Fundamentals of Actuarial Mathematics – Long Term

Solutions to Sample Multiple Choice Questions

July 1, 2022

Versions:

July 1, 2022	Original version for FAM-L.

These solutions are written to provide evidence that the stated answer is correct and to offer at least one approach to solving the problem. There may be additional approaches that can be employed.

Question 1.1

Answer C

Answer C is false. If the purchaser of a single premium immediate annuity has higher mortality than expected, this reduces the number of payments that will be paid. Therefore, the Actuarial Present Value will be less and the insurance company will benefit. Therefore, single premium life annuities do not need to be underwritten.

The other items are true.

A: Life insurance is typically underwritten to prevent adverse selection as higher mortality than expected will result in the Actuarial Present Value of the benefits being higher than expected.

B: In some cases, such as direct marketed products for low face amounts, there may be very limited underwriting. The actuary would assume that mortality will be higher than normal, but the expenses related to selling the business will be low and partially offset the extra mortality.

D: If the insured's occupation or hobby is hazardous, then the insured life may be rated.

E: If the purchaser of the pure endowment has higher mortality than expected, this reduces the number of endowments that will be paid. Therefore, the Actuarial Present Value will be less and the insurance company will benefit. Therefore, pure endowments do not need to be underwritten.

Question 1.2

Answer E

Insurers have an increased interest in combining savings and insurance products so Item E is false.

The other items are all true.

Question 2.1

Answer: B

Since $S_0(t) = 1 - F_0(t) = \left(1 - \frac{t}{\omega}\right)^{\frac{1}{4}}$, we have $\ln[S_0(t)] = \frac{1}{4}\ln\left[\frac{\omega - t}{\omega}\right]$.

Then $\mu_t = -\frac{d}{dt}\log S_0(t) = \frac{1}{4}\,\frac{1}{\omega - t}$, and $\mu_{65} = \frac{1}{180} = \frac{1}{4}\,\frac{1}{\omega - 65} \Rightarrow \omega = 110.$

$$e_{106} = \sum_{t=1}^{3} {}_t p_{106}, \text{ since } {}_4 p_{106} = 0$$

$${}_t p_{106} = \frac{S_0(106+t)}{S_0(106)} = \frac{\left(1 - \frac{106+t}{110}\right)^{1/4}}{\left(1 - \frac{106}{110}\right)^{1/4}} = \left(\frac{4-t}{4}\right)^{1/4}$$

$$e_{106} = \sum_{i=1}^{i=4} {}_t p_{106} = \frac{1}{4^{0.25}}\left(1^{0.25} + 2^{0.25} + 3^{0.25}\right) = 2.4786$$

Question 2.2

Answer: D

This is a mixed distribution for the population, since the vaccine will apply to all once available.

Available? (A)	$\Pr(A)$	${}_2p \mid A$	S = # of survivors: $E(S \mid A)$	$Var(S \mid A)$	$E(S^2 \mid A)$
Yes	0.2	0.9702	97,020	2,891	9,412,883,291
No	0.8	0.9604	96,040	3,803	9,223,685,403
			$E(S)$		$E(S^2)$
			96,236		9,261,524,981
			$Var(S)$	157,285	
			$SD(S)$	397	

As an example, the formulas for the "No" row are

Pr(No) = 1 – 0.2 = 0.8

${}_2p$ given No = (0.98 during year 1)(0.98 during year 2) = 0.9604.

$E(S \mid \text{No}), Var(S \mid \text{No})$ and $E(S^2 \mid \text{No})$ are just binomial, $n = 100{,}000$; p(success) = 0.9604

$E(S), E(S^2)$ are weighted averages,

$Var(S) = E(S^2) - E(S)^2$

Or, by the conditional variance formula:

$$Var(S) = Var[E(S \mid A)] + E[Var(S \mid A)]$$
$$= 0.2(0.8)(97{,}020 - 96{,}040)^2 + 0.2(2{,}891) + 0.8(3{,}803)$$
$$= 153{,}664 + 3{,}621 = 157{,}285$$

$StdDev(S) = 397$

Question 2.3

Answer: A

$$f_x(t) = -\frac{d}{dt}S_x(t) = -\frac{d}{dt}\left(e^{-\frac{B}{\ln c}(c^x)(c^t-1)}\right)$$

$$= -e^{-\frac{B}{\ln c}(c^x)(c^t-1)} \cdot \left(-\frac{B}{\ln c}\cdot c^x\right)\cdot c^t \cdot \ln c$$

$$= e^{-\frac{B}{\ln c}(c^x)(c^t-1)} \cdot Bc^{x+t}$$

$$= 0.00027 \times 1.1^{x+t} \cdot e^{-\frac{0.00027}{\ln(1.1)}(1.1^x)(1.1^t-1)}$$

$$f_{50}(10) = 0.00027 \times 1.1^{50+10} \cdot e^{-\frac{0.00027}{\ln(1.1)}(1.1^{50})(1.1^{10}-1)} = 0.04839$$

Alternative Solution:

$$f_x(t) = {}_tp_x \cdot \mu_{x+t}$$

Then we can use the formulas given for Makeham with $A = 0,\ B = 0.00027$ and $c = 1.1$

$$f_x(t) = \left(e^{-\frac{0.00027}{\ln(1.1)}(1.1^{50})(1.1^{10}-1)}\right)\left(0.00027 \times 1.1^{50+10}\right) = 0.04839$$

Question 2.4

Answer: E

$$\overset{\circ}{e}_{75:\overline{10}|} = \int_{t=0}^{t=10} {}_t p_{75} dt \quad \text{where} \quad {}_t p_x = \frac{{}_{t+x} p_0}{{}_x p_0} = \frac{1-\dfrac{(t+x)^2}{10000}}{1-\dfrac{x^2}{10000}} = \frac{10000-(t+x)^2}{10000-x^2} \quad \text{for } 0 < t < 100 - x$$

$$= \int_0^{10} \frac{10000-75^2-150t-t^2}{10000-75^2} dt$$

$$= \frac{1}{4375} \cdot \left[4375t - 75t^2 - \frac{t^3}{3} \right]_{t=0}^{t=10} = 8.21$$

Question 2.5

Answer: B

$$e_{40} = e_{40:\overline{20}|} + {}_{20} p_{40} \cdot e_{60}$$
$$= 18 + (1-0.2)(25)$$
$$= 38$$
$$e_{40} = e_{40:\overline{1}|} + p_{40} \cdot e_{41}$$
$$\Rightarrow e_{41} = \frac{e_{40} - e_{40:\overline{1}|}}{p_{40}} = \frac{e_{40} - p_{40}}{p_{40}} = \frac{38 - 0.997}{0.997} = 37.11434$$

Question 2.6

Answer: C

$$\mu_x = -\frac{d}{d_x} lnS_0(x) = -\frac{1}{3}\frac{d}{d_x} ln\left(1-\frac{x}{60}\right)$$
$$= \frac{1}{180}\left(1-\frac{x}{60}\right)^{-1} = \frac{1}{3(60-x)}$$

Therefore, $1000\mu_{35} = (1000)\dfrac{1}{3(25)} = \dfrac{1000}{75} = 13.3.$

Question 2.7

Answer: B

$$ {}_{20}q_{30} = \frac{S_0(30) - S_0(50)}{S_0(30)} = \frac{\left(1 - \frac{30}{250}\right) - \left(1 - \left[\frac{50}{100}\right]^2\right)}{1 - \frac{30}{250}} = \frac{\frac{220}{250} - \frac{3}{4}}{\frac{220}{250}} $$

$$ = \frac{440 - 375}{440} = \frac{65}{440} = \frac{13}{88} = 0.1477 $$

Question 2.8

Answer: C

The 20-year female survival probability = $e^{-20\mu}$
The 20-year male survival probability = $e^{-30\mu}$

We want 1-year female survival = $e^{-\mu}$

Suppose that there were M males and $3M$ females initially. After 20 years, there are expected to be $Me^{-30\mu}$ and $3Me^{-20\mu}$ survivors, respectively. At that time we have:

$$ \frac{3Me^{-20\mu}}{Me^{-30\mu}} = \frac{85}{15} \Rightarrow e^{10\mu} = \frac{85}{45} = \frac{17}{9} \Rightarrow e^{-\mu} = \left(\frac{9}{17}\right)^{1/10} = 0.938 $$

Question 3.1

Answer: B

Under constant force over each year of age, $l_{x+k} = (l_x)^{1-k}(l_{x+1})^k$ for x an integer and $0 \le k \le 1$.

$${}_{2|3}q_{[60]+0.75} = \frac{l_{[60]+2.75} - l_{[60]+5.75}}{l_{[60]+0.75}}$$

$$l_{[60]+0.75} = (80,000)^{0.25}(79,000)^{0.75} = 79,249$$
$$l_{[60]+2.75} = (77,000)^{0.25}(74,000)^{0.75} = 74,739$$
$$l_{[60]+5.75} = (67,000)^{0.25}(65,000)^{0.75} = 65,494$$

$${}_{2|3}q_{[60]+0.75} = \frac{l_{[60]+2.75} - l_{[60]+5.75}}{l_{[60]+0.75}} = \frac{74,739 - 65,494}{79,249} = 0.11679$$

$$1000\ {}_{2|3}q_{[60]+0.75} = 116.8$$

Question 3.2

Answer: D

$$l_{65+1} = 1000 - 40 = 960$$
$$l_{66+1} = 955 - 45 = 910$$
$$\mathring{e}_{[65]} = \int_0^1 {}_tp_{[65]}dt + p_{[65]}\int_0^1 {}_tp_{66}\,dt + p_{[65]}p_{66}\,\mathring{e}_{67}$$
$$15.0 = \left[1 - \left(\frac{1}{2}\right)\left(\frac{40}{1000}\right)\right] + \frac{960}{1000}\left[1 - \left(\frac{1}{2}\right)\left(\frac{50}{960}\right)\right] + \left(\frac{960}{1000}\right)\left(\frac{910}{960}\right)\mathring{e}_{67}$$
$$\mathring{e}_{67} = \frac{15(1000) - (980 + 935)}{910} = 14.37912$$
$$\mathring{e}_{[66]} = \int_0^1 {}_tp_{[66]}dt + p_{[66]}\,\mathring{e}_{67} = \left[1 - \left(\frac{1}{2}\right)\left(\frac{45}{955}\right)\right] + \left(\frac{910}{955}\right)\mathring{e}_{67}$$
$$\mathring{e}_{[66]} = \left[1 - \left(\frac{1}{2}\right)\left(\frac{45}{955}\right)\right] + \left(\frac{910}{955}\right)(14.37912) = 14.678$$

Note that because deaths are uniformly distributed over each year of age, $\int_0^1 {}_tp_x dt = 1 - 0.5q_x$.

Question 3.3

Answer: E

$$ {}_{2.2}q_{[51]+0.5} = \frac{l_{[51]+0.5} - l_{53.7}}{l_{[51]+0.5}} $$

$$ l_{[51]+0.5} = 0.5l_{[51]} + 0.5l_{[51]+1} = 0.5(97,000) + 0.5(93,000) = 95,000 $$

$$ l_{53.7} = 0.3l_{53} + 0.7l_{54} = 0.3(89,000) + 0.7(83,000) = 84,800 $$

$$ {}_{2.2}q_{[51]+0.5} = \frac{95,000 - 84,800}{95,000} = 0.1074 $$

$$ 10,000\,{}_{2.2}q_{[51]+0.5} = 1,074 $$

Question 3.4

Answer: B

Let S denote the number of survivors.

This is a binomial random variable with $n = 4000$ and success probability $\frac{21,178.3}{99,871.1} = 0.21206$

$$ E(S) = 4,000(0.21206) = 848.24 $$

The variance is $Var(S) = (0.21206)(1-0.21206)(4,000) = 668.36$

$$ StdDev(S) = \sqrt{668.36} = 25.853 $$

The 90% percentile of the standard normal is 1.282

Let S^* denote the normal distribution with mean 848.24 and standard deviation 25.853. Since S is discrete and integer-valued, for any integer s,

$$ \begin{aligned} \Pr(S \geq s) &= \Pr(S > s - 0.5) \approx \Pr(S^* > s - 0.5) \\ &= \Pr\left(\frac{S^* - 848.24}{25.853} > \frac{s - 0.5 - 848.24}{25.853}\right) \\ &= \Pr\left(Z > \frac{s - 0.5 - 848.24}{25.853}\right) \end{aligned} $$

For this probability to be at least 90%, we must have $\frac{s - 0.5 - 848.24}{25.853} < -1.282$

$$ \Rightarrow s < 815.6 $$

So $s = 815$ is the largest integer that works.

Question 3.5

Answer: E

Using UDD

$l_{63.4} = (0.6)66,666 + (0.4)(55,555) = 62,221.6$

$l_{65.9} = (0.1)(44,444) + (0.9)(33,333) = 34,444.1$

$${}_{3.4|2.5}q_{60} = \frac{l_{63.4} - l_{65.9}}{l_{60}} = \frac{62,221.6 - 34,444.1}{99,999} = 0.277778 \qquad \text{(a)}$$

Using constant force

$$l_{63.4} = l_{63}\left(\frac{l_{64}}{l_{63}}\right)^{0.4} = l_{63}^{0.6}\, l_{64}^{0.4}$$
$$= \left(66,666^{0.6}\right)\left(55,555^{0.4}\right)$$
$$= 61,977.2$$
$$l_{65.9} = l_{65}^{0.1}\, l_{66}^{0.9} = \left(44,444^{0.1}\right)\left(33,333^{0.9}\right)$$
$$= 34,305.9$$

$${}_{3.4|2.5}q_{60} = \frac{61,977.2 - 34,305.9}{99.999}$$
$$= 0.276716 \qquad \text{(b)}$$

$100,000(a - b) = 100,000(0.277778 - 0.276716) = 106$

Question 3.6

Answer: D

$e_{[61]} = e_{[61]:\overline{3}|} + {}_3p_{[61]}(e_{64})$

$p_{[61]} = 0.90,$

${}_2p_{[61]} = 0.9(0.88) = 0.792,$

${}_3p_{[61]} = 0.792(0.86) = 0.68112$

$$e_{[61]:\overline{3}|} = \sum_{k=1}^{3} {}_kp_{[61]} = 0.9 + 0.792 + 0.68112 = 2.37312$$

$e_{[61]} = 2.37312 + 0.68112e_{64} = 2.37312 + 0.68112(5.10) = 5.847$

Question 3.7

Answer: B

$$
\begin{aligned}
{}_{2.5}q_{[50]+0.4} &= 1 - {}_{2.5}p_{[50]+0.4} = 1 - {}_{2.9}p_{[50]} / \left(p_{[50]}\right)^{0.4} \\
&= 1 - \left\{p_{[50]}\ p_{[50]+1} \left(p_{52}\right)^{0.9}\right\} / \left(1 - q_{[50]}\right)^{0.4} \\
&= 1 - \left\{\left(1 - q_{[50]}\right)\left(1 - q_{[50]+1}\right)\left(1 - q_{52}\right)^{0.9}\right\} / \left(1 - q_{[50]}\right)^{0.4} \\
&= 1 - \left\{(1 - 0.0050)(1 - 0.0063)(1 - 0.0080)^{0.9}\right\} / (1 - 0.0050)^{0.4} \\
&= 0.01642
\end{aligned}
$$

$$1000\ {}_{2.5}q_{[50]+0.4} = 16.42$$

Question 3.8

Answer: B

$$E(N) = 1000\left({}_{40}p_{35} + {}_{40}p_{45}\right) = 1000\left(\frac{85,203.5}{99,556.7} + \frac{61,184.9}{99,033.9}\right) = 1473.65$$

$$Var(N) = 1000\ {}_{40}p_{35}\left(1 - {}_{40}p_{35}\right) + 1000\ {}_{40}p_{45}\left(1 - {}_{40}p_{45}\right) = 359.50$$

Since $1473.65 + 1.645\sqrt{359.50} = 1504.84$

$N = 1505$

Question 3.9

Answer: E

From the SULT, we have:

$$_{25}p_{20} = \frac{\ell_{45}}{\ell_{20}} = \frac{99,033.9}{100,000.0} = 0.99034$$

$$_{25}p_{45} = \frac{\ell_{70}}{\ell_{45}} = \frac{91,082.4}{99,033.9} = 0.91971$$

The expected number of survivors from the sons is 1980.68 with variance 19.133.

The expected number of survivors from fathers is 1839.42 with variance 147.687.

The total expected number of survivors is therefore 3820.10.

The standard deviation of the total expected number of survivors is therefore

$\sqrt{19.133 + 147.687} = \sqrt{166.82} = 12.916$

The 99th percentile equals $3820.10 + (2.326)(12.916) = 3850$

Question 3.10

Answer: C

The number of left-handed members at the end of each year k is:

$L_0 = 75$ and $L_1 = (75)(0.75)$

Thereafter, $L_k = L_{k-1} \times 0.75 + 35 \times 0.75 = 75 \times 0.75^k + 35 \times \left(0.75 + 0.75^2 + ...0.75^{k-1}\right)$

Similarly, the number of right-handed members after each year k is:

$R_0 = 25$ and $R_1 = (25)(0.5)$

Thereafter, $R_k = R_{k-1} \times 0.50 + 15 \times 0.50 = 25 \times 0.50^k + 15 \times \left(0.50 + 0.50^2 + ...0.50^{k-1}\right)$

At the end of year 5, the number of left-handed members is expected to be 89.5752, and the number of right-handed members is expected to be 14.8435.

The proportion of left-handed members at the end of year 5 is therefore

$$\frac{89.5752}{89.5752 + 14.8438} = 0.8578$$

Question 3.11

Answer: B

$$ {}_{2.5}q_{50} = {}_{2}q_{50} + {}_{2}p_{50}\ {}_{0.5}q_{52} \;\; = 0.02 + (0.98)\left(\frac{0.5}{2}\right)(0.04) \;\; = 0.0298 $$

Question 3.12

Answer: C

$$ {}_{3.5}p_{[61]} - {}_{3.5}p_{[60]+1} = {}_{0.5}p_{64}\left({}_{3}p_{[61]} - {}_{3}p_{[60]+1}\right) $$

$$ = \left(\frac{\ell_{65}}{\ell_{64}}\right)^{0.5}\left(\frac{\ell_{64}}{\ell_{[61]}} - \frac{\ell_{64}}{\ell_{[60]+1}}\right) $$

$$ = \left(\frac{4016}{5737}\right)^{0.5}\left(\frac{5737}{8654} - \frac{5737}{9600}\right) $$

$$ = 0.05466 $$

Question 3.13

Answer: B

$$ \mathring{e}_{[58]+2} = e_{[58]+2} + 0.5 $$

$$ e_{[58]+2} = p_{[58]+2}(1 + e_{61}) = p_{[58]+2}\left[1 + \frac{e_{60}}{p_{60}} - 1\right] $$

$$ = \frac{\ell_{61}}{\ell_{[58]+2}} \times \frac{e_{60}}{p_{60}} = \frac{2210}{3548} \times \frac{1}{(2210/3904)} = \frac{3904}{3549} = 1.100338 $$

$$ \mathring{e}_{[58]+2} = 1.100338 + 0.5 = 1.6 $$

Question 3.14

Answer: C

We need to determine ${}_{3|2.5}q_{90}$.

$$ {}_{3|2.5}q_{90} = \frac{l_{90+3} - l_{90+3+2.5}}{l_{90}} = \frac{l_{93} - l_{95.5}}{l_{90}} = \frac{l_{93} - (l_{95} - 0.5d_{95})}{l_{90}} = \frac{825 - [600 - 0.5(240)]}{1,000} = 0.3450 $$

where $l_{90} = 1,000$, $l_{93} = 825$, $l_{97} = \frac{d_{97}}{q_{97}} = \frac{72}{1} = 72$, $l_{96} = \frac{l_{97}}{p_{96}} = \frac{72}{0.2} = 360$,

$l_{95} = \frac{l_{96}}{p_{95}} = \frac{360}{1-0.4} = 600$, and $d_{95} = l_{95} - l_{96} = 600 - 360 = 240$.

Question 4.1

Answer: A

$E[Z] = 2 \cdot A_{40} - {}_{20}E_{40}\, A_{60} = (2)(0.36987) - (0.51276)(0.62567) = 0.41892$

$E[Z^2] = 0.24954$ which is given in the problem.

$Var(Z) = E[Z^2] - (E[Z])^2 = 0.24954 - 0.41892^2 = 0.07405$

$SD(Z) = \sqrt{0.07405} = 0.27212$

An alternative way to obtain the mean is $E[Z] = 2A^{\ 1}_{40:\overline{20|}} + {}_{20|}A_{40}$. Had the problem asked for the evaluation of the second moment, a formula is

$E[Z^2] = (2^2)\left({}^2A^{\ 1}_{40:\overline{20|}}\right) + (v^2)^{20}\left({}_{20}p_{40}\right)\left({}^2A_{60}\right)$

Question 4.2

Answer: D

Half-year	PV of Benefit	
1	$300,000v^{0.5} = (300,000)(1.09)^{-1} = 275,229$	$PV > 277,000$ if and only if (x) dies in the 2nd or 3rd half years.
2	$330,000v^{1} = (330,000)(1.09)^{-2} = 277,754$	
3	$360,000v^{1.5} = (360,000)(1.09)^{-3} = 277,986$	
4	$390,000v^{2} = (390,000)(1.09)^{-4} = 276,286$	

Under CF assumption, ${}_{0.5}p_x = {}_{0.5}p_{x+0.5} = (0.84)^{0.5} = 0.9165$ and ${}_{0.5}p_{x+1} = {}_{0.5}p_{x+1.5} = (0.77)^{0.5} = 0.8775$ Then the probability of dying in the 2nd or 3rd half-years is

$\left({}_{0.5}p_x\right)\left(1 - {}_{0.5}p_{x+0.5}\right) + \left(p_x\right)\left(1 - {}_{0.5}p_{x+1}\right) = (0.9165)(0.0835) + (0.84)(0.1225) = 0.1794$

Question 4.3

Answer: D

$$A_{60:\overline{3}|} = q_{60}v + (1-q_{60})q_{60+1}v^2 + (1-q_{60})(1-q_{60+1})v^3 = 0.86545$$

$$q_{60+1} = \frac{A_{60:\overline{3}|} - q_{60}v - (1-q_{60})v^3}{(1-q_{60})v^2 - (1-q_{60})v^3} = \frac{0.86545 - \dfrac{0.01}{1.05} - \dfrac{0.99}{1.05^3}}{\dfrac{0.99}{1.05^2} - \dfrac{0.99}{1.05^3}} = 0.017 \text{ when } v = 1/1.05.$$

The primes indicate calculations at 4.5% interest.

$$\begin{aligned} A'_{60:\overline{3}|} &= q_{60}v' + (1-q_{60})q_{60+1}v'^2 + (1-q_{60})(1-q_{60+1})v'^3 \\ &= \frac{0.01}{1.045} + \frac{0.99(0.017)}{1.045^2} + \frac{0.99(0.983)}{1.045^3} \\ &= 0.87777 \end{aligned}$$

Question 4.4

Answer: A

$$Var(Z) = E(Z^2) - E(Z)^2$$

$$\begin{aligned} E(Z) &= E\left[(1+0.2T)(1+0.2T)^{-2}\right] = E\left[(1+0.2T)^{-1}\right] \\ &= \int_0^{40} \frac{1}{(1+0.2t)} f_T(t)dt = \frac{1}{40}\int_0^{40} \frac{1}{1+0.2t}dt \\ &= \frac{1}{40}\ \frac{1}{0.2}\ \ln(1+0.2t)\Big|_0^{40} = \frac{1}{8}\ln(9) = 0.27465 \end{aligned}$$

$$\begin{aligned} E(Z^2) &= E\{(1+0.2T)^2[(1+0.2T)^{-2}]^2\} = E[(1+0.2T)^{-2}] \\ &= \int_0^{40} \frac{1}{(1+0.2t)^2} f_T(t) = \frac{1}{40}\ \frac{1}{0.2}\left[\frac{-1}{(1+0.2t)}\right]_0^{40} \\ &= \frac{1}{8}\left(1-\frac{1}{9}\right) = \frac{1}{9} = 0.11111 \end{aligned}$$

$$Var(Z) = 0.11111 - (0.27465)^2 = 0.03568$$

Question 4.5

Answer: C

The earlier the death (before year 30), the larger the loss. Since we are looking for the 95th percentile of the present value of benefits random variable, we must find the time at which 5% of the insureds have died. The present value of the death benefit for that insured is what is being asked for.

$$l_{45} = 99,033.9 \Rightarrow 0.95 l_{45} = 94,082.2$$
$$l_{65} = 94,579.7$$
$$l_{66} = 94,020.3$$

So, the time is between ages 65 and 66, i.e. time 20 and time 21.

$$l_{65} - l_{66} = 94,579.7 - 94,020.3 = 559.4$$
$$l_{65+t} - l_{66} = 94,579.7 - 94,082.2 = 497.5$$
$$497.5 / 559.4 = 0.8893$$

The time just before the last 5% of deaths is expected to occur is: 20 + 0.8893 = 20.8893

The present value of death benefits at this time is:

$$100,000 e^{-20.8893(0.05)} = 35,188$$

Question 4.6

Answer: B

Time	Age	q_x^{SULT}	Improvement factor	q_x
0	70	0.010413	100.00%	0.010413
1	71	0.011670	95.00%	0.011087
2	72	0.013081	90.25%	0.011806

$$v = 1/1.05 = 0.952381$$

$$EPV = 1,000[0.010413v + 0.989587(0.011087)v^2 + 0.989587(0.988913)(0.011806)v^3]$$
$$= 29.85$$

Question 4.7

Answer: B

$$Var(Z) = 0.10E[Z] \Rightarrow v^{50}\,{}_{25}p_x(1 - {}_{25}p_x) = 0.10 \cdot v^{25}\,{}_{25}p_x$$

$$\Rightarrow \frac{(1-0.57)}{(1+i)^{50}} = 0.10 \times \frac{1}{(1+i)^{25}}$$

$$\Rightarrow (1+i)^{25} = \frac{0.43}{0.10} = 4.3 \Rightarrow i = 0.06$$

Question 4.8

Answer: C

Let A_{51}^{SULT} designate A_{51} using the Standard Ultimate Life Table at 5%.

$$\begin{aligned}\text{APV (insurance)} &= 1000\left(\frac{1}{1.04}\right)\left(q_{50} + p_{50}\,A_{51}^{SULT}\right)\\ &= 1000\left(\frac{1}{1.04}\right)\left[0.001209 + (1-0.001209)(0.19780)\right]\\ &= 191.12\end{aligned}$$

Question 4.9

Answer: D

$$A_{35} = A_{35:\overline{15}|}^{1} + A_{35:\overset{\;\,1}{\overline{15}|}}\,A_{50}$$

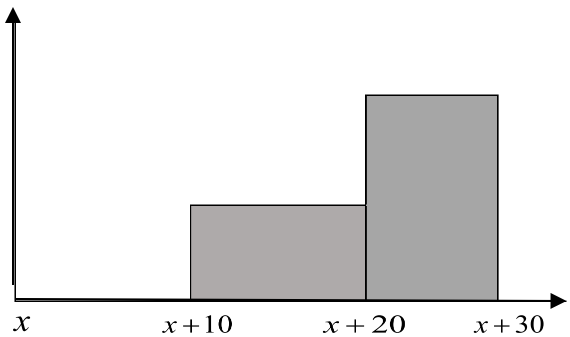
$$0.32 = 0.25 + 0.14\,A_{50}$$

$$A_{50} = \frac{0.07}{0.14} = 0.50$$

Question 4.10

Answer: D

Drawing the benefit payment pattern:



$$E[Z] = {}_{10}E_x \bullet \overline{A}_{x+10} + {}_{20}E_x \bullet \overline{A}_{x+20} - 2\ {}_{30}E_x \bullet \overline{A}_{x+30}$$

Question 4.11

Answer: A

$$Var(Z_2) = (1000)^2 \left[{}^2A_{x:\overline{n}|} - \left(A_{x:\overline{n}|} \right)^2 \right] = 15{,}000$$

$$= (1000)^2 \left({}^2A^1_{x:\overline{n}|} + {}^2A_{x:\overline{n}|}^{\ \ 1} \right) - (1000)^2 \left[A^1_{x:\overline{n}|} + A_{x\,\overline{n}|}^{\ \ 1} \right]^2$$

$$= (1000)^2\ {}^2A^1_{x:\overline{n}|} + (1000)^2\ {}^2A_{x:\overline{n}|}^{\ \ 1} - (1000)^2 \left(A^1_{x:\overline{n}|} \right)^2 - (1000)^2 \left(A_{x\,\overline{n}|}^{\ \ 1} \right)^2$$

$$- 2(1000)^2 \left(A^1_{x:\overline{n}|} \right) \left(A_{x\,\overline{n}|}^{\ \ 1} \right)$$

$$= (1000)^2 \left[{}^2A^1_{x:\overline{n}|} - \left(A^1_{x:\overline{n}|} \right)^2 \right] + \left(1000^2\ {}^2A_{x:\overline{n}|}^{\ \ 1} \right) - \left(1000 A_{x:\overline{n}|}^{\ \ 1} \right)^2$$

$$- \left(1000 A^1_{x:\overline{n}|} \right)^2 - (2) \left(1000 A^1_{x:\overline{n}|} \right) \left(1000 A_{x\,\overline{n}|}^{\ \ 1} \right)$$

$$= V\left(Z_1 \right) + (1000) \left(1000\ {}^2A_{x:\overline{n}|}^{\ \ 1} \right) - \left(1000 A_{x\,\overline{n}|}^{\ \ 1} \right)^2 - \left(1000 A^1_{x:\overline{n}|} \right)^2$$

$$-(2) \left(1000 A^1_{x:\overline{n}|} \right) \left(1000 A_{x\,\overline{n}|}^{\ \ 1} \right)$$

$$15{,}000 = Var\left(Z_1 \right) + (1000)(136) - (209)^2 - 2(528)(209)$$

Therefore, $Var\left(Z_1 \right) = 15{,}000 - 136{,}000 + 43{,}681 + 220{,}704 = 143{,}385.$

Question 4.12

Answer: C

$Z_3 = 2Z_1 + Z_2$ so that $\text{Var}(Z_3) = 4\text{Var}(Z_1) + \text{Var}(Z_2) + 4Cov(Z_1, Z_2)$

where $Cov(Z_1, Z_2) = \underbrace{E[Z_1 Z_2]}_{=0} - E[Z_1]E[Z_2] = -(1.65)(10.75)$

$$\begin{aligned}\text{Var}(Z_3) &= 4(46.75) + 50.78 - 4(1.65)(10.75)\\ &= 166.83\end{aligned}$$

Question 4.13

Answer: C

$$\begin{aligned}{}_{2|2}A_{65} = &\underbrace{v^3}_{\text{payment year 3}} \underbrace{{}_2p_{[65]}}_{\text{Lives 2 years}} \times \underbrace{q_{[65]+2}}_{\text{Die year 3}}\\ &+ \underbrace{v^4}_{\text{payment year 4}} \underbrace{{}_3p_{[65]}}_{\text{Lives 3 years}} \times \underbrace{q_{65+3}}_{\text{Die year 4}}\\ &= \left(\frac{1}{1.04}\right)^3 (0.92)(0.9)(0.12)\\ &+ \left(\frac{1}{1.04}\right)^4 (0.92)(0.9)(0.88)(0.14)\\ &= 0.088 + 0.087 = 0.176\end{aligned}$$

The actuarial present value of this insurance is therefore $2000 \times 0.176 = 352$.

Question 4.14

Answer: E

Out of 400 lives initially, we expect $400\,{}_{25}p_{60} = 400\dfrac{l_{85}}{l_{60}} = 400\left(\dfrac{61{,}184.9}{96{,}634.1}\right) = 253.26$ survivors

The standard deviation of the number of survivors is $\sqrt{400\,{}_{25}p_{60}\left(1-{}_{25}p_{60}\right)} = 9.639$

To ensure 86% funding, using the normal distribution table, we plan for
$253.26 + 1.08(9.639) = 263.67$

The initial fund must therefore be $F = (264)(5000)\left(\dfrac{1}{1.05}\right)^{25} = 389{,}800.$

Question 4.15

Answer: E

$$E[Z] = \int_0^\infty b_t \cdot v^t \cdot {}_t p_x \cdot \mu_{x+t}\ dt = \int_0^\infty e^{0.02t} \cdot e^{-0.06t} \cdot e^{-0.04t} \cdot 0.04\ dt$$

$$= 0.04\int_0^\infty e^{-0.08t} dt = \frac{0.04}{0.08} = \frac{1}{2}$$

$$E\left[Z^2\right] = \int_0^\infty \left(b_t \cdot v^t\right)^2 \cdot {}_t p_x \cdot \mu_{x+t}\ dt = \int_0^\infty \left(e^{0.04t}\right)\left(e^{-0.12t}\right)\left(0.04e^{-0.04}\right)dt = \frac{0.04}{0.12} = \frac{1}{3}$$

$$Var[Z] = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{12} = 0.0833$$

Question 4.16

Answer: D

$$A^{\ 1}_{[50]:\overline{3}|} = vq_{[50]} + v^2 p_{[50]} q_{[50]+1} + v^3 p_{[50]} p_{[50]+1} q_{52}$$

where: $v = \dfrac{1}{1.04}$

$$q_{[50]} = 0.7\left(0.045\right) = 0.0315$$
$$p_{[50]} = 1 - q_{[50]} = 0.9685$$
$$q_{[50]+1} = 0.8\left(0.050\right) = 0.040$$
$$p_{[50]+1} = 1 - q_{[50]+1} = 0.960$$
$$q_{52} = 0.055$$

So: $A^{\ 1}_{[50]:\overline{3}|} = 0.1116$

Question 4.17

Answer: A

The median of K_{48} is the integer m for which

$$P(K_{48} < m) \le 0.5 \text{ and } P(K_{48} > m) \le 0.5.$$

This is equivalent to finding m for which

$$\frac{l_{48+m}}{l_{48}} \ge 0.5 \text{ and } \frac{l_{48+m+1}}{l_{48}} \le 0.5.$$

Based on the SULT and $l_{48}(0.5) = (98,783.9)(0.5) = 49,391.95$, we have $m = 40$ since

$$l_{88} \ge 49,391.95 \text{ and } l_{89} \le 49,391.95.$$

So: $APV = 5000 A_{48} + 5000\,{}_{40}E_{48} A_{88} = 5000 A_{48} + 5000 \cdot {}_{20}E_{48} \cdot {}_{20}E_{68} \cdot A_{88}$

$$= 5000\left(0.17330\right) + 5000\left(0.35370\right)(0.20343)\left(0.72349\right) = 1126.79$$

Question 4.18

Answer: A

The present value random variable $\mathbf{PV} = 1{,}000{,}000e^{-0.05T}$, $2 \le T \le 10$ is a decreasing function of T so that its 90th percentile is

$1{,}000{,}000e^{-0.05p}$ where p is the solution to $\int_2^p 0.4t^{-2}dt = 0.10$.

$$\int_2^p 0.4t^{-2}dt = -0.4\left(\frac{t^{-1}}{-1}\right)\Bigg|_2^p = 0.4\left(\frac{1}{2} - \frac{1}{p}\right) = 0.10$$

$$p = 4$$

$$1{,}000{,}000e^{-0.05\times 4} = 81{,}873.08$$

Question 4.19

Answer: B

Superscript SULT refers to values from the SULT. Values without superscripts refer to this select life.

$$q_{80} = 0.8q_{80}^{SULT} = 0.0261264 \Rightarrow p_{80} = 0.9738736$$

$$A_{80} = vq_{80} + vp_{80}A_{81}^{SULT}$$

$$= (1.05)^{-1}(0.0261264) + (1.05)^{-1}(0.9738736)(0.60984) = 0.59051$$

$$100{,}000A_{80} = 59{,}051$$

Question 5.1

Answer: A

$$E(Y) = \bar{a}_{\overline{10|}} + e^{-\delta(10)} e^{-\mu(10)} \bar{a}_{x+10}$$

$$= \frac{\left(1 - e^{-0.6}\right)}{0.06} + e^{-0.7} \frac{1}{0.07}$$

$$= 14.6139$$

$$Y > E(Y) \Rightarrow \left(\frac{1 - e^{-0.06T}}{0.06}\right) > 14.6139$$

$$\Rightarrow T > 34.90$$

$$\Pr[Y > E(Y)] = \Pr(T > 34.90) = e^{-34.90(0.01)} = 0.705$$

Question 5.2

Answer: B

$$A_{x:\overline{n|}}^{\;\;\,1} = {}_nE_x$$

$$A_x = A_{x:\overline{n|}}^{1} + {}_nE_x A_{x+n}$$

$$0.3 = A_{x:\overline{n|}}^{1} + (0.35)(0.4) \Rightarrow A_{x:\overline{n|}}^{1} = 0.16$$

$$A_{x:\overline{n|}} = A_{x:\overline{n|}}^{1} + {}_nE_x = 0.16 + 0.35 = 0.51$$

$$\ddot{a}_{x:\overline{n|}} = \frac{1 - A_{x:\overline{n|}}}{d} = \frac{1 - 0.51}{(0.05/1.05)} = 10.29$$

$$a_{x:\overline{n|}} = \ddot{a}_{x:\overline{n|}} - 1 + {}_nE_x = 10.29 - 0.65 = 9.64$$

Question 5.3

Answer: C

$$\left(\overline{Ia}\right)_{40:\overline{t|}} = \int_0^t s\,{}_s p_{40} v^s ds \Rightarrow \frac{d\left(\overline{Ia}\right)_{40:\overline{t|}}}{dt} = t\,{}_t p_{40} v^t$$

At $t = 10.5$,

$$10.5\,{}_{10.5}E_{40} = 10.5\,{}_{10}p_{40}\,{}_{0.5}p_{50} v^{10.5}$$
$$= 10.5\,{}_{10}E_{40}\,{}_{0.5}p_{50} v^{0.5}$$
$$= 10.5 \times 0.60920 \times (1 - 0.5 \times 0.001209)(0.975900073)$$
$$= 6.239$$

Question 5.4

Answer: A

$\overset{\circ}{e}_{40} = \dfrac{1}{\mu} = 50$ So receive K for 50 years guaranteed and for life thereafter.

$$10{,}000 = K\left[\bar{a}_{\overline{50|}} + {}_{50|}\bar{a}_{40}\right]$$

$$\bar{a}_{\overline{50|}} = \int_0^{50} e^{-\delta t} = \frac{1 - e^{-50\delta}}{\delta} = \frac{1 - e^{-50(0.01)}}{0.01} = 39.35$$

$${}_{50|}\bar{a}_{40} = {}_{50}E_{40}\bar{a}_{40+50} = e^{-(\delta+\mu)50}\frac{1}{\mu+\delta} = e^{-1.5}\frac{1}{0.03} = 7.44$$

$$K = \frac{10{,}000}{39.35 + 7.44} = 213.7$$

Question 5.5

Answer: A

$$\ddot{a}_{45}^{S} = 1 + vp_{45}^{S}\ddot{a}_{46}^{SULT}$$

$$p_{45}^{S} = e^{-\int_0^1 \mu_{45+t}^{S}dt} = e^{-\int_0^1 (\mu_{45+t}^{SULT}+0.05)dt} = e^{-\int_0^1 (\mu_{45+t}^{SULT})dt} e^{-\int_0^1 (0.05)dt} = p_{45}^{SULT}\cdot e^{-0.05} = \left(\frac{98,957.6}{99,033.9}\right)e^{-0.05} = 0.9504966$$

$$\ddot{a}_{45}^{S} = 1 + vp_{45}^{S}\ddot{a}_{46}^{SULT} = 1+(1.05)^{-1}(0.9504966)(17.6706) = 17.00$$

$$100\ddot{a}_{45}^{S} = 1700$$

Question 5.6

Answer: D

Let Y_i be the present value random variable of the payment to life *i*.

$$E[Y_i] = \ddot{a}_x = \frac{1-A_x}{d} = 11.55 \qquad Var[Y_i] = \frac{{}^2A_x - \left(A_x\right)^2}{d^2} = \frac{0.22-0.45^2}{\left(0.05/1.05\right)^2} = 7.7175$$

Then $Y = \sum_{i=1}^{100} Y_i$ is the present value of the aggregate payments.

$E[Y] = 100E[Y_i] = 1155$ and $Var[Y] = 100Var[Y_i] = 771.75$

$$\Pr[Y \le F] = \Pr\left[Z \le \frac{F-1155}{\sqrt{771.75}}\right] = 0.95 \Rightarrow \frac{F-1155}{\sqrt{771.75}} = 1.645$$

$$\Rightarrow F = 1155 + 1.645\sqrt{771.75} = 1200.699$$

Question 5.7

Answer: C

$$\ddot{a}^{(2)}_{35:\overline{30|}} \approx \ddot{a}_{35:\overline{30|}} - \frac{(m-1)}{2m}\left(1 - v^{30}\,{}_{30}p_{35}\right)$$

$$\ddot{a}_{35:\overline{30|}} = \frac{1 - A_{35:\overline{30|}}}{d} = \frac{1 - A^{1}_{35:\overline{30|}} - {}_{30}E_{35}}{d}$$

$$= \frac{1 - \left(A_{35} - {}_{30}E_{35} \bullet A_{65}\right) - {}_{30}E_{35}}{d}$$

Since ${}_{30}E_{35} = v^{30}\,{}_{30}p_{35} = 0.2722,$ then

$$\ddot{a}_{35:\overline{30|}} = \frac{1 - \left(A_{35} - v^{30}\,{}_{30}p_{35} \bullet A_{65}\right) - v^{30}\,{}_{30}p_{35}}{d}$$

$$= \frac{1 - (0.188 - (0.2722)(0.498)) - 0.2722}{(0.04/1.04)}$$

$$= 17.5592$$

$$\ddot{a}^{(2)}_{35:\overline{30|}} \approx 17.5592 - \frac{1}{4}(1 - 0.2722) = 17.38$$

$$1000\,\ddot{a}^{(2)}_{35:\overline{30|}} \approx 1000 \times 17.38 = 17{,}380$$

Question 5.8

Answer: C

The expected present value is:

$$\ddot{a}_{\overline{5|}} + {}_{5}E_{55}\ddot{a}_{60} = 4.54595 + 0.77382 \times 14.9041 = 16.07904$$

The probability that the sum of the undiscounted payments will exceed the expected present value is the probability that at least 17 payments will be made. This will occur if (55) survives to age 71. The probability is therefore:

$${}_{16}p_{55} = \frac{\ell_{71}}{\ell_{55}} = \frac{90{,}134.0}{97{,}846.2} = 0.92118$$

Question 5.9

Answer: C

$$\ddot{a}_{[x]:\overline{n}|} = 1 + vp_{[x]}\ddot{a}_{x+1:\overline{n-1}|} = 1 + (1+k)\left(vp_x\ddot{a}_{x+1:\overline{n-1}|}\right) = 1 + (1+k)\left(\ddot{a}_{x:\overline{n}|} - 1\right)$$

Therefore, we have

$$k = \frac{\ddot{a}_{[x]:\overline{n}|} - 1}{\ddot{a}_{x:\overline{n}|} - 1} - 1 = \frac{21.167}{20.854} - 1 = 0.015$$

Question 6.1

Answer: D

The equation of value is given by

Actuarial Present Value of Premiums = Actuarial Present Value of Death Benefits.

The death benefit in the first year is $1000+P$. The death benefit in the second year is $1000+2P$.

The formula is $P\ddot{a}_{80:2} = 1000A^{\,1}_{80:\overline{2}|} + P(IA)^{\,1}_{80:\overline{2}|}$.

Solving for P we obtain $P = \dfrac{1000A^{\,1}_{80:\overline{2}|}}{\ddot{a}_{80:\overline{2}|} - (IA)^{\,1}_{80:\overline{2}|}}$.

$$\ddot{a}_{80:\overline{2}|} = 1 + p_{80}v = 1 + \frac{0.967342}{1.03} = 1.93917$$

$$1000A^{\,1}_{80:\overline{2}|} = 1000\left(vq_{80} + v^2 p_{80} q_{81}\right) = 1000\left(\frac{0.032658}{1.03} + \frac{(0.967342)(0.036607)}{1.03^2}\right) = 65.08552$$

$$(IA)^{\,1}_{80:\overline{2}|} = vq_{80} + 2v^2 p_{80} q_{81} = \frac{0.032658}{1.03} + (2)\frac{(0.967342)(0.036607)}{1.03^2} = 0.09846$$

$$P = \frac{65.08552}{1.93917 - 0.09846} = 35.36 \rightarrow D$$

Question 6.2

Answer: E

$$G\ddot{a}_{x:\overline{10}|} = 100,000A^{1}_{x:\overline{10}|} + G(IA)^{1}_{x:\overline{10}|} + 0.45G + 0.05G\ddot{a}_{x:\overline{10}|} + 200\ddot{a}_{x:\overline{10}|}$$

$$G = \frac{(100,000)(0.17094) + 200(6.8865)}{(1-0.05)(6.8865) - 0.96728 - 0.45} = 3604.23$$

Question 6.3

Answer: C

Let C be the annual contribution, then $C = \dfrac{{}_{20}E_{45}\,\ddot{a}_{65}}{\ddot{a}_{45:\overline{20}|}}$

Let K_{65} be the curtate future lifetime of (65). The required probability is

$$\Pr\left(\frac{C\ddot{a}_{45:\overline{20}|}}{{}_{20}E_{45}} > \ddot{a}_{\overline{K_{65}+1}|}\right) = \Pr\left(\frac{{}_{20}E_{45}\,\ddot{a}_{65}}{\ddot{a}_{45:\overline{20}|}}\frac{\ddot{a}_{45:\overline{20}|}}{{}_{20}E_{45}} > \ddot{a}_{\overline{K_{65}+1}|}\right) = \Pr\left(\ddot{a}_{65} > \ddot{a}_{\overline{K_{65}+1}|}\right) = \Pr\left(13.5498 > \ddot{a}_{\overline{K_{65}+1}|}\right)$$

Thus, since $\ddot{a}_{\overline{21}|} = 13.4622$ and $\ddot{a}_{\overline{22}|} = 13.8212$ we have

$$\Pr\left(\ddot{a}_{\overline{K_{65}+1}|} < 13.5498\right) = \Pr\left(K_{65}+1 \le 21\right) = 1 - {}_{21}p_{65} = 1 - \frac{l_{86}}{l_{65}} = 1 - \frac{57{,}656.7}{94{,}579.7} = 0.390$$

Question 6.4

Answer: E

Let X_i be the present value of a life annuity of 1/12 per month on life i for $i = 1, 2, \ldots, 200$.

Let $S = \sum_{i=1}^{200} X_i$ be the present value of all the annuity payments.

$$E[X_i] = \ddot{a}_{62}^{(12)} = \frac{1 - A_{62}^{(12)}}{d^{(12)}} = \frac{1 - 0.4075}{0.05813} = 10.19267$$

$$Var\left(X_i\right) = \frac{{}^2A_{62}^{(12)} - \left(A_{62}^{(12)}\right)^2}{\left(d^{(12)}\right)^2} = \frac{0.2105 - (0.4075)^2}{(0.05813)^2} = 13.15255$$

$E[S] = (200)(180)(10.19267) = 366{,}936.12$

$Var(S) = (200)(180)^2(13.15255) = 85{,}228{,}524$

With the normal approximation, for $\Pr(S \le M) = 0.90$

$M = E[S] + 1.282\sqrt{Var(S)} = 366{,}936.12 + 1.282\sqrt{85{,}228{,}524} = 378{,}771.45$

So $\pi = \dfrac{378{,}771.45}{200} = 1893.86$

Question 6.5

Answer: D

Let k be the policy year, so that the mortality rate during that year is q_{30+k-1}. The objective is to determine the smallest value of k such that

$$v^{k-1}\left({}_{k-1}p_{30}\right)(1000P_{30}) < v^{k}\left({}_{k-1}p_{30}\right)q_{30+k-1}(1000)$$
$$P_{30} < vq_{30+k-1}$$
$$\frac{0.07698}{19.3834} < \frac{q_{29+k}}{1.05}$$
$$q_{29+k} > 0.00417$$
$$29+k > 61 \Rightarrow k > 32$$

Therefore, the smallest value that meets the condition is 33.

Question 6.6

Answer: B

$$\text{Net Premium} = 10,000A_{62} / \ddot{a}_{62} = 10,000(0.31495)/14.3861 = 218.93$$

$$G = 218.93(1.03) = 225.50$$

Let ${}_0L^*$ be the present value of future loss at issue for one policy.

$$\begin{aligned}
{}_0L^* &= 10,000v^{K+1} - (G-5)\ddot{a}_{\overline{K+1}|} + 0.05G \\
&= 10,000v^{K+1} - (225.50-5)\frac{1-v^{K+1}}{d} + 0.05(225.50) \\
&= \left(10,000+4630.50\right)v^{K+1} - 4630.50 + 11.28 \\
&= 14,630.50v^{K+1} - 4619.22
\end{aligned}$$

$$E({}_0L^*) = 14,630.50A_{62} - 4619.22 = 14,630.50(0.31495) - 4619.22 = -11.34$$
$$Var({}_0L^*) = (14,630.50)^2\left({}^2A_{62} - A_{62}^2\right) = (14,630.50)^2\left(0.12506 - 0.31495^2\right) = 5,536,763$$

Let ${}_0L$ be the aggregate loss for 600 such policies.

$$E({}_0L) = 600E({}_0L^*) = 600(-11.34) = -6804$$
$$Var({}_0L) = 600Var({}_0L^*) = 600(5,536,763) = 3,322,057,800$$
$$StdDev({}_0L) = 3,322,057,800^{0.5} = 57,637$$

$$\Pr\left({}_0L < 40,000\right) = \Phi\left(\frac{40,000+6804}{57,637}\right) = \Phi\left(0.81\right) = 0.7910$$

Question 6.7

Answer: C

There are four ways to approach this problem. In all cases, let π denote the net premium.

The first approach is an intuitive result. The key is that in addition to the pure endowment, there is a benefit equal in value to a temporary interest only annuity due with annual payment π. However, if the insured survives the 20 years, the value of the annuity is not received.

$$\pi \ddot{a}_{40:\overline{20|}} = 100{,}000\ {}_{20}E_{40} + \pi \ddot{a}_{40:\overline{20|}} - {}_{20}p_{40}\ \ddot{a}_{\overline{20|}5\%}\pi$$

Based on this equation,

$$\pi = \frac{100{,}000\,{}_{20}E_{40}}{{}_{20}p_{40}\ddot{a}_{\overline{20|}}} = \frac{100{,}000v^{20}}{\ddot{a}_{\overline{20|}}} = \frac{100{,}000}{\ddot{s}_{\overline{20|}}} = \frac{100{,}000}{34.71925} = 2880$$

The second approach is also intuitive. If you set an equation of value at the end of 20 years, the present value of benefits is 100,000 for all the people who are alive at that time. The people who have died have had their premiums returned with interest. Therefore, the premiums plus interest that the company has are only the premiums for those alive at the end of 20 years. The people who are alive have paid 20 premiums. Therefore $\pi \ddot{s}_{\overline{20|}} = 100{,}000$.

The third approach uses random variables to derive the expected present value of the return of premium benefit. Let K be the curtate future lifetime of (40). The present value random variable is then

$$Y = \begin{cases} \pi \ddot{s}_{\overline{K+1|}} v^{K+1}, & K < 20 \\ 0, & K \geq 20 \end{cases}$$

$$= \begin{cases} \pi \ddot{a}_{\overline{K+1|}}, & K < 20 \\ 0, & K \geq 20 \end{cases}$$

$$= \begin{cases} \pi \ddot{a}_{\overline{K+1|}} - 0, & K < 20 \\ \pi \ddot{a}_{\overline{20|}} - \pi \ddot{a}_{\overline{20|}}, & K \geq 20 \end{cases}.$$

The first term is the random variable that corresponds to a 20-year temporary annuity. The second term is the random variable that corresponds to a payment with a present value of $\pi \ddot{a}_{\overline{20|}}$ contingent on surviving 20 years. The expected present value is then $\pi \ddot{a}_{40:\overline{20|}} - {}_{20}p_{40}\ \ddot{a}_{\overline{20|}}\pi$.

The fourth approach takes the most steps.

$$\pi \ddot{a}_{40:\overline{20|}} = 100,000\ {}_{20}E_{40} + \pi\sum_{k=0}^{19} v^{k+1}\ \ddot{s}_{\overline{k+1|}}\ {}_{k|}q_{40} = 100,000\,{}_{20}E_{40} + \pi\sum_{k=0}^{19} v^{k+1}\ \frac{(1+i)^{k+1}-1}{d}\ {}_{k|}q_{40}$$

$$= 100,000\,{}_{20}E_{40} + \frac{\pi}{d}\left(\sum_{k=0}^{19} {}_{k|}q_{40} - v^{k+1}\ {}_{k|}q_{40}\right) = 100,000\ {}_{20}E_{40} + \frac{\pi}{d}\left({}_{20}q_{40} - A^{1}_{40:\overline{20|}}\right)$$

$$= 100,000\ {}_{20}E_{40} + \frac{\pi}{d}\left({}_{20}q_{40} - 1 + d\ddot{a}_{40:\overline{20|}} + v^{20}\ {}_{20}p_{40}\right)$$

$$= 100,000\ {}_{20}E_{40} + \pi\ddot{a}_{40:\overline{20|}} - \pi\ {}_{20}p_{40}\ \frac{1-v^{20}}{d}$$

$$= 100,000\ {}_{20}E_{40} + \pi\ddot{a}_{40:\overline{20|}} - {}_{20}p_{40}\ \ddot{a}_{\overline{20|}6\%}\pi.$$

Question 6.8

Answer: B

$\ddot{a}_{60:\overline{10|}} = 7.9555$

$\ddot{a}_{60:\overline{20|}} = 12.3816$

Annual level amount $= \dfrac{40 + 5\ddot{a}_{60:\overline{10|}} + 5\ddot{a}_{60:\overline{20|}}}{\ddot{a}_{60}} = \dfrac{141.686}{14.9041} = 9.51$

Question 6.9

Answer: D

$\ddot{a}_{50:\overline{10|}} = 8.0550$

$A^{\ 1}_{50:\overline{20|}} = A_{50:\overline{20|}} - {}_{20}E_{50} = 0.38844 - 0.34824 = 0.04020$

$\ddot{a}_{50:\overline{20|}} = 12.8428$

APV of Premiums = APV Death Benefit + APV Commission and Taxes + APV Maintenance

$G\ddot{a}_{50:\overline{10|}} = 100,000A^{\ 1}_{50:\overline{20|}} + 0.12G\ddot{a}_{50:\overline{10|}} + 0.3G + 25\ddot{a}_{50:\overline{20|}} + 50$

$8.0550G = 4020 + 1.2666G + 371.07$

$6.7883G = 4391.07$

$\Rightarrow G = 646.86$

Question 6.10

Answer: D

$$\ddot{a}_{x:\overline{3}|} = \frac{\text{Actuarial PV of the benefit}}{\text{Level Annual Premium}} = \frac{152.85}{56.05} = 2.727$$

$$\ddot{a}_{x:\overline{3}|} = 1 + \frac{0.975}{1.06} + \frac{0.975\left(p_{x+1}\right)}{(1.06)^2} = 2.727$$

$$\Rightarrow p_{x+1} = 0.93$$

Actuarial PV of the benefit =

$$152.85 = 1,000\left[\frac{0.025}{1.06} + \frac{0.975(1-0.93)}{(1.06)^2} + \frac{0.975(0.93)\left(q_{x+2}\right)}{(1.06)^3}\right]$$

$$\Rightarrow q_{x+2} = 0.09 \Rightarrow p_{x+2} = 0.91$$

Question 6.11

Answer: C

For calculating P

$$A_{50} = vq_{50} + vp_{50}A_{51} = v(0.0048) + v(1-0.0048)(0.39788) = 0.38536$$

$$\ddot{a}_{50} = \left(1 - A_{50}\right)/\,d = 15.981$$

$$P = A_{50}\,/\,\ddot{a}_{50} = 0.02411$$

For this particular life,

$$A'_{50} = vq'_{50} + vp'_{50}A_{51} = v(0.048) + (1-0.048)(0.39788) = 0.41037$$

$$\ddot{a}'_{50} = \left(1 - A'_{50}\right)/\,d = 15.330$$

Expected PV of loss = $A'_{50} - P\ddot{a}'_{50} = 0.41037 - 0.02411(15.330) = 0.0408$

Question 6.12

Answer: E

1,020 in the solution is the 1,000 death benefit plus the 20 death benefit claim expense.

$$A_x = 1 - d\ddot{a}_x = 1 - d(12.0) = 0.320755$$

$$G\ddot{a}_x = 1{,}020A_x + 0.65G + 0.10G\ddot{a}_x + 8 + 2\ddot{a}_x$$

$$G = \frac{1{,}020A_x + 8 + 2\ddot{a}_x}{\ddot{a}_x - 0.65 - 0.10\ddot{a}_x} = \frac{1{,}020(0.320755) + 8 + 2(12.0)}{12.0 - 0.65 - 0.10(12.0)} = 35.38622$$

Let $Z = v^{K_x+1}$ denote the present value random variable for a whole life insurance of 1 on (x).

Let $Y = \ddot{a}_{\overline{K_x+1}|}$ denote the present value random variable for a life annuity-due of 1 on (x).

$$\begin{aligned} L &= 1{,}020Z + 0.65G + 0.10GY + 8 + 2Y - GY \\ &= 1{,}020Z + (2 - 0.9G)Y + 0.65G + 8 \\ &= 1{,}020v^{K_x+1} + (2 - 0.9G)\frac{1 - v^{K_x+1}}{d} + 0.65G + 8 \\ &= \left(1{,}020 + \frac{0.9G - 2}{d}\right)v^{K_x+1} + \frac{2 - 0.9G}{d} + 0.65G + 8 \end{aligned}$$

$$\begin{aligned} Var(L) &= \left[{}^2A_x - (A_x)^2\right]\left(1{,}020 + \frac{0.9G - 2}{d}\right)^2 \\ &= (0.14 - 0.320755^2)\left(1{,}020 + \frac{0.9(35.38622) - 2}{d}\right)^2 \\ &= 0.037116(2{,}394{,}161) \\ &= 88{,}861 \end{aligned}$$

Question 6.13

Answer: D

If $T_{45} = 10.5$, then $K_{45} = 10$ and $K_{45} + 1 = 11$.

$${}_0L = 10{,}000v^{K_{45}+1} - G(1 - 0.10)\ddot{a}_{\overline{K_{45}+1}|} + G(0.80 - 0.10) = 10{,}000v^{11} - 0.9G\ddot{a}_{\overline{11}|} + 0.7G$$

$$4953 = 10{,}000(0.58468) - 0.9G(8.72173) + 0.7G$$

$$G = (5846.8 - 4953)/(7.14956) = 125.01$$

$$\begin{aligned} E({}_0L) &= 10{,}000A_{45} - (1 - 0.1)G\ddot{a}_{45} + (0.8 - 0.1)G \\ &= (10{,}000)(0.15161) - (0.9)(125.01)(17.8162) + (0.7)(125.01) \end{aligned}$$

$$E({}_0L) = -400.87$$

Question 6.14

Answer: D

$$100,000A_{40} = P[\ddot{a}_{40:\overline{10}|} + 0.5\,{}_{10|}\ddot{a}_{40:\overline{10}|}]$$

$$P = \frac{100,000A_{40}}{\ddot{a}_{40:\overline{10}|} + 0.5\,{}_{10|}\ddot{a}_{40:\overline{10}|}} = \frac{100,000(0.12106)}{8.0863 + 0.5(4.9071)} = \frac{12,106}{10.53985} = 1148.59$$

where

$${}_{10|}\ddot{a}_{40:\overline{10}|} = {}_{10}E_{40}\left[\ddot{a}_{50:\overline{10}|}\right] = 0.60920[8.0550] = 4.9071$$

There are several other ways to write the right hand side of the first equation.

Question 6.15

Answer: B

Woolhouse: $\quad {}^{W}\ddot{a}_x^{(4)} = 3.4611 - \frac{3}{8} = 3.0861$

UDD:
$$\begin{aligned} {}^{UDD}\ddot{a}_x^{(4)} &= \alpha(4)\ddot{a}_x - \beta(4) \\ &= 1.00019(3.4611) - 0.38272 \\ &= 3.0790 \end{aligned}$$

and $\quad A_x = 1 - d\,\ddot{a}_x = 1 - (0.04762)(3.4611) = 0.83518$

$$P^{(W)} = \frac{1000(0.83518)}{3.0861} = 270.63$$

$$P^{(UDD)} = \frac{1000(0.83518)}{3.0790} = 271.25$$

$$\frac{P^{(UDD)}}{P^{(W)}} = \frac{271.25}{270.63} = 1.0023$$

Question 6.16

Answer: A

$$P_{30:\overline{20|}} = \frac{1}{\ddot{a}_{30:\overline{20|}}} - d \Rightarrow \frac{2,143}{100,000} + 0.05 = \frac{1}{\ddot{a}_{30:\overline{20|}}} \Rightarrow \ddot{a}_{30:\overline{20|}} = 14$$

$$A_{30:\overline{20|}} = 1 - d\,\ddot{a}_{30:\overline{20|}} = 1 - 0.05(14) = 0.3$$

$$G\ddot{a}_{30:\overline{20|}} = 100,000A_{30:\overline{20|}} + (200 + 50\ddot{a}_{30:\overline{20|}}) + (0.33G + 0.06G\,\ddot{a}_{30:\overline{20|}})$$

$$14G = 100,000(0.3) + [200 + 50(14)] + (0.33G + 0.84G)$$

$$12.83G = 30,900$$

$$G = 2408$$

Question 6.17

Answer: A

$$q_x^{\text{NS}} = q_{x+1}^{\text{NS}} = 1 - e^{-0.1} = 0.095$$

Then the annual premium for the non-smoker policies is P^{NS}, where

$$P^{\text{NS}}\left(1 + vp_x^{\text{NS}}\right) = 100,000vq_x^{\text{NS}} + 100,000v^2 p_x^{\text{NS}} q_{x+1}^{\text{NS}} + 30,000v^2 p_x^{\text{NS}} p_{x+1}^{\text{NS}}$$

$$P^{\text{NS}} = \frac{100,000(0.926)(0.095) + 100,000(0.857)(0.905)(0.095) + 30,000(0.857)(0.905)^2}{1 + (0.926)(0.905)}$$

$$P^{\text{NS}} = 20,251$$

And then $P^{\text{S}} = 40,502$.

$$q_x^S = q_{x+1}^S = 1.5(1 - e^{-0.1}) = 0.143$$

$$\begin{aligned} EPV(L^{\text{S}}) &= 100,000vq_x^{\text{S}} + 100,000v^2 p_x^{\text{S}} q_{x+1}^{\text{S}} + 30,000v^2 p_x^{\text{S}} p_{x+1}^{\text{S}} - P^{\text{S}} - P^{\text{S}} vp_x^{\text{S}} \\ &= 100,000(0.926)(0.143) + 100,000(0.857)(0.857)(0.143) \\ &\quad + 30,000(0.857)(0.857)^2 - 40,502 - 40,502(0.926)(0.857) \\ &= -30,017 \end{aligned}$$

Question 6.18

Answer: D

$$P = 30{,}000\ {}_{20|}\ddot{a}_{40} + PA^{\,1}_{40:\overline{20|}}$$

$$\Rightarrow \quad P = 30{,}000\ {}_{20|}\ddot{a}_{40} \,/\left(1 - A^{\,1}_{40:\overline{20|}}\right)$$

$$= 30{,}000(5.46429)/(1-0.0146346) = 166{,}363$$

Question 6.19

Answer: C

Let π be the annual premium, so that $\pi\ddot{a}_{50} = A_{50} + 0.01\ddot{a}_{50} + 0.19$

$$\Rightarrow \pi = \frac{A_{50}+0.19}{\ddot{a}_{50}} + 0.01 = \frac{0.18931+0.19}{17.0245} + 0.01 = 0.03228$$

Loss at issue: $L_0 = v^{k+1} - (\pi - 0.01)\ddot{a}_{\overline{k+1|}}\left(1-v^{k+1}\right)/d + 0.19$

$$\Rightarrow Var\left[L_0\right] = \left(1 + \frac{(\pi - 0.01)}{d}\right)^2 \left({}^2A_{50} - A_{50}^2\right)$$

$$= (2.15467)(0.05108 - 0.18931^2)$$

$$= (2.15467)(0.015242)$$

$$= 0.033$$

Question 6.20

Answer: B

EPV(premiums) = EPV(benefits)

$$P\left(1 + vp_x + v^2\,{}_2p_x\right) = P\left(vq_x + 2v^2 p_x\, q_{x+1}\right) + 10000\left(v^3\,{}_2p_x\, q_{x+2}\right)$$

$$P\left(1 + \frac{0.9}{1.04} + \frac{0.9\times 0.88}{1.04^2}\right) = P\left(\frac{0.1}{1.04} + \frac{2\times 0.9\times 0.12}{1.04^2}\right) + 10000\left(\frac{0.9\times 0.88\times 0.15}{1.04^3}\right)$$

$$2.5976P = 0.29588P + 1056.13$$

$$P = 459$$

Question 6.21

Answer: C

$$P\times \ddot{a}_{75:\overline{15|}} = 1000\left(A^{\;1}_{75:\overline{15|}} + 15\times P\times A_{75:\overline{15|}}^{\;\;\;\;1}\right) \rightarrow P = \frac{1000A^{\;1}_{75:\overline{15|}}}{\ddot{a}_{75:\overline{15|}} - 15\times A_{75:\overline{15|}}^{\;\;\;\;1}}$$

$$A^{\;1}_{75:\overline{15|}} = A_{75:\overline{15|}} - A_{75:\overline{15|}}^{\;\;\;\;1} = 0.7 - 0.11 = 0.59$$

$$\ddot{a}_{75:\overline{15|}} = \frac{1 - A_{75:\overline{15|}}}{d} = (1-0.7)/0.04 = 7.5$$

So $P = \dfrac{590}{7.5 - 15(0.11)} = 100.85$

Question 6.22

Answer: C

Let the monthly net premium $= \pi$

$$12\pi = \frac{100{,}000\bar{A}_{45}}{\ddot{a}^{(12)}_{45:\overline{20|}}}$$

$$\alpha(12) = 1.00020$$
$$\beta(12) = 0.46651$$
$$\frac{i}{\delta} = 1.02480$$

$$100{,}000\bar{A}_{45} = 100{,}000\frac{i}{\delta}A_{45} = (1.02480)(15{,}161) = 15{,}536.99$$

$$\begin{aligned}\ddot{a}^{(12)}_{45:\overline{20|}} &= \alpha(12)\ddot{a}_{45:\overline{20|}} - \beta(12)\left(1 - {}_{20}E_{45}\right)\\ &= 1.00020\left[12.9391\right] - 0.46651(1-0.35994)\\ &= 12.6431\end{aligned}$$

$$12\pi = \frac{15{,}536.99}{12.6431}$$
$$12\pi = 1228.891$$
$$\pi = 102.41$$

Question 6.23

Answer: D

$$G\ddot{a}_{x:\overline{30}|} = \text{APV}\left[\text{gross premium}\right] = \text{APV}\left[\text{Benefits + expenses}\right]$$
$$= FA_x + \left(30 + 30\ddot{a}_x\right) + G\left(0.6 + 0.10\ddot{a}_{x:\overline{30}|} + 0.10\ddot{a}_{x:\overline{15}|}\right)$$

$$G = \frac{FA_x + 30 + 30\ddot{a}_x}{\ddot{a}_{x:\overline{30}|} - 0.6 - 0.1\ddot{a}_{x:\overline{30}|} - 0.1\ddot{a}_{x:\overline{15}|}}$$
$$= \frac{FA_x + 30 + 30(15.3926)}{14.0145 - 0.6 - 0.1(14.0145) - 0.1(10.1329)}$$
$$= \frac{FA_x + 491.78}{10.9998}$$
$$= \frac{FA_x}{10.9998} + \frac{491.78}{10.9998} = \frac{FA_x}{10.9998} + 44.71$$
$$\Rightarrow h = 44.71$$

Question 6.24

Answer: E

In general, the loss at issue random variable can be expressed as:

$$L = \bar{Z}_x - P\bullet\bar{Y}_x = \bar{Z}_x - P\bullet\left(\frac{1-\bar{Z}_x}{\delta}\right) = \bar{Z}_x\bullet\left(1+\frac{P}{\delta}\right) - \frac{P}{\delta}$$

Using actuarial equivalence to determine the premium rate:

$$P = \frac{\bar{A}_x}{\bar{a}_x} = \frac{0.3}{(1-0.3)/0.07} = 0.03$$

$$Var(L) = \left(1+\frac{P}{\delta}\right)^2 \bullet Var\left(\bar{Z}_x\right) = \left(1+\frac{0.03}{0.07}\right)^2 \bullet Var\left(\bar{Z}_x\right) = 0.18$$
$$Var\left(\bar{Z}_x\right) = \frac{0.18}{\left(1+\frac{0.03}{0.07}\right)^2} = 0.088$$

$$Var(L^*) = \left(1+\frac{P^*}{\delta}\right)^2 \bullet Var\left(\bar{Z}_x\right) = \left(1+\frac{0.06}{0.07}\right)^2 (0.088) = 0.304$$

Question 6.25

Answer: C

Need $\text{EPV(Ben} + \text{Exp)} - \text{EPV(Prem)} = -800$

$$\text{EPV(Prem)} = G\ddot{a}_{55:\overline{10}|} = 8.0192G$$

$$\begin{aligned}
\text{EPV(Ben} + \text{Exp)} &= 12{,}000\ {}_{10|}\ddot{a}_{55}^{(12)} + 300\ddot{a}_{55} \\
&= 12{,}000\ {}_{10}E_{55}\ddot{a}_{65}^{(12)} + 300\ddot{a}_{55} \\
&= 12{,}000\ {}_{10}E_{55}\left(\ddot{a}_{65} - \frac{m-1}{2m}\right) + 300\ddot{a}_{55} \\
&= 12{,}000(0.59342)\left(13.5498 - 11/24\right) + 300(16.0599) \\
&= 98{,}042.83
\end{aligned}$$

Therefore, $98{,}042.83 - 8.0192G = -800$

$G = 12{,}326$

Question 6.26

Answer: D

EPV (Premiums) = $Pa_{90} = P(\ddot{a}_{90} - 1) = (4.1835)P$

$\text{EPV(Benefits)} = 1000A_{90} = 1000(0.75317) = 753.17$

Therefore,

$$P = \frac{753.17}{4.1835} = 180.03$$

Question 6.27

Answer: D

$$\text{EPV(Premiums)} = \text{EPV(Benefits)}$$

$$\begin{aligned}\text{EPV(Premiums)} &= 3P\,\bar{a}_x - 2P\,{}_{20}E_x\,\bar{a}_{x+20} \\ &= 3P\left(1/\mu+\delta\right) - 2P\left(e^{-20(\mu+\delta)}\right)\left(1/\mu+\delta\right) \\ &= 3P\left(1/0.09\right) - 2Pe^{-1.8} - 1/0.09 \\ &= 29.66P\end{aligned}$$

$$\begin{aligned}\text{EPV(Benefits)} &= 1{,}000{,}000\bar{A}_x - 500{,}000\,{}_{20}E_x\,\bar{A}_{x+20} \\ &= 1{,}000{,}000\left(\mu/\mu+\delta\right) - 500{,}000e^{-20(\mu+\delta)}\ \ \mu/\mu+\delta \\ &= 1{,}000{,}000\left(0.03/0.09\right) - 500{,}000e^{-1.8}\ \ 0.03/0.09 \\ &= 305{,}783.5\end{aligned}$$

$$29.66P = 305{,}783.5$$

$$P = \frac{305{,}783.5}{29.66}$$

$$P = 10{,}309.62$$

Question 6.28

Answer: B

$$G\ddot{a}_{40:\overline{5}|} = 1000A_{40} + 0.15G + 0.05G\ddot{a}_{40:\overline{5}|} + 5 + 5\ddot{a}_{40:\overline{5}|}$$

$$\ddot{a}_{40:\overline{5}|} = \ddot{a}_{40} - {}_5E_{40} \bullet \ddot{a}_{45} = 18.4578 - (0.78113)(17.8162) = 4.5410$$

$$G = \frac{121.06 + 5 + 5(4.5410)}{-0.15 + 0.95(4.5410)} = 35.73$$

Question 6.29

Answer: B

Per equivalence Principle:

$$G\ddot{a}_{35} = 100,000A_{35} + 0.4G + 150 + 0.1G\ddot{a}_{35} + 50\ddot{a}_{35}$$

$$1770\ddot{a}_{35} = 100,000\left(1 - d\ddot{a}_{35}\right) + 0.4(1770) + 150 + 0.1(1770)\ddot{a}_{35} + 50\ddot{a}_{35}$$

$$1770\ddot{a}_{35} = 100,000 + 708 + 150 + \ddot{a}_{35}\left(177 + 50 - 100,000\left(\frac{0.035}{1.035}\right)\right)$$

Solving for $\ddot{a}_{35}$, we have

$$\ddot{a} = \frac{100,858}{1770 + 3154.64} = \frac{100,858}{4924.64} = 20.48$$

Question 6.30

Answer: A

The loss at issue is given by:

$$L_0 = 100v^{K+1} + 0.05G + 0.05G\,\ddot{a}_{\overline{K+1|}} - G\,\ddot{a}_{\overline{K+1|}}$$

$$= 100v^{K+1} + 0.05G - 0.95G\left(\frac{1 - v^{K+1}}{d}\right)$$

$$= \left(100 + \frac{0.95G}{d}\right)v^{K+1} + 0.05G - 0.95\frac{G}{d}$$

Thus, the variance is

$$Var\left(L_0\right) = \left[100 + \frac{0.95\left(2.338\right)}{0.04/1.04}\right]^2\left({}^2A_x - \left(A_x\right)^2\right)$$

$$= \left[100 + \frac{0.95\left(2.338\right)}{0.04/1.04}\right]^2\left(0.17 - \left(1 - \frac{0.04}{1.04}(16.50)\right)^2\right)$$

$$= 908.1414$$

Question 6.31

Answer: D

$$\bar{A}_{35} = \left(1 - e^{-35(\mu+\delta)}\right) \times \left(\frac{\mu}{\mu+\delta}\right) + e^{-35(\mu+\delta)}\bar{A}_{70} = 0.063421 + 0.146257 = 0.209679$$

$$\bar{a}_{35} = \frac{1-\bar{A}_{35}}{\delta} = \frac{1-0.209679}{0.05} = 15.80642$$

$$\bar{P}_{35} = \frac{\bar{A}_{35}}{\bar{a}_{35}} = \frac{0.209679}{15.80642} = 0.0132654$$

The annual net premium for this policy is therefore $100,000 \times 0.0132654 = 1,326.54$

Question 6.32

Answer: C

Assuming UDD

Let P = monthly net premium

$$\text{EPV(premiums)} = 12P\ddot{a}_x^{(12)} \cong 12P\left[\alpha(12)\dot{a}_x - \beta(12)\right]$$

$$= 12P\left[1.00020(9.19) - 0.46651\right]$$

$$= 104.7039P$$

$$\text{EPV(benefits)} = 100,000\bar{A}_x$$

$$= 100,000\frac{i}{\delta}A_x = 100,000\frac{i}{\delta}\left(1 - d\ddot{a}_x\right)$$

$$= 100,000\frac{0.05}{\ln(1.05)}\left(1 - \frac{0.05}{1.05}(9.19)\right)$$

$$= 57,632.62$$

$$P = \frac{57,632.62}{104.7039} = 550.43$$

Question 6.33

Answer: B

The probability that the endowment payment will be made for a given contract is:

$$\begin{aligned} {}_{15}p_x &= \exp\left(-\int_0^{15} 0.02t\, dt\right) \\ &= \exp\left(-0.01t^2 \Big|_0^{15}\right) \\ &= \exp\left(-0.01(15)^2\right) \\ &= 0.1054 \end{aligned}$$

Because the premium is set by the equivalence principle, we have $E\left[{}_0L\right] = 0.$ Further,

$$\begin{aligned} \text{Var}\left({}_0L\right) &= 500\left[\left(10{,}000v^{15}\right)^2 \left({}_{15}p_x\right)\left(1 - {}_{15}p_x\right)\right] \\ &= 1{,}942{,}329{,}000 \end{aligned}$$

Then, using the normal approximation, the approximate probability that the aggregate losses exceed 50,000 is

$$P\left({}_0L > 50{,}000\right) = P\left(Z > \frac{50{,}000 - 0}{\sqrt{1{,}942{,}329{,}000}}\right) = P(Z > 1.13) = 0.13$$

Question 6.34

Answer: A

Let B be the amount of death benefit.

$$\text{EPV(Premiums)} = 500\ddot{a}_{61} = 500(14.6491) = 7324.55$$

$$\text{EPV(Benefits)} = \text{B} \cdot A_{61} = (0.30243)\text{B}$$

$$\text{EPV(Expenses)} = (0.12)(500) + (0.03)(500)\ddot{a}_{61} = (0.12)(500) + (0.03)(7324.55) = 279.74$$

$$\text{EPV(Premiums)} = \text{EPV(Benefits)} + \text{EPV(Expenses)}$$

$$7324.55 = (0.30243)\text{B} + 279.74$$

$$7044.81 = (0.30243)\text{B}$$

$$\text{B} = 23{,}294$$

Question 6.35

Answer: D

Let G be the annual gross premium. By the equivalence principle, we have

$$G\ddot{a}_{35} = 100,000A_{35} + 0.15G + 0.04G\ddot{a}_{35}$$

so that

$$G = \frac{100,000A_{35}}{0.96\ddot{a}_{35} - 0.15} = \frac{100,000(0.09653)}{0.96(18.9728) - 0.15} = 534.38$$

Question 6.36

Answer: B

By the equivalence principle,

$$4500\bar{a}_{x:\overline{20|}} = 100,000\bar{A}^{\,1}_{x:\overline{20|}} + R\bar{a}_{x:\overline{20|}}$$

where

$$\bar{A}^{\,1}_{x:\overline{20|}} = \frac{\mu}{\mu+\delta}\left(1 - e^{-20(\mu+\delta)}\right) = \frac{0.04}{0.12}\left(1 - e^{-20(0.12)}\right) = 0.3031$$

$$\bar{a}_{x:\overline{20|}} = \frac{1 - e^{-20(\mu+\delta)}}{\mu+\delta} = \frac{1 - e^{-20(0.12)}}{0.12} = 7.5774$$

Solving for R, we have

$$R = 4500 - 100,000\left(\frac{0.3031}{7.5774}\right) = 500$$

Question 6.37

Answer: D

By the equivalence principle, we have

$$G\ddot{a}_{35:\overline{10|}} = 50,000A_{35} + 100a_{35} + 100A_{35}$$

so

$$G = \frac{50,100A_{35} + 100(\ddot{a}_{35} - 1)}{\ddot{a}_{35:\overline{10|}}} = \frac{50,100(0.09653) + 100(17.9728)}{8.0926} = 819.69$$

Question 6.38

Answer: B

Let P be the annual net premium

$$P = \frac{1000\bar{A}_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} = \frac{1000(0.192)}{\ddot{a}_{x:\overline{n|}}}$$

where

$$\ddot{a}_{x:\overline{n|}} = \frac{1 - A_{x:\overline{n|}}}{d} = \frac{(1.05)}{(0.05)}\left(1 - A^{1}_{x:\overline{n|}} - A_{x:\overset{\;1}{\overline{n|}}}\right)$$

$$\bar{A}_{x:\overline{n|}} = \frac{i}{\delta}\left(A^{1}_{x:\overline{n|}}\right) + {}_{n}E_{x}$$

$$\Rightarrow 0.192 = \frac{0.05}{0.04879}\left(A^{1}_{x:\overline{n|}}\right) + 0.172$$

$$\Rightarrow A^{1}_{x:\overline{n|}} = 0.019516$$

$$\Rightarrow \ddot{a}_{x:\overline{n|}} = \frac{1.05}{0.05}(1 - 0.019516 - 0.172) = 16.978$$

Therefore, we have

$$P = \frac{1000(0.192)}{16.978} = 11.31$$

Question 6.39

Answer: A

Premium at issue for (40): $\dfrac{1000A_{40}}{\ddot{a}_{40}} = \dfrac{121.06}{18.4578} = 6.5587$

Premium at issue for (80): $\dfrac{1000A_{80}}{\ddot{a}_{80}} = \dfrac{592.93}{8.5484} = 69.3615$

Lives in force after ten years:

Issued at age 40: $10,000\,{}_{10}p_{40} = 10,000 \times \dfrac{98,576.4}{99,338.3} = 9923.30$

Issued at age 80: $10,000\,{}_{10}p_{80} = 10,000 \times \dfrac{41,841.1}{75,657.2} = 5530.35$

The total number of lives after ten years is therefore: $9923.30 + 5530.35 = 15,453.65$

The average premium after ten years is therefore:

$$\frac{(6.5587 \times 9923.30) + (69.3615 \times 5530.35)}{15,453.65} = 29.03$$

Question 6.40

Answer: C

Let P be the annual net premium at $x+1$. Also, let A_y^* be the expected present value for the special insurance described in the problem issued to (y).

$$P\ddot{a}_{x+1} = 1000\sum_{k=0}^{\infty}(1.03)^{k+1} v^{k+1} {}_{k|}q_{x+1} = 1000A_{x+1}^*$$

We are given

$$110\ddot{a}_x = 1000\sum_{k=0}^{\infty}(1.03)^{k+1} v^{k+1} {}_{k|}q_x = 1000A_x^*$$

Which implies that

$$110\left(1+vp_x\ddot{a}_{x+1}\right) = 1000\left(1.03vq_x + 1.03vp_xA_{x+1}^*\right)$$

Solving for A_{x+1}^*, we get

$$A_{x+1}^* = \frac{\frac{110}{1000}\left[1+v(0.95)(7)\right]-1.03v(0.05)}{1.03v(0.95)} = 0.8141032$$

Thus, we have

$$P = \frac{1000(0.8141032)}{7} = 116.3005$$

Question 6.41

Answer: B

Let P be the net premium for year 1.

Then:

$$P + 1.01Pvp_x = 100,000vq_x + (1.01)(100,000)v^2 p_x q_{x+1}$$

$$P\left[1+\frac{1.01}{1.05}0.99\right] = 100,000\left(\frac{0.01}{1.05} + \frac{(1.01)(0.99)(0.02)}{(1.05)^2}\right) \Rightarrow P = 1416.93$$

Question 6.42

Answer: D

The policy is fully discrete, so all cash flows occur at the start or end of a year.

Die Year 1 ==> $L_0 = 1000v - 315.80 = 625.96$

Die Year 2 ==> $L_0 = 1000v^2 - 315.80(1+v) = 273.71$

Survive Year 2 ==> $L_0 = 1000v^3 - 315.80(1+v+v^2) = -58.03$

There is a loss if death occurs in year 1 or year 2, otherwise the policy was profitable.

$\Pr\left(\text{death in year 1 or 2}\right) = 1 - e^{-2\mu} = 0.113$

Question 6.43

Answer: C

$APV\left(\text{expenses}\right) = 0.35G + 8 + 0.15Ga_{30:\overline{4}|} + 4a_{30:\overline{9}|}$

$= 0.20G + 4 + 0.15G\,\ddot{a}_{30:\overline{5}|} + 4\ddot{a}_{30:\overline{10}|}$

$G\,\ddot{a}_{30:\overline{5}|} = 0.20G + 4 + 0.15G\,\ddot{a}_{30:\overline{5}|} + 4\ddot{a}_{30:\overline{10}|} + 200{,}000A^{1}_{30:\overline{10}|}$

$$G = \frac{200{,}000A^{1}_{30:\overline{10}|} + 4 + 4\ddot{a}_{30:\overline{10}|}}{0.85\ddot{a}_{30:\overline{5}|} - 0.20}$$

$200{,}000\,A^{1}_{30:\overline{10}|} = 200{,}000\left[A_{30:\overline{10}|} - {}_{10}E_{30}\right]$

$= 200{,}000(0.61447 - 0.61152) = 590$

$$G = \frac{590 + 4 + 4\left(8.0961\right)}{0.85\left(4.5431\right) - 0.20} = 171.07$$

Questio# 6.44

Answer: D

Let P be the premium per 1 of insurance.

$$P\ddot{a}_{50:\overline{10|}} = P(IA)^{1}_{50:\overline{10|}} + {}_{10}E_{50}A_{60}$$

$$\ddot{a}_{50:\overline{10|}} = \ddot{a}_{50} - {}_{10}E_{50}\ddot{a}_{60} = 17.0 - 0.60 \times 15.0 = 8$$

$$A_{60} = 1 - d\ddot{a}_{60} = 1 - \left(\frac{0.05}{1.05}\right)15 = 0.285714$$

$$P\left(\ddot{a}_{50:\overline{10|}} - (IA)^{1}_{50:\overline{10|}}\right) = {}_{10}E_{50}A_{60}$$

$$P = \frac{{}_{10}E_{50}A_{60}}{\ddot{a}_{50:\overline{10|}} - (IA)^{1}_{50:\overline{10|}}} = \frac{0.6 \times 0.285714}{8 - 0.15} = 0.021838$$

$$100P = 2.18$$

Question 6.45

Answer: E

$$L_0 = 100{,}000v^T - 560\bar{a}_{\overline{T}|} = \left(100{,}000 + \frac{560}{\delta}\right)e^{-\delta T} - \frac{560}{\delta}$$

Since L_0 is a decreasing function of T, the 75th percentile of L_0 is $L_0(t)$ where t is such that $\Pr[T_{35} > t] = 0.75$.

$$\frac{\ell_{35+t}}{\ell_{35}} = 0.75$$

$$\ell_{35+t} = 0.75\ell_{35} = 0.75 \times 99{,}556.7 = 74{,}667.5$$

$$\ell_{81} < 74{,}667.5 < \ell_{80}$$

$$t = (80 - 35) + s$$

$$\ell_{80+s} = s\ell_{81} + (1-s)\ell_{80}$$

$$74{,}667.5 = 73{,}186.3s + 75{,}657.2(1-s)$$

$$s = 0.40054$$

$$t = 45.40054$$

$$L_0(45.40054) = \left(100{,}000 + \tfrac{560}{\ln(1.05)}\right)e^{-45.40054\ln(1.05)} - \tfrac{560}{\ln(1.05)} = 689.25$$

Question 6.46

Answer: E

Let P be the premium per 1 of insurance.

$$P\ddot{a}_{55:\overline{10}|} = 0.51213P + v^{10}\,{}_{10}p_{55}\,\ddot{a}_{65}$$

$$\ddot{a}_{55} = \ddot{a}_{55:\overline{10}|} + v^{10}\,{}_{10}p_{55}\,\ddot{a}_{65} \Rightarrow v^{10}\,{}_{10}p_{55}\,\ddot{a}_{65} = 12.2758 - 7.4575 = 4.8183$$

$$7.4575P = 0.51213P + 4.8183 \Rightarrow P = 0.693742738$$

$$300P = 208.12$$

Question 6.47

Answer: D

$$G\ddot{a}_{70:\overline{10}|} = 100{,}000\,{}_{10}E_{70}\ddot{a}_{80} + 0.05G\ddot{a}_{70:\overline{10}|} + 0.7G$$

$$7.6491G = (100{,}000)(0.50994)(8.5484) + 0.05G(7.6491) + 0.7G$$

$$\Rightarrow G = 66{,}383.54$$

Question 6.48

Answer: A

Actuarial present value of insured benefits:

$$100{,}000\left[\frac{0.95\times 0.02}{1.06^6} + \frac{0.95\times 0.98\times 0.03}{1.06^7} + \frac{0.95\times 0.98\times 0.97\times 0.04}{1.06^8}\right] = 5{,}463.32$$

$$\Rightarrow P\left(1 + \frac{0.95}{1.06^5}\right) = 5{,}463.32 \Rightarrow P = 3{,}195.12$$

Question 6.49

Answer: C

$$G\ddot{a}^{(12)}_{40:\overline{20}|} = 100{,}000\left(\frac{i}{\delta}\right)A_{40} + 200 + 0.04G\ddot{a}^{(12)}_{40:\overline{20}|}$$

$$\ddot{a}^{(12)}_{40:\overline{20}|} = \alpha(12)\ddot{a}_{40:\overline{20}|} - (1 - {}_{20}E_{40})\beta(12)$$
$$= 1.00020\cdot 12.9935 - (1 - 0.36663)\cdot 0.46651 = 12.700625$$

$$G = \frac{(100{,}000)(1.02480)(0.12106)+200}{0.96\times 12.700625} = 1033.92$$

$$\Rightarrow \text{G/12} = 86.16$$

Question 6.50

Answer: A

$$1,000P = 1,000\frac{A_{35}}{\ddot{a}_{35}} = \frac{96.53}{18.9728} = 5.0878$$

Benefits paid during July 2018:

$$10,000 \times 1,000 \times q_{35} = 10,000 \times 0.391 = 3910$$

Premiums payable during July 2018:

$$10,000 \times (1 - q_{35}) \times 5.0878 = 9,996.09 \times 5.0878 = 50,858.10$$

Cash flow during July 2018:

$$3910 - 50,858 = -46,948$$

Question 6.51

Answer: D

Under the Equivalence Principle

$$P\ddot{a}_{62:\overline{10}|} = 50,000\left(\ddot{a}_{62} - \ddot{a}_{62:\overline{10}|}\right) + P\left((IA)^{\,1}_{62:\overline{10}|}\right)$$

where $(IA)^{\,1}_{62:\overline{10}|} = 11A^{\,1}_{62:\overline{10}|} - \sum_{k=1}^{10} A^{\,1}_{62:\overline{k}|} = 11(0.091) - 0.4891 = 0.5119$

So $$P = \frac{50,000\left(\ddot{a}_{62} - \ddot{a}_{62:\overline{10}|}\right)}{\ddot{a}_{62:\overline{10}|} - (IA)^{\,1}_{62:\overline{10}|}} = \frac{50,000(12.2758 - 7.4574)}{7.4574 - 0.5119} = 34,687$$

Question 6.52

Answer: E

$$G\ddot{a}_{45:\overline{10}|} = HA_{45} + G + 0.05G\ddot{a}_{45:\overline{10}|} + 80 + 10\ddot{a}_{45} + 10\ddot{a}_{45:\overline{10}|}$$

$$G = \frac{HA_{45} + 80 + 10\left(\ddot{a}_{45} + \ddot{a}_{45:\overline{10}|}\right)}{0.95\ddot{a}_{45:\overline{10}|} - 1}$$

$$G = \frac{HA_{45} + 80 + 10(17.8162 + 8.0751)}{(0.95 \times 8.0751) - 1}$$

$$G = \frac{A_{45}}{(0.95 \times 8.0751) - 1}H + \frac{80 + 10(17.8162 + 8.0751)}{(0.95 \times 8.0751) - 1}$$

$$g = \frac{A_{45}}{(0.95 \times 8.0751) - 1}$$

$$f = \frac{80 + 10(17.8162 + 8.0751)}{(0.95 \times 8.0751) - 1} = 50.80$$

Question 6.53

Answer: D

$$G = 0.35G + 2000\left(\frac{0.1}{1.08} + \frac{0.9 \times 0.1}{1.08^2} + \frac{0.9 \times 0.9 \times 0.1}{1.08^3}\right)$$

$$0.65G = 468.107$$

$$G = 720.16$$

Question 6.54

Answer: A

On a unit basis, $Var(L_0)=\left(1+\frac{P}{d}\right)^2\left[{}^2A_{45}-(A_{45})^2\right]=\left(1+\frac{A_{45}}{d\ddot{a}_{45}}\right)^2\left[{}^2A_{45}-(A_{45})^2\right]$

$$=\left(\frac{d\ddot{a}_{45}+1-d\ddot{a}_{45}}{d\ddot{a}_{45}}\right)^2\left[{}^2A_{45}-(A_{45})^2\right]=\frac{{}^2A_{45}-(A_{45})^2}{(d\ddot{a})^2}$$

$$=\frac{0.03463-0.15161^2}{\left(\frac{0.05}{1.05}\times 17.8162\right)^2}=0.016178038$$

The standard deviation of $L_0=0.127193$

$(200,000)(\text{The standard deviation of } L_0)=25,439$

Question 7.1

Answer: C

$$
\begin{aligned}
{}_{10}V &= 50{,}000\left(A_{50} + {}_{10}E_{50}A_{60}\right) - (875)\left[\ddot{a}_{50:\overline{10|}}\right] \\
&= 50{,}000[0.18931 + (0.60182)(0.29028)] - 875[8.0550] \\
&= 11{,}152
\end{aligned}
$$

Question 7.2

Answer: C

${}_0V = 0$

${}_2V = 2000$

Year 1: $\left({}_0V + P\right)\left(1+i\right) = q_x\left(2000 + {}_1V\right) + p_x\ {}_1V$

$$
\begin{aligned}
&P(1.1) = 0.15(2000 + {}_1V) + 0.85({}_1V) \\
&1.1P - 300 = {}_1V
\end{aligned}
$$

Year 2: $\left({}_1V + P\right)\left(1+i\right) = q_{x+1}(2000 + {}_2V) + p_{x+1}(2000)$

$$
\begin{aligned}
&(1.1P - 300 + P)(1.1) = 0.165(2000 + 2000) + 0.835(2000) \\
&2.31P - 330 = 2330
\end{aligned}
$$

$$
P = \frac{2330 + 330}{2.31} = 1152
$$

Question 7.3

Answer: E

$i^{(4)} = 0.08$ means an interest rate of $j = 0.02$ per quarter. This problem can be done with two quarterly recursions or a single calculation.

Using two recursions:

$$_{10.75}V = \frac{[_{10.5}V + 60(1-0.1)](1.02) - \dfrac{800-706}{800}(1000)}{\dfrac{706}{800}}$$

$$753.72 = \frac{[_{10.5}V + 54](1.02) - 117.50}{0.8825} \Rightarrow {}_{10.5}V = 713.31$$

$$_{10.5}V = \frac{[_{10.25}V](1.02) - \dfrac{898-800}{898}(1000)}{\dfrac{800}{898}} \Rightarrow 713.31 = \frac{[_{10.25}V](1.02) - 109.13}{0.8909}$$

$$_{10.25}V = 730.02$$

Using a single step, $_{10.25}V$ is the EPV of cash flows through time 10.75 plus $_{0.5}E_{80.25}$ times the EPV of cash flows thereafter (that is, $_{10.75}V$).

$$_{10.25}V = (1000)\left[\frac{898-800}{898(1.02)} + \frac{800-706}{898(1.02)^2}\right] - (60)(1-0.1)\left[\frac{800}{898(1.02)}\right] + \left[\frac{706}{898(1.02)^2}\right](753.72) = 730$$

Question 7.4

Answer: B

EPV of benefits at issue $= 1000A_{40} + 4\ {}_{11}E_{40}(1000A_{51})$

$$= 121.06 + (4)(0.57949)(197.80) = 579.55$$

EPV of expenses at issue $= 100 + 10(\ddot{a}_{40} - 1) = 100 + 10(17.4578) = 274.58$

$\pi = (579.55 + 274.58) / \ddot{a}_{40} = 854.13 / 18.4578 = 46.27$

$G = 1.02\pi = 47.20$

EPV of benefits at time $1 = 1000A_{41} + 4\ {}_{10}E_{41} \times 1000A_{51}$

$$= 126.65 + (4)(0.60879)(197.80) = 608.32$$

EPV of expenses at time $1 = 10(\ddot{a}_{41}) = 10(18.3403) = 183.40$

Gross Prem Policy Value $= 608.32 + 183.40 - G\ddot{a}_{41} = 791.72 - 47.20(18.3403) = -73.94$

Question 7.5

Answer: E

${}_{4.5}V = v^{0.5}\ {}_{0.5}p_{x+4.5}\ {}_{5}V + v^{0.5}\ {}_{0.5}q_{x+4.5}\ b$, where $b = 10,000$ is the death benefit during year 5

$${}_{0.5}q_{x+4.5} = \frac{{}_{0.5}q_{x+4}}{1 - {}_{0.5}q_{x+4}} = \frac{0.5(0.04561)}{1 - 0.5(0.04561)} = 0.02334$$

$${}_{0.5}p_{x+4.5} = 0.97666$$

$${}_{5}V = \frac{({}_{4}V + P)(1.03) - q_{x+4}b}{p_{x+4}}$$

$${}_{5}V = \frac{(1,405.08 + 647.46)(1.03) - 0.04561(10,000)}{0.95439} = 1,737.25$$

$${}_{4.5}V = (1.03)^{-0.5}(0.97666)(1,737.25) + (1.03)^{-0.5}(0.02334)(10,000)$$

$$= 1,671.81 + 229.98 = 1,902$$

${}_{4.5}V$ can also be calculated recursively:

$${}_{0.5}q_{x+4} = 0.5(0.04561) = 0.02281$$

$${}_{4.5}V = \frac{(1,405.08 + 647.16)(1.03)^{0.5} - 0.02281(10,000)/(1.03)^{0.5}}{1 - 0.02281} = 1,902$$

The interest adjustment on the death benefit term is needed because the death benefit will not be paid for another one-half year.

Question 7.6

Answer: E

Gross premium = G

$$G\ddot{a}_{45} = 2000A_{45} + \underbrace{\left(1\left(\frac{2000}{1000}\right)+20\right)}_{22} + \underbrace{\left(0.5\left(\frac{2000}{1000}\right)+10\right)}_{11}\ddot{a}_{45} + 0.20G + 0.05G\ddot{a}_{45}$$

$$\left(0.95\ddot{a}_{45} - 0.20\right)G = 2000A_{45} + 22 + 11\ddot{a}_{45}$$

$$G = \frac{2000A_{45} + 22 + 11\ddot{a}_{45}}{0.95\ddot{a}_{45} - 0.20} = \frac{2000(0.15161) + 22 + 11(17.8162)}{0.95(17.8162) - 0.20} = 31.16$$

There are two ways to proceed. The first is to calculate the gross premium policy value (with equivalence principle gross premium and original assumptions, both of which do apply here) and the net premium policy value and take the difference.

The net premium is $\dfrac{2000A_{45}}{\ddot{a}_{45}} = \dfrac{2000(0.15161)}{17.8162} = 17.02$

The net premium policy value is $2000A_{55} - 17.02\ddot{a}_{55} = 2000(0.23524) - 17.02(16.0599) = 197.14$

The gross premium policy value is

$$2000A_{55} + [0.05(31.16) + 0.5(2000/1000) + 10]\ddot{a}_{55} - 31.16\ddot{a}_{55}$$
$$= 2000(0.23524) + (12.56 - 31.16)(16.0599) = 171.77$$

Expense reserve is 171.77 – 197.14 = –25

The second is to calculate the expense reserve directly based on the pattern of expenses. The first step is to determine the expense premium.

The present value of expenses is

$$[0.05G + 0.5(2000/1000) + 10]\ddot{a}_{45} + 0.20G + 1.0(2000/1000) + 20$$
$$= 12.558(17.8162) + 28.232 = 251.97$$

The expense premium is 251.97/17.8162=14.14

The expense reserve is the expected present value of future expenses less future expense premiums, that is,

$$[0.05G + 0.5(2000/1000) + 10]\ddot{a}_{55} - 14.14\ddot{a}_{55} = -1.582(16.0599) = -25$$

There is a shortcut with the second approach based on recognizing that expenses that are level throughout create no expense reserve (the level expense premium equals the actual expenses). Therefore, the expense reserve in this case is created entirely from the extra first year expenses. They occur only at issue so the expected present value is 0.20(31.16)+1.0(2000/1000)+20 = 28.232. The expense premium for those expenses is then 28.232/17.8162 = 1.585 and the expense reserve is the present value of future non-level expenses (0) less the present value of those future expense premiums, which is 1.585(16.0599) = 25 for a reserve of –25.

Question 7.7

Answer: D

$$\ddot{a}_{x+10} = (1 - A_{x+10}) / d = (1 - 0.4) / (0.05 / 1.05) = 12.6$$

$$\ddot{a}_{x+10}^{(12)} \approx 12.6 - 11/24 = 12.142$$

$${}_{10}V = 10,000 A_{x+10} + 100 \ddot{a}_{x+10} - 12 \ddot{a}_{x+10}^{(12)} (30)(1 - 0.05)$$

$${}_{10}V = 10,000(0.4) + 100(12.6) - 12(12.142)(28.50)$$

$${}_{10}V = 1107$$

Question 7.8

Answer: C

The simplest solution is recursive:

${}_0V = 0$ since the policy values are net premium policy values.

$$q_{[70]} = (0.7)(0.010413) = 0.007289$$

$${}_1V = \frac{(0 + 35.168)(1.05) - (1000)(0.007289)}{1 - 0.007289} = 29.86$$

Prospectively, $q_{[70]+1} = (0.8)(0.011670) = 0.009336$; $q_{[70]+2} = (0.9)(0.013081) = 0.011773$

$$A_{[70]+1} = (0.009336)v + (1 - 0.009336)(0.011773)v^2 + (1 - 0.009336)(1 - 0.011773)(0.47580)v^2 = 0.44197$$

$$\ddot{a}_{[70]+1} = \left(1 - A_{[70]+1}\right) / d = (1 - 0.44197) / (0.05 / 1.05) = 11.7186$$

$${}_1V = (1000)(0.44197) - (11.7186)(35.168) = 29.85$$

Question 7.9

Answer: A

Let $P = 0.00253$ be the monthly net premium per 1 of insurance.

$$
\begin{aligned}
{}_{10}V &= 100{,}000\left[\frac{i}{\delta} A^{\;1}_{55:\overline{10|}} + A_{55:\overset{\;\;1}{\overline{10|}}} - 12P\ddot{a}^{(12)}_{55:\overline{10|}}\right] \\
&= 100{,}000\left[1.02480(0.02471) + 0.59342 - (12)(0.00253)(7.8311)\right] \\
&\approx 38{,}100
\end{aligned}
$$

Where

$$
\begin{aligned}
A^{\;1}_{55:\overline{10|}} &= A_{55:\overline{10|}} - {}_{10}E_{55} = 0.61813 - 0.59342 = 0.02471 \\
A_{55:\overset{\;\;1}{\overline{10|}}} &= {}_{10}E_{55} = 0.59342 \\
\ddot{a}_{55:\overline{10|}} &= 8.0192 \\
\ddot{a}^{(12)}_{55:\overline{10|}} &= \alpha(12)\ddot{a}_{55:\overline{10|}} - \beta(12)\left[1 - {}_{10}E_{55}\right] \\
&= 1.00020(8.0192) - 0.46651(1 - 0.59342) = 7.8311
\end{aligned}
$$

Question 7.10

Answer: C

Use superscript *g* for gross premiums and gross premium policy values.

Use superscript *n* (representing "net") for net premiums and net premium policy values.

Use superscript *e* for expense premiums and expense reserves.

$P^g = 977.60$ (given)

$$P^e = \frac{0.58P^g + 450 + \left(0.02P^g + 50\right)\ddot{a}_{45}}{\ddot{a}_{45}}$$

$$= \frac{0.58(977.60) + 450 + [0.02(977.60) + 50]17.8162}{17.8162} = 126.64$$

Alternatively,

$$P^n = \frac{100{,}000A_{45}}{\ddot{a}_{45}} = 850.97 \qquad P^e = P^g - P^n = 126.63$$

$${}_5V^e = \left(0.02P^g + 50\right)\ddot{a}_{50} - P^e\ddot{a}_{50} = [0.02(977.60) + 50](17.0245) - 126.64(17.0245) = -972$$

Alternatively,

$${}_5V^n = 100{,}000A_{50} - P^n\ddot{a}_{50}$$

$$= 100{,}000(0.18931) - 850.97(17.0245) = 4443.66$$

$${}_5V^g = 100{,}000A_{50} + \left(50 + 0.02P^g - P^g\right)\ddot{a}_{50}$$

$$= 100{,}000(0.18931) + [50 + 0.02(977.60) - 977.60](17.0245) = 3471.93$$

$${}_5V^e = {}_5V^g - {}_5V^n = -972$$

Question 7.11

Answer: B

$$L = 10{,}000v^{K_{45}+1} - P\ddot{a}_{\overline{K_{45}+1|}} = 10{,}000v^{11} - P\ddot{a}_{\overline{11|}}$$

$$4450 = 10{,}000(0.58468) - 8.7217P$$

$$P = (5{,}846.8 - 4{,}450) / 8.7217 = 160.15$$

$$A_{55} = 1 - d\ddot{a}_{55} = 1 - (0.05/1.05)(13.4205) = 0.36093$$

$${}_{10}V = 10{,}000A_{55} - P\ddot{a}_{55} = (10{,}000)(0.36093) - (160.15)(13.4205) = 1{,}460$$

Question 7.12

Answer: E

In the final year: $({}_{24}V + P)(1+\text{i}) = b_{25}(q_{68}) + 1(p_{68})$

Since $b_{25} = 1$, this reduces to $({}_{24}V + P)(1+i) = 1 \Rightarrow (0.6+P)(1.04) = 1 \Rightarrow P = 0.36154$

Looking back to the 12th year: $({}_{11}V + P)(1+i) = b_{12}(q_{55}) + {}_{12}V(p_{55})$

$\Rightarrow (5.36154)(1.04) = 14(0.15) + {}_{12}V(0.85) \Rightarrow {}_{12}V = 4.089$

Question 7.13

Answer: A

This first solution recognizes that the full preliminary term reserve at the end of year 10 for a 30 year endowment insurance on (40) is the same as the net premium policy value at the end of year 9 for a 29 year endowment insurance on (41). Then, using superscripts of FPT for full preliminary term reserve and NLP for net premium policy value to distinguish the symbols, we have

$$1000\,{}_{10}V^{FPT} = 1000\,{}_{9}V^{NLP} = 1000(A_{50:\overline{20|}} - P_{41:\overline{29|}}\ddot{a}_{50:\overline{20|}})$$
$$= 1000[0.38844 - 0.01622(12.8428)] = 180$$

or $= 1000\left(1 - \dfrac{\ddot{a}_{50:\overline{20|}}}{\ddot{a}_{41:\overline{29|}}}\right) = 1000\left(1 - \dfrac{12.8428}{15.6640}\right) = 180$

where

$$\ddot{a}_{41:\overline{29|}} = \ddot{a}_{41} - {}_{29}E_{41}\,\ddot{a}_{70}$$
$$= 18.3403 - (0.2228726)(12.0083)$$
$$= 15.6640$$

$$A_{41:\overline{29|}} = 1 - d(15.6640) = 0.254095$$

$${}_{29}E_{41} = v^{29}\left(\frac{l_{70}}{l_{41}}\right) = (0.242946)\left(\frac{91{,}082.4}{99{,}285.9}\right) = 0.2228726$$

$$P_{41:\overline{29|}} = \frac{0.254095}{15.6640} = 0.01622$$

Alternatively, working from the definition of full preliminary term reserves as having ${}_1V^{FPT} = 0$ and the discussion of modified reserves in the Notation and Terminology Study Note, let α be the valuation premium in year 1 and β be the valuation premium thereafter. Then (with some of the values taken from above),

$$\alpha = 1000vq_{40} = 0.5019$$

APV (valuation premiums) = APV (benefits)

$$\alpha + {}_1E_{40}(\ddot{a}_{41:\overline{29}|})\beta = 1000A_{40:\overline{30}|}$$

$$0.5019 + 0.95188(15.6640)\beta = 242.37$$

$$\beta = \frac{242.37 - 0.5019}{14.9102} = 16.22$$

Where

$${}_1E_{40} = (1 - 0.000527)v = 0.95188$$

$$A_{40:\overline{30}|} = A_{40} + {}_{20}E_{40}({}_{10}E_{60})(1 - A_{70})$$

$$= 0.12106 + 0.36663(0.57864)(1 - 0.42818) = 0.24237$$

$${}_{10}V^{FPT} = 1000A_{50:\overline{20}|} - \beta\ddot{a}_{50:\overline{20}|} = 1000(0.38844) - 16.22(12.8427) = 180$$

Question 7.14

Answer: A

$$\left({}_5V + 0.96G - 50\right)(1.05) = q_{50}(100{,}200) + p_{50}\ {}_6V$$

$$\left(5500 + 0.96G - 50\right)(1.05) = (0.009)(100{,}200) + (1 - 0.009)(7100)$$

$$(1.05)(0.96)G + 5722.5 = 7937.9$$

$$(1.05)(0.96)G = 2215.4$$

$$G = 2197.8$$

Question 7.15

Answer: E

$${}_{15.6}V\left(1+i\right)^{0.4} = {}_{0.4}p_{x+15.6}\ \ {}_{16}V + {}_{0.4}q_{x+15.6}\ \ 100$$

$${}_{15.6}V(1.05)^{0.4} = 0.957447(49.78) + 0.042553(100)$$

$${}_{15.6}V = 50.91$$

Question 7.16

Answer: D

APV future benefits

$$=1000\left[0.04v+0.05\times 0.96v^2+0.96\times 0.95\times(0.06+0.94\times 0.683)v^3\right]=630.25$$

APV future premiums $=130(1+0.96v)=248.56$

$$E[{}_3L]=630.25-248.56=381.69$$

Question 7.17

Answer: D

$$\frac{V\left[{}_{10}L\right]}{V\left[{}_{11}L\right]}=\frac{\left(1+\frac{p}{d}\right)^2\left({}^2A_{x+10}-A_{x+10}^2\right)}{\left(1+\frac{p}{d}\right)^2\left({}^2A_{x+11}-A_{x+11}^2\right)}$$

$$A_{x+10}=vq_{x+10}+vp_{x+10}\ A_{x+11}$$

$$=(0.90703)^{1/2}(0.02067)+(0.90703)^{1/2}(1-0.02067)(0.52536)=0.50969$$

$${}^2A_{x+10}=v^2q_{x+10}+v^2p_{x+10}\ {}^2A_{x+11}$$

$$=(0.90703)(0.02067)+(0.90703)(1-0.02067)(0.30783)=0.29219$$

$$\Rightarrow\frac{\text{Var}\left({}_kL\right)}{\text{Var}\left({}_{k+1}L\right)}=\frac{(0.29219)-(0.50969)^2}{(0.30783)-(0.52536)^2}=\frac{0.03241}{0.03183}=1.018$$

Question 7.18

Answer: A

$$ {}_1V_x = A_{x+1} - P_x\,\ddot{a}_{x+1} = 1 - d\ddot{a}_{x+1} - P_x\,\ddot{a}_{x+1} $$

$$ = 1 - \underbrace{(P_x + d)}\,\ddot{a}_{x+1} = 1 - \ddot{a}_{x+1} \big/ \ddot{a}_x $$

$$ \Rightarrow \ddot{a}_x\left(1 - {}_1V_x\right) = \ddot{a}_{x+1} $$

Since $\ddot{a}_x = 1 + vp_x\ddot{a}_{x+1}$ substituting we get

$$ \ddot{a}_x\left(1 - {}_1V_x\right) = \frac{\ddot{a}_x - 1}{vp_x} \Rightarrow \ddot{a}_x\left(1 - {}_1V_x\right)vp_x = \ddot{a}_x - 1 $$

Solving for $\ddot{a}_x$, we get $\ddot{a}_x = \dfrac{1}{1-\left(1-{}_1V_x\right)vp_x} = \dfrac{1}{1-(1-0.012)\left(\dfrac{1}{1.04}\right)(1-0.009)}$

$$ = 17.07942 $$

Question 7.19

Answer: D

Let G be the annual gross premium.

Using the equivalence principle, $0.90G\ddot{a}_{40} - 0.40G = 100{,}000A_{40} + 300$

So $G = \dfrac{100{,}000(0.12106) + 300}{0.90(18.4578) - 0.40} = 765.2347$

The gross premium policy value after the first year and immediately after the second premium and associated expenses are paid is

$$ 100{,}000A_{41} - 0.90G\left(\ddot{a}_{41} - 1\right) $$

$$ = 12{,}665 - 0.90(765.2347)(17.3403) $$

$$ = 723 $$

Question 7.20

Answer: A

If G denotes the gross premium, then

$$G = \frac{1000A_{35} + 30\ddot{a}_{35} + 270}{0.96\ddot{a}_{35} - 0.26} = \frac{1000(0.09653) + 30(18.9728) + 270}{0.96(18.9728) - 0.26} = 52.12$$

So that,

$$R = 1000A_{36} + (30 - 0.96G)\ddot{a}_{36}$$
$$= 1000(0.10101) + (30 - 0.96 \times 52.12)(18.8788) = -277.23$$

Note that $S = 0$ as per definition of FPT reserve.

Question 7.21

Answer: D

$$\pi = \frac{1000 \quad {}_{10|}\ddot{a}_{55}}{\ddot{a}_{55:\overline{10|}} - (IA)^{1}_{55:\overline{10|}}} = \frac{1000(0.59342)(13.5498)}{8.0192 - 0.14743} = 1021.46$$

$${}_9V = 1000 \quad {}_{1|}\ddot{a}_{64} + 10\pi A^{1}_{64:\overline{1|}} - \pi\ddot{a}_{64:\overline{1|}}$$

$$= 1000\frac{1}{1.05}\left(\frac{94,579.7}{95,082.5}\right)13.5498 + 10(1021.46)\frac{1}{1.05}(0.005288) - 1021.46$$

$$= 11,866$$

Question 7.22

Answer: C

$$V\left[L_0\#1\right] = \left(B_1 + \frac{P_1}{d}\right)^2\left({}^2A_x - A_x^2\right) = 20.55 \qquad ==> \left(8 + \frac{1.25(1.06)}{0.06}\right)^2\left({}^2A_x - A_x^2\right) = 20.55$$

$${}^2A_x - A_x^2 = \frac{20.55}{\left(8 + \frac{1.25(1.06)}{0.06}\right)^2} = 0.0227$$

$$V\left[L_0\#2\right] = \left(12 + \frac{1.875(1.06)}{0.06}\right)^2\left({}^2A_x - A_x^2\right) = \left(12 + \frac{1.875(1.06)}{0.06}\right)^2(0.0227) = 46.24$$

Question 7.23

Answer: D

We have Present Value of Modified Premiums = Present Value of level net premiums

$$vq_x + \beta\left(\ddot{a}_{25:\overline{20|}} - 1\right) + P \cdot {}_{20}E_{25} \cdot \ddot{a}_{45:\overline{20|}} = P\ddot{a}_{25:\overline{40|}}$$

$$\Rightarrow \beta = \frac{P\left(\ddot{a}_{25:\overline{40|}}\right) - P \cdot {}_{20}E_{25} \cdot \ddot{a}_{45:\overline{20|}} - vq_x}{\ddot{a}_{25:\overline{20|}} - 1} = \frac{P\,\ddot{a}_{25:\overline{20|}} - vq_x}{\ddot{a}_{25:\overline{20|}} - 1}$$

We are given that $P = 0.0216$

$$\Rightarrow \beta = \frac{0.0216(11.087) - (1.04)^{-1}(0.005)}{11.087 - 1} = 0.023265$$

For insurance of 10,000, $\beta = 233$.

Question 7.24

Answer: C

$P^g = P^n + P^e$ where P^e is the expense loading

$$P^n = 1{,}000{,}000\frac{A_{50}}{\ddot{a}_{50}} = 1{,}000{,}000\left(\frac{0.18931}{17.0245}\right) = 11{,}119.86$$

$$P^e = P^g - P^n = 11{,}800 - 11{,}120 = 680$$

Question 7.25

Answer: B

$${}_3V^{FPT} = 100{,}000\,A_{[55]+3} - 100{,}000\,P_{[55]+1}\,\ddot{a}_{[55]+3}$$

$$= 100{,}000\,A_{58} - 100{,}000\frac{A_{[55]+1}}{\ddot{a}_{[55]+1}}\ddot{a}_{58}$$

$$= 100{,}000\left(0.27 - \frac{0.24}{\frac{1-0.24}{d}} \cdot \frac{1-0.27}{d}\right)$$

$$= 3947.37$$

Question 7.26

Answer: D

$$ {}_1V = \left({}_0V + P\right)(1+i) - \left(25{,}000 + {}_1V - {}_1V\right)q_x = P(1+i) - (25{,}000)q_x $$

$$ {}_2V = \left({}_1V + P\right)(1+i) - \left(50{,}000 + {}_2V - {}_2V\right)q_{x+1} = 50{,}000 $$

$$ \left(\left(P(1+i) - 25{,}000q_x\right) + P\right)(1+i) - 50{,}000q_{x+1} = 50{,}000 $$

$$ \left(\left(P(1.05) - 25{,}000(0.15)\right) + P\right)(1.05) - 50{,}000(0.15) = 50{,}000 $$

Solving for P, we get

$$ P = \frac{61{,}437.50}{2.1525} = 28{,}542.39 $$

Question 7.27

Answer: B

Since G is determined using the equivalence principle, ${}_0V = 0$

$$ \text{Then, } {}_1V^e = \frac{\left(0 + \overbrace{G - 187}^{P^e} - 0.25G - 10\right)(1.03)}{0.992} = -38.7 $$

$$ \Rightarrow 0.75G = \frac{-38.7(0.992)}{1.03} + 187 + 10 = 159.72 $$

$$ \Rightarrow G = 212.97 $$

Question 7.28

Answer: D

$_{20}V = 0 ==> 1000A_{65} = (P+W) \times \ddot{a}_{65}$

At issue, present value of benefits must equal present value of premium, so:

$1000A_{45} = P\ddot{a}_{45} + W\,_{20}E_{45} \times \ddot{a}_{65}$

$354.77 = (P+W)(13.5498) \Rightarrow P+W = 26.182674 \Rightarrow P = 26.182674 - W$

$151.61 = 17.8162P + W(0.35994)(13.5498)$

$151.61 = 17.8162(26.182674 - W) + W(0.35994)(13.5498)$

$\Rightarrow W = 24.33447$

Question 7.29

Answer: E

$$V_{10} = 2,290 = B\left(1 - \frac{\ddot{a}_{x+10}}{\ddot{a}_x}\right) = B\left(1 - \frac{11.4}{14.8}\right) \Rightarrow B = 9,968.24$$

$$G\ddot{a}_x = 25 + 5\ddot{a}_x + B \times A_x$$

$$A_x = 1 - d\ddot{a}_x = 1 - \left(\frac{0.04}{1.04} \times 14.8\right) = 0.430769231$$

$$G \times 14.8 = 25 + 5 \times 14.8 + 9,968.24 \times 0.430769231$$

$$\Rightarrow G = 296.82$$

$${}_{10}V^g = 9,968.24A_{x+10} + 5\ddot{a}_{x+10} - 296.82\ddot{a}_{x+10}$$

$$A_{x+10} = 1 - d\ddot{a}_{x+10} = 1 - \left(\frac{0.04}{1.04} \times 11.4\right) = 0.561538462$$

$${}_{10}V^g = 9,968.24 \times 0.561538462 + 5 \times 11.4 - 296.82 \times 11.4$$

$$\Rightarrow {}_{10}V^g = 2,270.80$$

Alternatively, the expense net premium is based on the extra expenses in year 1, so

$$P^e = (30 - 5) / 14.8 = 1.68919$$

$${}_{10}V^e = 0 - 1.68919(11.4) = -19.26$$

$${}_{10}V^g = {}_{10}V^n + {}_{10}V^e = 2290 - 19.26 = 2270.74$$

Question 7.30

Answer: E

$$L_{10} = 10,000A_{35} = 965.30$$

$$L_{10}^* = 10,000$$

$$L_{10}^* - L_{10} = 10,000 - 965.30 = 9034.70$$

Question 7.31

Answer: E

Future expenses at $x+2 = 0.08G+5$

Expense load at $x+2 = P^e$

$-23.64 = (0.08G+5) - P^e$

$\Rightarrow P^e = 58.08$

$1000P_{x:\overline{3}|} = 368.05 - 58.08 = 309.97$

Question 7.32

Answer: B

$$L_A = v^T - 0.10\bar{a}_{\overline{T}|} = \left(1+\frac{10}{6}\right)v^T - \frac{10}{6}$$

$$Var[L_A] = \left(1+\frac{10}{6}\right)^2 Var[v^T] = 0.455 \Rightarrow Var[v^T] = 0.06398$$

$$L_B = 2v^T - 0.16\bar{a}_{\overline{T}|} = \left(2+\frac{16}{6}\right)v^T - \frac{16}{6}$$

$$Var[L_B] = \left(2+\frac{16}{6}\right)^2 Var[v^T] = \left(2+\frac{16}{6}\right)^2 (0.06398) = 1.39$$

Question 18.1

Answer: A

$$\hat{S}(10.8) = \hat{S}(10.0)\left(\frac{50-10-1}{50-10}\right) = 0.897$$

Then, $\hat{S}(10.4) = 0.897$ because $\hat{S}$ is constant between deaths.

Question 18.2

Answer: D

$\hat{S}(1) = 0.8$

$$Var[S(1)] = \frac{S(1)(1-S(1)}{n} \approx \frac{(0.8)(0.2)}{1000} = 0.01265^2$$

$\Rightarrow$ 95% CI is approximately $(0.8 \pm 1.96(0.01265)) = (0.775, 0.825)$

Question 18.3

Answer: D

$$\hat{H}(1.5) = \frac{1}{90} + \frac{3}{81} = 0.048148$$

$$\hat{S}(1.5) = e^{-\hat{H}(1.5)} = 0.9530$$

Question 18.4

Answer: D

$$\hat{S}(21.0) = \frac{59}{60} \times \frac{59-8+1-1}{59-8+1} \times \frac{51-6+7-2}{51-6+7} \times \frac{50-7+7-1}{50-7+7} \times \frac{49-6+5-1}{49-6+5}$$
$$= 0.8724$$

$$Var[\hat{S}(21.0)] \approx 0.8742^2\left(\frac{1}{60 \times 59} + \frac{1}{52 \times 51} + \frac{1}{52 \times 50} + \frac{1}{50 \times 49} + \frac{1}{48 \times 47}\right) = 0.02280$$

➔ The upper limit of the 80% linear confidence interval is $0.8724 + 1.282(0.2280)^{0.5} = 0.936.$

Question # 18.5
Answer: D

$$\hat{S}(30)=\frac{100-10-14-16}{100}=0.60,\quad \hat{S}(40)=\frac{100-10-14-16-20}{100}=0.40$$

Use linear interpolation to find $\hat{S}(32)$

$$\hat{S}(32)=\left(\frac{40-32}{40-30}\right)\hat{S}(30)+\left(\frac{32-30}{40-30}\right)\hat{S}(40)+=0.8(0.60)+0.2(0.40)=0.56$$

Question 18.6

Answer: A

The contribution from Life 1 is ${}_{20}p_{70}$. With Gompertz and the selected parameters, the contribution is ${}_{20}p_{70} = \exp\left[-\frac{B}{\ln c}c^{70}(c^{20}-1)\right] = \exp\left[-\frac{0.000003}{\ln 1.1}1.1^{70}(1.1^{20}-1)\right] = 0.86730.$

The contribution from Life 2 is ${}_{19}p_{70} - {}_{20}p_{70}$. The contribution is

$${}_{19}p_{70} - {}_{20}p_{70} = \exp\left[-\frac{B}{\ln c}c^{70}(c^{19}-1)\right] - 0.86730 = \exp\left[-\frac{0.000003}{\ln 1.1}1.1^{70}(1.1^{19}-1)\right] - 0.86730$$

$= 0.88058 - 0.86730 = 0.01328.$

The contribution to the likelihood is 0.86730(0.01328) = 0.01152.

Question 18.7

Answer: D

The contribution from Life 1 is ${}_{14.5}p_{60}$. With Gompertz and the selected parameters, the contribution is ${}_{14.5}p_{60} = \exp\left[-\frac{B}{\ln c}c^{60}(c^{14.5}-1)\right] = \exp\left[-\frac{0.000004}{\ln 1.12}1.12^{60}(1.12^{14.5}-1)\right] = 0.87619.$

The contribution from Life 2 is ${}_{14.5}p_{60} \times \mu_{74.5}$. The contribution is

${}_{14.5}p_{60} \times \mu_{74.5} = 0.87619 \times 0.000004 \times 1.12^{74.5} = 0.01627.$

The contribution to the likelihood is 0.87619(0.01627) = 0.01426.

The contribution to the log-likelihood is ln(0.01426) = –4.25.