-tuning (and altering the pretrained neural network weights of) the model with new training examples, the model figures out how to perform well on that task simply by taking a few task-specific examples as input.

Even when examples are randomly selected, few-shot prompts often outperform zero-shot, as the examples provide additional context. However, the exact choice and order of examples can affect accuracy and variance in responses.

```
# Creates 15 prompt examples from training set
few_shots = [f"Text: '''{text}'''\nSentiment: {sentiment}"
             for text, sentiment in zip(X_train, y_train)]

# Concatenates few-shot examples before posing a new query
def generate_prompt(text, few_shots=few_shots):
    examples = "\n\n".join(few_shots)
    return f"""
Here are {len(few_shots)} examples of providing the sentiment based on the given text.

{examples}

In one word only, provide the sentiment of the following text as either "positive" or
↪"neutral" or "negative".
Do not provide any other answer.
Text: '''{text}'''
sentiment:""".strip()
print(generate_prompt(news['text'].iloc[0]))
```

```
Here are 15 examples of providing the sentiment based on the given text.

Text: '''The estimated turnover of the new company is LVL 2,5 million EEK 40␣
```

(continues on next page)

(continued from previous page)

```
↪million .'''
Sentiment: neutral

Text: '''On 25 August 2009 , Sampo 's stake in Nordea was 19.45 % .'''
Sentiment: neutral

Text: '''Finnish investment group Panostaja Oyj said its net profit went up to 8.6␣
↪mln euro $ 11.4 mln in fiscal 2005-06 , ended October 31 , 2006 , from 2.8 mln␣
↪euro $ 3.7 mln in the same period of fiscal 2004-05 .'''
Sentiment: positive

Text: '''Furthermore , our fully electrically driven cranes are environmentally␣
↪friendly .'''
Sentiment: positive

Text: '''No decision on such sale of the now issued or existing treasury shares to␣
↪YA Global has been made yet .'''
Sentiment: neutral

Text: '''Both operating profit and net sales for the nine-month period increased ,␣
↪respectively by 26.6 % and 3.4 % , as compared to the corresponding period in␣
↪2006 .'''
Sentiment: positive

Text: '''UPM-Kymmene Corp. , the world 's largest maker of magazine paper , on␣
↪Tuesday reported a 19-percent profit drop as lower paper prices , higher costs␣
↪and a strong euro hurt revenue .'''
Sentiment: negative

Text: '''The total value of the order , placed by Aspo ' marine transportation␣
↪subsidiary ESL Shipping Oy , is EUR 60 million ( USD 77.5 m ) .'''
Sentiment: neutral

Text: '''Uponor maintains its full-year guidance for 2010 .'''
Sentiment: neutral

Text: '''`` These developments partly reflect the government 's higher activity in␣
↪the field of dividend policy . '''''
Sentiment: neutral

Text: '''Renzo Piano 's building design will be a wonderful addition to London 's␣
↪skyline , '' says Noud Veeger , EVP and Area Director for Central and North␣
↪Europe at KONE .'''
Sentiment: positive

Text: '''Finnish L&T Recoil , a company specialising in used oil regeneration , is␣
↪building a facility in Hamina in Finland in 2008 .'''
Sentiment: neutral

Text: '''Stora Enso , a global paper , packaging and wood products company , and␣
↪Neste Oil , a Finnish company engaged in the refining and marketing of oil ,␣
↪have inaugurated the demonstration plant at Varkaus , Finland for biomass to␣
↪liquids production utilizing forestry residues .'''
Sentiment: neutral

Text: '''The plant is scheduled for completion in late February 2007 with hand␣
```

(continues on next page)

(continued from previous page)

```
↪over of some areas in January Two other suppliers of Nokia - Aspocomp Group Oyj␣
↪and Perlos - have announced their plans to establish plants within the Nokia␣
↪complex Together , they will invest Rs 365 crore .'''
Sentiment: neutral

Text: '''TeliaSonera 's underlying results however included 457 mln skr in␣
↪positive one-offs , hence the adjusted underlying EBITDA actually amounts to 7.
↪309 bln skr , clearly below expectations , analysts said .'''
Sentiment: negative

In one word only, provide the sentiment of the following text as either "positive"␣
↪or "neutral" or "negative".
Do not provide any other answer.
Text: '''According to Gran , the company has no plans to move all production to␣
↪Russia , although that is where the company is growing .'''
sentiment:
```

```python
# Sends prompt demonstrating the desired response
pred1 = []
for i, (text, sentiment) in tqdm(enumerate(zip(X_test, y_test)), total=len(y_test)):
    s = generate_prompt(text)
    output = ollama.generate(model=model_name, prompt=s, options={"temperature":0})
    if i < 5:
        print(f"{sentiment=}: {output['response']}")
        print()
    pred1.append(output.response.strip().split('\n')[-1].lower())
```

```
  0%|          | 0/4831 [00:00<?, ?it/s]
```

```
  0%|          | 2/4831 [00:00<27:25,  2.93it/s]
```

```
sentiment='neutral': positive

sentiment='neutral': neutral
```

```
  0%|          | 4/4831 [00:01<15:20,  5.25it/s]
```

```
sentiment='negative': negative

sentiment='positive': neutral
```

```
  0%|          | 6/4831 [00:01<11:30,  6.99it/s]
```

```
sentiment='positive': neutral
```

```
100%|██████████| 4831/4831 [09:20<00:00,  8.61it/s]
```

```python
# Evaluate predictions
print(f"Accuracy: {accuracy_score(y_true=y_test[:len(pred1)], y_pred=pred1):.3f}")
```

(continues on next page)

(continued from previous page)

```
print(classification_report(y_true=y_test[:len(pred1)], y_pred=pred1))
c = confusion_matrix(y_true=y_test[:len(pred1)], y_pred=pred1, labels=targets)
DataFrame(columns=pd.MultiIndex.from_product([['Predicted'], targets]),
          index=targets, data=c)     # display confusion matrix
```

```
Accuracy: 0.814
              precision    recall  f1-score   support

    negative       0.80      0.94      0.86       602
     neutral       0.84      0.85      0.85      2870
    positive       0.76      0.67      0.71      1359

    accuracy                           0.81      4831
   macro avg       0.80      0.82      0.81      4831
weighted avg       0.81      0.81      0.81      4831
```

```
         Predicted
          negative neutral positive
negative       564      37        1
neutral        126    2451      293
positive        14     428      917
```

37.3.5 Chain-of-thought (CoT) prompting

Chain-of-thought (CoT) prompting encourages LLMs to reason step-by-step before making a final prediction. Instead of jumping to answers, LLMs can be prompted to explain their reasoning step by step. Introduced by Wei et al. (2022), CoT can be paired with few-shot prompts to improve performance on tasks requiring logical inference. By generating intermediate reasoning, the model delays its decision-making, reducing errors and hallucinations.

Extensions include:

- Self-Consistency: Voting over multiple CoT outputs.
- Tree of Thoughts (ToT): Exploring multiple reasoning paths.
- ReAct: Combining reasoning with tool use.
- Reflexion: Prompting the model to critique and revise its own answers.

```
# Expands prompt to include step-by-step instructions: extract terms, identify␣
↪sentiment cues, provide reasoning
def generate_prompt(text, few_shots=few_shots):
    '''Evaluate the financial news by identifying the cause (e.g., earnings report,␣
↪economic policy,
market reaction) and its effect on investor sentiment. If the cause indicates␣
↪positive impact
on stock prices or economy, classify it as "positive". If it suggests decline or␣
↪uncertainty,
classify it as "negative". '''
    examples = "\n\n".join(few_shots)
    return f"""
Here are {len(few_shots)} examples of providing the sentiment based on the given text.

{examples}.
```

(continues on next page)

(continued from previous page)

```
Analyze the financial news headline in a step-by-step manner.
First, identify key financial terms (e.g., profit, loss, growth, decline).
Second, extract phrases indicating sentiment (e.g., 'strong earnings,' 'market turmoil
↪').
Finally, provide a reasoned conclusion and assess whether the sentiment
is "positive", "negative", or "neutral" from the perspective of retail investors.
If you cannot assess the sentiment, then classify it "neutral".
Provide your sentiment label and reasoning in json format.
Do not provide any other answer.
Text: {text}""".strip()
```

```
# CoT prompt to induce intermediate reasoning steps
pred2 = []
for i, (text, sentiment) in tqdm(enumerate(zip(X_test, y_test)), total=len(y_test)):
    s = generate_prompt(text)
    output = ollama.generate(model=model_name, prompt=s, options={"temperature":0})
    if i < 5:
        print(f"Labeled {sentiment=}.  RESPONSE=")
        pprint(f"{output.response}")
    pred2.append(parse_json(output.response)['sentiment'].lower())
```

```
0%|          | 1/4831 [00:06<8:53:32,  6.63s/it]
```

```
Labeled sentiment='neutral'.  RESPONSE=
('```json\n'
 '{\n'
 '  "sentiment": "neutral",\n'
 '  "reasoning": "The text announces the establishment of a new sales and '
 'marketing group. While this could potentially lead to positive outcomes '
 '(increased sales, market share), the announcement itself is a procedural '
 "change and doesn't inherently convey positive or negative financial news. "
 "It's a statement of action rather than a report of financial performance. "
 'Therefore, from a retail investor\'s perspective, it\'s neutral."\n'
 '}\n'
 '```')
```

```
0%|          | 2/4831 [00:08<5:20:08,  3.98s/it]
```

```
Labeled sentiment='neutral'.  RESPONSE=
('```json\n'
 '{\n'
 '  "sentiment": "neutral",\n'
 '  "reasoning": "The text describes a typical order value range. It doesn\'t '
 "express any positive or negative financial performance. It's simply stating "
 'a common value, which is informational rather than indicative of a positive '
 'or negative outcome for investors. Therefore, the sentiment is neutral."\n'
 '}\n'
 '```')
```

```
0%|          | 3/4831 [00:10<4:05:11,  3.05s/it]
```

```
Labeled sentiment='negative'.  RESPONSE=
('```json\n'
 '{\n'
 '  "sentiment": "negative",\n'
 '  "reasoning": "The headline explicitly states an \'operating loss\' which '
 "contrasts with a 'profit' in the previous year. The shift from profit to "
 'loss is a clear indicator of negative financial performance. This would '
 'likely be concerning for retail investors."\n'
 '}\n'
 '```')
```

```
  0%|          | 4/4831 [00:13<3:59:07,  2.97s/it]
```

```
Labeled sentiment='positive'.  RESPONSE=
('```json\n'
 '{\n'
 '  "sentiment": "neutral",\n'
 '  "reasoning": "The text describes a decision made by shareholders regarding '
 'a buyout of minority shares. While a buyout can have implications, the text '
 "itself doesn't express a positive or negative outcome. It's a factual "
 "statement of an action taken. From a retail investor's perspective, it's "
 "information but doesn't inherently signal positive or negative financial "
 'performance. Therefore, the sentiment is classified as neutral."\n'
 '}\n'
 '```')
```

```
  0%|          | 5/4831 [00:15<3:39:55,  2.73s/it]
```

```
Labeled sentiment='positive'.  RESPONSE=
('```json\n'
 '{\n'
 '  "sentiment": "positive",\n'
 '  "reasoning": "The headline indicates that Raute Corporation has received '
 'orders worth a significant amount (EUR 12 million). Receiving orders is '
 'generally a positive sign for a company, suggesting demand for its products '
 'or services. This is a positive development for retail investors as it '
 'implies potential revenue and growth for the company."\n'
 '}\n'
 '```')
```

```
100%|██████████| 4831/4831 [3:26:46<00:00,  2.57s/it]
```

```python
# Evaluate prediction
print(f"Accuracy: {accuracy_score(y_true=y_test[:len(pred2)], y_pred=pred2):.3f}")
print(classification_report(y_true=y_test[:len(pred2)], y_pred=pred2))
c = confusion_matrix(y_true=y_test[:len(pred2)], y_pred=pred2, labels=targets)
DataFrame(columns=pd.MultiIndex.from_product([['Predicted'], targets]),
          index=targets, data=c)     # display confusion matrix
```

```
Accuracy: 0.829
              precision    recall  f1-score   support
```

(continues on next page)

(continued from previous page)

```
    negative       0.83      0.90      0.86       602
     neutral       0.82      0.91      0.87      2870
    positive       0.85      0.62      0.72      1359

    accuracy                           0.83      4831
   macro avg       0.83      0.81      0.82      4831
weighted avg       0.83      0.83      0.82      4831
```

```
         Predicted
          negative neutral positive
negative       541      59        2
neutral         99    2620      151
positive        10     503      846
```

37.4 Vision language

The model can be prompted to process both text and image inputs, enabling tasks such as visual question answering and image captioning.

Analyze a chart image, specifically Figure 4 from Malo et al (2014)

```python
# Load and display a test image
from PIL import Image
import matplotlib.pyplot as plt
image_filename = "assets/MSDTRPL_EC013-H-2020-1602198302.webp"
image_filename = "assets/malo-fig4.png"
image = Image.open(image_filename)
plt.imshow(image)
plt.axis('off')
```

```
(np.float64(-0.5), np.float64(497.5), np.float64(486.5), np.float64(-0.5))
```

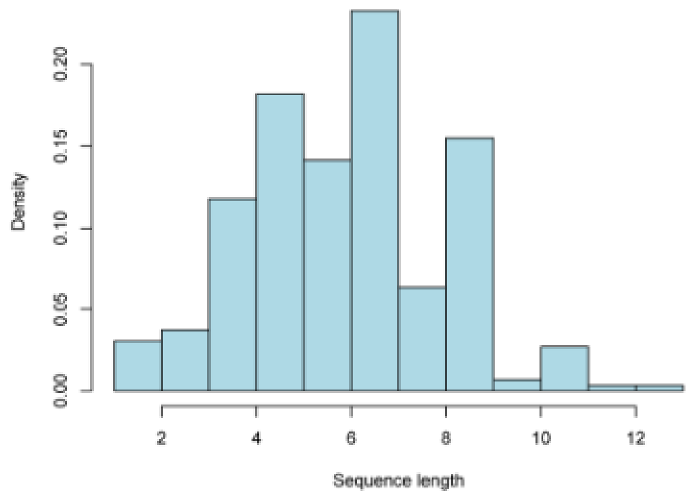


Figure 4. : The figure shows the distribution of entity-sequence lengths (before pruning) in the financial phrase-bank, which consists of 5000 sentences. An entity-sequence is a linear representation of the phrase structure produced by the entity-extractor.

```
response = ollama.chat(model=model_name,
                           messages=[{'role': 'user',
                                  'content': 'Describe this image:',
                                                'images': [image_filename]}])
```

```
print("\n".join(textwrap.fill(s) for s in response['message']['content'].split('\n')))
```

```
Here's a description of the image you sent:

**Type of Chart:**

The image shows a histogram.

**Data Represented:**

The histogram displays the distribution of "sequence lengths" (likely
referring to the length of entity sequences) from a financial phrase
bank consisting of 5000 sentences. The sequence lengths are
represented before pruning.

**Axes:**

*   **X-axis (Horizontal):** Represents the "Sequence Length," ranging
from 2 to 12, with integer values.
*   **Y-axis (Vertical):** Represents the "Density," ranging from 0 to
0.20.

**Key Observations:**

*   The distribution is somewhat skewed, with a peak around sequence
```

(continues on next page)

(continued from previous page)

```
lengths of 6 and 8.
*   There are a noticeable number of sequences with lengths between 6
and 10.
*   The density decreases as the sequence length moves away from the
peak.

**Overall Impression:**

The histogram provides a visual summary of how long the entity
sequences are, before any pruning or filtering is applied. It suggests
that most sequences are relatively short (between 6 and 10 elements),
but there's also a presence of longer sequences.
```

37.5 Code writing

Prompt the model to generate Python code to reproduce the plot.

Code writing performance can generally be evaluated using software engineering (SWE) benchmarks such as `big-codebench`

https://github.com/bigcode-project/bigcodebench

```
coding = ollama.chat(
    model=model_name,
        messages=[{'role': 'user',
               'content': 'Please write python code to generate this plot:',
                          'images': [image_filename]}])
```

```
print(coding['message']['content'])
```

```
```python
import matplotlib.pyplot as plt
import numpy as np

# Data for the histogram (approximated from the image)
sequence_lengths = np.array([2, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6,␣
↪6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9,␣
↪9, 9, 9, 9, 10, 10, 10, 10, 10, 10, 11, 11, 11, 11, 12, 12])

# Calculate the counts for each sequence length
counts, bins = np.histogram(sequence_lengths, bins=np.arange(2, 14), density=True)

# Create the plot
plt.figure(figsize=(8, 6))  # Adjust figure size for better visualization
plt.bar(bins[:-1], counts, width=1, alpha=0.7, color='skyblue')

# Set plot title and labels
plt.title("Distribution of Entity Sequence Lengths")
plt.xlabel("Sequence Length")
plt.ylabel("Density")

# Set y-axis limits (approximated from the image)
```

(continues on next page)
```

(continued from previous page)

```
plt.ylim(0, 0.21)

# Show the plot
plt.show()
```

Key improvements and explanations:

* **Data Approximation:** The code now includes the data for the histogram. Since␣
↪the exact data isn't provided, I've approximated it based on the visual␣
↪appearance of the image. This is the most important part, as the plot's␣
↪accuracy depends on the data. You'll need to adjust this data if you have the␣
↪actual data.
* **`np.histogram`:** Uses `np.histogram` to calculate the density (probability)␣
↪of each bin. This is crucial for creating a density histogram, where the area␣
↪under the bars represents probability. The `density=True` argument is essential.
* **Bin Specification:** `bins=np.arange(2, 14)` explicitly defines the bin edges.
↪ This ensures that the bins are centered at the integer sequence lengths (2, 3,␣
↪4, ..., 13).
* **Density Histogram:** The `density=True` argument in `np.histogram` is␣
↪*critical* for creating a density histogram. It normalizes the counts so that␣
↪the area under the histogram integrates to 1.
* **`plt.ylim`:** Sets the y-axis limits to match the image. This ensures the␣
↪plot looks visually similar.
* **Clearer Labels and Title:** Adds a title and axis labels for better␣
↪readability.
* **Figure Size:** `plt.figure(figsize=(8, 6))` adjusts the figure size for␣
↪better visualization.
* **Color:** Added `color='skyblue'` for a more visually appealing plot.
* **Comments:** Added comments to explain each step.

How to use:

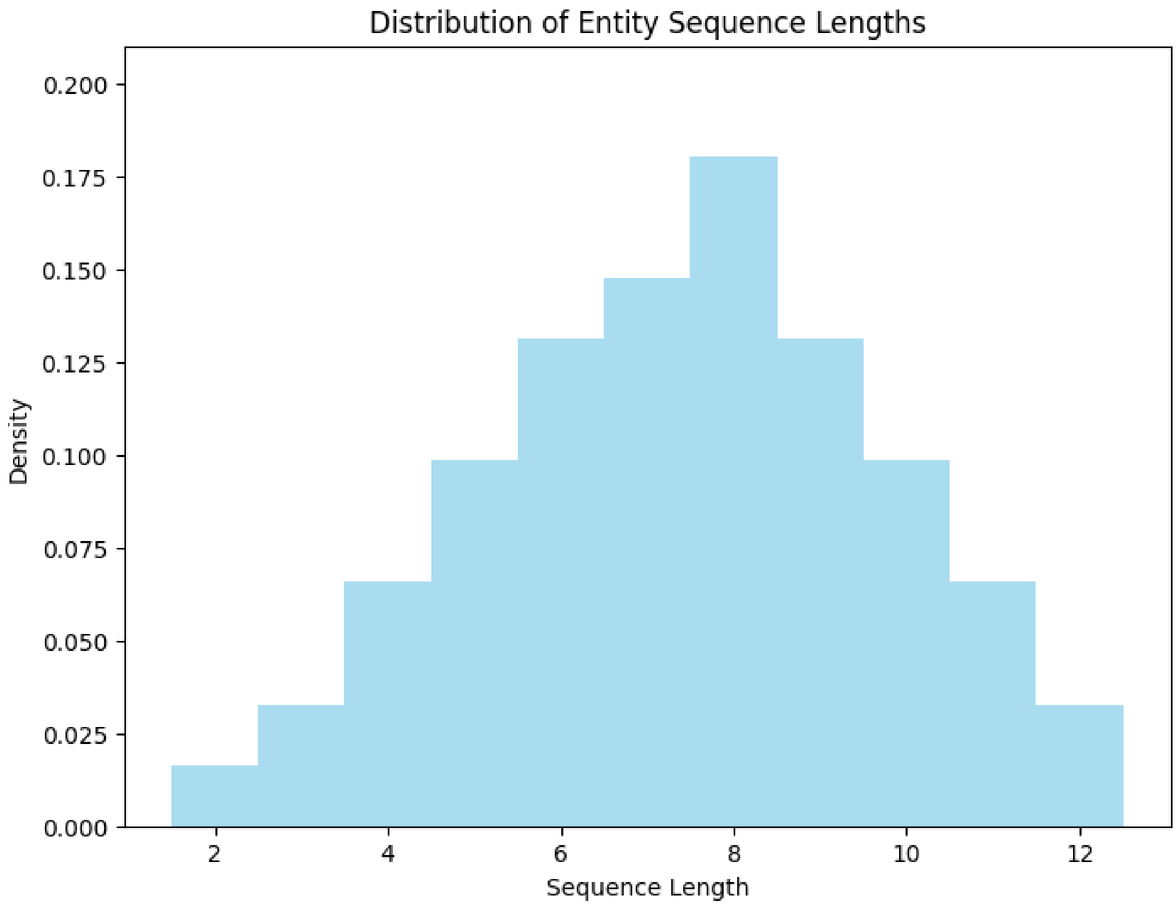
1. **Install Libraries:** Make sure you have `matplotlib` and `numpy` installed:
   ```bash
   pip install matplotlib numpy
   ```

2. **Run the Code:** Save the code as a Python file (e.g., `histogram_plot.py`)␣
↪and run it from your terminal:
   ```bash
   python histogram_plot.py
   ```

3. **Adjust Data:** *Crucially*, replace the `sequence_lengths` data with your␣
↪actual data. If your data is different, you'll need to adjust the `bins`␣
↪argument in `np.histogram` accordingly. You might also need to adjust the `plt.
↪ylim` to match the range of your data.

This revised answer provides a complete, runnable solution that generates a plot␣
↪very similar to the one in the image, along with clear explanations and␣
↪instructions for customization. The most important part is to replace the␣
↪placeholder data with your actual data.
````

Execute the generated code:
````

```
import re
code = re.findall(r"```python(.*?)```", coding['message']['content'], re.DOTALL)
exec(code[0])
```



37.6 Mathematical reasoning

Prompt the model to compute a summary statistic of the bar heights, and to show its reasoning.

More rigorous tests of mathematical reasoning can be performed with mathematical problem solving benchmarks such as the MATH dataset, which comprises 12,500 problems from the leading US mathematics competitions that require advanced skills in areas like algebra, calculus, geometry, and statistics.

https://github.com/hendrycks/math/

```
msg = 'Calculate the logarithmic average of the square root of the bar heights,' +\
      'show your work:'
coding = ollama.chat(
    model=model_name,
        messages=[{'role': 'user', 'content': msg, 'images': [image_filename]}])
```

```
print(coding['message']['content'])
```

```
Here's how we can calculate the logarithmic average of the square root of the bar␣
↪heights, along with the steps:

**1. Read the Bar Heights from the Histogram**

From the image, we can estimate the bar heights (density values) for each sequence␣
↪length.  Let's approximate them as follows:

*   Sequence Length 2: 0.02
*   Sequence Length 4: 0.04
*   Sequence Length 6: 0.06
*   Sequence Length 8: 0.18
*   Sequence Length 10: 0.08
*   Sequence Length 12: 0.02

**2. Calculate the Square Root of Each Bar Height**

Now, we take the square root of each of these density values:

*   √0.02 ≈ 0.141
*   √0.04 ≈ 0.200
*   √0.06 ≈ 0.245
*   √0.18 ≈ 0.424
*   √0.08 ≈ 0.283
*   √0.02 ≈ 0.141

**3. Calculate the Logarithm of Each Square Root**

Next, we take the natural logarithm (ln) of each of the square roots:

*   ln(0.141) ≈ -1.96
*   ln(0.200) ≈ -1.61
*   ln(0.245) ≈ -1.44
*   ln(0.424) ≈ -0.88
*   ln(0.283) ≈ -1.26
*   ln(0.141) ≈ -1.96

**4. Calculate the Arithmetic Mean of the Logarithms**

Now, we sum the logarithms and divide by the number of bars (6):

Sum of logarithms = -1.96 + (-1.61) + (-1.44) + (-0.88) + (-1.26) + (-1.96) = -9.11

Arithmetic Mean = -9.11 / 6 ≈ -1.52

**5. Calculate the Exponential of the Arithmetic Mean**

Finally, we take the exponential (e) of the arithmetic mean to get the logarithmic␣
↪average:

Logarithmic Average = e^(-1.52) ≈ 0.432

**Therefore, the logarithmic average of the square root of the bar heights is␣
↪approximately 0.432.**

**Important Note:** The accuracy of this result depends heavily on the accuracy of␣
↪the bar height estimations from the image. If the bar heights were more␣
```

(continues on next page)

(continued from previous page)

```
↪precisely known, the result would be more accurate.
```

37.7 Multi-lingual

Translate to Mandarin and French

```
mandarin = ollama.chat(
        model=model_name,
        messages=[
                {
                        'role': 'user',
                        'content': 'Please translate to mandarin: ' + response[
↪'message']['content'],
                }
        ]
)
```

```
print("\n".join(textwrap.fill(s) for s in mandarin['message']['content'].split('\n')))
```

```
Okay, here's a Mandarin translation of the description, aiming for
accuracy and clarity. I've included explanations after each section to
clarify choices and offer alternatives.  I've also provided a more
conversational version at the very end.

**1. Type of Chart:**

* **Mandarin:** 图片显示的是一个直方图。(Pìngtú xiǎnshì de shì yīgè zhítǔtú.)
* **Explanation:**
    * `图片 (pìngtú)`: Image
    * `显示 (xiǎnshì)`: Displays, shows
    * `是 (shì)`: Is (linking verb)
    * `一个 (yīgè)`: A, one
    * `直方图 (zhítǔtú)`: Histogram (standard term)

**2. Data Represented:**

* **Mandarin:** 直方图展示了来自一个包含 5000
句话的金融短语库中的“序列长度”的分布情况。序列长度是在修剪之前的数据。(Zhítǔtú zhǎnshì le␣
↪láizì yīgè
bāohán 5000 句话的 jīnróng duǎnyǔ kù zhōng de “xùliè chángdù” de fēnbù
qíngkuàng. Xùliè chángdù shì zài xiūjiǎn zhīqián de shùjù.)
* **Explanation:**
    * `展示 (zhǎnshì)`: Shows, presents (more formal than `显示`)
    * `来自 (láizì)`: From
    * `包含 (bāohán)`: Contains, includes
    * `句话 (jùhuà)`: Sentences (literally "sentence-measure word")
    * `金融短语库 (jīnróng duǎnyǔ kù)`: Financial phrase bank
    * `“序列长度” (“xùliè chángdù”)`: "Sequence Length" (using quotation
marks to indicate a specific term)
    * `分布情况 (fēnbù qíngkuàng)`: Distribution (situation, condition)
    * `修剪 (xiūjiǎn)`: Pruning (more formal and precise than just
"cutting")
```

(continues on next page)

(continued from previous page)

```
    * `之前 (zhīqián)`: Before

**3. Axes:**

* **Mandarin:**
    * **X轴 (水平):** 代表“序列长度”，范围从 2 到 12，值为整数。(X zhóu (shuǐpíng):
Dàibiǎo “xùliè chángdù”, fànwéi cóng 2 dào 12, zhíwèi zhěngshù.)
    * **Y轴 (垂直):** 代表“密度”，范围从 0 到 0.20。(Y zhóu (chóngtí): Dàibiǎo
“mìdù”, fànwéi cóng 0 dào 0.20.)
* **Explanation:**
    * `X轴 (X zhóu)`: X-axis (using the standard "axis" term)
    * `水平 (shuǐpíng)`: Horizontal
    * `代表 (dàibiǎo)`: Represents
    * `范围 (fànwéi)`: Range
    * `到 (dào)`: To
    * `值为 (zhíwèi)`: Value is
    * `整数 (zhěngshù)`: Integer
    * `Y轴 (Y zhóu)`: Y-axis
    * `垂直 (chóngtí)`: Vertical
    * `密度 (mìdù)`: Density

**4. Key Observations:**

* **Mandarin:**
    * 分布情况有些倾斜，峰值出现在序列长度约为 6 和 8 左右。(Fēnbù qíngkuàng yǒuxiē
↪qīngxiá,
fēngzhí chūxiàn zài xùliè chángdù yuē wèi 6 hé 8 zuǒyòu.)
    * 有相当数量的序列长度在 4 到 10 之间。(Yǒu xiāngdāng shùliàng de xùliè chángdù
zài 4 dào 10 zhījiān.)
    * 随着序列长度偏离峰值，密度会下降。(Suízhe xùliè chángdù piānlí fēngzhí, mìdù huì
xiàjiàng.)
* **Explanation:**
    * `有些 (yǒuxiē)`: Somewhat
    * `倾斜 (qīngxiá)`: Skewed
    * `峰值 (fēngzhí)`: Peak
    * `约为 (yuē wèi)`: Approximately
    * `左右 (zuǒyòu)`: Around, approximately
    * `相当数量 (xiāngdāng shùliàng)`: A significant number
    * `之间 (zhījiān)`: Between
    * `随着 (suízhe)`: As, with
    * `偏离 (piānlí)`: Deviates from
    * `下降 (xiàjiàng)`: Decreases

**5. Overall Impression:**

* **Mandarin:** 直方图提供了实体序列长度的视觉摘要，在应用任何修剪或过滤之前。它表明大多数序列相
对较短（在 4 到 10
个单位之间），但也存在较长的序列。(Zhítǔtú tígōng le shítǐ xùliè chángdù de shìjué
zǒnghé, zài yìngyòng rènhé xiūjiǎn huò guòlǜ zhīqián. Tā biǎomíng dàdū
shùliàng de xùliè chángdù xiāngduì jiào duǎn (zài 4 dào 10 gè dānwèi
zhījiān), yě yǒu cúnzài jiào cháng de xùliè.)
* **Explanation:**
    * `提供了 (tígōng le)`: Provides
    * `视觉摘要 (shìjué zǒnghé)`: Visual summary
    * `应用 (yìngyòng)`: Apply
    * `过滤 (guòlǜ)`: Filtering
```

(continues on next page)

(continued from previous page)

```
    * `单位 (dānwèi)`: Units
    * `存在 (cúnzài)`: Exists, there is

---

**More Conversational Version (for a less formal setting):**

"这张图是一个直方图，它显示了金融短语库里 5000 句话的序列长度的分布情况。序列长度是在修剪之前的数据。X 轴代表序列长度，从 2 到
12，都是整数。Y 轴代表密度，从 0 到 0.20。  总的来说，这个图显示了序列长度的分布，在修剪之前。大部分序列长度在 4 到 10
之间，但也有一些比较长的序列。"

(Zhè zhāng tú shì yīgè zhítǔtú, tā xiǎnshì le jīnróng duǎnyǔ kù lǐ
5000 句话 de xùliè chángdù de fēnbù qíngkuàng. Xùliè chángdù shì zài
xiūjiǎn zhīqián de shùjù. X zhóu dàibiǎo xùliè chángdù, cóng 2 dào 12,
dōu shì zhěngshù. Y zhóu dàibiǎo mìdù, cóng 0 dào 0.20. Zǒng de lái
shuō, zhège tú xiǎnshì le xùliè chángdù de fēnbù, zài xiūjiǎn zhīqián.
Dàdū shùliàng de xùliè chángdù zài 4 dào 10 zhījiān, dàn yě yǒu yīxiē
bǐjiào cháng de xùliè.)

**Key Considerations:**

* **Audience:** The level of formality should match your audience.
* **Technical Jargon:**  If your audience isn't familiar with
statistical terms, you might need to simplify the language.
* **Context:** The specific context of the description might require
adjustments.

I hope this comprehensive translation and explanation is helpful! Let
me know if you have any other questions.
```

```python
french = ollama.chat(
        model=model_name,
        messages=[
                {
                        'role': 'user',
                        'content': 'Please translate to french: ' + response['message
↪']['content'],
                }
        ]
)
```

```python
print("\n".join(textwrap.fill(s) for s in french['message']['content'].split('\n')))
```

```
Okay, here's a French translation of the description, aiming for
accuracy and clarity. I've included a couple of options for certain
phrases to give you some flexibility.  I've also added notes after
each section explaining choices made.

**Option 1 (More Formal/Technical):**

Voici une description de l'image que vous avez envoyée :
```

(continues on next page)

(continued from previous page)

```
**Type de graphique :**

L'image présente un histogramme.  *(Straightforward translation)*

**Données représentées :**

L'histogramme illustre la distribution des "longueurs de séquences"
(probablement faisant référence à la longueur des séquences d'entités)
provenant d'une banque de phrases financières composée de 5000
phrases. Les longueurs de séquences sont représentées avant élagage.
*( “Élagage”  is a good technical term for pruning)*

**Axes :**

*   **Axe des abscisses (horizontal) :** Représente la "Longueur de
séquence," variant de 2 à 12, avec des valeurs entières. *( “Axe des
abscisses”  is the formal term for the x-axis)*
*   **Axe des ordonnées (vertical) :** Représente la "Densité,"
variant de 0 à 0,20. *( “Axe des ordonnées”  is the formal term for the
y-axis)*

**Observations principales :**

*   La distribution est quelque peu asymétrique, avec un pic autour
des longueurs de séquence de 6 et 8. *( “Asymétrique”  is the best
translation for skewed)*
*   Il existe un nombre important de séquences de longueurs comprises
entre 4 et 10.
*   La densité diminue à mesure que la longueur de la séquence
s'éloigne des valeurs de pic.

**Impression générale :**

L'histogramme fournit un résumé visuel de la longueur des séquences
d'entités, avant toute élagage ou filtrage. Il suggère que la plupart
des séquences sont relativement courtes (entre 4 et 10 unités), mais
qu'il existe également une présence de séquences plus longues.

**Option 2 (Slightly Less Formal):**

Voici une description de l'image que vous avez envoyée :

**Type de graphique :**

L'image montre un histogramme. *( “Montre”  is a simpler verb than
 “présente” )

**Données représentées :**

L'histogramme illustre la distribution des "longueurs de séquences"
(probablement la longueur des séquences d'entités) provenant d'une
banque de phrases financières contenant 5000 phrases. Les longueurs de
séquences sont représentées avant l'élagage. *( “Contenant”  is a
simpler alternative to  “composée de” )

**Axes :**
```

(continues on next page)

(continued from previous page)

```
*   **Axe horizontal :** Représente la "Longueur de séquence," allant
de 2 à 12, avec des valeurs entières. *(Omitting "des abscisses" makes
it less formal)*
*   **Axe vertical :** Représente la "Densité," allant de 0 à 0,20.
*(Omitting "des ordonnées" makes it less formal)*

**Observations principales :**

*   La distribution est un peu asymétrique, avec un pic autour des
longueurs de séquence de 6 et 8. *( “Un peu”  is a simpler way to say
 “somewhat” )
*   On observe un nombre important de séquences de longueurs comprises
entre 4 et 10.  *( “On observe”  is a more conversational way to say
 “there are” )
*   La densité diminue à mesure que la longueur de la séquence
s'éloigne des valeurs maximales. *( “Valeurs maximales”  is a simpler
alternative to  “valeurs de pic” )

**Impression générale :**

L'histogramme donne un aperçu visuel de la longueur des séquences
d'entités, avant tout élagage ou filtrage. Il laisse entendre que la
plupart des séquences sont relativement courtes (entre 4 et 10
unités), mais qu'il existe aussi des séquences plus longues. *( “Laisse
entendre”  is a more conversational way to say  “suggests” )

**Key Considerations & Notes:**

*   **"Pruning":**  "Élagage" is the most accurate technical term for
pruning in this context.
*   **"Skewed":** "Asymétrique" is the best translation for "skewed"
in a statistical context.
*   **Formality:** Option 2 is slightly less formal and might be more
suitable for a broader audience.
*   **Context:** The best option will depend on the intended audience
and the overall tone of the document.
*   **"Sequence Length":**  I'm keeping "longueur de séquence" as it's
the most direct translation.  You could potentially rephrase it
depending on the specific terminology used in your field.



To help me refine the translation further, could you tell me:

*   Who is the intended audience for this translation?
*   What is the overall tone of the document?
```

References:

Malo, P., Sinha, A., Takala, P., Korhonen, P. and Wallenius, J. (2014): “Good debt or bad debt: Detecting semantic orientations in economic texts.” Journal of the American Society for Information Science and Technology.

Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Brian Ichter, Fei Xia, Ed Chi, Quoc Le and Denny Zhou, 2023, Chain-of-Thought Prompting Elicits Reasoning in Large Language Models, https://arxiv.org/abs/2201.11903

Greg Durrett, 2023, “CS388 Natural Language Processing course materisl”, retrieved from https://www.cs.utexas.edu/~gdurrett/courses/online-course/materials.html

https://llama.meta.com/docs/how-to-guides/prompting/

https://www.promptingguide.ai/techniques/cot

https://medium.com/age-of-awareness/chain-of-thought-in-ai-7f45c3d2c12a

CHAPTER

THIRTYEIGHT

LLM AGENTS

The true sign of intelligence is not knowledge but imagination - Albert Einstein

We introduce the capabilities of LLM agents using chatbots with memory, retrieval-augmented generation (RAG), and multi-agent collaboration. Employing Microsoft' s Phi-4-mini model, a ChromaDB vector store and sentence embedding models via Ollama, we demonstrate how LLMs can be enhanced to improve factual accuracy and support long-context reasoning. The final application centers around measuring the value of corporate philanthropy, illustrating how agents can ground their reasoning in retrieved knowledge and engage in multi-role dialogue to develop actionable plans.

```
# By: Terence Lim, 2020-2025 (terence-lim.github.io)
import torch
from transformers import AutoModelForCausalLM, AutoProcessor, GenerationConfig,␣
↪logging
import io
import soundfile
import matplotlib.pyplot as plt
from PIL import Image
from tqdm import tqdm
from pprint import pprint
import warnings
import textwrap
def wrap(text):
    """Helper to wrap text for pretty printing"""
    return "\n".join([textwrap.fill(s, width=80) for s in text.split('\n')])
torch.random.manual_seed(0)
logging.set_verbosity_error()
# display gpu and memory
gpu_stats = torch.cuda.get_device_properties(0)
max_memory = round(gpu_stats.total_memory / 1024 / 1024 / 1024, 3)
print(f"GPU = {gpu_stats.name}. Max memory = {max_memory} GB.")
```

```
GPU = NVIDIA GeForce RTX 3080 Laptop GPU. Max memory = 15.739 GB.
```

38.1 Microsoft Phi-4 model

The Microsoft Phi-4 family is designed for compactness and strong reasoning across modalities. Released in February 2024, Phi-4-multimodal is a 5.6B parameter model which processes audio, vision, and language simultaneously within the same representation space. Its capabilities include:

- Instruction following: Instruct-tuned for multi-turn conversation, summarization, Q&A
- Math and code reasoning: Fine-tuned on mathematical and coding data
- Multilingual: Supports multiple languages with cross-lingual reasoning
- Vision: integrates a visual encoder that converts image inputs into embeddings, supporting tasks such as
  - OCR (Optical Character Recognition)
  - chart and table understanding
  - image-based reasoning (e.g., figures, diagrams)
- Audio Capabilities: a specialized audio encoder for speech inputs, supporting:
  - Automatic Speech Recognition (ASR)
  - Multilingual speech translation

https://azure.microsoft.com/en-us/blog/empowering-innovation-the-next-generation-of-the-phi-family/

```
# load and configures Phi-4-mini-instruct using HuggingFace
model_path = "microsoft/Phi-4-mini-instruct"
model_path = "microsoft/Phi-4-multimodal-instruct"

with warnings.catch_warnings():  # ignore small sample warnings
    warnings.simplefilter("ignore")
    processor = AutoProcessor.from_pretrained(model_path, trust_remote_code=True)

    print(processor.tokenizer)

    model = AutoModelForCausalLM.from_pretrained(
        model_path,
        trust_remote_code=True,
        torch_dtype='auto',
        _attn_implementation='flash_attention_2',
        #    _attn_implementation='eager',      # 'flash_attention_2',
    ).cuda()
```

```
GPT2TokenizerFast(name_or_path='microsoft/Phi-4-multimodal-instruct', vocab_
↪size=200019, model_max_length=131072, is_fast=True, padding_side='right',␣
↪truncation_side='right', special_tokens={'bos_token': '<|endoftext|>', 'eos_token
↪': '<|endoftext|>', 'unk_token': '<|endoftext|>', 'pad_token': '<|endoftext|>'},␣
↪clean_up_tokenization_spaces=False, added_tokens_decoder={
        199999: AddedToken("<|endoftext|>", rstrip=False, lstrip=False, single_
↪word=False, normalized=False, special=True),
        200010: AddedToken("<|endoftext10|>", rstrip=False, lstrip=False, single_
↪word=False, normalized=False, special=True),
        200011: AddedToken("<|endoftext11|>", rstrip=False, lstrip=False, single_
↪word=False, normalized=False, special=True),
        200018: AddedToken("<|endofprompt|>", rstrip=False, lstrip=False, single_
↪word=False, normalized=False, special=True),
        200019: AddedToken("<|assistant|>", rstrip=True, lstrip=False, single_
```

(continues on next page)

(continued from previous page)

```
↪word=False, normalized=False, special=True),
        200020: AddedToken("<|end|>", rstrip=True, lstrip=False, single_word=False,
↪ normalized=False, special=True),
        200021: AddedToken("<|user|>", rstrip=True, lstrip=False, single_
↪word=False, normalized=False, special=True),
        200022: AddedToken("<|system|>", rstrip=True, lstrip=False, single_
↪word=False, normalized=False, special=True),
        200023: AddedToken("<|tool|>", rstrip=True, lstrip=False, single_
↪word=False, normalized=False, special=False),
        200024: AddedToken("<|/tool|>", rstrip=True, lstrip=False, single_
↪word=False, normalized=False, special=False),
        200025: AddedToken("<|tool_call|>", rstrip=True, lstrip=False, single_
↪word=False, normalized=False, special=False),
        200026: AddedToken("<|/tool_call|>", rstrip=True, lstrip=False, single_
↪word=False, normalized=False, special=False),
        200027: AddedToken("<|tool_response|>", rstrip=True, lstrip=False, single_
↪word=False, normalized=False, special=False),
        200028: AddedToken("<|tag|>", rstrip=True, lstrip=False, single_word=False,
↪ normalized=False, special=True),
}
)
```

```
Loading checkpoint shards:   0%|          | 0/3 [00:00<?, ?it/s]
```

Memory usage

```
def cuda_memory():
    """Show GPU memory free"""
    if torch.cuda.is_available():
        device = torch.device('cuda')
        total_memory = torch.cuda.get_device_properties(device).total_memory
        reserved_memory = torch.cuda.memory_reserved(device)
        allocated_memory = torch.cuda.memory_allocated(device)
        free_memory = total_memory - reserved_memory
        print(f"Total memory: {total_memory / (1024**3):.2f} GB")
        print(f"Reserved memory: {reserved_memory / (1024**3):.2f} GB")
        print(f"Allocated memory: {allocated_memory / (1024**3):.2f} GB")
        print(f"Free memory: {free_memory / (1024**3):.2f} GB")

cuda_memory()
```

```
Total memory: 15.74 GB
Reserved memory: 10.45 GB
Allocated memory: 10.42 GB
Free memory: 5.29 GB
```

Generate model response

```
generation_args = {
    "max_new_tokens": 512,
    #"return_full_text": False,
    "temperature": 0.6,    # nonzero so that chat responses can vary slightly
    "do_sample": True,     # False,
}
```

(continues on next page)

(continued from previous page)

```
generation_config = GenerationConfig.from_pretrained(model_path,
                                                     'generation_config.json')
```

Helpers to format content to prompt for response

```
def create_content(query, image=None, audio=None):
    """Formats a message content string"""
    image_prompt = '' if image is None else '<|image_1|>'
    audio_prompt = '' if audio is None else '<|audio_1|>'
    prompt = f'{image_prompt}{audio_prompt}{query}'
    return prompt

def create_prompt(query, image=None, audio=None):
    """Generate a single prompt for inference response"""
    user_prompt = '<|user|>'
    assistant_prompt = '<|assistant|>'
    prompt_suffix = '<|end|>'
    content = create_content(query, image=image, audio=audio)
    prompt = f'{user_prompt}{content}{prompt_suffix}{assistant_prompt}'
    return prompt

def pipe(query, image=None, audio=None, verbose=False, **kwargs):
    """Pipeline to format and send query, and generate and decode response"""
    if isinstance(query, list):  # query input is given as a list of messages
        prompt = processor.tokenizer.apply_chat_template(query,
                                                          tokenize=False,
                                                          add_generation_prompt=True)
        # remove last <|endoftext|> if it is there, which is used for training, not␣
↪inference.
        # For training, make sure to add <|endoftext|> in the end.
        if prompt.endswith('<|endoftext|>'):
            prompt = prompt.rstrip('<|endoftext|>')
    elif isinstance(query, str): # query input is given as a string
        prompt = create_prompt(query, image=image, audio=audio)
    else:
        raise Exception('Invalid prompt format, must be str or list of messages')
    if verbose:
        print(prompt)
    inputs = processor(prompt, images=image, audios=audio, return_tensors='pt').to(
↪'cuda:0')

    with warnings.catch_warnings():  # ignore small sample warnings                ␣
↪
        warnings.simplefilter("ignore")

        generate_ids = model.generate(
            **inputs,
            generation_config=generation_config,
            # https://huggingface.co/microsoft/Phi-4-multimodal-instruct/discussions/
↪46
            num_logits_to_keep=1,
            **kwargs,
    )
    generate_ids = generate_ids[:, inputs['input_ids'].shape[1] :]
    response = processor.batch_decode(
        generate_ids, skip_special_tokens=True, clean_up_tokenization_spaces=False)[0]
```

(continues on next page)

(continued from previous page)

```
    return response
```

38.2 Chatbot

A chatbot simulates ongoing dialogue between a user and an LLM. By appending previous user inputs and model responses into the prompt, the chatbot maintains conversational memory. This enables it to handle follow-up questions, build on earlier answers, and maintain topic continuity over multiple turns. During a chat session, the prompt is iteratively expanded to include each previous exchange. This accumulated memory allows the model to generate responses that take prior context into account.

Simulate user queries

```
# Simulate a sequence of user queries to be processed
sequence_user_queries = [
    "What are the challenges to measuring the value of corporate philanthropy?",
    "Please suggest some solutions.",
    "Thank you, please concisely summarize into an action plan."
]
```

38.2.1 Memory

During a chat session, the prompt is iteratively expanded to include each previous exchange. This accumulated memory allows the model to generate responses that take prior context into account. The final response is displayed, along with the complete prompt history used in that turn—demonstrating how memory accumulation influences generation.

```
# Prompt grows over time with user and assistant turns.
memory = []    # memory of past AI-assistant turns
for turn in range(len(sequence_user_queries)):

    # Add initial user query
    query = sequence_user_queries[0]
    messages = [{"role": "user", "content": query}]

    # 3. Loop through past turns of AI response and user prompts
    for response, query in zip(memory, sequence_user_queries[1:len(memory)+1]):

        # Add AI assistant's response
        messages.append({"role": "assistant", "content": response})

        # Add user's next query
        messages.append({"role": "user", "content": query})

    # 4. Generate response and save to memory
    output = pipe(messages, **generation_args)
    memory.append(output)
    print(f'----------- {turn=} -----------')
    print('QUERY:', query)
    print('RESPONSE:')
    print(wrap(memory[-1]))
    print()
```

```
 33%|███▎      | 1/3 [00:21<00:42, 21.09s/it]
```

```
----------- turn=0 -----------
QUERY: What are the challenges to measuring the value of corporate philanthropy?
RESPONSE:
Measuring the value of corporate philanthropy presents several challenges,
including:

1. Lack of standardized metrics: There is no universally accepted way to measure
the impact of corporate philanthropy. Different organizations may use different
metrics, making it difficult to compare and assess the value of their
philanthropic efforts.

2. Difficulty in quantifying social impact: Corporate philanthropy often aims to
address complex social issues, making it challenging to quantify the impact.
Social impact can be long-term and may not be immediately visible, requiring a
long-term perspective to assess the value of corporate philanthropy.

3. Lack of transparency: Corporate philanthropy can be seen as a way for
companies to boost their public image, rather than a genuine effort to make a
positive impact. This lack of transparency can make it difficult to assess the
true value of corporate philanthropy.

4. Difficulty in attributing outcomes: It can be challenging to attribute
specific outcomes to corporate philanthropy, as social issues are often
influenced by multiple factors. This makes it difficult to determine the direct
impact of corporate philanthropy on a particular issue.

5. Conflicts of interest: Corporate philanthropy can sometimes be driven by a
company's strategic interests, rather than a genuine desire to make a positive
impact. This can create conflicts of interest and make it difficult to assess
the true value of corporate philanthropy.

6. Lack of accountability: Without clear guidelines and accountability
mechanisms, it can be challenging to ensure that corporate philanthropy is
effective and achieves its intended goals. This can result in wasted resources
and missed opportunities to make a positive impact.

7. Limited resources: Corporate philanthropy is often limited by a company's
financial resources. This can make it difficult to measure the value of
corporate philanthropy, as it may not be possible to address all the social
issues that a company wants to support.

Overall, measuring the value of corporate philanthropy requires a comprehensive
approach that takes into account the complex nature of social issues and the
various factors that influence corporate philanthropy. It is essential for
companies to be transparent, accountable, and focused on creating a positive
social impact.
```

```
 67%|██████▋   | 2/3 [00:43<00:21, 21.72s/it]
```

```
----------- turn=1 -----------
QUERY: Please suggest some solutions.
RESPONSE:
To address the challenges of measuring the value of corporate philanthropy, the
```

(continues on next page)

(continued from previous page)

```
following solutions can be considered:

1. Develop standardized metrics: Establishing universally accepted metrics and
guidelines for measuring the impact of corporate philanthropy can help
organizations compare and assess their efforts more effectively. These metrics
could include both quantitative and qualitative measures, such as the number of
people served, the amount of money donated, and the long-term impact on
communities.

2. Focus on social impact: Companies should prioritize measuring the social
impact of their philanthropic efforts, rather than just financial outcomes. This
can be done by setting clear goals and objectives, tracking progress over time,
and using both quantitative and qualitative data to assess the impact on the
communities and individuals they serve.

3. Increase transparency: Companies should be transparent about their
philanthropic efforts, including the reasons behind their giving, the selection
process for grants, and the outcomes achieved. This can help build trust with
stakeholders and ensure that corporate philanthropy is genuinely focused on
making a positive impact.

4. Improve attribution methods: To better attribute outcomes to corporate
philanthropy, companies can use a combination of data analysis, case studies,
and stakeholder feedback. This can help identify the direct and indirect impact
of their efforts and better understand how their contributions are making a
difference.

5. Address conflicts of interest: Companies should be mindful of potential
conflicts of interest and ensure that their philanthropic efforts are not solely
driven by a desire to boost their public image. This can be achieved by
involving external stakeholders, such as nonprofit partners or community
leaders, in the decision-making process.

6. Establish accountability mechanisms: Companies should establish clear
accountability mechanisms, such as third-party evaluations or independent
audits, to ensure that their philanthropic efforts are effective and achieving
their intended goals. This can help build trust with stakeholders and provide a
basis for continuous improvement.

7. Leverage partnerships: Companies can collaborate with nonprofit
organizations, community groups, and other stakeholders to maximize the impact
of their philanthropic contributions. By leveraging the expertise and resources
of others, companies can address complex social issues more effectively and
ensure that their efforts are making a meaningful difference.

By implementing these solutions, companies can improve their ability to measure
the value of corporate philanthropy and demonstrate the positive social impact
of their efforts.
```

```
100%|██████████| 3/3 [00:59<00:00, 19.80s/it]
```

```
----------- turn=2 -----------
QUERY: Thank you, please concisely summarize into an action plan.
RESPONSE:
Action Plan for Measuring the Value of Corporate Philanthropy:
```

(continues on next page)

(continued from previous page)

```
1. Develop standardized metrics:
   - Establish universal guidelines and metrics for measuring the impact of
corporate philanthropy.
   - Include both quantitative and qualitative measures, such as the number of
people served, amount of money donated, and long-term community impact.

2. Focus on social impact:
   - Set clear goals and objectives for philanthropic efforts.
   - Track progress over time using both quantitative and qualitative data.
   - Assess the impact on communities and individuals served.

3. Increase transparency:
   - Be transparent about philanthropic efforts, including selection processes,
outcomes, and reasons behind giving.
   - Share information with stakeholders to build trust and demonstrate genuine
impact.

4. Improve attribution methods:
   - Use data analysis, case studies, and stakeholder feedback to better
attribute outcomes to corporate philanthropy.
   - Identify direct and indirect impacts of philanthropic efforts.

5. Address conflicts of interest:
   - Involve external stakeholders, such as nonprofit partners or community
leaders, in decision-making processes.
   - Ensure philanthropic efforts are not solely driven by a desire to boost
public image.

6. Establish accountability mechanisms:
   - Implement third-party evaluations, independent audits, or other
accountability mechanisms to ensure effectiveness and achievement of goals.
   - Use findings to continuously improve philanthropic efforts.

7. Leverage partnerships:
   - Collaborate with nonprofit organizations, community groups, and other
stakeholders to maximize impact.
   - Leverage their expertise and resources to address complex social issues
more effectively.

By following this action plan, companies can improve their ability to measure
the value of corporate philanthropy and demonstrate the positive social impact
of their efforts.
```

Show the final response from the last turn of the Chatbot

```
print('FINAL ANSWER:')
print(wrap(output))
```

```
FINAL ANSWER:
Action Plan for Measuring the Value of Corporate Philanthropy:
```

(continues on next page)

(continued from previous page)

```
1. Develop standardized metrics:
   - Establish universal guidelines and metrics for measuring the impact of
corporate philanthropy.
   - Include both quantitative and qualitative measures, such as the number of
people served, amount of money donated, and long-term community impact.

2. Focus on social impact:
   - Set clear goals and objectives for philanthropic efforts.
   - Track progress over time using both quantitative and qualitative data.
   - Assess the impact on communities and individuals served.

3. Increase transparency:
   - Be transparent about philanthropic efforts, including selection processes,
outcomes, and reasons behind giving.
   - Share information with stakeholders to build trust and demonstrate genuine
impact.

4. Improve attribution methods:
   - Use data analysis, case studies, and stakeholder feedback to better
attribute outcomes to corporate philanthropy.
   - Identify direct and indirect impacts of philanthropic efforts.

5. Address conflicts of interest:
   - Involve external stakeholders, such as nonprofit partners or community
leaders, in decision-making processes.
   - Ensure philanthropic efforts are not solely driven by a desire to boost
public image.

6. Establish accountability mechanisms:
   - Implement third-party evaluations, independent audits, or other
accountability mechanisms to ensure effectiveness and achievement of goals.
   - Use findings to continuously improve philanthropic efforts.

7. Leverage partnerships:
   - Collaborate with nonprofit organizations, community groups, and other
stakeholders to maximize impact.
   - Leverage their expertise and resources to address complex social issues
more effectively.

By following this action plan, companies can improve their ability to measure
the value of corporate philanthropy and demonstrate the positive social impact
of their efforts.
```

38.3 Retrieval-augmented generation

One major challenge in standard LLMs is **hallucination**, where the model produces text that sounds believable but is actually false or unsupported by facts. Because LLMs are trained to model word sequences rather than factual accuracy, they can sometimes generate convincing yet misleading statements—especially when no clear answer exists in their training data. For example, Hicks et al. (2024) argue that LLMs are "bullshit machines" that prioritize fluent language over truth. Similarly, Mahowald et al. (2023) caution against assuming that skill with language equates to actual reasoning or understanding.

RAG addresses this issue by anchoring the model's responses in retrieved documents, making the output more factually reliable. In a common implementation known as prompt-based in-context retrieval, the system first retrieves information from an external knowledge base, and then it generates a response based on that retrieved content. The LLM is prompted with the relevant document and instructed to answer solely based on that information, similar to reading a reference article before answering a question, allowing for better factual accuracy and handling of long-context tasks. RAG is increasingly used in several practical domains:

- Search-Augmented LLMs improve accuracy in fact-checking and open-domain Q&A tasks.
- Enterprise Applications enable better handling of legal, financial, and medical documents.
- Personalized AI Assistants can use retrieval to support domain-specific queries using private or proprietary knowledge bases (e.g., for customer service).

For evaluating **closed-domain question-answering (QA)** LLMs, benchmarks such as SQuAD (Stanford Question Answering Dataset) HotpotQA are used, focusing on a system's ability to answer questions within specific documents, including tasks like complex document comprehension and multi-hop reasoning. This task differs from **open-domain QA**, where LLMs are expected to draw knowledge from a vast, unrestricted knowledge base.

More advanced data structures have also been employed for RAG, such as RAPTOR by Sarthi et al (2024), which organizes data and recursive summaries in a tree structure, integrating information across lengthy documents for better performance on complex, multi-step reasoning tasks

The LangChain framework offers flexible tools for loading and preprocessing input documents of many formats. A textbook is read and divided into chunks of roughly 1,000 characters with some overlap. These chunks provide manageable input sizes for retrieval while maintaining continuity. The text itself outlines how companies can measure philanthropic value through social, business, and investor-oriented metrics, and forms the knowledge base for grounding agent responses.

```
# LangChain to loads and split documents into chunks for retrieval
from langchain_community.document_loaders import TextLoader,␣
↪UnstructuredMarkdownLoader
from langchain_text_splitters import RecursiveCharacterTextSplitter
```

```
# Loads markdown file (MVCP.md) and split into overlapping chunks
loader = TextLoader('assets/MVCP.md')
document = loader.load()
text_splitter = RecursiveCharacterTextSplitter(chunk_size=1000, chunk_overlap=200)
chunked_documents = text_splitter.split_documents(document)
```

```
print('Number of chunks:', len(chunked_documents))
print(wrap(str(chunked_documents[:2])))
```

```
Number of chunks: 196
[Document(metadata={'source': 'assets/MVCP.md'}, page_content='MEASURING THE
VALUE OF CORPORATE PHILANTHROPY: SOCIAL IMPACT, BUSINESS BENEFITS, AND INVESTOR
RETURNS\nby\nTerence Lim, Ph.D.\nReport Author and Manager, Standards and
Measurement,\nCommittee Encouraging Corporate Philanthropy\n(through the
2008-2009 Goldman Sachs Public Service Program)\n\nHow to measure the value and
results of corporate philanthropy remains\none of corporate giving
professionals' greatest challenges. Social and\nbusiness benefits are often
long-term or intangible, which make\nsystematic measurement complex. And yet:
Corporate philanthropy faces\nincreasing pressures to show it is as
strategic,cost-effective, and value-enhancing\nas possible. The industry faces a
critical need to assess current practices and\nmeasurement trends, clarify the
demands practitioners face for impact evidence,\nand identify the most promising
steps forward in order to make progress on these\nchallenges.'),
Document(metadata={'source': 'assets/MVCP.md'}, page_content='This report aims
to meet that need, by providing the corporate\nphilanthropic community with a
```

(continues on next page)

(continued from previous page)

```
review of recent measurement studies, models,\nand evidence drawn from
complementary business disciplines as well as the social\nsector. Rather than
present an other compendium of narrative accounts and case\nstudies, we endeavor
to generalize the most valuable concepts and to recognize\nthe strengths and
limitations of various measurement approaches. In conjunction\nwith the
annotated references that follow, the analysis herein should provide
an\nexcellent starting point for companies wishing to adapt current
methodologies in\nthe field to their own corporate giving programs.\nTo realize
meaningful benefits, philanthropy cannot be treated as just\nanother "check in
the box," but rather must be executed no less professionally,\nproactively, and
strategically than other core business activities. Our hope is\nthat this work
will enlighten giving professionals, CEOs, and the investor')]
```

38.3.1 ChromaDB vector database

ChromaDB is an open-source vector store used to store and retrieve text embeddings. It enables fast and efficient semantic search, allowing the system to find the most relevant document chunks based on a given query.

For compatibility, ChromaDB requires SQLite version 3.35 or higher. If the system uses an older version, apply this workaround patch: Install pysqlite3-binary, then enter the following three lines in sqlite_version.py to swap the packages.

```python
# kludge to hack ChromaDB to use lower version of SQLite
with open('sqlite_version.py', 'w') as f:
    f.write('''__import__('pysqlite3')
import sys
sys.modules['sqlite3'] = sys.modules.pop('pysqlite3')
    '''.strip())
import sqlite_version
```

```python
# ChromaDB vector store to store and retrieve document embeddings.
import chromadb
client = chromadb.Client()
try:
    client.delete_collection(name="docs")
except:
    pass
#collection = client.get_or_create_collection(name="docs")
collection = client.create_collection(name="docs")
```

The sentence embeddings for both documents and queries are generated using the `mxbai-embed-large` model served via **Ollama**. This embedding model, which reached state-of-the-art performance in April 2024, was trained on over 700 million high-quality sentence pairs and fine-tuned on 30 million triplets. These embeddings are stored in ChromaDB to support rapid document retrieval.

```python
# Ollama for generating sentence embeddings using mxbai-embed-large model
# !ollama pull mxbai-embed-large
import ollama

# store chunk embeddings in a vector database
for i, d in enumerate(chunked_documents):
    response = ollama.embeddings(model="mxbai-embed-large", prompt=d.page_content)
    embedding = response["embedding"]
    collection.add(
```

(continues on next page)

(continued from previous page)

```
        ids=[str(i)],
        embeddings=[embedding],
        documents=[d.page_content],
    )
```

38.3.2 Retrieval

The retrieval pipeline consists of four main steps:

- A user query is written in natural language.
- The query is embedded using the same model as the document chunks.
- The top 5 most semantically similar chunks are retrieved from the ChromaDB vector store.
- A new prompt is constructed by combining the user' s question with the retrieved content for generation.

```
# A sample question prompt
query = "What are the challenges to measuring the value of corporate philanthropy?"
```

Embed the prompt query

```
# generate an embedding for the query
response = ollama.embeddings(
  prompt=query,
  model="mxbai-embed-large"
)
len(response['embedding'])    # vector length
```

```
1024
```

Retrieve the 5 most similar document chunks from the vector database

```
# ses ChromaDB to find top 5 most similar chunks
results = collection.query(
  query_embeddings=[response["embedding"]],
    n_results=5,
)
for data in results['documents'][0]:
  print(textwrap.fill(data))
```

```
MEASURING THE VALUE OF CORPORATE PHILANTHROPY: SOCIAL IMPACT, BUSINESS
BENEFITS, AND INVESTOR RETURNS by Terence Lim, Ph.D. Report Author and
Manager, Standards and Measurement, Committee Encouraging Corporate
Philanthropy (through the 2008-2009 Goldman Sachs Public Service
Program)  How to measure the value and results of corporate
philanthropy remains one of corporate giving professionals' greatest
challenges. Social and business benefits are often long-term or
intangible, which make systematic measurement complex. And yet:
Corporate philanthropy faces increasing pressures to show it is as
strategic,cost-effective, and value-enhancing as possible. The
industry faces a critical need to assess current practices and
measurement trends, clarify the demands practitioners face for impact
```

(continues on next page)

(continued from previous page)

```
evidence, and identify the most promising steps forward in order to
make progress on these challenges.
detailed insights into the related measurement process which can help
demonstrate understanding of what drives long term business success
quality of management and superior potential to create financial
value. The value of corporate philanthropy is measurable; as with many
elements of business, however, it cannot always be measured as
precisely as we would like. What gets measured gets managed goes the
old adage; indeed measurement plays a crucial role in enabling
companies to reach their full potential both philanthropically and as
more successful and sustainable enterprises overall.
This report aims to meet that need, by providing the corporate
philanthropic community with a review of recent measurement studies,
models, and evidence drawn from complementary business disciplines as
well as the social sector. Rather than present an other compendium of
narrative accounts and case studies, we endeavor to generalize the
most valuable concepts and to recognize the strengths and limitations
of various measurement approaches. In conjunction with the annotated
references that follow, the analysis herein should provide an
excellent starting point for companies wishing to adapt current
methodologies in the field to their own corporate giving programs. To
realize meaningful benefits, philanthropy cannot be treated as just
another "check in the box," but rather must be executed no less
professionally, proactively, and strategically than other core
business activities. Our hope is that this work will enlighten giving
professionals, CEOs, and the investor
Corporate philanthropy is as vital as ever to business and society but
it faces steep pressures to demonstrate that it is also cost effective
and aligned with corporate needs. Indeed many corporate giving
professionals cite measurement as their primary management challenge.
Social and business benefits are often long-term, intangible or both
and a systematic measurement of these results can be complex. Social
change takes time. The missions and intervention strategies involved
are diverse. For these reasons, the field of corporate philanthropy
has been unable to determine a shared definition or method of
measurement for social impact. Similarly, the financial value of
enhancing intangibles such as a company's reputational and human
capital cannot be measured directly and may not be converted into
tangible bottom line profits in the near term. Corporate givers and
grant recipients often useless formal anecdotal methods to convey
impact. While stories
### Summary   The attractiveness of these ROI methods for calculating
corporate philanthropy's social returns is in bringing businesslike
quantitative frameworks to evaluating and comparing the effectiveness
of diverse social programs, and aggregating their social impact.
However these sophisticated methodologies place heavy demands on data
collection assumptions and value judgments underlying the analysis.
Funders must assemble data and calculations on the program's monetary
benefits and make subjective judgments on the relative value of
different types of social changes. Corporate funders need to be
knowledgeable and thoughtful about these limitations and typically
should not rely solely on ROI when evaluating grants. Proponents of
these methods note that the benefits of ROI analysis lie more in
encouraging funders to lay bare the assumptions and trade-offs that
may already be involved in their grant making decisions.
```

38.3.3 Generation

Construct the prompt combining the query and supporting context retrieved. Then send to the LLM, which generates a grounded response.

```
# assembles relevant chunks into a prompt alongside the original question
prompt = f"""
Only use relevant information in the following text delimited by triple quotes:

'''{data}.'''

Respond to this prompt: {query}."""
```

```
# send a conversation-style chat-template prompt incorporating the retrieved text
messages = [
    {"role": "user", "content": prompt},
]
output = pipe(messages, **generation_args)
print('QUERY:', query)
print('RESPONSE:')
print(wrap(output))
```

```
QUERY: What are the challenges to measuring the value of corporate philanthropy?
RESPONSE:
The challenges to measuring the value of corporate philanthropy include:

1. Social and business benefits are often long-term or intangible, making
systematic measurement complex.
2. Corporate philanthropy faces increasing pressures to show it is as strategic,
cost-effective, and value-enhancing as possible.
3. The industry faces a critical need to assess current practices and
measurement trends.
4. There is a need to clarify the demands practitioners face for impact
evidence.
5. Identifying the most promising steps forward to make progress on these
challenges.
6. Demonstrating understanding of what drives long-term business success and
quality of management.
7. Measuring the financial value of enhancing intangibles such as a company's
reputational and human capital can be complex and may not be directly
convertible into tangible bottom-line profits in the near term.
8. The measurement of social impact lacks a shared definition or method, and
social change takes time.
9. Missions and intervention strategies in corporate philanthropy are diverse.
10. The use of sophisticated ROI methodologies for evaluating and comparing the
effectiveness of diverse social programs places heavy demands on data collection
assumptions and value judgments underlying the analysis.
```

38.4 Multi-agents

In multi-agent workflows, multiple LLM agents play different roles and collaborate to accomplish a shared objective. In this demonstration, three role-playing agents work together to simulate human-like decision-making processes:

- A Chief Executive Officer (CEO) asks strategic questions about measuring philanthropic value.
- A Chief Giving Officer (CGO) synthesizes responses into an evolving plan.
- An AI Assistant (nicknamed CorGi) uses RAG to answer the CEO's questions using only retrieved text.

The agents interact with each other over several turns, collaboratively building and refining a plan grounded in the corporate philanthropy document.

38.4.1 Tool calling

Traditional LLMs are limited in that they cannot retrieve live data or execute real-world functions unless explicitly programmed. Without tools, they may also make computational errors or respond inconsistently to ambiguous prompts. Recent advances focus on enabling tool use, where LLMs can:

- Make function calls.
- Use APIs to gather real-time data.
- Perform calculations or database lookups.

The main approaches include:

- Allow the model to generate function calls instead of plain text.
- Zero-Shot Tool Use: Models learn to use tools on-the-fly via structured prompts.
- Fine-Tuned Tool Use: Models are trained to recognize and call specific functions during generation.

AI Assistant agent:

The AI Research Assistant agent responds to queries by leveraging RAG. Its prompt instructs it to only use retrieved documents to answer and to reply "I don't know" if the information isn't found in the text. This strict constraint helps minimize hallucination and ensures that all responses are grounded in the knowledge base.

```
# AI assistant agent answers using only retrieved text chunks
def rag_agent(query, n_results=5):
    """Role of a RAG-based question-answering agent"""
    response = ollama.embeddings(prompt=query, model="mxbai-embed-large")

    results = collection.query(query_embeddings=[response["embedding"]],
                               n_results=n_results)
    text = "\n".join(results['documents'][0])

    rag_prompt=f"""
You are a helpful AI research assistant.
Use only the information in the text to succintly answer the question.
Reply "I don't know" if the information is not found in the text.
Text: {text}.
Question: {query}"""

    # generate a response combining the prompt and data
    output = pipe(rag_prompt, **generation_args)
    return output, results['ids'][0]
```

Reply to a query using grounded knowledge in the document

```
pprint(rag_agent("What is corporate philanthropy?"))
```

```
('Corporate philanthropy refers to the charitable giving and activities of a '
 "company, often aimed at enhancing the company's reputation, addressing "
 'employee concerns, and potentially contributing to business value through '
 'innovation, market knowledge, and development of new technologies.',
 ['102', '1', '87', '140', '142'])
```

Query about a concept not present in the document

```
pprint(rag_agent("What is corporate greed?"))
```

```
('The text does not provide information about what corporate greed is.',
 ['137', '140', '136', '130', '117'])
```

38.4.2 Role playing

Chief Giving Officer agent:

This agent summarizes CorGi's outputs into a coherent strategic plan that can be reviewed by leadership.

```
# CGO (Chief Giving Officer) agent summarizes answers into a growing plan
def cgo_agent(text):
    """Role of an information-summarizing Chief Giving Officer"""
    cgo_prompt = f"""
You are the chief giving officer of a company.
You want to describe an action plan for measuring the value
of your company's corporate philanthropy program.
Use bullet points to summarize the plan using only the
relevant and distinct information in the text.
Text: {text}""".strip()
    output = pipe(cgo_prompt, **generation_args)
    return output
```

Chief Executive Officer agent:

This agent reads the current version of the plan and generates new questions to improve or expand it.

```
# CEO (Chief Executive Officer) agent asks new questions to improve the corporate␣
↪giving plan
def ceo_agent(text, verbose=False):
    """Role as an inquisitive Chief Executive Officer"""
    ceo_prompt = f"""
You are the chief executive officer of a company.
You want to know about the plan for measuring the value
of your company's corporate philanthropy program.
Please ask a simple and general question for an idea to
improve the your company's plan that is different than the text.
Text: {text}""".strip()
    output = pipe(ceo_prompt, verbose=verbose, **generation_args)
    return output
```

Over multiple turns, this role-playing setup allows the system to iteratively build a more comprehensive and data-backed plan.

```
# iterate over 10 turns to co-develop a full plan
memory = '* Measure the amount of monetary donations.'
docs = []
for turn in range(10):
    print(f"Turn {turn+1}...")
    question = ceo_agent(memory)
    print('CEO >>>', wrap(question))
    print('\n------------------------------\n')
    answer, doc = rag_agent(question)
    docs.extend(doc)    # for diagnostics: track the RAG chunks retrieved
    print('Corgi-AI >>>', wrap(answer))
    print('\n------------------------------\n')
    memory = cgo_agent("\n".join([memory, answer]))
    print('CGO >>>', wrap(memory))
    print('\n------------------------------')
```

```
Turn 1...
CEO >>> How might we also consider non-monetary contributions, such as volunteer␣
↪hours
or in-kind donations, in evaluating the overall impact of our corporate
philanthropy program?

------------------------------

Corgi-AI >>> I don't know

------------------------------

CGO >>> - Measure the amount of monetary donations.
- Track the number of volunteer hours contributed.
- Assess the impact of the donations through beneficiary feedback.
- Evaluate the visibility and reach of the philanthropy program through media
coverage and social media engagement.
- Analyze the correlation between philanthropic activities and company
reputation.
- Compare the company’ s philanthropic efforts with industry standards.
- Review the alignment of the philanthropy program with the company’ s mission
and values.
- Monitor the long-term outcomes of the philanthropic initiatives.
- Collect data on the satisfaction levels of the stakeholders involved in the
program.
- Assess the effectiveness of the philanthropic program in achieving the
company’ s social responsibility goals.
- Evaluate the cost-effectiveness of the philanthropy program in relation to the
benefits received.

------------------------------
Turn 2...
CEO >>> How can we incorporate community engagement and local stakeholder input␣
↪into the
evaluation of our company's philanthropy program?

------------------------------
```

(continues on next page)

(continued from previous page)

```
Corgi-AI >>> I don't know

------------------------------

CGO >>> - Measure the amount of monetary donations.
- Track the number of volunteer hours contributed.
- Assess the impact of the donations through beneficiary feedback.
- Evaluate the visibility and reach of the philanthropy program through media
coverage and social media engagement.
- Analyze the correlation between philanthropic activities and company
reputation.
- Compare the company’ s philanthropic efforts with industry standards.
- Review the alignment of the philanthropy program with the company’ s mission
and values.
- Monitor the long-term outcomes of the philanthropic initiatives.
- Collect data on the satisfaction levels of the stakeholders involved in the
program.
- Assess the effectiveness of the philanthropy program in achieving the
company’ s social responsibility goals.
- Evaluate the cost-effectiveness of the philanthropy program in relation to the
benefits received.

------------------------------
Turn 3...
CEO >>> Considering the diverse facets of our corporate philanthropy program, what
innovative metrics could we implement to better understand its cultural and
employee engagement impact within our organization?

------------------------------

Corgi-AI >>> To better understand the cultural and employee engagement impact␣
↪within the
organization, the company could implement metrics such as internal surveys to
assess employee needs and identity, measure the extent to which philanthropic
programs are meeting these needs, and evaluate the relative importance different
employee segments attach to intrinsic needs. Additionally, the company could
analyze the benefits of philanthropy in terms of employee engagement, customer
loyalty, reputation capital, and market opportunities, and identify key
intermediate outcomes that could yield desired business behaviors and benefits.
These metrics could be developed by leveraging models and evidence from related
business disciplines.

------------------------------

CGO >>> - Measure the amount of monetary donations.
- Track the number of volunteer hours contributed.
- Assess the impact of the donations through beneficiary feedback.
- Evaluate the visibility and reach of the philanthropy program through media
coverage and social media engagement.
- Analyze the correlation between philanthropic activities and company
reputation.
- Review the alignment of the philanthropy program with the company’ s mission
and values.
- Monitor the long-term outcomes of the philanthropic initiatives.
- Collect data on the satisfaction levels of the stakeholders involved in the
program.
```

(continues on next page)

(continued from previous page)

```
- Assess the effectiveness of the philanthropy program in achieving the
company’ s social responsibility goals.
- Evaluate the cost-effectiveness of the philanthropy program in relation to the
benefits received.
- Implement internal surveys to assess employee needs and identity.
- Measure the extent to which philanthropic programs are meeting these needs.
- Evaluate the relative importance different employee segments attach to
intrinsic needs.
- Analyze the benefits of philanthropy in terms of employee engagement, customer
loyalty, reputation capital, and market opportunities.
- Identify key intermediate outcomes that could yield desired business behaviors
and benefits.

------------------------------
Turn 4...
CEO >>> How can we integrate community engagement and environmental sustainability␣
↪into
our corporate philanthropy program to further align with our company's values
and enhance our overall impact?

------------------------------

Corgi-AI >>> I don't know.

------------------------------

CGO >>> - Measure the amount of monetary donations.
- Track the number of volunteer hours contributed.
- Assess the impact of the donations through beneficiary feedback.
- Evaluate the visibility and reach of the philanthropy program through media
coverage and social media engagement.
- Analyze the correlation between philanthropic activities and company
reputation.
- Review the alignment of the philanthropy program with the company’ s mission
and values.
- Monitor the long-term outcomes of the philanthropic initiatives.
- Collect data on the satisfaction levels of the stakeholders involved in the
program.
- Assess the effectiveness of the philanthropy program in achieving the
company’ s social responsibility goals.
- Evaluate the cost-effectiveness of the philanthropy program in relation to the
benefits received.
- Measure the extent to which philanthropic programs are meeting employee needs.
- Evaluate the relative importance different employee segments attach to
intrinsic needs.
- Analyze the benefits of philanthropy in terms of employee engagement, customer
loyalty, reputation capital, and market opportunities.
- Identify key intermediate outcomes that could yield desired business behaviors
and benefits.

------------------------------
Turn 5...
CEO >>> How can we integrate employee feedback and participation to further␣
↪enhance the
effectiveness of our corporate philanthropy program?
```

(continues on next page)

(continued from previous page)

```
------------------------------

Corgi-AI >>> To further enhance the effectiveness of the corporate philanthropy␣
↪program,
integrating employee feedback and participation can be achieved by using
internal surveys to assess the extent to which the philanthropic program meets
employee needs and creates a greater sense of identity between employee and
employer. This assessment should consider the relative importance that different
employee segments attach to different intrinsic needs. Additionally, improving
employees' sense of status, prestige, belonging within the work group and
organization, and emotional rewards inherent in their work can be achieved
through corporate philanthropic initiatives. These initiatives can also help
employee recruitment, as evidenced by the 2004 corporate community involvement
survey by Deloitte LLP, which found that 72% of employed Americans trying to
decide between two jobs offering the same location job description pay and
benefits would choose to work for the company that also supports charitable
causes. By systematically measuring the impact of corporate philanthropy,
companies can provide data-based evidence of its positive effects and make a
more persuasive case for why companies should engage in philanthropic causes.

------------------------------

CGO >>> - Measure the amount of monetary donations
- Track the number of volunteer hours contributed
- Assess the impact of the donations through beneficiary feedback
- Evaluate the visibility and reach of the philanthropy program through media
coverage and social media engagement
- Analyze the correlation between philanthropic activities and company
reputation
- Review the alignment of the philanthropy program with the company’ s mission
and values
- Monitor the long-term outcomes of the philanthropic initiatives
- Collect data on the satisfaction levels of the stakeholders involved in the
program
- Assess the effectiveness of the philanthropy program in achieving the
company’ s social responsibility goals
- Evaluate the cost-effectiveness of the philanthropy program in relation to the
benefits received
- Measure the extent to which philanthropic programs are meeting employee needs
- Evaluate the relative importance different employee segments attach to
intrinsic needs
- Analyze the benefits of philanthropy in terms of employee engagement, customer
loyalty, reputation capital, and market opportunities
- Identify key intermediate outcomes that could yield desired business behaviors
and benefits
- Use internal surveys to assess the extent to which the philanthropic program
meets employee needs and creates a greater sense of identity between employee
and employer
- Consider the relative importance that different employee segments attach to
different intrinsic needs
- Improve employees' sense of status, prestige, belonging within the work group
and organization, and emotional rewards inherent in their work through corporate
philanthropic initiatives
- Leverage the positive impact of corporate philanthropy on employee recruitment
- Systematically measure the impact of corporate philanthropy to provide data-
based evidence of its positive effects and make a more persuasive case for why
```

(continues on next page)

(continued from previous page)

```
companies should engage in philanthropic causes.

------------------------------
Turn 6...
CEO >>> How can we incorporate the perspectives of the communities we aim to serve␣
↪into
the evaluation of our corporate philanthropy program's effectiveness?

------------------------------

Corgi-AI >>> The text does not provide specific information on how to incorporate␣
↪the
perspectives of the communities served into the evaluation of a corporate
philanthropy program's effectiveness.

------------------------------

CGO >>> - Measure the amount of monetary donations
- Track the number of volunteer hours contributed
- Assess the impact of donations through beneficiary feedback
- Evaluate the visibility and reach of the philanthropy program through media
coverage and social media engagement
- Analyze the correlation between philanthropic activities and company
reputation
- Review the alignment of the philanthropy program with the company’ s mission
and values
- Monitor the long-term outcomes of the philanthropic initiatives
- Collect data on the satisfaction levels of the stakeholders involved in the
program
- Assess the effectiveness of the philanthropy program in achieving the
company’ s social responsibility goals
- Evaluate the cost-effectiveness of the philanthropy program in relation to the
benefits received
- Measure the extent to which philanthropic programs are meeting employee needs
- Evaluate the relative importance different employee segments attach to
intrinsic needs
- Analyze the benefits of philanthropy in terms of employee engagement, customer
loyalty, reputation capital, and market opportunities
- Identify key intermediate outcomes that could yield desired business behaviors
and benefits
- Use internal surveys to assess the extent to which the philanthropic program
meets employee needs and creates a greater sense of identity between employee
and employer
- Leverage the positive impact of corporate philanthropy on employee recruitment
- Systematically measure the impact of corporate philanthropy to provide data-
based evidence of its positive effects and make a more persuasive case for why
companies should engage in philanthropic causes.

------------------------------
Turn 7...
CEO >>> How can we integrate the evaluation of employee well-being and␣
↪satisfaction as a
key performance indicator in our corporate philanthropy program's success
measurement?

------------------------------
```

(continues on next page)

(continued from previous page)

```
Corgi-AI >>> To integrate the evaluation of employee well-being and satisfaction␣
↪as a key
performance indicator in the corporate philanthropy program's success
measurement, we can use internal surveys to assess the extent to which the
philanthropic program is meeting employee needs and creating a greater sense of
identity between employee and employer. This assessment should take into account
the relative importance that different employee segments attach to different
intrinsic needs.

------------------------------

CGO >>> - Measure the amount of monetary donations
- Track the number of volunteer hours contributed
- Assess the impact of donations through beneficiary feedback
- Evaluate the visibility and reach of the philanthropy program through media
coverage and social media engagement
- Analyze the correlation between philanthropic activities and company
reputation
- Review the alignment of the philanthropy program with the company’ s mission
and values
- Monitor the long-term outcomes of the philanthropic initiatives
- Collect data on the satisfaction levels of the stakeholders involved in the
program
- Assess the effectiveness of the philanthropy program in achieving the
company’ s social responsibility goals
- Evaluate the cost-effectiveness of the philanthropy program in relation to the
benefits received
- Measure the extent to which philanthropic programs are meeting employee needs
- Evaluate the relative importance different employee segments attach to
intrinsic needs
- Analyze the benefits of philanthropy in terms of employee engagement, customer
loyalty, reputation capital, and market opportunities
- Identify key intermediate outcomes that could yield desired business behaviors
and benefits
- Use internal surveys to assess the extent to which the philanthropic program
meets employee needs and creating a greater sense of identity between employee
and employer
- Leverage the positive impact of corporate philanthropy on employee recruitment
- Integrate the evaluation of employee well-being and satisfaction as a key
performance indicator in the corporate philanthropy program's success
measurement.

------------------------------
Turn 8...
CEO >>> How can we incorporate a system to measure the long-term social and
environmental impact of our corporate philanthropy initiatives on the
communities and ecosystems we support?

------------------------------

Corgi-AI >>> To incorporate a system to measure the long-term social and␣
↪environmental impact
of corporate philanthropy initiatives, we can review recent measurement studies,
models, and evidence from complementary business disciplines and the social
sector. We can generalize the most valuable concepts and recognize the strengths
```

(continues on next page)

(continued from previous page)

```
and limitations of various measurement approaches. We can also consult resources
like the TRASI database, which identifies 150 different approaches currently
used to measure the social impact of programs. Additionally, we can use social
performance and financial performance literature to understand the link between
corporate social performance and financial performance. Finally, we can adapt
current methodologies in the field to our own corporate giving programs.

------------------------------

CGO >>> - Measure the amount of monetary donations
- Track the number of volunteer hours contributed
- Assess the impact of donations through beneficiary feedback
- Evaluate the visibility and reach of the philanthropy program through media
coverage and social media engagement
- Analyze the correlation between philanthropic activities and company
reputation
- Review the alignment of the philanthropy program with the company’s mission
and values
- Monitor the long-term outcomes of the philanthropic initiatives
- Collect data on the satisfaction levels of the stakeholders involved in the
program
- Assess the effectiveness of the philanthropy program in achieving the
company’s social responsibility goals
- Evaluate the cost-effectiveness of the philanthropy program in relation to the
benefits received
- Measure the extent to which philanthropic programs are meeting employee needs
- Evaluate the relative importance different employee segments attach to
intrinsic needs
- Analyze the benefits of philanthropy in terms of employee engagement, customer
loyalty, reputation capital, and market opportunities
- Identify key intermediate outcomes that could yield desired business behaviors
and benefits
- Use internal surveys to assess the extent to which the philanthropic program
meets employee needs and creating a greater sense of identity between employee
and employer
- Leverage the positive impact of corporate philanthropy on employee recruitment
- Integrate the evaluation of employee well-being and satisfaction as a key
performance indicator in the corporate philanthropy program's success
measurement.
- Incorporate a system to measure the long-term social and environmental impact
of corporate philanthropy initiatives
- Review recent measurement studies, models, and evidence from complementary
business disciplines and the social sector
- Generalize the most valuable concepts and recognize the strengths and
limitations of various measurement approaches
- Consult resources like the TRASI database, which identifies 150 different
approaches currently used to measure the social impact of programs
- Use social performance and financial performance literature to understand the
link between corporate social performance and financial performance
- Adapt current methodologies in the field to our own corporate giving programs.

------------------------------
Turn 9...
CEO >>> How can we integrate employee perspectives more deeply into our corporate
philanthropy program's success measurement to ensure alignment with our
company's culture and values while also fostering employee engagement and
```

(continues on next page)

(continued from previous page)

```
satisfaction?

------------------------------

Corgi-AI >>> To integrate employee perspectives more deeply into our corporate
↪philanthropy
program's success measurement, we should consider the following steps:

1. Conduct internal surveys to assess the extent to which the philanthropic
program is meeting employee needs and creating a greater sense of identity
between employee and employer.
2. Take into account the relative importance that different employee segments
attach to different intrinsic needs.
3. Analyze the conversations and analyses summarized above to identify potential
areas for improvement and alignment with our company's culture and values.
4. Consider complementary disciplines such as human resources, marketing, risk
management, and capital budgeting to improve measurement methods and identify
long-term financial benefits.
5. Document success in attracting top candidates based on their exposure to the
company's corporate philanthropy program.

By following these steps, we can better understand and measure the impact of our
corporate philanthropy program on employee engagement, satisfaction, and
alignment with our company's culture and values.

------------------------------

CGO >>> - Measure the monetary donations and volunteer hours.
- Assess the impact through beneficiary feedback, media coverage, social media
engagement, and company reputation.
- Review the alignment with the company's mission and values, long-term
outcomes, and stakeholder satisfaction.
- Analyze the philanthropy program's effectiveness in achieving social
responsibility goals, cost-effectiveness, and meeting employee needs.
- Evaluate the importance of philanthropy for different employee segments,
benefits in terms of engagement, loyalty, reputation, and market opportunities.
- Identify key intermediate outcomes for business behaviors and benefits.
- Use internal surveys to assess meeting employee needs and creating a sense of
identity.
- Leverage philanthropy's impact on employee recruitment.
- Integrate employee well-being and satisfaction as KPIs.
- Measure the long-term social and environmental impact.
- Review measurement studies, models, and evidence from complementary
disciplines.
- Consult resources like the TRASI database for social impact measurement
approaches.
- Understand the link between corporate social and financial performance.
- Adapt current methodologies to our corporate giving programs.
- Conduct internal surveys for employee needs and identity.
- Consider intrinsic needs of different employee segments.
- Analyze conversations and analyses for improvement and alignment.
- Include complementary disciplines for measurement and long-term benefits.
- Document success in attracting top candidates through the program.

------------------------------
Turn 10...
```

(continues on next page)

(continued from previous page)

```
CEO >>> How can we incorporate a more holistic approach to measure the intangible
benefits, such as employee morale and community relations, in addition to the
tangible outcomes of our corporate philanthropy program?

------------------------------

Corgi-AI >>> To incorporate a more holistic approach to measure the intangible␣
↪benefits of
corporate philanthropy, such as employee morale and community relations, in
addition to tangible outcomes, we should consider the following strategies:

1. Develop clear strategies by which philanthropic initiatives contribute
towards strategic business needs, such as improved employee engagement, customer
loyalty, reputational risk, and growth opportunities. This involves
understanding the mechanisms by which these business benefits are expected to be
achieved.

2. Utilize complementary disciplines such as human resources, marketing, risk
management, and capital budgeting to improve measurement methods. This includes
applying modeling approaches for valuing future cash flows, analyzing scenarios,
and calibrating expected monetary profits linked to the behaviors of loyal
customers and engaged employees.

3. Conduct measurement studies, models, and gather evidence from complementary
business disciplines and the social sector. This can provide a starting point
for adapting current methodologies in the field to corporate giving programs.

4. Recognize the strengths and limitations of various measurement approaches and
generalize the most valuable concepts from recent measurement studies and
evidence.

5. Treat philanthropy as a professional, proactive, and strategic activity, not
just another "check in the box." This involves executing corporate philanthropy
with the same level of professionalism and strategic planning as other core
business activities.

By implementing these strategies, companies can better articulate and quantify
the long-term financial benefits and intangible assets created by philanthropic
initiatives, thereby making a more persuasive business case for their investment
in corporate philanthropy.

------------------------------

CGO >>> - Develop strategies linking philanthropic initiatives to strategic␣
↪business
needs (employee engagement, customer loyalty, reputational risk, growth
opportunities).
- Utilize complementary disciplines (human resources, marketing, risk
management, capital budgeting) for improved measurement methods.
- Conduct measurement studies, models, and gather evidence from complementary
business disciplines and the social sector.
- Recognize strengths and limitations of measurement approaches, applying
valuable concepts from recent studies.
- Treat philanthropy as a professional, strategic activity.
- Articulate and quantify long-term financial benefits and intangible assets
created by philanthropy.
```

(continues on next page)

(continued from previous page)

```
- Leverage internal surveys to assess employee needs, identity, and benefits
(engagement, loyalty, reputation, market opportunities).
- Evaluate philanthropy's impact on employee recruitment and well-being.
- Measure long-term social and environmental impact.
- Integrate employee well-being and satisfaction as KPIs.
- Document success in attracting top candidates through the program.
- Consult resources like the TRASI database for social impact measurement
approaches.
- Understand the link between corporate social and financial performance.
- Adapt current methodologies to corporate giving programs.
- Analyze conversations and analyses for improvement and alignment.

------------------------------
```

```python
# Display the final turn
print("FINAL ANSWER")
print(wrap(memory))
```

```
FINAL ANSWER
- Develop strategies linking philanthropic initiatives to strategic business
needs (employee engagement, customer loyalty, reputational risk, growth
opportunities).
- Utilize complementary disciplines (human resources, marketing, risk
management, capital budgeting) for improved measurement methods.
- Conduct measurement studies, models, and gather evidence from complementary
business disciplines and the social sector.
- Recognize strengths and limitations of measurement approaches, applying
valuable concepts from recent studies.
- Treat philanthropy as a professional, strategic activity.
- Articulate and quantify long-term financial benefits and intangible assets
created by philanthropy.
- Leverage internal surveys to assess employee needs, identity, and benefits
(engagement, loyalty, reputation, market opportunities).
- Evaluate philanthropy's impact on employee recruitment and well-being.
- Measure long-term social and environmental impact.
- Integrate employee well-being and satisfaction as KPIs.
- Document success in attracting top candidates through the program.
- Consult resources like the TRASI database for social impact measurement
approaches.
- Understand the link between corporate social and financial performance.
- Adapt current methodologies to corporate giving programs.
- Analyze conversations and analyses for improvement and alignment.
```

For attribution, show the source reference documents which supported the responses.

```python
# source document IDs of text chunks retrieved are tracked for attribution
print("Source documents reference id's:")
print(set(docs))
```

```
Source documents reference id's:
{'87', '190', '95', '84', '5', '12', '195', '86', '97', '0', '8', '102', '103', '4
↪', '88', '151', '152', '1', '9'}
```

38.5 Multimodal

38.5.1 Multi-lingual

LLM agents built on the Phi-4-multimodal model can take advantage of its multilingual capabilities, supporting over 20 languages including English, French, German, Hindi, Italian, Portuguese, Spanish, and Thai. To evaluate language-specific performance, benchmarks and leaderboards based on MMLU and other open datasets are available. For example:

https://huggingface.co/spaces/uonlp/open_multilingual_llm_leaderboard

```
# Translate english to french
french = pipe(f"Please translate to French: {memory}", max_new_tokens=512)
print(wrap(french))
```

```
- Développer des stratégies reliant les initiatives philanthropiques aux besoins
stratégiques des affaires (engagement des employés, fidélité des clients, risque
de réputation, opportunités de croissance).
- Utiliser des disciplines complémentaires (ressources humaines, marketing,
gestion des risques, budgétisation des capitaux) pour des méthodes de mesure
améliorées.
- Conduire des études de mesure, des modèles et rassembler des preuves des
disciplines commerciales complémentaires et du secteur social.
- Reconnaître les forces et les limites des approches de mesure, en appliquant
des concepts précieux des études récentes.
- Traiter la philanthropie comme une activité professionnelle et stratégique.
- Articuler et quantifier les avantages financiers à long terme et les actifs
intangibles créés par la philanthropie.
- Utiliser des enquêtes internes pour évaluer les besoins, l'identité et les
avantages des employés (engagement, fidélité, réputation, opportunités de
marché).
- Évaluer l'impact de la philanthropie sur la recrutement des employés et leur
bien-être.
- Mesurer l'impact à long terme social et environnemental.
- Intégrer le bien-être et la satisfaction des employés comme indicateurs de
performance (KPIs).
- Documenter le succès dans l'attraction de candidats exceptionnels grâce au
programme.
- Consulter des ressources comme la base de données TRASI pour les approches de
mesure de l'impact social.
- Comprendre la relation entre la performance sociale et financière de
l'entreprise.
- Adapter les méthodologies actuelles aux programmes de dons d'entreprise.
- Analyser les conversations et les analyses pour l'amélioration et
l'alignement.
```

```
# Translate back to english
print(wrap(pipe(f"Please translate to English: {french}", max_new_tokens=512)))
```

```
- Develop strategies that link philanthropic initiatives to the strategic
business needs (employee engagement, customer loyalty, reputation risk, growth
opportunities).
- Use complementary disciplines (human resources, marketing, risk management,
capital budgeting) for improved measurement methods.
- Conduct measurement studies, models, and gather evidence of the complementary
```

(continues on next page)

(continued from previous page)

```
commercial disciplines and the social sector.
- Recognize the strengths and limitations of measurement approaches, applying
valuable concepts from recent studies.
- Treat philanthropy as a professional and strategic activity.
- Articulate and quantify the long-term financial benefits and intangible assets
created by philanthropy.
- Use internal surveys to evaluate the needs, identity, and benefits of
employees (engagement, loyalty, reputation, market opportunities).
- Evaluate the impact of philanthropy on employee recruitment and well-being.
- Measure the long-term social and environmental impact.
- Integrate employee well-being and satisfaction as performance indicators
(KPIs).
- Document the success in attracting exceptional candidates through the program.
- Consult resources like the TRASI database for social impact measurement
approaches.
- Understand the relationship between the social performance and financial
performance of the company.
- Adapt current methodologies to corporate donation programs.
- Analyze conversations and analyses for improvement and alignment.
```

38.5.2 Vision language

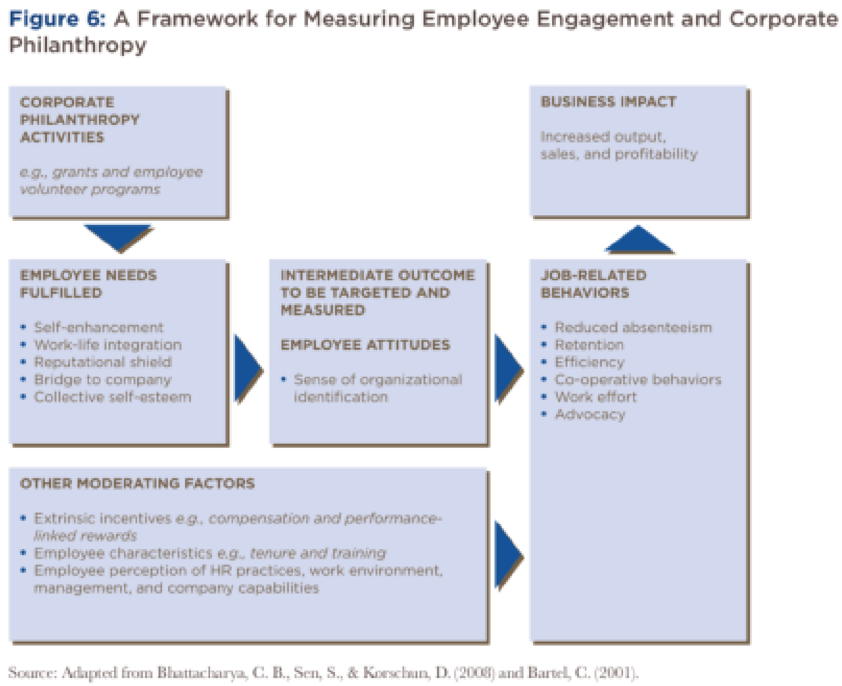
Perform optical character recognition (OCR) and chart and table interpretation of figure 6 from the MVCP document.

Multimodal evaluation of agents for image and video analysis can be performed with more comprehensive benchmarks and datasets, such as:

https://github.com/bradyfu/awesome-multimodal-large-language-models

```
# Load and display a test image
image = Image.open("assets/lim-fig6.png")
plt.imshow(image)
plt.axis('off')
```

```
(np.float64(-0.5), np.float64(953.5), np.float64(785.5), np.float64(-0.5))
```




```
print(wrap(pipe("Describe what is presented in this image", image=image, max_new_tokens=512)))
```

```
The image is a diagram titled "Figure 6: A Framework for Measuring Employee
Engagement and Corporate Philanthropy." It is divided into several sections,
each with a specific focus. The top left section is labeled "CORPORATE
PHILANTHROPY ACTIVITIES" and includes examples such as grants and employee
volunteer programs. The top right section is labeled "BUSINESS IMPACT" and lists
outcomes like increased output, sales, and profitability. Below these sections,
there are three main columns. The first column on the left is labeled "EMPLOYEE
NEEDS FULFILLED" and includes points such as self-enhancement, work-life
integration, reputational shield, bridge to company, and collective self-esteem.
The middle column is labeled "INTERMEDIATE OUTCOME TO BE TARGETED AND MEASURED"
and includes "EMPLOYEE ATTITUDES" and "Sense of organizational identification."
The third column on the right is labeled "JOB-RELATED BEHAVIORS" and lists
outcomes like reduced absenteeism, retention, efficiency, co-operative
behaviors, work effort, and advocacy. At the bottom, there is a section labeled
"OTHER MODERATING FACTORS" which includes extrinsic incentives, employee
characteristics, and employee perception of HR practices, work environment,
management, and company capabilities. The source of the diagram is cited as
adapted from Bhattacharya, C. B., Sen, S., & Korschun, D. (2008) and Bartel, C.
(2001).
```

Probing the model’s image reasoning capabilities:

```
# Load and display a test image
image = Image.open("assets/MSDTRPL_EC013-H-2020-1602198302.webp")
plt.imshow(image)
plt.axis('off')
```

```
(np.float64(-0.5), np.float64(999.5), np.float64(562.5), np.float64(-0.5))
```

```
print(wrap(pipe("What is shown in this image?", image=image, max_new_tokens=512)))
```

```
The image depicts a large crowd of people, many of whom are holding up their
hands. The crowd appears to be in a state of excitement or celebration, with
some individuals raising their hands in the air. The people are dressed in a
variety of clothing, including suits and casual attire. The background is filled
with more people, suggesting that this is a public event or gathering.
```

```
print(wrap(pipe("Identify the movie that this scene is taken from, and explain your reasoning",
                image=image, max_new_tokens=128)))
```

```
This scene is from the movie 'Forrest Gump.' The reasoning behind this
identification includes the distinctive style of the crowd, the casual yet
significant attire of the individuals, and the overall chaotic yet focused
atmosphere typical of a pivotal moment in the film. The scene captures a moment
of intense public reaction, which is a recurring theme in the movie.
```

```
print(wrap(pipe("Please give your best estimate of the number of people in this image",
                image=image, max_new_tokens=32)))
```

```
There are approximately 30 people visible in the image.
```

```
print(wrap(pipe("Briefly recount a specific instance of corporate philanthropy in the plot of the movie Trading Places?",
          max_new_tokens=512)))
```

```
In the movie "Trading Places," a notable instance of corporate philanthropy
occurs when Louis Winthorpe III, played by Eddie Murphy, and Billy Ray
Valentine, played by Dan Akroyd, are swapped due to a bet. Louis, who is a
successful commodities trader, is given a chance to live a life of luxury and
privilege, while Billy, a struggling street trader, is given Louis' life.

During this period, Louis, who is now living a life of luxury, decides to use
his newfound wealth to help those in need. He donates a significant amount of
money to a local homeless shelter, which is a clear act of corporate
philanthropy. This act not only helps the shelter but also serves as a turning
point in Louis' character development, as he begins to understand the value of
giving back to the community.

This instance of corporate philanthropy in the movie highlights the importance
of using wealth and resources to help those in need, and it also serves as a
reminder that even those who are successful can make a positive impact on
society.
```

38.5.3 Audio

Finally, perform automatic speech recognition (ASR) and speech translation on an audio input.

Rigorous open benchmark datasets for evaluation are also available, e.g.

https://github.com/huggingface/open_asr_leaderboard

```python
# Perform speech-to-text on an audio file
url = "assets/pork-bellies!.mp3"
audio, samplerate = soundfile.read(url)
response = pipe("Transcribe the audio to text",
                audio=[(audio, samplerate)], max_new_tokens=512)
print(wrap(response))
```

```
Pork bellies! I have a hunch something very exciting is going to happen in the
pork belly market this morning.
```

```python
from playsound import playsound
playsound(url)    # play the original sound file
```

References:

Patrick Lewis, Ethan Perez, Aleksandra Piktus, Fabio Petroni, Vladimir Karpukhin, Naman Goyal†, Heinrich Küttler, Mike Lewis†, Wen-tau Yih, Tim Rocktäschel, Sebastian Riedel, Douwe Kiela, 2021, Retrieval-Augmented Generation for Knowledge-Intensive NLP Tasks

Danqi Chen, Adam Fisch, Jason Weston, Antoine Bordes. (2017), "Reading Wikipedia to Answer Open-Domain Questions."

Ori Ram, Yoav Levine, Itay Dalmedigos, Dor Muhlgay, Amnon Shashua, Kevin Leyton-Brown, Yoav Shoham (2023), "In-Context Retrieval-Augmented Language Models."

Kyle Mahowald and Anna A. Ivanova and Idan A. Blank and Nancy Kanwisher and Joshua B. Tenenbaum and Evelina Fedorenko, 2024, Dissociating language and thought in large language models, https://arxiv.org/abs/2301.06627

Hicks, Michael Townsen, Humphries, James and Slater, Joe, 2024, "ChatGPT is bullshit" , Ethics and Information Technology Volume 26, Issue 2, https://doi.org/10.1007/s10676-024-09775-5

Parth Sarthi and Salman Abdullah and Aditi Tuli and Shubh Khanna and Anna Goldie and Christopher D. Manning, 2024, RAPTOR: Recursive Abstractive Processing for Tree-Organized Retrieval, https://arxiv.org/abs/2401.18059

https://github.com/NirDiamant/RAG_Techniques

Lim, Terence, 2010, Measuring the value of corporate philanthropy: Social impact, business benefits, and investor returns. published by CECP.