

**Fundamentals of Actuarial Mathematics –
Long-Term**

Sample Questions

July 1st, 2022

This note contains sample questions for the Fundamentals of Actuarial Mathematics – Long-Term (FAM-L) exam. The questions are sorted by the Society of Actuaries’ recommended resources for this exam. All question numbers follow the format of X.Y, where X identifies the source and Y is the question number from that source. X refers to the chapter of *Actuarial Mathematics for Life Contingent Risks, 3rd Edition (AMLCR)*.

Many of these questions are based on questions that have previously appeared on MLC exams from 2012 through 2017. These are identified by a parenthetical expression at the end of such questions. Questions that have been modified have been modified to:

- Replace the Illustrative Life Table (ILT) which was used on the MLC exam with the Standard Ultimate Life Table (SULT), which will be used with the FAM-L exam
- Change language to reflect the current terms used on the FAM-L exam. For example, consistent with *AMCLR*, we now use “policy value” instead of “reserve” in many cases where prior exams would have used “reserve.”

Multiple-choice questions from MLC exams in 2012 and 2013, which are included in these sample questions, were intended to average six minutes each. The multiple-choice questions for the MLC exam in 2014 and later were intended to average five minutes each. The multiple-choice questions on the FAM-L exam are intended to average five minutes each, therefore, the questions based on the 2012 and 2013 MLC exams may be slightly longer than those on the FAM-L exam. That being said, these questions are representative of the types of questions that might be asked of candidates sitting for the FAM-L exam. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on the FAM-L exam.

There are also some additional sample questions that are not directly based on prior exam questions from 2012 through 2017.

Versions:

July 1, 2022 Original version for FAM-L

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1.1 Determine which of the following statements is NOT true with regard to underwriting.

- (A) Life insurance policies are typically underwritten to prevent adverse selection.
- (B) The distribution method affects the level of underwriting.
- (C) Single premium immediate annuities are typically underwritten to prevent adverse selection.
- (D) Underwriting may result in an insured life being classified as a rated life due to the insured's occupation or hobby.
- (E) A pure endowment does not need to be underwritten to prevent adverse selection.

1.2. Over the last 30 years, life insurance products and the management of the associated risks have radically changed and become more complex.

Determine which of the following is NOT a reason for this change.

- (A) More sophisticated policyholders.
- (B) More competition among life insurance companies.
- (C) More computational power.
- (D) More complex risk management techniques.
- (E) Separation of the savings elements and the protection elements of life insurance products.

2.1. You are given:

(i) $S_0(t) = \left(1 - \frac{t}{\omega}\right)^{\frac{1}{4}}$, for $0 \leq t \leq \omega$

(ii) $\mu_{65} = \frac{1}{180}$

Calculate e_{106} , the curtate expectation of life at age 106.

(A) 2.2

(B) 2.5

(C) 2.7

(D) 3.0

(E) 3.2

[Question 3 on the Fall 2012 exam]

2.2 Scientists are searching for a vaccine for a disease. You are given:

- (i) 100,000 lives age x are exposed to the disease
- (ii) Future lifetimes are independent, except that the vaccine, if available, will be given to all at the end of year 1
- (iii) The probability that the vaccine will be available is 0.2
- (iv) For each life during year 1, $q_x = 0.02$
- (v) For each life during year 2, $q_{x+1} = 0.01$ if the vaccine has been given, and $q_{x+1} = 0.02$ if it has not been given

Calculate the standard deviation of the number of survivors at the end of year 2.

- (A) 100
- (B) 200
- (C) 300
- (D) 400
- (E) 500

[Question 20 on the Spring 2013 exam]

2.3. You are given that mortality follows Gompertz Law with $B = 0.00027$ and $c = 1.1$.

Calculate $f_{50}(10)$.

- (A) 0.048
- (B) 0.050
- (C) 0.052
- (D) 0.054
- (E) 0.056

2.4. You are given ${}_tq_0 = \frac{t^2}{10,000}$ for $0 < t < 100$.

Calculate $\overset{\circ}{e}_{75:\overline{10}|}$.

- (A) 6.6
- (B) 7.0
- (C) 7.4
- (D) 7.8
- (E) 8.2

2.5. You are given the following:

(i) $e_{\overline{40:20}|} = 18$

(ii) $e_{60} = 25$

(iii) ${}_{20}q_{40} = 0.2$

(iv) $q_{40} = 0.003$

Calculate e_{41} .

(A) 36.1

(B) 37.1

(C) 38.1

(D) 39.1

(E) 40.1

2.6. You are given the survival function:

$$S_0(x) = \left(1 - \frac{x}{60}\right)^{\frac{1}{3}}, \quad 0 \leq x \leq 60.$$

Calculate $1000\mu_{35}$.

(A) 5.6

(B) 6.7

(C) 13.3

(D) 16.7

(E) 20.1

[Question 2 on the Spring 2016 exam]

2.7. You are given the following survival function of a newborn:

$$S_0(x) = \begin{cases} 1 - \frac{x}{250}, & 0 \leq x < 40 \\ 1 - \left(\frac{x}{100}\right)^2, & 40 \leq x \leq 100 \end{cases}$$

Calculate the probability that (30) dies within the next 20 years.

- (A) 0.13
- (B) 0.15
- (C) 0.17
- (D) 0.19
- (E) 0.21

[Question 2 on the Fall 2016 exam]

2.8. In a population initially consisting of 75% females and 25% males, you are given:

- (i) For a female, the force of mortality is constant and equals μ
- (ii) For a male, the force of mortality is constant and equals 1.5μ
- (iii) At the end of 20 years, the population is expected to consist of 85% females and 15% males

Calculate the probability that a female survives one year.

- (A) 0.89
- (B) 0.92
- (C) 0.94
- (D) 0.96
- (E) 0.99

[Question 3 on the Fall 2016 exam]

3.1. You are given:

(i) An excerpt from a select and ultimate life table with a select period of 3 years:

| x | $l_{[x]}$ | $l_{[x]+1}$ | $l_{[x]+2}$ | l_{x+3} | $x+3$ |
|-----|-----------|-------------|-------------|-----------|-------|
| 60 | 80,000 | 79,000 | 77,000 | 74,000 | 63 |
| 61 | 78,000 | 76,000 | 73,000 | 70,000 | 64 |
| 62 | 75,000 | 72,000 | 69,000 | 67,000 | 65 |
| 63 | 71,000 | 68,000 | 66,000 | 65,000 | 66 |

(ii) Deaths follow a constant force of mortality over each year of age

Calculate $1000 {}_{2|3}q_{[60]+0.75}$.

- (A) 104
- (B) 117
- (C) 122
- (D) 135
- (E) 142

[Question 2 on the Fall 2012 exam]

3.2. You are given:

(i) The following extract from a mortality table with a one-year select period:

| x | $l_{[x]}$ | $d_{[x]}$ | l_{x+1} | $x+1$ |
|-----|-----------|-----------|-----------|-------|
| 65 | 1000 | 40 | — | 66 |
| 66 | 955 | 45 | — | 67 |

(ii) Deaths are uniformly distributed over each year of age

$$\overset{\circ}{e}_{[65]} = 15.0$$

Calculate $\overset{\circ}{e}_{[66]}$.

- (A) 14.1
- (B) 14.3
- (C) 14.5
- (D) 14.7
- (E) 14.9

[Question 19 on the Spring 2013 exam]

3.3. You are given:

(i) An excerpt from a select and ultimate life table with a select period of 2 years:

| x | $l_{[x]}$ | $l_{[x]+1}$ | l_{x+2} | $x+2$ |
|-----|-----------|-------------|-----------|-------|
| 50 | 99,000 | 96,000 | 93,000 | 52 |
| 51 | 97,000 | 93,000 | 89,000 | 53 |
| 52 | 93,000 | 88,000 | 83,000 | 54 |
| 53 | 90,000 | 84,000 | 78,000 | 55 |

(ii) Deaths are uniformly distributed over each year of age

Calculate $10,000 {}_{2.2}q_{[51]+0.5}$.

- (A) 705
- (B) 709
- (C) 713
- (D) 1070
- (E) 1074

[Question 3 on the Fall 2013 exam]

3.4. The SULT Club has 4000 members all age 25 with independent future lifetimes. The mortality for each member follows the Standard Ultimate Life Table.

Calculate the largest integer N , using the normal approximation, such that the probability that there are at least N survivors at age 95 is at least 90%.

- (A) 800
- (B) 815
- (C) 830
- (D) 845
- (E) 860

[A modified version of Question 24 on the Fall 2013 exam]

3.5. You are given:

| x | l_x |
|-----|--------|
| 60 | 99,999 |
| 61 | 88,888 |
| 62 | 77,777 |
| 63 | 66,666 |
| 64 | 55,555 |
| 65 | 44,444 |
| 66 | 33,333 |
| 67 | 22,222 |

$a = {}_{3.4|2.5}q_{60}$ assuming a uniform distribution of deaths over each year of age

$b = {}_{3.4|2.5}q_{60}$ assuming a constant force of mortality over each year of age

Calculate $100,000(a - b)$.

- (A) -24
- (B) 9
- (C) 42
- (D) 73
- (E) 106

[Question 25 on the Fall 2013 exam]

3.6. You are given the following extract from a table with a 3-year select period:

| x | $q_{[x]}$ | $q_{[x]+1}$ | $q_{[x]+2}$ | q_{x+3} | $x+3$ |
|-----|-----------|-------------|-------------|-----------|-------|
| 60 | 0.09 | 0.11 | 0.13 | 0.15 | 63 |
| 61 | 0.10 | 0.12 | 0.14 | 0.16 | 64 |
| 62 | 0.11 | 0.13 | 0.15 | 0.17 | 65 |
| 63 | 0.12 | 0.14 | 0.16 | 0.18 | 66 |
| 64 | 0.13 | 0.15 | 0.17 | 0.19 | 67 |

$$e_{64} = 5.10$$

Calculate $e_{[61]}$.

- (A) 5.30
- (B) 5.39
- (C) 5.68
- (D) 5.85
- (E) 6.00

[Question 2 on the Spring 2014 exam]

3.7. For a mortality table with a select period of two years, you are given:

| x | $q_{[x]}$ | $q_{[x]+1}$ | q_{x+2} | $x+2$ |
|-----|-----------|-------------|-----------|-------|
| 50 | 0.0050 | 0.0063 | 0.0080 | 52 |
| 51 | 0.0060 | 0.0073 | 0.0090 | 53 |
| 52 | 0.0070 | 0.0083 | 0.0100 | 54 |
| 53 | 0.0080 | 0.0093 | 0.0110 | 55 |

The force of mortality is constant between integral ages.

Calculate $1000 {}_{2.5}q_{[50]+0.4}$.

- (A) 15.2
- (B) 16.4
- (C) 17.7
- (D) 19.0
- (E) 20.2

[Question 20 on the Fall 2014 exam]

3.8. A club is established with 2000 members, 1000 of exact age 35 and 1000 of exact age 45.

You are given:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) Future lifetimes are independent
- (iii) N is the random variable for the number of members still alive 40 years after the club is established

Using the normal approximation, without the continuity correction, calculate the smallest n such that $\Pr(N \geq n) \leq 0.05$.

- (A) 1500
- (B) 1505
- (C) 1510
- (D) 1515
- (E) 1520

[A modified version of Question 1 on the Spring 2015 exam]

3.9. A father-son club has 4000 members, 2000 of which are age 20 and the other 2000 are age 45. In 25 years, the members of the club intend to hold a reunion.

You are given:

- (i) All lives have independent future lifetimes.
- (ii) Mortality follows the Standard Ultimate Life Table.

Using the normal approximation, without the continuity correction, calculate the 99th percentile of the number of surviving members at the time of the reunion.

- (A) 3810
- (B) 3820
- (C) 3830
- (D) 3840
- (E) 3850

[A modified version of Question 1 on the Fall 2015 exam]

- 3.10.** A group of 100 people start a Scissor Usage Support Group. The rate at which members enter and leave the group is dependent on whether they are right-handed or left-handed.

You are given the following:

- (i) The initial membership is made up of 75% left-handed members (L) and 25% right-handed members (R)
- (ii) After the group initially forms, 35 new (L) and 15 new (R) join the group at the start of each subsequent year
- (iii) Members leave the group only at the end of each year
- (iv) $q^L = 0.25$ for all years
- (v) $q^R = 0.50$ for all years

Calculate the proportion of the Scissor Usage Support Group's expected membership that is left-handed at the start of the group's 6th year, before any new members join for that year.

- (A) 0.76
- (B) 0.81
- (C) 0.86
- (D) 0.91
- (E) 0.96

[Question 2 on the Fall 2015 exam]

3.11. For the country of Bienna, you are given:

- (i) Bienna publishes mortality rates in biennial form, that is, mortality rates are of the form:
 ${}_2q_{2x}$, for $x = 0, 1, 2, \dots$
- (ii) Deaths are assumed to be uniformly distributed between ages $2x$ and $2x + 2$, for $x = 0, 1, 2, \dots$
- (iii) ${}_2q_{50} = 0.02$
- (iv) ${}_2q_{52} = 0.04$

Calculate the probability that (50) dies during the next 2.5 years.

- (A) 0.02
- (B) 0.03
- (C) 0.04
- (D) 0.05
- (E) 0.06

[Question 1 on the Fall 2016 exam]

- 3.12. X and Y are both age 61. X has just purchased a whole life insurance policy. Y purchased a whole life insurance policy one year ago.

Both X and Y are subject to the following 3-year select and ultimate table:

| x | $\ell_{[x]}$ | $\ell_{[x]+1}$ | $\ell_{[x]+2}$ | ℓ_{x+3} | $x+3$ |
|-----|--------------|----------------|----------------|--------------|-------|
| 60 | 10,000 | 9,600 | 8,640 | 7,771 | 63 |
| 61 | 8,654 | 8,135 | 6,996 | 5,737 | 64 |
| 62 | 7,119 | 6,549 | 5,501 | 4,016 | 65 |
| 63 | 5,760 | 4,954 | 3,765 | 2,410 | 66 |

The force of mortality is constant over each year of age.

Calculate the difference in the probability of survival to age 64.5 between X and Y.

- (A) 0.035
- (B) 0.045
- (C) 0.055
- (D) 0.065
- (E) 0.075

[Question 2 on the Spring 2017 exam]

3.13. A life is subject to the following 3-year select and ultimate table:

| $[x]$ | $\ell_{[x]}$ | $\ell_{[x]+1}$ | $\ell_{[x]+2}$ | ℓ_{x+3} | $x+3$ |
|-------|--------------|----------------|----------------|--------------|-------|
| 55 | 10,000 | 9,493 | 8,533 | 7,664 | 58 |
| 56 | 8,547 | 8,028 | 6,889 | 5,630 | 59 |
| 57 | 7,011 | 6,443 | 5,395 | 3,904 | 60 |
| 58 | 5,853 | 4,846 | 3,548 | 2,210 | 61 |

You are also given:

- (i) $e_{60} = 1$
- (ii) Deaths are uniformly distributed over each year of age

Calculate $\ddot{e}_{[58]+2}$.

- (A) 1.5
- (B) 1.6
- (C) 1.7
- (D) 1.8
- (E) 1.9

[Question 1 on the Fall 2017 exam]

3.14. You are given the following information from a life table:

| x | l_x | d_x | p_x | q_x |
|-----|-------|-------|-------|-------|
| 95 | — | — | — | 0.40 |
| 96 | — | — | 0.20 | — |
| 97 | — | 72 | — | 1.00 |

You are also given:

(i) $l_{90} = 1000$ and $l_{93} = 825$

(ii) Deaths are uniformly distributed over each year of age.

Calculate the probability that (90) dies between ages 93 and 95.5.

(A) 0.195

(B) 0.220

(C) 0.345

(D) 0.465

(E) 0.668

[Question 1 on the Spring 2014 exam]

4.1. For a special whole life insurance policy issued on (40), you are given:

- (i) Death benefits are payable at the end of the year of death
- (ii) The amount of benefit is 2 if death occurs within the first 20 years and is 1 thereafter
- (iii) Z is the present value random variable for the payments under this insurance
- (iv) $i = 0.03$
- (v)

| x | A_x | ${}_{20}E_x$ |
|-----|---------|--------------|
| 40 | 0.36987 | 0.51276 |
| 60 | 0.62567 | 0.17878 |

- (vi) $E[Z^2] = 0.24954$

Calculate the standard deviation of Z .

- (A) 0.27
- (B) 0.32
- (C) 0.37
- (D) 0.42
- (E) 0.47

[Question 14 on the Fall 2012 exam]

4.2. For a special 2-year term insurance policy on (x) , you are given:

- (i) Death benefits are payable at the end of the half-year of death
- (ii) The amount of the death benefit is 300,000 for the first half-year and increases by 30,000 per half-year thereafter
- (iii) $q_x = 0.16$ and $q_{x+1} = 0.23$
- (iv) $i^{(2)} = 0.18$
- (v) Deaths are assumed to follow a constant force of mortality between integral ages
- (vi) Z is the present value random variable for this insurance

Calculate $\Pr(Z > 277,000)$.

- (A) 0.08
- (B) 0.11
- (C) 0.14
- (D) 0.18
- (E) 0.21

[Question 15 on the Fall 2012 exam]

4.3. You are given:

- (i) $q_{60} = 0.01$
- (ii) Using $i = 0.05$, $A_{60:\overline{3}|} = 0.86545$

Using $i = 0.045$ calculate $A_{60:\overline{3}|}$.

- (A) 0.866
- (B) 0.870
- (C) 0.874
- (D) 0.878
- (E) 0.882

[Question 7 on the Spring 2013 exam]

4.4 For a special increasing whole life insurance on (40), payable at the moment of death, you are given:

- (i) The death benefit at time t is $b_t = 1 + 0.2t$, $t \geq 0$
 - (ii) The interest discount factor at time t is $v(t) = (1 + 0.2t)^{-2}$, $t \geq 0$
 - (iii) ${}_t p_{40} \mu_{40+t} = \begin{cases} 0.025, & 0 \leq t < 40 \\ 0, & \text{otherwise} \end{cases}$
 - (iv) Z is the present value random variable for this insurance
- Calculate $\text{Var}(Z)$.

- (A) 0.036
- (B) 0.038
- (C) 0.040
- (D) 0.042
- (E) 0.044

[Question 8 on the Spring 2013 exam]

4.5. For a 30-year term life insurance of 100,000 on (45), you are given:

- (i) The death benefit is payable at the moment of death
- (ii) Mortality follows the Standard Ultimate Life Table
- (iii) $\delta = 0.05$
- (iv) Deaths are uniformly distributed over each year of age

Calculate the 95th percentile of the present value of benefits random variable for this insurance.

- (A) 30,200
- (B) 31,200
- (C) 35,200
- (D) 36,200
- (E) 37,200

[A modified version of Question 11 on the Fall 2017 exam]

4.6. For a 3-year term insurance of 1000 on (70), you are given:

- (i) q_{70+k}^{SULT} is the mortality rate from the Standard Ultimate Life Table, for $k = 0, 1, 2$
- (ii) q_{70+k} is the mortality rate used to price this insurance, for $k = 0, 1, 2$
- (iii) $q_{70+k} = (0.95)^k q_{70+k}^{SULT}$, for $k = 0, 1, 2$
- (iv) $i = 0.05$

Calculate the single net premium.

- (A) 29.05
- (B) 29.85
- (C) 30.65
- (D) 31.45
- (E) 32.25

[A modified version of Question 13 on the Fall 2013 exam]

4.7. For a 25-year pure endowment of 1 on (x), you are given:

- (i) Z is the present value random variable at issue of the benefit payment
- (ii) $Var(Z) = 0.10E[Z]$
- (iii) ${}_{25}p_x = 0.57$

Calculate the annual effective interest rate.

- (A) 5.8%
- (B) 6.0%
- (C) 6.2%
- (D) 6.4%
- (E) 6.6%

[Question 6 on the Fall 2017 exam]

- 4.8.** For a whole life insurance of 1000 on (50), you are given:
- (i) The death benefit is payable at the end of the year of death
 - (ii) Mortality follows the Standard Ultimate Life Table
 - (iii) $i = 0.04$ in the first year, and $i = 0.05$ in subsequent years
- Calculate the actuarial present value of this insurance.

- (A) 187
- (B) 189
- (C) 191
- (D) 193
- (E) 195

[A modified version of Question 5 on the Spring 2014 exam]

- 4.9.** You are given:
- (i) $A_{35:\overline{15}|} = 0.39$
 - (ii) $A_{35:\overline{15}|}^1 = 0.25$
 - (iii) $A_{35} = 0.32$

Calculate A_{50} .

- (A) 0.35
- (B) 0.40
- (C) 0.45
- (D) 0.50
- (E) 0.55

[Question 4 on the Fall 2014 exam]

4.10. The present value random variable for an insurance policy on (x) is expressed as:

$$Z = \begin{cases} 0, & \text{if } T_x \leq 10 \\ v^{T_x}, & \text{if } 10 < T_x \leq 20 \\ 2v^{T_x}, & \text{if } 20 < T_x \leq 30 \\ 0, & \text{thereafter} \end{cases}$$

Determine which of the following is a correct expression for $E[Z]$.

- (A) ${}_{10|}\bar{A}_x + {}_{20|}\bar{A}_x - {}_{30|}\bar{A}_x$
- (B) $\bar{A}_x + {}_{20}E_x \bar{A}_{x+20} - 2 {}_{30}E_x \bar{A}_{x+30}$
- (C) ${}_{10}E_x \bar{A}_x + {}_{20}E_x \bar{A}_{x+20} - 2 {}_{30}E_x \bar{A}_{x+30}$
- (D) ${}_{10}E_x \bar{A}_{x+10} + {}_{20}E_x \bar{A}_{x+20} - 2 {}_{30}E_x \bar{A}_{x+30}$
- (E) ${}_{10}E_x \left[\bar{A}_{x+10} + {}_{10}E_{x+10} \bar{A}_{x+20} - {}_{10}E_{x+20} \bar{A}_{x+30} \right]$

[Question 4 on the Spring 2015 exam]

4.11. You are given:

- (i) Z_1 is the present value random variable for an n -year term insurance of 1000 issued to (x)
- (ii) Z_2 is the present value random variable for an n -year endowment insurance of 1000 issued to (x)
- (iii) For both Z_1 and Z_2 the death benefit is payable at the end of the year of death
- (iv) $E[Z_1] = 528$
- (v) $Var(Z_2) = 15,000$
- (vi) $A_{x:n|} = 0.209$
- (vii) ${}^2A_{x:n|} = 0.136$

Calculate $Var(Z_1)$.

- (A) 143,400
- (B) 177,500
- (C) 211,200
- (D) 245,300
- (E) 279,300

[Question 5 on the Spring 2015 exam]

4.12. For three fully discrete insurance products on the same (x) , you are given:

- (i) Z_1 is the present value random variable for a 20-year term insurance of 50
- (ii) Z_2 is the present value random variable for a 20-year deferred whole life insurance of 100
- (iii) Z_3 is the present value random variable for a whole life insurance of 100.
- (iv) $E[Z_1] = 1.65$ and $E[Z_2] = 10.75$
- (v) $Var(Z_1) = 46.75$ and $Var(Z_2) = 50.78$

Calculate $Var(Z_3)$.

- (A) 62
- (B) 109
- (C) 167
- (D) 202
- (E) 238

[Question 4 on the Fall 2015 exam]

4.13. For a 2-year deferred, 2-year term insurance of 2000 on $[65]$, you are given:

(i) The following select and ultimate mortality table with a 3-year select period:

| x | $q_{[x]}$ | $q_{[x]+1}$ | $q_{[x]+2}$ | q_{x+3} | $x+3$ |
|-----|-----------|-------------|-------------|-----------|-------|
| 65 | 0.08 | 0.10 | 0.12 | 0.14 | 68 |
| 66 | 0.09 | 0.11 | 0.13 | 0.15 | 69 |
| 67 | 0.10 | 0.12 | 0.14 | 0.16 | 70 |
| 68 | 0.11 | 0.13 | 0.15 | 0.17 | 71 |
| 69 | 0.12 | 0.14 | 0.16 | 0.18 | 72 |

(ii) $i = 0.04$

(iii) The death benefit is payable at the end of the year of death

Calculate the actuarial present value of this insurance.

(A) 260

(B) 290

(C) 350

(D) 370

(E) 410

[Question 9 on the Fall 2015 exam]

- 4.14.** A fund is established for the benefit of 400 workers all age 60 with independent future lifetimes. When they reach age 85, the fund will be dissolved and distributed to the survivors.

The fund will earn interest at a rate of 5% per year.

The initial fund balance, F , is determined so that the probability that the fund will pay at least 5000 to each survivor is 86%, using the normal approximation.

Mortality follows the Standard Ultimate Life Table.

Calculate F .

- (A) 350,000
- (B) 360,000
- (C) 370,000
- (D) 380,000
- (E) 390,000

[A modified version of Question 3 on the Spring 2016 exam]

- 4.15.** For a special whole life insurance on (x) , you are given:

- (i) Death benefits are payable at the moment of death
- (ii) The death benefit at time t is $b_t = e^{0.02t}$, for $t \geq 0$
- (iii) $\mu_{x+t} = 0.04$, for $t \geq 0$
- (iv) $\delta = 0.06$
- (v) Z is the present value at issue random variable for this insurance

Calculate $Var(Z)$.

- (A) 0.020
- (B) 0.036
- (C) 0.052
- (D) 0.068
- (E) 0.083

[Question 4 on the Fall 2016 exam]

- 4.16. You are given the following extract of ultimate mortality rates from a two-year select and ultimate mortality table:

| x | q_x |
|-----|-------|
| 50 | 0.045 |
| 51 | 0.050 |
| 52 | 0.055 |
| 53 | 0.060 |

The select mortality rates satisfy the following:

- $q_{[x]} = 0.7q_x$
- $q_{[x]+1} = 0.8q_{x+1}$

You are also given that $i = 0.04$.

Calculate $A^1_{[50]:\overline{3}|}$.

- (A) 0.08
- (B) 0.09
- (C) 0.10
- (D) 0.11
- (E) 0.12

[Question 5 on the Fall 2016 exam]

4.17. For a special whole life policy on (48), you are given:

- (i) The policy pays 5000 if the insured's death is before the median curtate future lifetime at issue and 10,000 if death is after the median curtate future lifetime at issue
- (ii) Mortality follows the Standard Ultimate Life Table
- (iii) Death benefits are paid at the end of the year of death
- (iv) $i = 0.05$

Calculate the actuarial present value of benefits for this policy.

- (A) 1130
- (B) 1160
- (C) 1190
- (D) 1220
- (E) 1250

[A modified version of Question 6 on the Fall 2016 exam]

- 4.18. You are given that T , the time to first failure of an industrial robot, has a density $f(t)$ given by

$$f(t) = \begin{cases} 0.1, & 0 \leq t < 2 \\ 0.4t^{-2}, & 2 \leq t < 10 \end{cases}$$

with $f(t)$ undetermined on $[10, \infty)$.

Consider a supplemental warranty on this robot that pays 100,000 at the time T of its first failure if $2 \leq T \leq 10$, with no benefits payable otherwise.

You are also given that $\delta = 5\%$.

Calculate the 90th percentile of the present value of the future benefits under this warranty.

- (A) 82,000
- (B) 84,000
- (C) 87,000
- (D) 91,000
- (E) 95,000

[Question 5 on the Spring 2017 exam]

4.19. (80) purchases a whole life insurance policy of 100,000.

You are given:

- (i) The policy is priced with a select period of one year
- (ii) The select mortality rate equals 80% of the mortality rate from the Standard Ultimate Life Table
- (iii) Ultimate mortality follows the Standard Ultimate Life Table
- (iv) $i = 0.05$

Calculate the actuarial present value of the death benefits for this insurance

- (A) 58,950
- (B) 59,050
- (C) 59,150
- (D) 59,250
- (E) 59,350

[A modified version of Question 5 on the Fall 2017 exam]

5.1. You are given:

- (i) $\delta_t = 0.06, \quad t \geq 0$
- (ii) $\mu_x(t) = 0.01, \quad t \geq 0$
- (iii) Y is the present value random variable for a continuous annuity of 1 per year, payable for the lifetime of (x) with 10 years certain

Calculate $\Pr(Y > E[Y])$.

- (A) 0.705
- (B) 0.710
- (C) 0.715
- (D) 0.720
- (E) 0.725

[Question 21 on the Spring 2013 exam]

5.2. You are given:

- (i) $A_x = 0.30$
- (ii) $A_{x+n} = 0.40$
- (iii) $A_{\overline{x:n}|} = 0.35$
- (iv) $i = 0.05$

Calculate $a_{\overline{x:n}|}$.

- (A) 9.3
- (B) 9.6
- (C) 9.8
- (D) 10.0
- (E) 10.3

[Question 1 on the Fall 2013 exam]

5.3. You are given:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) Deaths are uniformly distributed over each year of age
- (iii) $i = 0.05$

Calculate $\frac{d}{dt}(\bar{Ia})_{40:\overline{1}|}$ at $t = 10.5$.

- (A) 5.8
- (B) 6.0
- (C) 6.2
- (D) 6.4
- (E) 6.6

[A modified version of Question 19 on the Fall 2017 exam]

5.4. (40) wins the SOA lottery and will receive both:

- A deferred life annuity of K per year, payable continuously, starting at age $40 + \overset{\circ}{e}_{40}$ and
- An annuity certain of K per year, payable continuously, for $\overset{\circ}{e}_{40}$ years

You are given:

- (i) $\mu = 0.02$
- (ii) $\delta = 0.01$
- (iii) The actuarial present value of the payments is 10,000

Calculate K .

- (A) 214
- (B) 216
- (C) 218
- (D) 220
- (E) 222

[A modified version of Question 5 on the Fall 2013 exam]

5.5. For an annuity-due that pays 100 at the beginning of each year that (45) is alive, you are given:

- (i) Mortality for standard lives follows the Standard Ultimate Life Table
- (ii) The force of mortality for standard lives age $45+t$ is represented as μ_{45+t}^{SULT}
- (iii) The force of mortality for substandard lives age $45+t$, μ_{45+t}^S , is defined as:

$$\mu_{45+t}^S = \begin{cases} \mu_{45+t}^{SULT} + 0.05, & \text{for } 0 \leq t < 1 \\ \mu_{45+t}^{SULT}, & \text{for } t \geq 1 \end{cases}$$

- (iv) $i = 0.05$

Calculate the actuarial present value of this annuity for a substandard life age 45.

- (A) 1700
- (B) 1710
- (C) 1720
- (D) 1730
- (E) 1740

[A modified version of Question 4 on the Fall 2017 exam]

5.6. For a group of 100 lives age x with independent future lifetimes, you are given:

- (i) Each life is to be paid 1 at the beginning of each year, if alive
- (ii) $A_x = 0.45$
- (iii) ${}^2A_x = 0.22$
- (iv) $i = 0.05$
- (v) Y is the present value random variable of the aggregate payments.

Using the normal approximation to Y , calculate the initial size of the fund needed in order to be 95% certain of being able to make the payments for these life annuities.

- (A) 1170
- (B) 1180
- (C) 1190
- (D) 1200
- (E) 1210

[Question 6 on the Spring 2014 exam]

5.7. You are given:

- (i) $A_{35} = 0.188$
- (ii) $A_{65} = 0.498$
- (iii) ${}_{30}p_{35} = 0.883$
- (iv) $i = 0.04$

Calculate $1000\ddot{a}_{35:\overline{30}|}^{(2)}$ using the two-term Woolhouse approximation.

- (A) 17,060
- (B) 17,310
- (C) 17,380
- (D) 17,490
- (E) 17,530

[Question 7 on the Spring 2015 exam]

5.8. For an annual whole life annuity-due of 1 with a 5-year certain period on (55), you are given:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) $i = 0.05$

Calculate the probability that the sum of the undiscounted payments actually made under this annuity will exceed the expected present value, at issue, of the annuity.

- (A) 0.88
- (B) 0.90
- (C) 0.92
- (D) 0.94
- (E) 0.96

[A modified version of Question 6 on the Spring 2017 exam]

5.9. For a select and ultimate mortality model with a one-year select period, you are given:

- (i) $p_{[x]} = (1 + k)p_x$, for some constant k
- (ii) $\ddot{a}_{x:\overline{n}|} = 21.854$
- (iii) $\ddot{a}_{[x]:\overline{n}|} = 22.167$

Calculate k .

- (A) 0.005
- (B) 0.010
- (C) 0.015
- (D) 0.020
- (E) 0.025

[Question 5 on the Spring 2016 exam]

6.1. You are given the following information about a special fully discrete 2-payment, 2-year term insurance on (80):

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) $i = 0.03$
- (iii) The death benefit is 1000 plus a return of all premiums paid without interest
- (iv) Level premiums are calculated using the equivalence principle

Calculate the net premium for this special insurance.

- (A) 32
- (B) 33
- (C) 34
- (D) 35
- (E) 36

[A modified version of Question 22 on the Fall 2012 exam]

6.2. For a fully discrete 10-year term life insurance policy on (x) , you are given:

- (i) Death benefits are 100,000 plus the return of all gross premiums paid without interest
- (ii) Expenses are 50% of the first year's gross premium, 5% of renewal gross premiums and 200 per policy expenses each year
- (iii) Expenses are payable at the beginning of the year
- (iv) $A^1_{x:\overline{10}|} = 0.17094$
- (v) $(IA)^1_{x:\overline{10}|} = 0.96728$
- (vi) $\ddot{a}_{x:\overline{10}|} = 6.8865$

Calculate the gross premium using the equivalence principle.

- (A) 3200
- (B) 3300
- (C) 3400
- (D) 3500
- (E) 3600

[Question 25 on the Fall 2012 exam]

- 6.3.** S, now age 65, purchased a 20-year deferred whole life annuity-due of 1 per year at age 45. You are given:
- (i) Equal annual premiums, determined using the equivalence principle, were paid at the beginning of each year during the deferral period
 - (ii) Mortality at ages 65 and older follows the Standard Ultimate Life Table
 - (iii) $i = 0.05$
 - (iv) Y is the present value random variable at age 65 for S's annuity benefits
- Calculate the probability that Y is less than the actuarial accumulated value of S's premiums.
- (A) 0.35
 - (B) 0.37
 - (C) 0.39
 - (D) 0.41
 - (E) 0.43

[A modified version of Question 20 on the Fall 2012 exam]

- 6.4.** For whole life annuities-due of 15 per month on each of 200 lives age 62 with independent future lifetimes, you are given:
- (i) $i = 0.06$
 - (ii) $A_{62}^{(12)} = 0.4075$ and ${}^2A_{62}^{(12)} = 0.2105$
 - (iii) π is the single premium to be paid by each of the 200 lives
 - (iv) S is the present value random variable at time 0 of total payments made to the 200 lives

Using the normal approximation, calculate π such that $\Pr(200\pi > S) = 0.90$.

- (A) 1850
- (B) 1860
- (C) 1870
- (D) 1880
- (E) 1890

[Question 19 on the Fall 2012 exam]

- 6.5.** For a fully discrete whole life insurance of 1000 on (30), you are given:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) $i = 0.05$
- (iii) The premium is the net premium

Calculate the first year for which the expected present value at issue of that year's premium is less than the expected present value at issue of that year's benefit.

- (A) 21
- (B) 25
- (C) 29
- (D) 33
- (E) 37

[A modified version of Question 1 on the Spring 2013 exam]

6.6. For fully discrete whole life insurance policies of 10,000 issued on 600 lives with independent future lifetimes, each age 62, you are given:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) $i = 0.05$
- (iii) Expenses of 5% of the first year gross premium are incurred at issue
- (iv) Expenses of 5 per policy are incurred at the beginning of each policy year
- (v) The gross premium is 103% of the net premium.
- (vi) ${}_0L$ is the aggregate present value of future loss at issue random variable

Calculate $\Pr({}_0L < 40,000)$, using the normal approximation.

- (A) 0.75
- (B) 0.79
- (C) 0.83
- (D) 0.87
- (E) 0.91

[A modified version of Question 15 on the Spring 2013 exam]

- 6.7. For a special fully discrete 20-year endowment insurance on (40), you are given:
- (i) The only death benefit is the return of annual net premiums accumulated with interest at 5% to the end of the year of death
 - (ii) The endowment benefit is 100,000
 - (iii) Mortality follows the Standard Ultimate Life Table
 - (iv) $i = 0.05$
- Calculate the annual net premium.

- (A) 2680
- (B) 2780
- (C) 2880
- (D) 2980
- (E) 3080

[A modified version of Question 3 on the Spring 2013 exam]

6.8. For a fully discrete whole life insurance on (60), you are given:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) $i = 0.05$
- (iii) The expected company expenses, payable at the beginning of the year, are:
 - 50 in the first year
 - 10 in years 2 through 10
 - 5 in years 11 through 20
 - 0 after year 20

Calculate the level annual amount that is actuarially equivalent to the expected company expenses.

- (A) 7.5
- (B) 9.5
- (C) 11.5
- (D) 13.5
- (E) 15.5

[Question 2 on the Spring 2013 exam]

6.9. For a fully discrete 20-year term insurance of 100,000 on (50), you are given:

- (i) Gross premiums are payable for 10 years
- (ii) Mortality follows the Standard Ultimate Life Table
- (iii) $i = 0.05$
- (iv) Expenses are incurred at the beginning of each year as follows:

| | Year 1 | Years 2-10 | Years 11-20 |
|-------------------------------|--------|------------|----------------|
| Commission as % of premium | 40% | 10% | Not applicable |
| Premium taxes as % of premium | 2% | 2% | Not applicable |
| Maintenance expenses | 75 | 25 | 25 |

- (v) Gross premiums are calculated using the equivalence principle

Calculate the gross premium for this insurance.

- (A) 617
- (B) 627
- (C) 637
- (D) 647
- (E) 657

[A modified version of Question 9 on the Fall 2013 exam]

6.10. For a fully discrete 3-year term insurance of 1000 on (x) , you are given:

- (i) $p_x = 0.975$
- (ii) $i = 0.06$
- (iii) The actuarial present value of the death benefit is 152.85
- (iv) The annual net premium is 56.05

Calculate p_{x+2} .

- (A) 0.88
- (B) 0.89
- (C) 0.90
- (D) 0.91
- (E) 0.92

[A modified version of Question 15 on the Fall 2013 exam]

6.11. For fully discrete whole life insurances of 1 issued on lives age 50, the annual net premium, P , was calculated using the following:

(i) $q_{50} = 0.0048$

(ii) $i = 0.04$

(iii) $A_{51} = 0.39788$

A particular life has a first-year mortality rate 10 times the rate used to calculate P . The mortality rates for all other years are the same as the ones used to calculate P .

Calculate the expected present value of the loss at issue random variable for this life, based on the premium P .

(A) 0.025

(B) 0.033

(C) 0.041

(D) 0.049

(E) 0.057

[A modified version of Question 16 on the Fall 2013 exam]

6.12. For a fully discrete whole life insurance of 1000 on (x) , you are given:

(i) The following expenses are incurred at the beginning of each year:

| | Year 1 | Years 2+ |
|----------------------|--------|----------|
| Percent of premium | 75% | 10% |
| Maintenance expenses | 10 | 2 |

(ii) An additional expense of 20 is paid when the death benefit is paid

(iii) The gross premium is determined using the equivalence principle

(iv) $i = 0.06$

(v) $\ddot{a}_x = 12.0$

(vi) ${}^2A_x = 0.14$

Calculate the variance of the loss at issue random variable.

(A) 14,600

(B) 33,100

(C) 51,700

(D) 70,300

(E) 88,900

[Question 18 on the Fall 2013 exam]

- 6.13.** For a fully discrete whole life insurance of 10,000 on (45), you are given:
- (i) Commissions are 80% of the first year premium and 10% of subsequent premiums. There are no other expenses
 - (ii) Mortality follows the Standard Ultimate Life Table
 - (iii) $i = 0.05$
 - (iv) ${}_0L$ denotes the loss at issue random variable
 - (v) If $T_{45} = 10.5$, then ${}_0L = 4953$

Calculate $E[{}_0L]$.

- (A) -580
- (B) -520
- (C) -460
- (D) -400
- (E) -340

[A modified version of Question 19 on the Fall 2013 exam]

- 6.14.** For a special fully discrete whole life insurance of 100,000 on (40), you are given:
- (i) The annual net premium is P for years 1 through 10, $0.5P$ for years 11 through 20, and 0 thereafter
 - (ii) Mortality follows the Standard Ultimate Life Table
 - (iii) $i = 0.05$

Calculate P .

- (A) 850
- (B) 950
- (C) 1050
- (D) 1150
- (E) 1250

[A modified version of Question 8 on the Spring 2014 exam]

- 6.15.** For a fully discrete whole life insurance of 1000 on (x) with net premiums payable quarterly, you are given:
- (i) $i = 0.05$
 - (ii) $\ddot{a}_x = 3.4611$
 - (iii) $P^{(W)}$ and $P^{(UDD)}$ are the annualized net premiums calculated using the 2-term Woolhouse (W) and the uniform distribution of deaths (UDD) assumptions, respectively

Calculate $\frac{P^{(UDD)}}{P^{(W)}}$.

- (A) 1.000
- (B) 1.002
- (C) 1.004
- (D) 1.006
- (E) 1.008

[A modified version of Question 9 on the Spring 2014 exam]

6.16. For a fully discrete 20-year endowment insurance of 100,000 on (30), you are given:

(i) $d = 0.05$

(ii) Expenses, payable at the beginning of each year, are:

| | First Year | | Renewal Years | |
|--------------------|--------------------|------------|--------------------|------------|
| | Percent of Premium | Per Policy | Percent of Premium | Per Policy |
| Taxes | 4% | 0 | 4% | 0 |
| Sales Commission | 35% | 0 | 2% | 0 |
| Policy Maintenance | 0% | 250 | 0% | 50 |

(iii) The net premium is 2143

Calculate the gross premium using the equivalence principle.

(A) 2410

(B) 2530

(C) 2800

(D) 3130

(E) 3280

[Question 10 on the Spring 2014 exam]

6.17. An insurance company sells special fully discrete two-year endowment insurance policies to smokers (S) and non-smokers (NS) age x . You are given:

- (i) The death benefit is 100,000; the maturity benefit is 30,000
- (ii) The level annual premium for non-smoker policies is determined by the equivalence principle
- (iii) The annual premium for smoker policies is twice the non-smoker annual premium
- (iv) $\mu_{x+t}^{\text{NS}} = 0.1, \quad t > 0$
- (v) $q_{x+k}^{\text{S}} = 1.5q_{x+k}^{\text{NS}}$ for $k = 0, 1$
- (vi) $i = 0.08$

Calculate the expected present value of the loss at issue random variable on a smoker policy.

- (A) $-30,000$
- (B) $-29,000$
- (C) $-28,000$
- (D) $-27,000$
- (E) $-26,000$

[Question 18 on the Spring 2013 exam]

- 6.18.** For a 20-year deferred whole life annuity-due with annual payments of 30,000 on (40), you are given:
- (i) The single net premium is refunded without interest at the end of the year of death if death occurs during the deferral period
 - (ii) Mortality follows the Standard Ultimate Life Table
 - (iii) $i = 0.05$

Calculate the single net premium for this annuity.

- (A) 162,000
- (B) 164,000
- (C) 165,200
- (D) 166,400
- (E) 168,800

[A modified version of Question 6 on the Fall 2014 exam]

- 6.19.** For a fully discrete whole life insurance of 1 on (50), you are given:
- (i) Expenses of 0.20 at the start of the first year and 0.01 at the start of each renewal year are incurred
 - (ii) Mortality follows the Standard Ultimate Life Table
 - (iii) $i = 0.05$
 - (iv) Gross premiums are determined using the equivalence principle.

Calculate the variance of L_0 , the gross loss-at-issue random variable.

- 0.023
- 0.028
- 0.033
- 0.038
- 0.043

[A modified version of Question 7 on the Fall 2014 exam]

6.20. For a special fully discrete 3-year term insurance on (75), you are given:

- (i) The death benefit during the first two years is the sum of the net premiums paid without interest
- (ii) The death benefit in the third year is 10,000
- (iii)

| x | p_x |
|-----|-------|
| 75 | 0.90 |
| 76 | 0.88 |
| 77 | 0.85 |

- (iv) $i = 0.04$

Calculate the annual net premium.

- (A) 449
- (B) 459
- (C) 469
- (D) 479
- (E) 489

[Question 8 on the Fall 2014 exam]

- 6.21.** For a special fully discrete 15-year endowment insurance on (75), you are given:
- (i) The death benefit is 1000
 - (ii) The endowment benefit is the sum of the net premiums paid without interest
 - (iii) $d = 0.04$
 - (iv) $A_{75:\overline{15}|} = 0.70$
 - (v) $A_{75:\overline{15}|}^{\frac{1}{d}} = 0.11$

Calculate the annual net premium.

- (A) 80
- (B) 90
- (C) 100
- (D) 110
- (E) 120

[Question 9 on the Fall 2014 exam]

- 6.22.** For a whole life insurance of 100,000 on (45) with premiums payable monthly for a period of 20 years, you are given:
- (i) The death benefit is paid immediately upon death
 - (ii) Mortality follows the Standard Ultimate Life Table
 - (iii) Deaths are uniformly distributed over each year of age
 - (iv) $i = 0.05$

Calculate the monthly net premium.

- (A) 98
- (B) 100
- (C) 102
- (D) 104
- (E) 106

[A modified version of Question 10 on the Fall 2014 exam]

6.23. For fully discrete 30-payment whole life insurance policies on (x) , you are given:

(i) The following expenses payable at the beginning of the year:

| | 1 st Year | Years 2 – 15 | Years 16 – 30 | Years 31 and after |
|--------------------|----------------------|-----------------|------------------|-----------------------|
| Per policy | 60 | 30 | 30 | 30 |
| Percent of premium | 80% | 20% | 10% | 0% |

(ii) $\ddot{a}_x = 15.3926$

(iii) $\ddot{a}_{x:\overline{15}|} = 10.1329$

(iv) $\ddot{a}_{x:\overline{30}|} = 14.0145$

(v) Annual gross premiums are calculated using the equivalence principle

(vi) The annual gross premium is expressed as $kF + h$, where F is the death benefit and k and h are constants for all F

Calculate h .

(A) 30.3

(B) 35.1

(C) 39.9

(D) 44.7

(E) 49.5

[Question 11 on the Fall 2014 exam]

6.24. For a fully continuous whole life insurance of 1 on (x) , you are given:

- (i) L is the present value of the loss at issue random variable if the premium rate is determined by the equivalence principle
- (ii) L^* is the present value of the loss at issue random variable if the premium rate is 0.06
- (iii) $\delta = 0.07$
- (iv) $\bar{A}_x = 0.30$
- (v) $Var(L) = 0.18$

Calculate $Var(L^*)$.

- (A) 0.18
- (B) 0.21
- (C) 0.24
- (D) 0.27
- (E) 0.30

[Question 8 on the Spring 2015 exam]

6.25. For a fully discrete 10-year deferred whole life annuity-due of 1000 per month on (55), you are given:

- (i) The premium, G , will be paid annually at the beginning of each year during the deferral period
- (ii) Expenses are expected to be 300 per year for all years, payable at the beginning of the year
- (iii) Mortality follows the Standard Ultimate Life Table
- (iv) $i = 0.05$
- (v) Using the two-term Woolhouse approximation, the expected loss at issue is -800

Calculate G .

- (A) 12,110
- (B) 12,220
- (C) 12,330
- (D) 12,440
- (E) 12,550

[A modified version of Question 9 on the Spring 2015 exam]

6.26. For a special fully discrete whole life insurance policy of 1000 on (90), you are given:

- (i) The first year premium is 0
- (ii) P is the renewal premium
- (iii) Mortality follows the Standard Ultimate Life Table
- (iv) $i = 0.05$
- (v) Premiums are calculated using the equivalence principle

Calculate P .

- (A) 150
- (B) 160
- (C) 170
- (D) 180
- (E) 190

[A modified version of Question 10 on the Spring 2015 exam]

6.27. For a special fully continuous whole life insurance on (x) , you are given:

(i) Premiums and benefits:

| | First 20 years | After 20 years |
|--------------|----------------|----------------|
| Premium Rate | $3P$ | P |
| Benefit | 1,000,000 | 500,000 |

(ii) $\mu_{x+t} = 0.03, t \geq 0$

(iii) $\delta = 0.06$

Calculate P using the equivalence principle.

(A) 10,130

(B) 10,190

(C) 10,250

(D) 10,310

(E) 10,370

[Question 11 on the Spring 2015 exam]

6.28. For a fully discrete 5-payment whole life insurance of 1000 on (40), you are given:

(i) Expenses incurred at the beginning of the first five policy years are as follows:

| | Year 1 | | Years 2-5 | |
|--------------------|--------------------|------------|--------------------|------------|
| | Percent of premium | Per policy | Percent of premium | Per policy |
| Sales Commissions | 20% | 0 | 5% | 0 |
| Policy Maintenance | 0% | 10 | 0% | 5 |

(ii) No expenses are incurred after Year 5

(iii) Mortality follows the Standard Ultimate Life Table

(iv) $i = 0.05$

Calculate the gross premium using the equivalence principle.

- (A) 31
- (B) 36
- (C) 41
- (D) 46
- (E) 51

[A modified version of Question 12 on the Spring 2015 exam]

6.29. (35) purchases a fully discrete whole life insurance policy of 100,000.

You are given:

- (i) The annual gross premium, calculated using the equivalence principle, is 1770
- (ii) The expenses in policy year 1 are 50% of premium and 200 per policy
- (iii) The expenses in policy years 2 and later are 10% of premium and 50 per policy
- (iv) All expenses are incurred at the beginning of the policy year
- (v) $i = 0.035$

Calculate \ddot{a}_{35} .

- (A) 20.0
- (B) 20.5
- (C) 21.0
- (D) 21.5
- (E) 22.0

[Question 7 on the Fall 2015 exam]

6.30. For a fully discrete whole life insurance of 100 on (x) , you are given:

- (i) The first year expense is 10% of the gross annual premium
- (ii) Expenses in subsequent years are 5% of the gross annual premium
- (iii) The gross premium calculated using the equivalence principle is 2.338
- (iv) $i = 0.04$
- (v) $\ddot{a}_x = 16.50$
- (vi) ${}^2A_x = 0.17$

Calculate the variance of the loss at issue random variable.

- (A) 900
- (B) 1200
- (C) 1500
- (D) 1800
- (E) 2100

[Question 8 on the Fall 2015 exam]

6.31. For a fully continuous whole life insurance policy of 100,000 on (35), you are given:

- (i) The density function of the future lifetime of a newborn:

$$f(t) = \begin{cases} 0.01e^{-0.01t}, & 0 \leq t < 70 \\ g(t), & t \geq 70 \end{cases}$$

- (ii) $\delta = 0.05$

- (iii) $\bar{A}_{70} = 0.51791$

Calculate the annual net premium rate for this policy.

- (A) 1000
(B) 1110
(C) 1220
(D) 1330
(E) 1440

[Question 10 on the Fall 2015 exam]

6.32. For a whole life insurance of 100,000 on (x), you are given:

- (i) Death benefits are payable at the moment of death
(ii) Deaths are uniformly distributed over each year of age
(iii) Premiums are payable monthly
(iv) $i = 0.05$
(v) $\ddot{a}_x = 9.19$

Calculate the monthly net premium.

- (A) 530
(B) 540
(C) 550
(D) 560
(E) 570

[Question 11 on the Fall 2015 exam]

6.33. An insurance company sells 15-year pure endowments of 10,000 to 500 lives, each age x , with independent future lifetimes. The single premium for each pure endowment is determined by the equivalence principle.

- (i) You are given:
- (ii) $i = 0.03$
- (iii) $\mu_x(t) = 0.02t, \quad t \geq 0$
- (iv) ${}_0L$ is the aggregate loss at issue random variable for these pure endowments.

Using the normal approximation without continuity correction, calculate $\Pr({}_0L > 50,000)$.

- (A) 0.08
- (B) 0.13
- (C) 0.18
- (D) 0.23
- (E) 0.28

[Question 12 on the Fall 2013 exam]

- 6.34.** For a fully discrete whole life insurance policy on (61), you are given:
- (i) The annual gross premium using the equivalence principle is 500
 - (ii) Initial expenses, incurred at policy issue, are 15% of the premium
 - (iii) Renewal expenses, incurred at the beginning of each year after the first, are 3% of the premium
 - (iv) Mortality follows the Standard Ultimate Life Table
 - (v) $i = 0.05$

Calculate the amount of the death benefit.

- (A) 23,300
- (B) 23,400
- (C) 23,500
- (D) 23,600
- (E) 23,700

[A modified version of Question 17 on the Spring 2015 exam]

- 6.35.** For a fully discrete whole life insurance policy of 100,000 on (35), you are given:
- (i) First year commissions are 19% of the annual gross premium
 - (ii) Renewal year commissions are 4% of the annual gross premium
 - (iii) Mortality follows the Standard Ultimate Life Table
 - (iv) $i = 0.05$

Calculate the annual gross premium for this policy using the equivalence principle.

- (A) 410
- (B) 450
- (C) 490
- (D) 530
- (E) 570

[A modified version of Question 7 on the Spring 2016 exam]

- 6.36.** For a fully continuous 20-year term insurance policy of 100,000 on (50), you are given:
- (i) Gross premiums, calculated using the equivalence principle, are payable at an annual rate of 4500
 - (ii) Expenses at an annual rate of R are payable continuously throughout the life of the policy
 - (iii) $\mu_{50+t} = 0.04$, for $t > 0$
 - (iv) $\delta = 0.08$
- Calculate R .

- (A) 400
- (B) 500
- (C) 600
- (D) 700
- (E) 800

[Question 8 on the Spring 2016 exam]

- 6.37.** For a fully discrete whole life insurance policy of 50,000 on (35), with premiums payable for a maximum of 10 years, you are given:
- (i) Expenses of 100 are payable at the end of each year including the year of death
 - (ii) Mortality follows the Standard Ultimate Life Table
 - (iii) $i = 0.05$
- Calculate the annual gross premium using the equivalence principle.

- (A) 790
- (B) 800
- (C) 810
- (D) 820
- (E) 830

[A modified version of Question 9 on the Spring 2016 exam]

- 6.38.** For an n -year endowment insurance of 1000 on (x) , you are given:
- (i) Death benefits are payable at the moment of death
 - (ii) Premiums are payable annually at the beginning of each year
 - (iii) Deaths are uniformly distributed over each year of age
 - (iv) $i = 0.05$
 - (v) ${}_nE_x = 0.172$
 - (vi) $\bar{A}_{x:\overline{n}|} = 0.192$

Calculate the annual net premium for this insurance.

- (A) 10.1
- (B) 11.3
- (C) 12.5
- (D) 13.7
- (E) 14.9

[Question 10 on the Spring 2016 exam]

- 6.39.** XYZ Insurance writes 10,000 fully discrete whole life insurance policies of 1000 on lives age 40 and an additional 10,000 fully discrete whole life policies of 1000 on lives age 80.

XYZ used the following assumptions to determine the net premiums for these policies:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) $i = 0.05$

During the first ten years, mortality did follow the Standard Ultimate Life Table.

Calculate the average net premium per policy in force received at the beginning of the eleventh year.

- (A) 29
- (B) 32
- (C) 35
- (D) 38
- (E) 41

[A modified version of Question 11 on the Spring 2016 exam]

- 6.40.** For a special fully discrete whole life insurance, you are given:

- (i) The death benefit is $1000(1.03)^k$ for death in policy year k , for $k = 1, 2, 3, \dots$
- (ii) $q_x = 0.05$
- (iii) $i = 0.06$
- (iv) $\ddot{a}_{x+1} = 7.00$
- (v) The annual net premium for this insurance at issue age x is 110

Calculate the annual net premium for this insurance at issue age $x + 1$.

- (A) 110
- (B) 112
- (C) 116
- (D) 120
- (E) 122

[Question 17 on the Spring 2016 exam]

6.41. For a special fully discrete 2-year term insurance on (x) , you are given:

- (i) $q_x = 0.01$
 - (ii) $q_{x+1} = 0.02$
 - (iii) $i = 0.05$
 - (iv) The death benefit in the first year is 100,000
 - (v) Both the benefits and premiums increase by 1% in the second year
- Calculate the annual net premium in the first year.

- (A) 1410
- (B) 1417
- (C) 1424
- (D) 1431
- (E) 1438

[Question 9 on the Fall 2016 exam]

6.42. For a fully discrete 3-year endowment insurance of 1000 on (x) , you are given:

- (i) $\mu_{x+t} = 0.06$, for $0 \leq t \leq 3$
 - (ii) $\delta = 0.06$
 - (iii) The annual premium is 315.80
 - (iv) L_0 is the present value random variable for the loss at issue for this insurance
- Calculate $\Pr[L_0 > 0]$.

- (A) 0.03
- (B) 0.06
- (C) 0.08
- (D) 0.11
- (E) 0.15

[Question 10 on the Fall 2016 exam]

6.43. For a fully discrete, 5-payment 10-year term insurance of 200,000 on (30), you are given:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) The following expenses are incurred at the beginning of each respective year:

| | Year 1 | | Years 2 - 10 | |
|-------------|--------------------|------------|--------------------|------------|
| | Percent of Premium | Per Policy | Percent of Premium | Per Policy |
| Taxes | 5% | 0 | 5% | 0 |
| Commissions | 30% | 0 | 10% | 0 |
| Maintenance | 0% | 8 | 0% | 4 |

- (iii) $i = 0.05$
- (iv) $\ddot{a}_{30:\overline{5}|} = 4.5431$

Calculate the annual gross premium using the equivalence principle.

- (A) 150
- (B) 160
- (C) 170
- (D) 180
- (E) 190

[A modified version of Question 11 on the Fall 2016 exam]

6.44. For a special fully discrete 10-year deferred whole life insurance of 100 on (50), you are given:

- (i) Premiums are payable annually, at the beginning of each year, only during the deferral period
- (ii) For deaths during the deferral period, the benefit is equal to the return of all premiums paid, without interest
- (iii) $i = 0.05$
- (iv) $\ddot{a}_{50} = 17.0$
- (v) $\ddot{a}_{60} = 15.0$
- (vi) ${}_{10}E_{50} = 0.60$
- (vii) $(IA)_{50:\overline{10}|}^1 = 0.15$

Calculate the annual net premium for this insurance.

- (A) 1.3
- (B) 1.6
- (C) 1.9
- (D) 2.2
- (E) 2.5

[Question 7 on the Spring 2017 exam]

6.45. For a fully continuous whole life insurance of 100,000 on (35), you are given:

- (i) The annual rate of premium is 560
- (ii) Mortality follows the Standard Ultimate Life Table
- (iii) Deaths are uniformly distributed over each year of age
- (iv) $i = 0.05$

Calculate the 75th percentile of the loss at issue random variable for this policy.

- (A) 610
- (B) 630
- (C) 650
- (D) 670
- (E) 690

[A modified version of Question 8 on the Spring 2017 exam]

- 6.46.** For a special 10-year deferred whole life annuity-due of 300 per year issued to (55), you are given:
- (i) Annual premiums are payable for 10 years
 - (ii) If death occurs during the deferral period, all premiums paid are returned without interest at the end of the year of death
 - (iii) $\ddot{a}_{55} = 12.2758$
 - (iv) $\ddot{a}_{55:\overline{10}|} = 7.4575$
 - (v) $(IA)_{55:\overline{10}|}^1 = 0.51213$

Calculate the level net premium.

- (A) 195
- (B) 198
- (C) 201
- (D) 204
- (E) 208

[Question 7 on the Fall 2017 exam]

6.47. For a 10-year deferred whole life annuity-due with payments of 100,000 per year on (70), you are given:

- (i) Annual gross premiums of G are payable for 10 years
- (ii) First year expenses are 75% of premium
- (iii) Renewal expenses for years 2 and later are 5% of premium during the premium paying period
- (iv) Mortality follows the Standard Ultimate Life Table
- (v) $i = 0.05$

Calculate G using the equivalence principle.

- (A) 64,900
- (B) 65,400
- (C) 65,900
- (D) 66,400
- (E) 66,900

[A modified version of Question 8 on the Fall 2017 exam]

6.48. For a special fully discrete 5-year deferred 3-year term insurance of 100,000 on (x) you are given:

- (i) There are two premium payments, each equal to P . The first is paid at the beginning of the first year and the second is paid at the end of the 5-year deferral period
- (ii) The following probabilities:
- (iii) ${}_5p_x = 0.95$
- (iv) $q_{x+5} = 0.02$, $q_{x+6} = 0.03$, $q_{x+7} = 0.04$
- (v) $i = 0.06$

Calculate P using the equivalence principle.

- (A) 3195
- (B) 3345
- (C) 3495
- (D) 3645
- (E) 3895

[Question 9 on the Fall 2017 exam]

6.49. For a special whole life insurance of 100,000 on (40), you are given:

- (i) The death benefit is payable at the moment of death
 - (ii) Level gross premiums are payable monthly for a maximum of 20 years
 - (iii) Mortality follows the Standard Ultimate Life Table
 - (iv) $i = 0.05$
 - (v) Deaths are uniformly distributed over each year of age
 - (vi) Initial expenses are 200
 - (vii) Renewal expenses are 4% of each premium including the first
 - (viii) Gross premiums are calculated using the equivalence principle
- Calculate the monthly gross premium.

- (A) 66
- (B) 76
- (C) 86
- (D) 96
- (E) 106

[A modified version of Question 10 on the Fall 2017 exam]

- 6.50.** On July 15, 2017, XYZ Corp buys fully discrete whole life insurance policies of 1,000 on each of its 10,000 workers, all age 35. It uses the death benefits to partially pay the premiums for the following year.

You are given:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) $i = 0.05$
- (iii) The insurance is priced using the equivalence principle

Calculate XYZ Corp's expected net cash flow from these policies during July 2018.

- (A) -47,000
- (B) -48,000
- (C) -49,000
- (D) -50,000
- (E) -51,000

[A modified version of Question 13 on the Fall 2017 exam]

6.51. For a special 10-year deferred whole life annuity-due of 50,000 on (62), you are given:

- (i) Level annual net premiums are payable for 10 years
- (ii) A death benefit, payable at the end of the year of death, is provided only over the deferral period and is the sum of the net premiums paid without interest
- (iii) $\ddot{a}_{62} = 12.2758$
- (iv) $\ddot{a}_{62:\overline{10}|} = 7.4574$
- (v) $A^1_{62:\overline{10}|} = 0.0910$
- (vi) $\sum_{k=1}^{10} A^1_{62:\overline{k}|} = 0.4891$

Calculate the net premium for this special annuity.

- (A) 34,400
- (B) 34,500
- (C) 34,600
- (D) 34,700
- (E) 34,800

[A modified version of Question 14 on the Fall 2013 exam]

6.52. For a fully discrete 10-payment whole life insurance of H on (45) , you are given:

(i) Expenses payable at the beginning of each year are as follows:

| Expense Type | First Year | Years 2-10 | Years 11+ |
|--------------|------------|------------|-----------|
| Per policy | 100 | 20 | 10 |
| % of Premium | 105% | 5% | 0% |

(ii) Mortality follows the Standard Ultimate Life Table

(iii) $i = 0.05$

(iv) The gross annual premium, calculated using the equivalence principle, is of the form $G = gH + f$, where g is the premium rate per 1 of insurance and f is the per policy fee

Calculate f .

- (A) 42.00
- (B) 44.20
- (C) 46.40
- (D) 48.60
- (E) 50.80

[A modified version of Question 11 on the Spring 2017 exam]

- 6.53.** A warranty pays 2000 at the end of the year of the first failure if a washing machine fails within three years of purchase. The warranty is purchased with a single premium, G , paid at the time of purchase of the washing machine.

You are given:

- (i) 10% of the washing machines that are working at the start of each year fail by the end of that year
- (ii) $i = 0.08$
- (iii) The sales commission is 35% of G
- (iv) G is calculated using the equivalence principle

Calculate G .

- (A) 630
- (B) 660
- (C) 690
- (D) 720
- (E) 750

[Question 12 on the Spring 2017 exam]

- 6.54.** For a fully discrete whole life insurance of 200,000 on (45), you are given:

- (i) Mortality follows the Standard Ultimate Life Table.
- (ii) $i = 0.05$
- (iii) The annual premium is determined using the equivalence principle.

Calculate the standard deviation of L_0 , the present value random variable for the loss at issue.

- (A) 25,440
- (B) 30,440
- (C) 35,440
- (D) 40,440
- (E) 45,440

[A modified version of Question 12 on the Fall 2017 exam]

7.1. For a special fully discrete whole life insurance on (40), you are given:

- (i) The death benefit is 50,000 in the first 20 years and 100,000 thereafter
- (ii) Level net premiums of 875 are payable for 20 years
- (iii) Mortality follows the Standard Ultimate Life Table
- (iv) $i = 0.05$

Calculate $_{10}V$, the net premium policy value at the end of year 10 for this insurance.

- (A) 11,090
- (B) 11,120
- (C) 11,150
- (D) 11,180
- (E) 11,210

[A modified version of Question 4 on the Fall 2012 exam]

7.2. A special fully discrete 2-year endowment insurance with a maturity value of 2000 is issued to (x). The death benefit is 2000 plus the net premium policy value at the end of the year of death. For year 2, the net premium policy value is the net premium policy value just before the maturity benefit is paid.

You are given:

- (i) $i = 0.10$
- (ii) $q_x = 0.150$ and $q_{x+1} = 0.165$

Calculate the level annual net premium.

- (A) 1070
- (B) 1110
- (C) 1150
- (D) 1190
- (E) 1230

[A modified version of Question 5 on the Fall 2012 exam]

7.3. For a whole life insurance of 1000 with semi-annual premiums on (80), you are given:

- (i) A gross premium of 60 is payable every 6 months starting at age 80
- (ii) Commissions of 10% are paid each time a premium is paid
- (iii) Death benefits are paid at the end of the quarter of death
- (iv) ${}_tV$ denotes the gross premium policy value at time t , $t \geq 0$
- (v) ${}_{10.75}V = 753.72$
- (vi)

| t | l_{90+t} |
|------|------------|
| 0 | 1000 |
| 0.25 | 898 |
| 0.50 | 800 |
| 0.75 | 706 |

- (vii) $i^{(4)} = 0.08$

Calculate ${}_{10.25}V$.

- (A) 680
- (B) 690
- (C) 700
- (D) 710
- (E) 730

[A modified version of Question 17 on the Fall 2012 exam]

- 7.4. For a special fully discrete whole life insurance on (40), you are given:
- (i) The death benefit is 1000 during the first 11 years and 5000 thereafter
 - (ii) Expenses, payable at the beginning of the year, are 100 in year 1 and 10 in years 2 and later
 - (iii) π is the level annual premium, determined using the equivalence principle
 - (iv) $G = 1.02 \times \pi$ is the level annual gross premium
 - (v) Mortality follows the Standard Ultimate Life Table
 - (vi) $i = 0.05$
 - (vii) ${}_{11}E_{40} = 0.57949$

Calculate the gross premium policy value at the end of year 1 for this insurance.

- (A) -82
- (B) -74
- (C) -66
- (D) -58
- (E) -50

[A modified version of Question 18 on the Fall 2012 exam]

7.5. For a fully discrete whole life insurance of 10,000 on (x) , you are given:

- (i) Deaths are uniformly distributed over each year of age
- (ii) The net premium is 647.46
- (iii) The net premium policy value at the end of year 4 is 1405.08
- (iv) $q_{x+4} = 0.04561$
- (v) $i = 0.03$

Calculate the net premium policy value at the end of 4.5 years.

- (A) 1570
- (B) 1680
- (C) 1750
- (D) 1830
- (E) 1900

[A modified version of Question 9 on the Spring 2013 exam]

7.6. For a fully discrete whole life insurance policy of 2000 on (45), you are given:

- (i) The gross premium is calculated using the equivalence principle
- (ii) Expenses, payable at the beginning of the year, are:

| | % of Premium | Per 1000 | Per Policy |
|---------------|--------------|----------|------------|
| First year | 25% | 1.5 | 30 |
| Renewal years | 5% | 0.5 | 10 |

- (iii) Mortality follows the Standard Ultimate Life Table
- (iv) $i = 0.05$

Calculate the expense reserve at the end of policy year 10.

- (A) -2
- (B) -8
- (C) -12
- (D) -21
- (E) -25

[A modified version of Question 16 on the Spring 2013 exam]

7.7. For a whole life insurance of 10,000 on (x) , you are given:

- (i) Death benefits are payable at the end of the year of death
- (ii) A premium of 30 is payable at the start of each month
- (iii) Commissions are 5% of each premium
- (iv) Expenses of 100 are payable at the start of each year
- (v) $i = 0.05$
- (vi) $1000A_{x+10} = 400$
- (vii) ${}_{10}V$ is the gross premium policy value at the end of year 10 for this insurance

Calculate ${}_{10}V$ using the two-term Woolhouse formula for annuities.

- (A) 950
- (B) 980
- (C) 1010
- (D) 1110
- (E) 1140

[Question 22 on the Spring 2013 exam]

7.8. For a fully discrete whole life insurance of 1000 on a select life [70], you are given:

- (i) Ultimate mortality follows the Standard Ultimate Life Table
- (ii) During the three-year select period, $q_{[x]+k} = (0.7 + 0.1k)q_{x+k}$, $k = 0, 1, 2$
- (iii) $i = 0.05$
- (iv) The net premium for this insurance is 35.168

Calculate ${}_1V$, the net premium policy value at the end of year 1 for this insurance.

- (A) 25.25
- (B) 27.30
- (C) 29.85
- (D) 31.60
- (E) 33.35

[A modified version of Question 6 on the Fall 2013 exam]

7.9. For a semi-continuous 20-year endowment insurance of 100,000 on (45), you are given:

- (i) Net premiums of 253 are payable monthly
- (ii) Mortality follows the Standard Ultimate Life Table
- (iii) Deaths are uniformly distributed over each year of age
- (iv) $i = 0.05$

Calculate ${}_{10}V$, the net premium policy value at the end of year 10 for this insurance.

- (A) 38,100
- (B) 38,300
- (C) 38,500
- (D) 38,700
- (E) 38,900

[A modified version of Question 7 on the Fall 2013 exam]

7.10. For a fully discrete whole life insurance of 100,000 on (45), you are given:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) $i = 0.05$
- (iii) Commission expenses are 60% of the first year's gross premium and 2% of renewal gross premiums
- (iv) Administrative expenses are 500 in the first year and 50 in each renewal year
- (v) All expenses are payable at the start of the year
- (vi) The gross premium, calculated using the equivalence principle, is 977.60

Calculate ${}_5V^e$, the expense reserve at the end of year 5 for this insurance.

- (A) -1070
- (B) -1020
- (C) -970
- (D) -920
- (E) -870

[A modified version of Question 8 on the Fall 2013 exam]

7.11. For a fully discrete whole life insurance of 10,000 on (45), you are given:

- (i) $i = 0.05$
- (ii) ${}_0L$ denotes the loss at issue random variable based on the net premium
- (iii) If $K_{45} = 10$, then ${}_0L = 4450$
- (iv) $\ddot{a}_{55} = 13.4205$

Calculate ${}_{10}V$, the net premium policy value at the end of year 10 for this insurance.

- (A) 1010
- (B) 1460
- (C) 1820
- (D) 2140
- (E) 2300

[A modified version of Question 17 on the Fall 2013 exam]

7.12. For a special fully discrete 25-year endowment insurance on (44), you are given:

- (i) The death benefit is $(26 - k)$ for death in year k , for $k = 1, 2, 3 \dots 25$
- (ii) The endowment benefit in year 25 is 1
- (iii) Net premiums are level
- (iv) $q_{55} = 0.15$
- (v) $i = 0.04$
- (vi) $_{11}V$, the net premium policy value at the end of year 11, is 5.00
- (vii) $_{24}V$, the net premium policy value at the end of year 24, is 0.60

Calculate $_{12}V$, the net premium policy value at end of year 12.

- (A) 3.63
- (B) 3.74
- (C) 3.88
- (D) 3.98
- (E) 4.09

[A modified version of Question 13 on the Spring 2014 exam]

7.13. For a fully discrete 30-year endowment insurance of 1000 on (40), you are given:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) $i = 0.05$

Calculate the full preliminary term (FPT) reserve for this policy at the end of year 10.

- (A) 180
- (B) 185
- (C) 190
- (D) 195
- (E) 200

[A modified version of Question 14 on the Spring 2014 exam]

7.14. For a fully discrete whole life insurance of 100,000 on (45), you are given:

- (i) The gross premium policy value at duration 5 is 5500 and at duration 6 is 7100
- (ii) $q_{50} = 0.009$
- (iii) $i = 0.05$
- (iv) Renewal expenses at the start of each year are 50 plus 4% of the gross premium.
- (v) Claim expenses are 200.

Calculate the gross premium.

- (A) 2200
- (B) 2250
- (C) 2300
- (D) 2350
- (E) 2400

[A modified version of Question 13 on the Fall 2014 exam]

7.15. For a fully discrete whole life insurance of 100 on (x), you are given:

- (i) $q_{x+15} = 0.10$
- (ii) Deaths are uniformly distributed over each year of age
- (iii) $i = 0.05$
- (iv) ${}_tV$ denotes the net premium policy value at time t
- (v) ${}_{16}V = 49.78$

Calculate ${}_{15.6}V$.

- (A) 49.7
- (B) 50.0
- (C) 50.3
- (D) 50.6
- (E) 50.9

[A modified version of Question 14 on the Fall 2014 exam]

7.16. For a fully discrete 5-payment whole life insurance of 1000 on (80), you are given:

- (i) The gross premium is 130
- (ii) $q_{80+k} = 0.01(k+1), \quad k = 0, 1, 2, \dots, 5$
- (iii) $v = 0.95$
- (iv) $1000A_{86} = 683$
- (v) ${}_3L$ is the prospective loss random variable at time 3, based on the gross premium

Calculate $E[{}_3L]$.

- (A) 330
- (B) 350
- (C) 360
- (D) 380
- (E) 390

[Question 15 on the Fall 2014 exam]

7.17. For a fully discrete whole life insurance of 1 on (x) , you are given:

- (i) $q_{x+10} = 0.02067$
- (ii) $v^2 = 0.90703$
- (iii) $A_{x+11} = 0.52536$
- (iv) ${}^2A_{x+11} = 0.30783$
- (v) ${}_kL$ is the prospective loss random variable at time k

Calculate $\frac{\text{Var}({}_{10}L)}{\text{Var}({}_{11}L)}$.

- (A) 1.006
- (B) 1.010
- (C) 1.014
- (D) 1.018
- (E) 1.022

[Question 16 on the Fall 2014 exam]

7.18. For a fully discrete whole life insurance of 1 on (x) , you are given:

- (i) The net premium policy value at the end of the first year is 0.012
- (ii) $q_x = 0.009$
- (iii) $i = 0.04$

Calculate \ddot{a}_x .

- (A) 17.1
- (B) 17.6
- (C) 18.1
- (D) 18.6
- (E) 19.1

[A modified version of Question 14 on the Spring 2015 exam]

- 7.19.** For a fully discrete whole life insurance of 100,000 on (40) you are given:
- (i) Expenses incurred at the beginning of the first year are 300 plus 50% of the first year premium
 - (ii) Renewal expenses, incurred at the beginning of the year, are 10% of each of the renewal premiums
 - (iii) Mortality follows the Standard Ultimate Life Table
 - (iv) $i = 0.05$
 - (v) Gross premiums are calculated using the equivalence principle

Calculate the gross premium policy value for this insurance immediately after the second premium and associated renewal expenses are paid.

- (A) 200
- (B) 340
- (C) 560
- (D) 720
- (E) 1060

[A modified version of Question 18 on the Spring 2015 exam]

7.20. For a fully discrete whole life insurance of 1000 on (35), you are given:

- (i) First year expenses are 30% of the gross premium plus 300
- (ii) Renewal expenses are 4% of the gross premium plus 30
- (iii) All expenses are incurred at the beginning of the policy year
- (iv) Gross premiums are calculated using the equivalence principle
- (v) The gross premium policy value at the end of the first policy year is R
- (vi) Using the Full Preliminary Term Method, the modified reserve at the end of the first policy year is S
- (vii) Mortality follows the Standard Ultimate Life Table
- (viii) $i = 0.05$

Calculate $R - S$.

- (A) -280
- (B) -140
- (C) 0
- (D) 140
- (E) 280

[A modified version of Question 15 on the Fall 2015 exam]

7.21. A special fully discrete 10-payment 10-year deferred whole life annuity-due on (55) of 1000 per year provides for a return of premiums without interest in the event of death within the first 10 years. You are given:

- (i) Annual net premiums are level
- (ii) Mortality follows the Standard Ultimate Life Table
- (iii) $i = 0.05$
- (iv) $(IA)_{55:\overline{10}|}^1 = 0.14743$

Calculate ${}_9V$, the net premium policy value at the end of year 9.

- (A) 11,540
- (B) 11,650
- (C) 11,760
- (D) 11,870
- (E) 11,980

[A modified version of Question 16 on the Fall 2015 exam]

7.22. For two fully discrete whole life insurance policies on (x) , you are given:

(i)

| | Death Benefit | Annual Net Premium | Variance of Loss at Issue |
|----------|------------------|-----------------------|------------------------------|
| Policy 1 | 8 | 1.250 | 20.55 |
| Policy 2 | 12 | 1.875 | W |

(ii) $i = 0.06$

(iii) The two policies are priced using the same mortality table.

Calculate W .

(A) 30.8

(B) 38.5

(C) 46.2

(D) 53.9

(E) 61.6

[Question 12 on the Spring 2016 exam]

7.23. For a 40-year endowment insurance of 10,000 issued to (25), you are given:

- (i) $i = 0.04$
- (ii) $p_{25} = 0.995$
- (iii) $\ddot{a}_{25:\overline{20}|} = 11.087$
- (iv) $\ddot{a}_{25:\overline{40}|} = 16.645$
- (v) The annual level net premium is 216
- (vi) A modified premium reserving method is used for this policy, where the modified premiums are:
 - A first year premium equal to the first year net cost of insurance,
 - Level premiums of β for years 2 through 20, and
 - Level premiums of 216 thereafter.

Calculate β .

- (A) 140
- (B) 170
- (C) 200
- (D) 230
- (E) 260

[Question 15 on the Spring 2016 exam]

- 7.24.** For a fully discrete whole life insurance policy of 1,000,000 on (50), you are given:
- (i) The annual gross premium, calculated using the equivalence principle, is 11,800
 - (ii) Mortality follows the Standard Ultimate Life Table
 - (iii) $i = 0.05$

Calculate the expense loading, P^e , for this policy.

- (A) 480
- (B) 580
- (C) 680
- (D) 780
- (E) 880

[A modified version of Question 8 on the Fall 2016 exam]

7.25. For a fully discrete whole life insurance policy of 100,000 on [55], a professional skydiver, you are given:

- (i) Level premiums are paid annually
- (ii) Mortality follows a 2-year select and ultimate table
- (iii) $i = 0.04$
- (iv) The following table of values for $A_{[x]+t}$:

| x | $A_{[x]}$ | $A_{[x]+1}$ | A_{x+2} |
|-----|-----------|-------------|-----------|
| 55 | 0.23 | 0.24 | 0.25 |
| 56 | 0.25 | 0.26 | 0.27 |
| 57 | 0.27 | 0.28 | 0.29 |
| 58 | 0.29 | 0.30 | 0.31 |

Calculate the Full Preliminary Term reserve at time 3.

- (A) 2700
- (B) 3950
- (C) 5200
- (D) 6450
- (E) 7800

[A modified version of Question 12 on the Fall 2016 exam]

7.26. For a special fully discrete 2-year endowment insurance on (x) , you are given:

- (i) The death benefit for year k is $25,000k$ plus the net premium policy value at the end of year k , for $k = 1, 2$. For year 2, this net premium policy value is the net premium policy value just before the maturity benefit is paid
- (ii) The maturity benefit is 50,000
- (iii) $p_x = p_{x+1} = 0.85$
- (iv) $i = 0.05$
- (v) P is the level annual net premium

Calculate P .

- (A) 27,650
- (B) 27,960
- (C) 28,200
- (D) 28,540
- (E) 28,730

[A modified version of Question 13 on the Fall 2016 exam]

7.27. The gross annual premium, G , for a fully discrete 5-year endowment insurance of 1000 issued on (x) is calculated using the equivalence principle. You are given:

(i) $1000P_{x:\overline{5}|} = 187.00$

(ii) The expense reserve at the end of the first year, ${}_1V^e = -38.70$

(iii) $q_x = 0.008$

(iv) Expenses, payable at the beginning of the year, are:

| Year | Percent of Premium | Per Policy |
|---------|--------------------|------------|
| First | 25% | 10 |
| Renewal | 5% | 5 |

(v) $i = 0.03$

Calculate G .

(A) 200

(B) 213

(C) 226

(D) 239

(E) 252

[Question 17 on the Fall 2016 exam]

7.28. For a special fully discrete whole life insurance of 1,000 on (45), you are given:

(i) The net premiums for year k are:

$$\begin{cases} P, & k = 1, 2, \dots, 20 \\ P + W, & k = 21, 22, \dots \end{cases}$$

(ii) Mortality follows the Standard Ultimate Life Table

(iii) $i = 0.05$

(iv) ${}_{20}V$, the net premium policy value at the end of the 20th year, is 0

Calculate W .

(A) 12

(B) 16

(C) 20

(D) 24

(E) 28

[A modified version of Question 15 on the Fall 2017 exam]

7.29. For a fully discrete whole life insurance of B on (x) , you are given:

- (i) Expenses, incurred at the beginning of each year, equal 30 in the first year and 5 in subsequent years
- (ii) The net premium policy value at the end of year 10 is 2290
- (iii) Gross premiums are calculated using the equivalence principle
- (iv) $i = 0.04$
- (v) $\ddot{a}_x = 14.8$
- (vi) $\ddot{a}_{x+10} = 11.4$

Calculate ${}_{10}V^g$, the gross premium policy value at the end of year 10.

- (A) 2190
- (B) 2210
- (C) 2230
- (D) 2250
- (E) 2270

[A modified version of Question 16 on the Fall 2017 exam]

7.30. Ten years ago J, then age 25, purchased a fully discrete 10-payment whole life policy of 10,000.

All actuarial calculations for this policy were based on the following:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) $i = 0.05$
- (iii) The equivalence principle

In addition:

- (i) L_{10} is the present value of future losses random variable at time 10.
- (ii) At the end of policy year 10, the interest rate used to calculate L_{10} is changed to 0%.

Calculate the increase in $E[L_{10}]$ that results from this change.

- (A) 5035
- (B) 6035
- (C) 7035
- (D) 8035
- (E) 9035

[A modified version of Question 18 on the Fall 2017 exam]

7.31. For a fully discrete 3-year endowment insurance of 1000 on (x) , you are given:

(i) Expenses, payable at the beginning of the year, are:

| Year(s) | Percent of Premium | Per Policy |
|---------|--------------------|------------|
| 1 | 20% | 15 |
| 2 and 3 | 8% | 5 |

(ii) The expense reserve at the end of year 2 is -23.64

(iii) The gross annual premium calculated using the equivalence principle is $G = 368.05$

(iv) $G = 1000P_{x:\overline{3}|} + P^e$, where P^e is the expense loading

Calculate $P_{x:\overline{3}|}$.

(A) 0.290

(B) 0.295

(C) 0.300

(D) 0.305

(E) 0.310

[Question 16 on the Spring 2014 exam]

7.32 For two fully continuous whole life insurance policies on (x) , you are given:

(i)

| | Death Benefit | Annual Premium Rate | Variance of the Present Value of Future Loss at t |
|----------|------------------|------------------------|--|
| Policy A | 1 | 0.10 | 0.455 |
| Policy B | 2 | 0.16 | - |

(ii) $\delta = 0.06$

Calculate the variance of the present value of future loss at t for Policy B.

- (A) 0.9
- (B) 1.4
- (C) 2.0
- (D) 2.9
- (E) 3.4

[Question 12 on the Spring 2014 exam]

18.1. An insurer is modelling time to death of lives insured at age x using the Kaplan-Meier estimator. You are given the following information.

- (i) There were 100 policies in force at time 0
- (ii) There were no new policies entering the study
- (iii) At time 10.0, immediately after a death, there were 50 policies remaining in force
- (iv) The Kaplan-Meier estimate of the survival function for retirement at time 10 is $\hat{S}(10.0) = 0.92$
- (v) The next death after time 10.0 occurred when there was one death at time 10.8
- (vi) During the period from time 10.0 to time 10.8, a total of 10 policies terminated for reasons other than death

Calculate $\hat{S}(10.8)$, the Kaplan-Meier estimate of the survival function $S(10.8)$.

- (A) 0.897
- (B) 0.903
- (C) 0.909
- (D) 0.910
- (E) 0.920

18.2. In a study of 1,000 people with a particular illness, 200 died within one year of diagnosis. Calculate a 95% (linear) confidence interval for the one-year empirical survival function.

- (A) (0.745, 0.855)
- (B) (0.755, 0.845)
- (C) (0.765, 0.835)
- (D) (0.775, 0.825)
- (E) (0.785, 0.815)

18.3. A cohort of 100 newborns is observed from birth. During the first year, 10 drop out of the study and one dies at time 1. Eight more drop out during the next six months, then, at time 1.5, three deaths occur.

Calculate $\hat{S}(1.5)$, the Nelson-Aalen estimator of the survival function, $S(1.5)$.

- (A) 0.950
- (B) 0.951
- (C) 0.952
- (D) 0.953
- (E) 0.954

18.4. You are given the following data based on 60 lives at time 0:

| j | $t_{(j)}$ | Deaths at $t_{(j)}$ | <i>Exits in</i> $(t_{(j)}^+, t_{(j+1)}^-)$ | <i>Entrants in</i> $(t_{(j)}^+, t_{(j+1)}^-)$ |
|-----|-----------|---------------------|---|--|
| 0 | | | 0 | 0 |
| 1 | 5.3 | 1 | 8 | 1 |
| 2 | 8.6 | 1 | 6 | 7 |
| 3 | 13.2 | 2 | 7 | 7 |
| 4 | 16.1 | 1 | 6 | 5 |
| 5 | 21.0 | 1 | 6 | 4 |

Calculate the upper limit of the 80% linear confidence interval for $S(21.0)$ using the Kaplan-Meier estimate and Greenwood's approximation.

- (A) 0.872
- (B) 0.893
- (C) 0.915
- (D) 0.936
- (E) 0.958

- 18.5.** In a mortality study, the following grouped death data were collected from 100 lives, all studied beginning at age 40.

| Age last birthday at death | Number of deaths |
|----------------------------|------------------|
| 40–49 | 10 |
| 50–59 | 14 |
| 60–69 | 16 |
| 70–79 | 20 |
| 80 and higher | 40 |

There were no terminations other than death.

Calculate $\hat{S}_{40}(32)$ using the ogive empirical distribution function.

- (A) 0.44
- (B) 0.48
- (C) 0.52
- (D) 0.56
- (E) 0.60

- 18.6** You are doing a mortality study of insureds between ages 70 and 90. Two specific lives contributed this data to the study:

| Life | Age at Entry | Age at Exit | Cause of exit |
|------|--------------|-----------------------|---------------|
| 1 | 70.0 | 90.0 | End of study |
| 2 | 70.0 | Between 89.0 and 90.0 | Death |

You assume mortality follows Gompertz law $\mu_x = B \times c^x$ and plan to use maximum likelihood estimation.

L is the likelihood function associated with these two lives.

L^* denotes the value of L if the Gompertz parameters are $B = 0.000003$ and $c = 1.1$.

Calculate L^* .

- (A) 0.0115
- (B) 0.0131
- (C) 0.0147
- (D) 0.0163
- (E) 0.0179

- 18.7** You are doing a mortality study of insureds between ages 60 and 90. Two specific lives contributed this data to the study:

| Life | Age at Entry | Age at Exit | Cause of exit |
|------|--------------|-------------|---------------|
| 1 | 60.0 | 74.5 | Policy lapsed |
| 2 | 60.0 | 74.5 | Death |

You assume mortality follows Gompertz law $\mu_x = B \times c^x$ and plan to use maximum likelihood estimation.

L is the log-likelihood function (using natural logs) associated with these two lives.

L^* denotes the value of L if the Gompertz parameters are $B = 0.000004$ and $c = 1.12$.

Calculate L^* .

- (A) -4.67
- (B) -4.53
- (C) -4.39
- (D) -4.25
- (E) -4.11